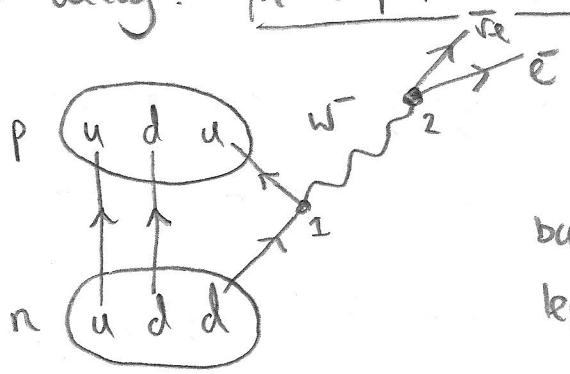
HIGH ENERGY & PARTICLE PHYSICS

(i) β^- decay: $\boxed{n \to p + e^- + \bar{\nu}_e}$



vertex 1:

$d \to u + W^-$

charge: $-\frac{1}{3}e \to \frac{2}{3}e - 1e$ ✓

baryon #: $\frac{1}{3} \to \frac{1}{3} + 0$ ✓

lepton #: $0 \to 0 + 0$ ✓

vertex 2: $W^- \to e^- + \bar{\nu}_e$

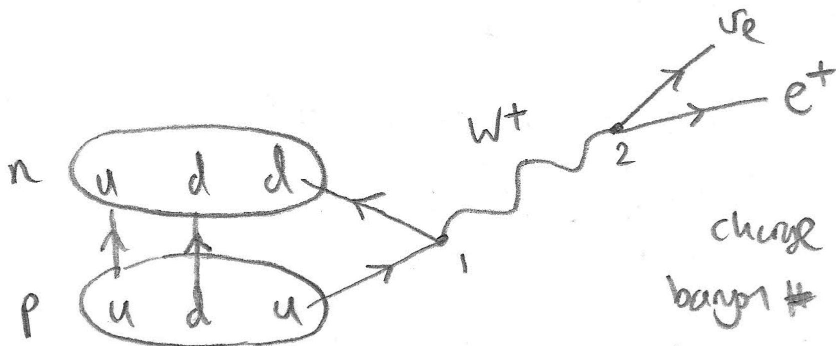
charge: $-e \to -e + 0$ ✓

baryon #: $0 \to 0 + 0$ ✓

lepton #: $0 \to 1 - 1$ ✓

Note $\bar{\nu}_e$ would have a lepton # of +1 so it must be an anti neutrino.

(ii) β^+ decay: $\boxed{p \to n + e^+ + \nu_e}$



vertex 1:

$u \to d + W^+$

charge: $\frac{2}{3}e \to -\frac{1}{3}e + 1e$ ✓

baryon #: $\frac{1}{3} \to \frac{1}{3} + 0$ ✓

lepton #: $0 \to 0 + 0$ ✓

vertex 2: $W^+ \to \nu_e + e^+$

charge: $1e \to 0 + 1e$ ✓

baryon #: $0 \to 0 + 0$ ✓

lepton #: $0 \to +1 - 1$ ✓

e^+ is the antiparticle of e^-, so has lepton # of -1.

(iii) $\pi^- + p \to n + \pi^- + \pi^+$

Quark content: $\bar{u}d + uud \to udd + \bar{u}d + u\bar{d}$

$$\pi^- + p \rightarrow n + \pi^- + \pi^+$$

$$\bar{u}d + udu \rightarrow udd + \bar{u}d + u\bar{d}$$

charge /e: $-\frac{2}{3}-\frac{1}{3} + \frac{2}{3}-\frac{1}{3}+\frac{2}{3} \rightarrow \frac{2}{3}-\frac{1}{3}-\frac{1}{3} \; -\frac{2}{3}-\frac{1}{3} + \frac{2}{3}+\frac{1}{3}$

$$-1 + 1 \rightarrow 0 \quad -1 + 1 \quad ✓$$

baryon #: $-\frac{1}{3}+\frac{1}{3} + \frac{1}{3}+\frac{1}{3}+\frac{1}{3} \rightarrow \frac{1}{3}+\frac{1}{3}+\frac{1}{3} + -\frac{1}{3}+\frac{1}{3} + \frac{1}{3}-\frac{1}{3}$

$$0 + 1 \rightarrow 1 + 0 + 0 \quad ✓$$

(iv) $K^+ \rightarrow \pi^+ + \pi^0$ kaon decay to pions

$u\bar{s} \rightarrow u\bar{d} + d\bar{d}$ Quark content of these Mesons.

