High Energy processes are those where the kinetic or potential energies of associated interactions are comparable or in excess of the rest mass energy of the constituent particles i.e. $E \sim mc^2$. In practice this means typical energies of GeV. $1\text{GeV} = 1.6 \times 10^{-10}\,\text{J}$

The **Standard Model of Particle Physics** describes the fundamental constituents of all matter, and how these distinct entities interact. These can be grouped into **fermions** (particles with half integer *spin*, that by the *Pauli Exclusion principle* cannot share the same *quantum state*) and **bosons** (integer spin particles that mediate the four fundamental interactions: *gravity*, *electromagnetism*, the *weak* force, and the *strong* force. Also the *Higgs* field, which confers *mass*).

Fermions are either **leptons** (*electron, muon, tau* e^-, μ^-, τ^- + associated *neutrinos* ν_e, ν_μ, ν_τ) or **quarks** (*up, down, charm, strange, top, bottom* u, d, c, s, t, b). For every particle x there is an **antiparticle** $\bar{x}$, which has the same mass, but *opposite charge*. For historical reasons, lepton antiparticles (such as the positron) are written as: e^+, μ^+, τ^+.

All leptons have the same magnitude of charge $e = 1.602 \times 10^{-19}\,\text{C}$. Quarks have *fractional* charge: u, c, t have charges of $+\frac{2}{3}e$ whereas d, s, b have charges of $-\frac{1}{3}e$. All quarks have spin 1/2. Quark masses, in MeV/c^2 are:

u	d	c	s	t	b
2.4	4.8	1,275	95	172,440	4,180

You don't get free quarks. They spontaneously form into groups called **hadrons**. **Baryons** are *quark triplets*, **mesons** are *quark + antiquark* pairs. It is possible or other groupings too e.g. a **pentaquark** (four quarks + one antiquark).

Baryon examples: $p = uud$ **proton ;** $n = udd$ **neutron;** $\Xi^0 = uss$ **;** $\Lambda^0 = uds$ **;** $\Sigma^- = dds$ **;** $\Omega^- = sss$

Meson examples: $\pi^+ = u\bar{d}$ **pion ;** $\pi^- = \bar{u}d$ **;** $\pi^0 = d\bar{d}$ **;** $K^+ = u\bar{s}$ **kaon.**

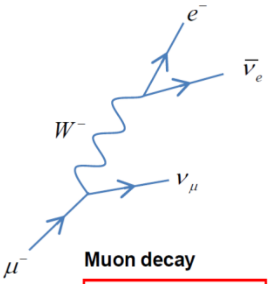
Bosons: *Photons* γ mediate the *electromagnetic* force. They are massless and chargeless. *Gluons* mediate the *strong* force between quarks. They are massless, and manifest in eight distinct combinations of red,green,blue (and anti versions) of a property called *'colour charge*.' Colour charge is the source of the strong interaction.

The *Weak* force (involved in radioactive decay) is mediated by the W^+, W^-, Z^0 bosons. Interestingly these have *mass* as well, in the case of W , electric charge. $W^\pm$ have masses of $80.39\,\text{GeV}/c^2$ and Z^0 has a mass of $91.19\,\text{GeV}/c^2$.

It is hypothesized that a 'graviton' mediates gravity, but currently a particle physics theory of gravity in not fully worked out.

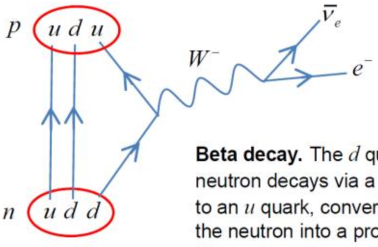
Relative strengths of forces are: Strong (60) : Electromagnetism (1) : Weak (10^{-4}) : Gravity (10^{-41}).

Feynman diagrams are a graphical method to represent particle physics interactions. They represent (typically the dominant) term in an infinite series, which when summed can determine the probability of a particular interaction. At each *vertex*, charge, baryon number (number of quarks), and lepton number are *conserved*. Note antiparticles yield negative numbers in this accounting. Interactions occur 'bottom top' (or alternatively 'left to right' - *I'll use 'bottom to top' as my convention*), which represents *causality* or 'the arrow of time.' The perpendicular dimension has no particular meaning as the length and angles of the arrows have no particular significance.



