Upper and lower bounds

When a measurement of a physical quantity (e.g. the length of an object) is made, the measurement will not be exact.
Depending on the precision of the measuring device, there will always be a degree of uncertainty about what the measurement actually is.

If a quantity being measured is quoted as being *precise* to *N* decimal places (or 'nearest ....') this means the true measurement lies somewhere within a range of values which can be expressed by an *inequality* determined by the measurement rounded up, and down, based on the quoted precision.

Example 1: The mass of a man is *M* = 72kg correct to the nearest gram. Since 1 gram = 0.001kg, this means

$$72-0.0005 \le M < 72+0.0005$$

$$\therefore \boxed{71.9995 \le M < 72.0005}$$ ← The left and right sides of the **inequality** for M are the *lower and upper bounds* for *M*.

Example 2: 100 planks of length *L* = 1m (correct to the nearest 5cm) are attached end-to-end. How long is the resulting structure?

$$1-0.025 \le L < 1+0.025$$

$$0.975\text{m} \le L < 1.025\text{m}$$

$$\therefore \boxed{97.5\text{m} \le 100L < 102.5\text{m}}$$

Example 3: An athlete runs *D* = 400m (to the nearest metre) in *T* = 50 seconds to the nearest 10th of a second. What is the athletes' average speed (S) in metres per second?

$$399.5\text{m} \le D < 400.5\text{m}$$

$$49.95\text{s} \le T < 50.05\text{s}$$

$$S = \frac{D}{T}$$

$$\therefore \frac{399.5}{50.05}\text{ms}^{-1} < S < \frac{400.5}{49.95}\text{ms}^{-1}$$

$$\therefore \boxed{7.982\text{ms}^{-1} < S < 8.018\text{ms}^{-1}}$$

Example 4: The electrical power *P* (in watts) drawn by a light bulb of resistance *R* = 2304 ohms (to the nearest 20 ohms) powered by mains with average voltage *V* = 240 volts (to the nearest 10 volts) is given by the formula

$$P = \frac{V^2}{R}$$

Calculate upper and lower bounds for *P*.

$$2304-10 \le R < 2304+10$$

$$2294 \le R < 2314$$

$$240-5 \le V < 240+5$$

$$235 \le V < 245$$

$$P = \frac{V^2}{R}$$

$$\therefore \frac{235^2}{2314} < P < \frac{245^2}{2294}$$

$$\therefore \boxed{23.9 < P < 26.2}$$

Note in both these examples, a *mixture* of upper and lower bounds in the fraction means an equality is impossible. i.e. the upper and lower bounds are unreachable limits.