charge /e: $\frac{2}{3}+\frac{1}{3} \rightarrow \frac{2}{3}+\frac{1}{3} + -\frac{1}{3}+\frac{1}{3}$

$$1 \rightarrow 1 + 0 \quad ✓$$

(v) $M_{K^\pm} c^2 + B_{K^\pm} = M_u c^2 + M_{\bar{s}} c^2$

$B_{K^\pm} = 2.4 + 95 - 0.5261 \times 938.27$ (MeV)

$$\boxed{B_{K^\pm} = -396.2 \text{ MeV}}$$

what does a -ve binding energy mean? Is it like a -ve gravitational potential energy? Interestingly, it appears most of the mass-energy of the kaon is <u>not</u> the mass-energy of the quarks - it is an energy held within the fields associated with the strong force.

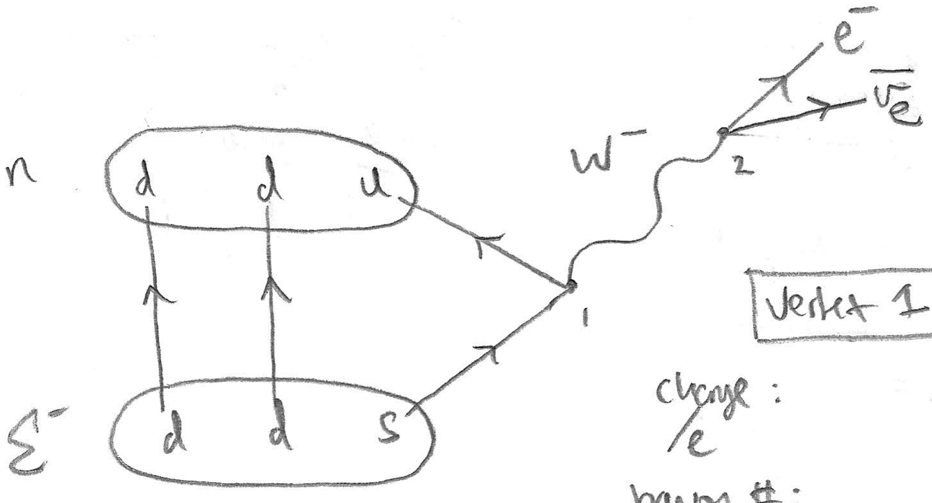
$$B_{\pi^\pm} = m_u c^2 + M_{\bar{d}} c^2 - M_{\pi^\pm} c^2$$

$$= 2.4 + 4.8 - 0.14875 \times 938.27 \text{ MeV}$$

$$= \boxed{-132.4 \text{ MeV}}$$

(vi) $\Sigma^- \rightarrow n + e^- + \bar{\nu}_e$

$dds \rightarrow udd + e^- + \bar{\nu}_e$



$\boxed{\text{Vertex 1}: \; s \rightarrow W^- + u}$

change /e : $-\frac{1}{3} \rightarrow -1 + \frac{2}{3}$ ✓

baryon #: $\frac{1}{3} \rightarrow 0 + \frac{1}{3}$ ✓

lepton #: $0 \rightarrow 0 + 0$ ✓

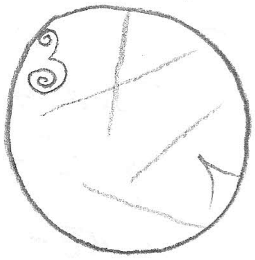
vertex 2 : $\boxed{W^- \rightarrow e^- + \bar{\nu}_e}$

change /e : $-1 \rightarrow -1 + 0$ ✓

baryon #: $0 \rightarrow 0 + 0$ ✓

lepton #: $0 \rightarrow 1 - 1$ ✓

vii) **CLOUD CHAMBER**



* Sealed environment containing a supersaturated vapour of water or alcohol.
* A high energy charged particle will leave a trail of ionized vapour molecules. These will act as 'condensation centres' about which droplets form.
  ie resulting in a 'cloud track' that can be photographed.

BUBBLE CHAMBER * High energy particle leaves a trail of bubbles, that can be photographed. Bubbles form on ionized molecules.

A bubble chamber is superheated, ie a liquid held above its boiling point under pressure. (whereas a cloud chamber is supersaturated). Reduce the pressure carefully, and bubbles form first on ionized molecules.