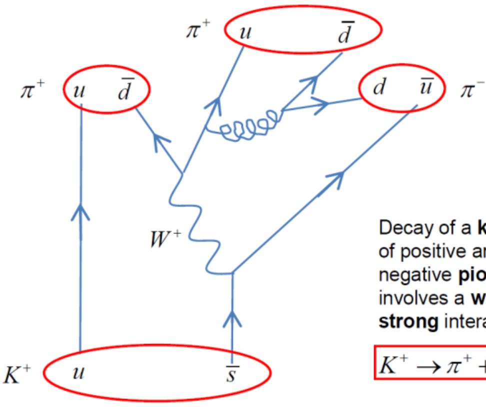
Muon decay

$\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e$



Beta decay. The d quark of a neutron decays via a W- boson to an u quark, converting the neutron into a proton. The W- boson decays into an electron and electron anti-neutrino.

$n \rightarrow p + e^- + \bar{\nu}_e$



Decay of a **kaon** to a pair of positive and one negative **pions**. Note this involves a **weak** *and* a **strong** interaction

$K^+ \rightarrow \pi^+ + \pi^+ + \pi^-$

Useful physical constants:

Electron mass $m_e = 9.109\times10^{-31}\,\text{kg}$; Proton mass $m_p = 1.6726\times10^{-27}\,\text{kg}$; Neutron mass $m_n = 1.6749\times10^{-27}\,\text{kg}$.

Speed of light $c = 2.998\times10^{8}\,\text{ms}^{-1}$; Planck's constant: $h = 6.626\times10^{-34}\,\text{Js}$; Electron charge $e = 1.602\times10^{-19}\,\text{C}$.

Atomic mass unit $u = 1.661\times10^{-27}\,\text{kg}$; $1\,\text{GeV}/c^2 = 1.7826619\times10^{-27}\,\text{kg} = 1.073544u$. Atomic mass unit is $\frac{1}{12}$ of the mass of the nucleus of Carbon-12.

Electron mass: $m_e = 0.51100\,\text{MeV/c}^2$; $m_p = 0.93827\,\text{GeV}/c^2$; $m_n = 0.93957\,\text{GeV}/c^2$; $u = 0.93149\,\text{GeV}/c^2$

Question 1

(i) The Feynman diagram for beta(-) decay implies the following decays: (a) $d \to u + W^-$ and (b) $W^- \to e^- + \bar{\nu}_e$. Show that charge, baryon number, lepton number are conserved in both.

(ii) Sketch the Feynman diagram for beta(+) decay, and show as in (i) that charge, baryon number and lepton number are conserved at each interaction vertex.

(iii) From the quark content of the particles, show that total baryon number and charge are conserved for the interaction: $\pi^- + p \to n + \pi^- + \pi^+$.

(iv) A kaon decays to pi mesons via the interaction $K^+ \to \pi^+ + \pi^0$. By considering the quark composition of these mesons, show that charge is conserved.

(v) Calculate the 'quark binding energy' in (a) K^+ and (b) π^+ given the respective rest masses are $m_{K^+} = 0.52616m_p$ and $m_{\pi^+} = 0.14875m_p$. To simplify calculations, use MeV/c^2 as the mass unit.

(vi) Sketch the Feynmann diagram for the decay $\Sigma^- \to n + e^- + \bar{\nu}_e$ and perform the same conservation law check as in (i) and (ii).

(vii) Two methods of detecting particles are cloud chambers and bubble chambers. Use your textbook to briefly summarize how both techniques work.

(viii) The energy of a gamma ray may be sufficient to create a matter + antimatter pair such as μ^-, μ^+ . Calculate the energy in J, and minimum wavelength of a gamma ray that can create such a pair of muons. The mass of a muon is $105.66\,\text{MeV}/c^2$.

(ix) A gamma ray produces an electron, positron pair, with both moving at $u = 0.8c$. Use the relativistic formula $E = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}} mc^2$ to determine the energy of the gamma ray in MeV.

(x) Accelerating charges radiate, i.e. lose energy via electromagnetic waves. This process is called *Bremsstrahlung*. Explain via a suitable diagram how a single high energy electron could result in a 'shower' of positrons and electrons, which enables the original source electron to be detected.

