

## Percentages %

### Definition

$x\%$  means  $x/100$ . So 12%, 0.12 and  $12/100$  are equivalent ways of writing the same thing. In *statistics* and *probability*, percentages are often used as one is typically concerned with numbers between 0 (something will not happen) and 1 (something is certain to happen). The percentage form is often easier to interpret as there are less zeros before the number of significance. For example, compare 1.23% to 0.0123.

### Typical problems

Percentage problems are best turned into *simple algebraic equations* to prevent confusion. Find out the unknown quantity, call this  $x$  (or some other letter) and then write down an equation. Solve this to find  $x$

I bought £100 worth of shares and made a 20% profit when I sold them. What profit £ $P$  did I make?

$$P + 100 = 100 \times 1.2$$

$$P = 20$$

I sold £150 worth of shares at a profit of 15%. What was the cost £ $C$  of the shares when I bought them?

$$1.15 \times C = 150$$

$$C = \frac{150}{1.15}$$

$$C = 130.43$$

My car insurance this year was £300. This is 30% less than what I paid a decade ago. What did I pay (£ $x$ ) a decade ago?

$$0.7x = 300 \quad \therefore x = 300 \div 0.7$$

$$x = 428.57$$

My cat is 14% lighter than she was ( $W$  kg) when I bought her. She now weighs 4kg. What was her original weight?

$$0.86W = 4 \quad \therefore W = \frac{4}{0.86} = 4.65$$

### Simple interest

If you invest £ $x$  at  $r\%$  simple interest per year ("per annum") every year you will receive  $r\%$  of the *original investment*.

e.g. if you invest £100 at 2% simple interest you will gain £2 every year.

So after ten years you will have £120.

### Compound interest

In this type of interest you receive a percentage of what is in the account, which *includes* previously accumulated interest.

e.g. if you invest £100 at 1% compound interest you will have

$$£100 \times 1.01 = £101 \text{ after one year}$$

$$£100 \times 1.01^2 = £102.01 \text{ after two years}$$

$$£100 \times 1.01^3 = £103.0301 \text{ after three years}$$

$$£100 \times 1.01^N \text{ after } N \text{ years}$$

If I invest £100 at 2% simple interest and another £100 at 1% compound interest, how many years ( $n$ ) will it take for me to have more money in the compound interest account?

$$1.01^n \times 100 = 100 + 2n$$

We can't solve this equation exactly, so we must guess values (or use an iterative numerical method like *Newton-Raphson*). The answer is about 127.1 years.

So after the 128<sup>th</sup> year, the compound interest account will offer better returns. But then you might not want to wait that long!