(viii) $\gamma \rightarrow \mu^- + \mu^+$

If the kinetic energy of muon pair is zero
ie all the γ energy $\rightarrow$ mass - energy of the muons

$E_\gamma = 2m_\mu c^2$ $\qquad$ $E_\gamma = \frac{hc}{\lambda}$ for a single photon.

So $\frac{hc}{\lambda} = 2m_\mu c^2$

$$\Rightarrow \boxed{\lambda = \frac{hc}{2m_\mu c^2}}$$

(Note if $E > 2m_\mu c^2$ then clearly λ is smaller
so minimum λ is $\frac{hc}{2m_\mu c^2}$).

$$\lambda = \frac{6.63 \times 10^{-34} \times 2.998 \times 10^8}{2 \times 105.66 \times 10^6 \times 1.602 \times 10^{-19}}$$

$$\boxed{\lambda = 5.87 \times 10^{-15} \text{ m}}$$

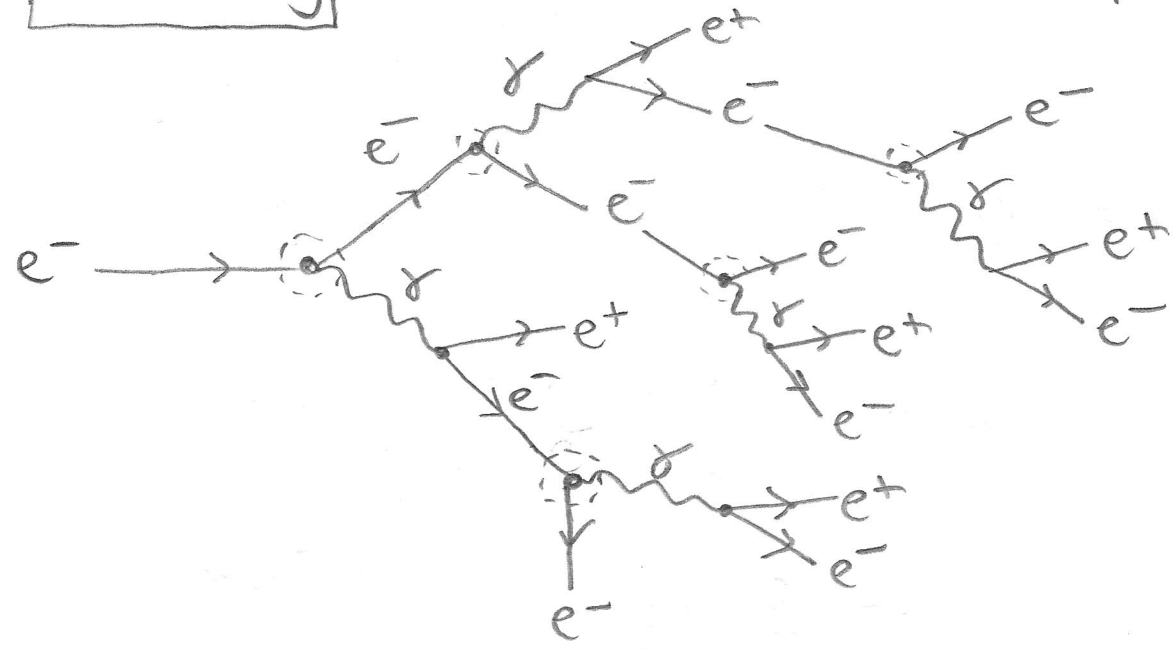
(ix) Total energy of $e^- + e^+$ is $\underbrace{\left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}} \times 2 m_e c^2}$

Total energy of $e^- + e^+$ is $(KE + Mc^2) \times 2$

$= \left(1 - 0.8^2\right)^{-\frac{1}{2}} \times 2 \times 0.51100$ MeV

$= \boxed{1.70 \text{ MeV}}$

(x) $\boxed{\text{Bremsstrahlung}}$ (radiation from rapidly accelerated particles)



e^- could be accelerated, say be interaction with the dense outer shell electrons of Pb (lead) atoms. This acceleration causes an emission of a γ ray.
The γ ray then decays to form an e^+, e^- pair.
If the e^- has sufficient energy, it may produce a γ ray to and create more pairs etc.

e^+ could in principle do the same — but perhaps attraction to e^- (and ∴ annihilation) in Pb might be more likely than repulsion from a Pb nucleus?

Either way, lots of extra e^- are produced, which enables the same e^- to be detected.

Note KE of e^- must be $> 2m_ec^2$ for at least one pair to be produced. ie $> 2 \times 0.51100$ MeV
A 100 MeV e^- could make up to 97 pairs.