(xi) For u, d, s quarks, the *Koide* mass ratio $\dfrac{m_u + m_d + m_s}{\left(\sqrt{m_u} + \sqrt{m_d} + \sqrt{m_s}\right)^2} \approx \dfrac{k}{9}$. Find integer k .

Question 2 Strangeness is -1 × the number of strange quarks in a hardron. The Ω^- particle has a strangeness of -3.

(i) Consider the interaction $K^- + p \to \Omega^- + K^+ + K^0$. In order that charge, baryon number and strangeness are conserved, determine the quark content of K^0 .

(ii) $\Omega^- \to \Xi^0 + \pi^-$ decays via a weak interaction. Draw a Feynman diagram for this process, and show that at every vertex, charge and baryon number is conserved. Also show that strangeness is *not* conserved.

Question 3 A τ^- lepton (mass $1776.86\,\mathrm{MeV}/c^2$) is moving at $0.2c$. It has a lifetime of about 2.9×10^{-13} s, so will mostly likely decay into other particles rather than result in *Bremsstrahlung* and a shower of positrons and electrons. Since it has such a large mass, τ^- can decay into hadrons. About 10.8% of the time, $\tau^- \to \pi^- + \nu_\tau$.

(i) Draw a Feynman diagram for $\tau^- \to \pi^- + \nu_\tau$, which occurs via a weak interaction.

(ii) Explain which conservation law would be violated if the tau neutrino was *not* produced.

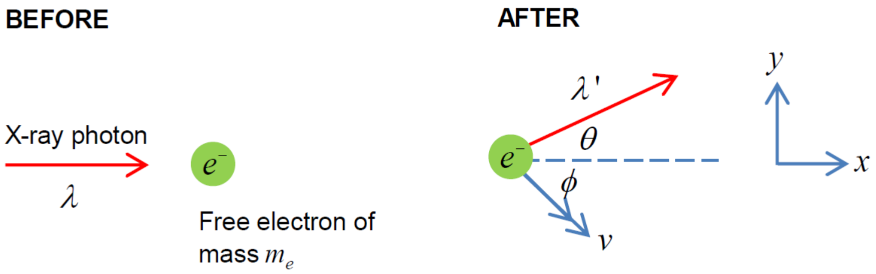
(iii) Using the relativistic formula $E = \left(1-\frac{u^2}{c^2}\right)^{-\frac{1}{2}} mc^2$, calculate the *kinetic* energy of the τ^-, in MeV.

(iv) The pion produced has a speed of $u = 0.99c$. What is the energy of the tau neutrino in MeV? $m_{\pi^+} = 0.14875 m_p$

(v) Accounting for relativistic effects, momentum of a particle of mass m is $p = \left(1-\frac{u^2}{c^2}\right)^{-\frac{1}{2}} mu$. Show that: $E^2 - p^2c^2 = m^2c^4$. If we can effectively neglect the mass of the tau neutrino, how much momentum does it have?

(vi) Evaluate the *Koide formulae* for leptons and quarks : (a) $\dfrac{m_e + m_\mu + m_\tau}{\left(\sqrt{m_e}+\sqrt{m_\mu}+\sqrt{m_\tau}\right)^2}$; (b) $\dfrac{m_c + m_b + m_t}{\left(\sqrt{m_c}+\sqrt{m_b}+\sqrt{m_t}\right)^2}$

Question 4 The *Compton effect* describes the scattering of X-rays off electrons. It is a classic demonstration of the wave-particle duality of *photons*. The scattering event can be analyzed like a collision problem involving particles, but the net result is the X-ray is not only scattered by angle θ, but *also* subject to a wavelength increase of $\Delta\lambda = \dfrac{h}{m_e c}(1-\cos\theta)$.