(xi) koide mass ratio $\dfrac{M_u + M_d + M_s}{(\sqrt{M_u} + \sqrt{M_d} + \sqrt{M_s})^2}$

$= k/4$ what is integer k ?

$\Rightarrow \quad k = \dfrac{2.4 + 4.8 + 95}{(\sqrt{2.4} + \sqrt{4.8} + \sqrt{95})^2} \times 4$

$= \boxed{5.06}$ so $\boxed{k \simeq 5}$

So $\dfrac{M_u + M_d + M_s}{(\sqrt{M_u} + \sqrt{M_d} + \sqrt{M_s})^2} \simeq 5/4 = \boxed{\frac{5}{6} \times \frac{2}{3}}$

2/ $\Omega^- = sss$

(i) $K^- + p \longrightarrow \Omega^- + K^+ + K^0$

$\bar{u}s + uud \longrightarrow sss + u\bar{s} + ?$

Charge/e: $-\frac{2}{3} + -\frac{1}{3} + \frac{2}{3} + \frac{2}{3} - \frac{1}{3} \rightarrow +3 \times -\frac{1}{3} + \frac{2}{3} + \frac{1}{3} + ?$

$-1 + 1 \rightarrow -1 + 1 + 0$

So charge of K^0 must be 0 (obvious from it's superscript!)

Baryon #: $-\frac{1}{3} + \frac{1}{3} + 1 \longrightarrow 1 + \frac{1}{3} - \frac{1}{3} + 0$

So baryon # of K^0 must be 0

Strangeness: $-1 + 0 \rightarrow -3 + 1 + ?$

? must be +1 is strangeness of K^0.

So in summary: * K^0 must have an $\bar{s}$ quark for its strangeness to be +1. (And no other strange quarks)

* To have a baryon # of 0 it must be a meson ie a quark, antiquark pair

* charge of $\bar{s}$ is $+\frac{1}{3}e$, so to have a net zero charge, remaining quark must have a charge of $-\frac{1}{3}e$.

options are: $\boxed{d, b}$ (Since s not allowed).

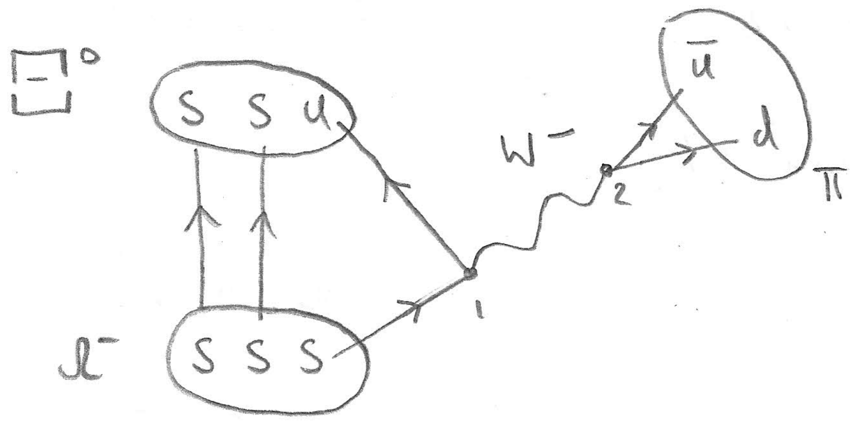
b is too massive to be created in the interaction

so ∴ $\boxed{d}$ So $\boxed{K^0 = \bar{s}d}$

(ii) $\boxed{\begin{array}{l} \Omega^- \rightarrow \Xi^0 + \pi^- \\ sss \rightarrow uss + \bar{u}d \end{array}}$

Vertex 2:

$\boxed{W^- \rightarrow \bar{u} + d}$



charge/e: $-1 \rightarrow -\frac{2}{3} + -\frac{1}{3}$ ✓

Baryon#: $0 \rightarrow -\frac{1}{3} + \frac{1}{3}$ ✓

Strangeness: $0 \rightarrow 0 + 0$ ✓

Vertex 1:

$\boxed{s \rightarrow u + W^-}$

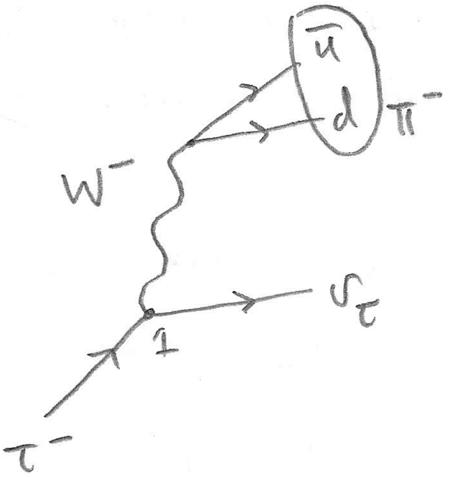
charge/e: $-\frac{1}{3} \rightarrow \frac{2}{3} - 1$ ✓

Baryon #: $\frac{1}{3} \rightarrow \frac{1}{3} + 0$ ✓

$\boxed{\text{Strangeness: } -1 \rightarrow 0 + 0 \quad \text{X Not conserved}}$

Strangeness not conserved in weak interactions.

3/ (i) $\boxed{\tau^- \to \pi^- + \nu_\tau}$



(ii) lepton number must be conserved at vertex 1

$\tau^- \to W^- + \nu_\tau$

lepton #: $1 \to 0 + 1$ ✓

If the ν_τ was not produced, lepton # would not be conserved since W^- is a boson, and ∴ has a lepton # of 0.

(iii) $\boxed{E = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}} mc^2}$ <u>total</u> energy of τ^-

∴ <u>kinetic</u> energy is $E - mc^2$

$$E_k = \left[\left(1 - 0.2^2\right)^{-\frac{1}{2}} - 1\right] \times 1776.86 \text{ MeV}$$

$$= \boxed{36.64 \text{ MeV}}$$

(iv) In $\tau^- \to \pi^- + \nu_\tau$, the pion has a speed of 0.1c

so $E_{\nu_\tau} = E_{\tau^-} - E_{\pi^-}$ (using $E = \gamma mc^2$) $\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}}$

$$= \left(1 - 0.2^2\right)^{-\frac{1}{2}} \times 1776.86 - \left(1 - 0.99^2\right)^{-\frac{1}{2}} \times 0.14875 \times 938.27$$

$$= 1813.50 - 989.37 \quad \text{MeV}$$

$$= \boxed{824.13 \text{ MeV}}$$

(MeV)

(v) Relativistic momentum: $\boxed{p = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}} mu}$

$\boxed{E = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}} mc^2}$ Total energy

so $E^2 - p^2 c^2 = \left(1 - \frac{u^2}{c^2}\right)^{-1} m^2 c^4 - \left(1 - \frac{u^2}{c^2}\right)^{-1} m^2 u^2 c^2$

$= \left(1 - \frac{u^2}{c^2}\right)^{-1} m^2 c^4 \left(1 - \frac{u^2}{c^2}\right)$

$= m^2 c^4.$

So $\boxed{E^2 - p^2 c^2 = m^2 c^4}$ Energy - momentum invariant.

if $m = 0 \Rightarrow E = pc$ so $\boxed{p = E/c}$

$\therefore \; p_{\nu_\tau} = \boxed{824.13 \text{ MeV}/c}$

$= 824.13 \times 10^6 \times 1.602 \times 10^{-19} / 2.998 \times 10^8$

$= \boxed{4.40 \times 10^{-19} \text{ kgms}^{-1}}$

(vi) a) $\frac{M_e + M_\mu + M_\tau}{\left(\sqrt{M_e} + \sqrt{M_\mu} + \sqrt{M_\tau}\right)^2} = \frac{0.511 + 105.66 + 1776.86}{\left(\sqrt{0.511} + \sqrt{105.66} + \sqrt{1776.86}\right)^2}$

$= 0.667 \simeq \boxed{2/3}$

Koide formula for leptons.

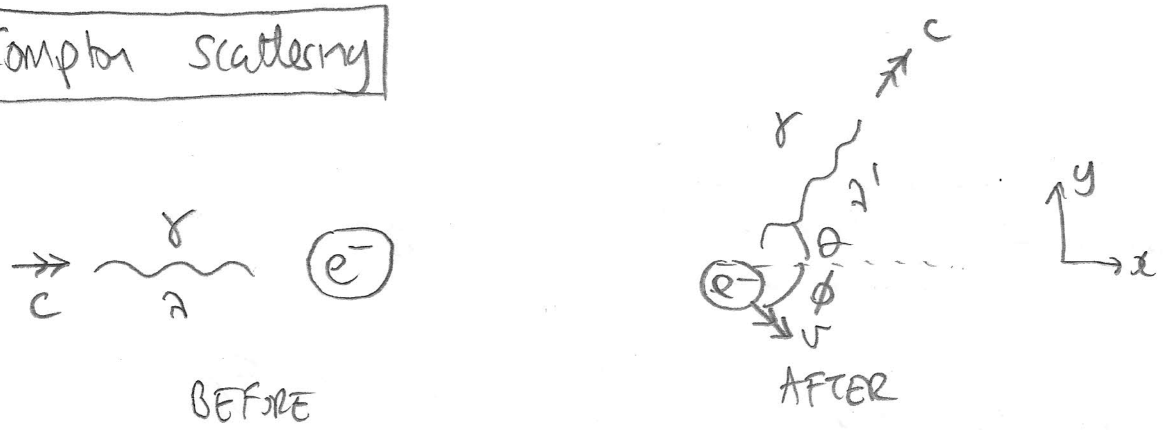
b) $$\frac{m_c + m_b + m_t}{\left(\sqrt{m_c} + \sqrt{m_b} + \sqrt{m_t}\right)^2} = \frac{1275 + 4180 + 172440}{\left(\sqrt{1275} + \sqrt{4180} + \sqrt{172440}\right)^2}$$

$$= 0.669 \approx \boxed{2/3}$$

Koide formula for heavy quarks.

Are these formula serendipity? or do they hint at a deeper unifying mathematics of the Standard Model.....

4/ $\boxed{\text{Compton Scattering}}$



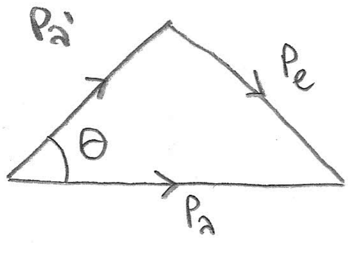
(i) Energy - momentum invariant (for photon of mass $m = 0$)

$$E_\lambda^2 - P_\lambda^2 c^2 = 0 \Rightarrow P_\lambda = E_\lambda / c$$

$$\Rightarrow P_\lambda = \frac{hc}{\lambda c}$$

$$\Rightarrow \boxed{P_\lambda = h/\lambda}$$ de-Broglie.

(ii) Momentum conservation $\underline{P}_\lambda = \underline{P}_{\lambda'} + \underline{P}_e$



By cosine rule:

$$\boxed{P_e^2 = P_\lambda^2 + P_{\lambda'}^2 - 2 P_\lambda P_{\lambda'} \cos\theta}$$

(iii) Conservation of energy

$$M_e c^2 + E_\lambda = E_{\lambda'} + E_e \qquad \therefore \boxed{E_e = M_e c^2 + E_\lambda - E_{\lambda'}}$$

$$\boxed{E_\lambda = P_\lambda c \qquad E_{\lambda'} = P_{\lambda'} c}$$

and $\boxed{E_e^2 - P_e^2 c^2 = M_e^2 c^4}$ } Energy-momentum invariants

So
$$P_e^2 = \frac{E_e^2}{c^2} - M_e^2 c^2$$

$$P_e^2 = \frac{\left(M_e c^2 + (P_\lambda - P_{\lambda'})c\right)^2}{c^2} - M_e^2 c^2$$

$$P_e^2 = \frac{M_e^2 c^4 + (P_\lambda - P_{\lambda'})^2 c^2 + 2M_e c^2 (P_\lambda - P_{\lambda'})c - M_e^2 c^2}{c^2}$$

$$P_e^2 = \cancel{M_e^2 c^2} + (P_\lambda - P_{\lambda'})^2 + 2M_e c (P_\lambda - P_{\lambda'}) - \cancel{M_e^2 c^2}$$

$$\therefore \boxed{P_e^2 = (P_\lambda - P_{\lambda'})^2 + 2M_e c (P_\lambda - P_{\lambda'})}$$

(iv) Equating with $P_e^2 = P_\lambda^2 + P_{\lambda'}^2 - 2P_\lambda P_{\lambda'} \cos\theta$:

$$\cancel{P_\lambda^2} + \cancel{P_{\lambda'}^2} - 2P_\lambda P_{\lambda'} \cos\theta = \cancel{P_\lambda^2} + \cancel{P_{\lambda'}^2} - 2P_\lambda P_{\lambda'} + 2M_e c (P_\lambda - P_{\lambda'})$$

$$2P_\lambda P_{\lambda'} (1 - \cos\theta) = 2M_e c (P_\lambda - P_{\lambda'})$$

Now use $P_\lambda = h/\lambda$ and $P_{\lambda'} = h/\lambda'$

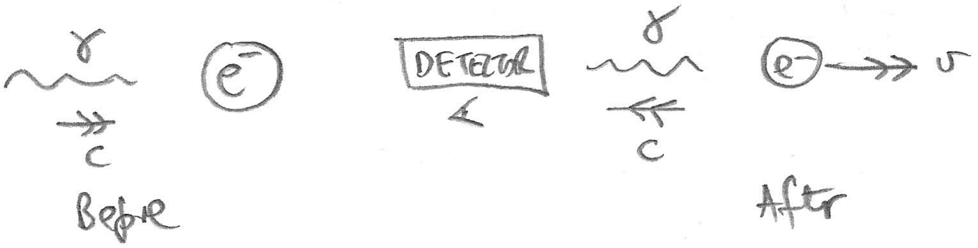
$$\Rightarrow \quad \frac{h^2}{\lambda\lambda'} (1 - \cos\theta) = M_e c h \left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

$$\therefore \boxed{\frac{h}{M_e c} (1 - \cos\theta) = \lambda' - \lambda}$$

v) $\boxed{\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)}$

Maximum $\Delta\lambda$ is when $\cos\theta = -1$ ie $\boxed{\theta = 180^\circ}$

ie direct recoil of the x-ray



$\therefore$ $\boxed{\Delta\lambda_{max} = \frac{2h}{m_e c}}$

So $\frac{\Delta\lambda_{max}}{\lambda} = \frac{2h}{m_e c \lambda}$

$E_\lambda = \frac{hc}{\lambda}$ Planck

So : $\frac{1}{\lambda} = \frac{E_\lambda}{hc}$

$\therefore$ $\frac{\Delta\lambda_{max}}{\lambda} = \frac{2h}{m_e c}\frac{E_\lambda}{hc}$

$\therefore$ $\frac{\Delta\lambda_{max}}{\lambda} = \frac{2E_\lambda}{m_e c^2}$

$[m_e = 0.51100\ MeV/c^2]$

So $\boxed{E_\lambda = \frac{1}{2} m_e c^2 \frac{\Delta\lambda_{max}}{\lambda}}$

if $\frac{\Delta\lambda_{max}}{\lambda} = 0.1$ $\Rightarrow$ $E_\lambda = \frac{1}{2} \times 511\ keV \times 0.1$

$\boxed{E_\lambda = 25.6\ keV}$

(vi) To conserve momentum:

$//x:$ $$p_\lambda = p_{\lambda'}\cos\theta + p_e\cos\phi$$

$//y:$ $$0 = p_{\lambda'}\sin\theta - p_e\sin\phi$$

so $$p_e\sin\phi = p_{\lambda'}\sin\theta$$
$$p_e\cos\phi = p_\lambda - p_{\lambda'}\cos\theta$$

$$\therefore \tan\phi = \frac{p_{\lambda'}\sin\theta}{p_\lambda - p_{\lambda'}\cos\theta} = \frac{\sin\theta}{p_\lambda/p_{\lambda'} - \cos\theta}$$

Now $$\frac{p_\lambda}{p_{\lambda'}} = \frac{h}{\lambda}\,\frac{\lambda'}{h} = \frac{\lambda'}{\lambda}$$

$$\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)$$

so $$\frac{\lambda'}{\lambda} - 1 = \frac{h}{m_e c\lambda}(1-\cos\theta)$$

so $$\frac{p_\lambda}{p_{\lambda'}} = \frac{\lambda'}{\lambda} = 1 + \frac{h}{m_e c\lambda}(1-\cos\theta)$$

$$\therefore \boxed{\tan\phi = \frac{\sin\theta}{1 + \frac{h}{m_e c\lambda}(1-\cos\theta) - \cos\theta}}$$

Now $p_e\sin\phi = p_{\lambda'}\sin\theta$ $\therefore$ $p_e = \frac{h}{\lambda'}\sin\theta/\sin\phi$

$$\lambda' = \lambda + \frac{h}{m_e c}(1-\cos\theta)$$

$$\therefore \quad P_e = \frac{h\sin\theta}{\sin\phi}\left(\lambda + \frac{h}{m_e c}(1-\cos\theta)\right)^{-1}$$

Now $\quad P_e = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} m_e v$

$$P_e^{-2} = \left(1 - \frac{v^2}{c^2}\right)\frac{1}{m_e^2 v^2}$$

$$\frac{1}{P_e^2} = \frac{1}{m_e^2 v^2} - \frac{1}{m_e^2 c^2}$$

$$\therefore \quad v^2 = \left(\frac{m_e^2}{P_e^2} + \frac{1}{c^2}\right)^{-1}$$

$$v = \left(\frac{m_e^2}{P_e^2} + \frac{1}{c^2}\right)^{-\frac{1}{2}}$$

So $\quad v = \left(\frac{m_e^2}{P_e^2} + \frac{1}{c^2}\right)^{-\frac{1}{2}}$ $\quad \leftarrow$ ie $v = c\left(\left(\frac{m_e c}{P_e}\right)^2 + 1\right)^{-\frac{1}{2}}$

where $\quad P_e = \frac{h\sin\theta}{\sin\phi}\left(\lambda + \frac{h}{m_e c}(1-\cos\theta)\right)^{-1}$

and $\quad \tan\phi = \frac{\sin\theta}{1 + \frac{h}{m_e c\lambda}(1-\cos\theta) - \cos\theta}$

Perhaps it is preferable to determine p_e in terms of λ, θ directly (rather than via ϕ).

From (iii) $$p_e^2 = (p_\lambda - p_{\lambda'})^2 + 2m_ec(p_\lambda - p_{\lambda'}) \quad (*)$$

$$p_\lambda = \frac{h}{\lambda} \quad \text{and} \quad p_{\lambda'} = \frac{h}{\lambda'} \qquad \lambda' = \lambda + \frac{h}{m_ec}(1-\cos\theta)$$

so $$\boxed{p_e = \sqrt{(p_\lambda - p_{\lambda'})^2 + 2m_ec(p_\lambda - p_{\lambda'})}}$$

Now when $\cos\theta = -1$ $(\theta = 180^\circ)$, λ' is <u>maximized</u>

so since $p_{\lambda'} = h/\lambda'$ this means $p_{\lambda'}$ is <u>minimized</u>

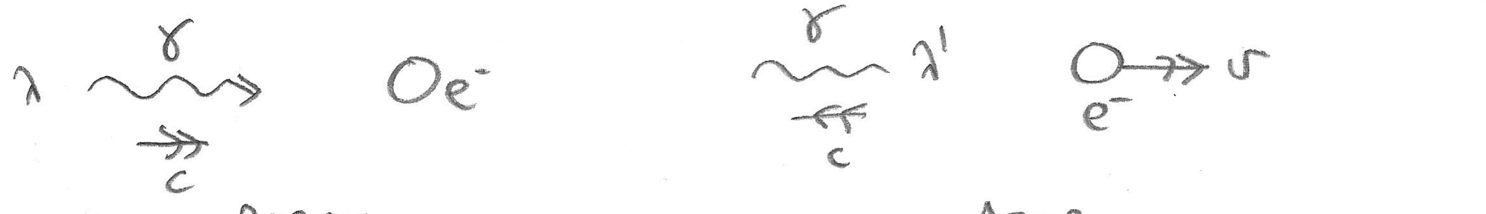
$\therefore$ $p_\lambda - p_{\lambda'}$ is maximized, which means p_e is maximized. (from (*))

$$\lambda'_{max} = \lambda + \frac{2h}{m_ec}$$

$$\text{So } p_\lambda - p_{\lambda'} = \frac{h}{\lambda} - \frac{h}{\lambda + \frac{2h}{m_ec}} = \frac{h}{\lambda}\left(1 - \frac{1}{1+\frac{2h}{\lambda m_ec}}\right)$$

So p_e max is $$\sqrt{\frac{h^2}{\lambda^2}\left(1-\left(1+\frac{2h}{\lambda m_ec}\right)^{-1}\right)^2 + \frac{2m_ech}{\lambda}\left(1-\left(1+\frac{2h}{\lambda m_ec}\right)^{-1}\right)}$$

Although a better way of finding it is to go back to conservation of energy



BEFORE AFTER

$$\boxed{\lambda' = \lambda + \frac{2h}{M_e c}}$$

(From $\Delta\lambda = \frac{h}{M_e c}(1-\cos\theta)$
and $\theta = +180^\circ$)

Conservation of energy

$$\frac{hc}{\lambda} + M_e c^2 = \frac{hc}{\lambda'} + \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} M_e c^2$$

$$\therefore \quad \left(\frac{hc/\lambda - hc/\lambda' + M_e c^2}{M_e c^2}\right)^{-2} = 1 - \frac{v^2}{c^2}$$

$$\therefore \quad \boxed{v = c\sqrt{1 - \left(\frac{M_e c^2}{M_e c^2 + \frac{hc}{\lambda} - \frac{hc}{\lambda'}}\right)^2}}$$