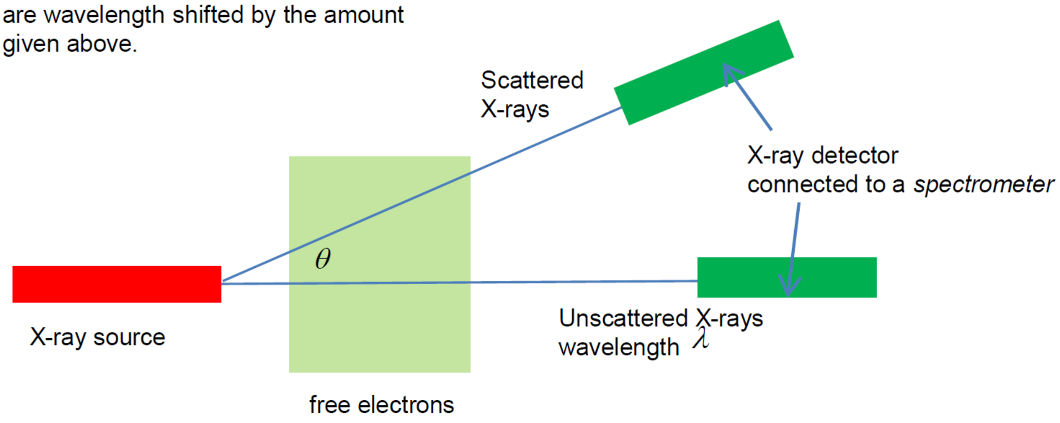
(i) Use the energy-momentum invariant, and Planck's law, to show that the momentum of a photon $p_\lambda = h/\lambda$, i.e. the de Broglie relation.

(ii) Let $p_\lambda = h/\lambda$ be the momentum of the X-ray photon before collision, and $p_{\lambda'} = h/\lambda'$ after the collision with the electron. By drawing a suitable triangle and using the cosine rule, or otherwise, show that conservation of momentum implies: $p_e^2 = p_\lambda^2 + p_{\lambda'}^2 - 2p_\lambda p_{\lambda'}\cos\theta$.

(iii) Use conservation of energy and the energy-momentum invariant (for both electron and photons) to show that: $p_e^2 = \left(p_\lambda - p_{\lambda'}\right)^2 + 2m_e c\left(p_\lambda - p_{\lambda'}\right)$. Don't forget the rest mass of the electron before the X-ray scatters!

(iv) By eliminating p_e^2 between the equations in (ii) and (iii), prove that:

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta).$$



If one places an X-ray detector at angle θ from an incident beam, one should detect scattered X-rays which are wavelength shifted by the amount given above.

(v) At what angle should one place a detector to measure the largest wavelength shift? Calculate the energy (in keV) of X-rays to result in a 10% (maximum) wavelength shift.

(vi) Derive the following expressions for the recoil speed v and angle ϕ of the electron in terms of λ, h, m_e, θ:

$$\tan\phi = \frac{\sin\theta}{1+\frac{h}{m_e c\lambda}(1-\cos\theta)-\cos\theta}; \qquad v = c\left(1+\left(\frac{m_e c}{p_e}\right)^2\right)^{-\frac{1}{2}}$$

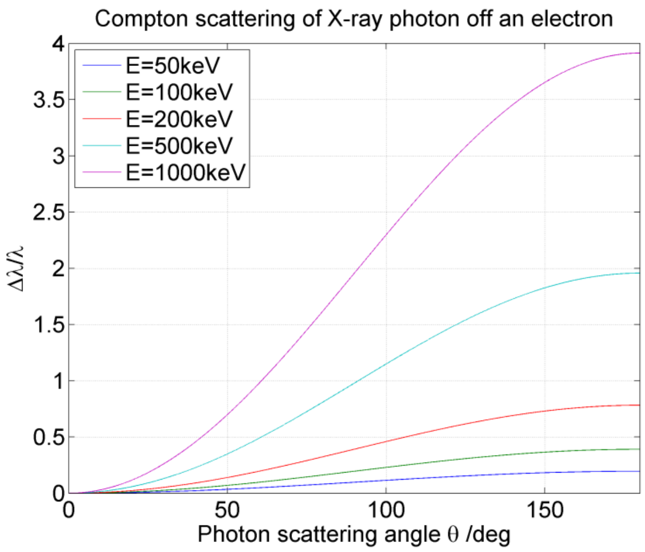
$$p_e = \sqrt{\left(p_\lambda - p_{\lambda'}\right)^2 + 2m_e c\left(p_\lambda - p_{\lambda'}\right)}$$

$$p_\lambda = h/\lambda; \quad p_\lambda = h/\lambda'; \quad \lambda' = \lambda + \frac{h}{m_e c}(1-\cos\theta)$$

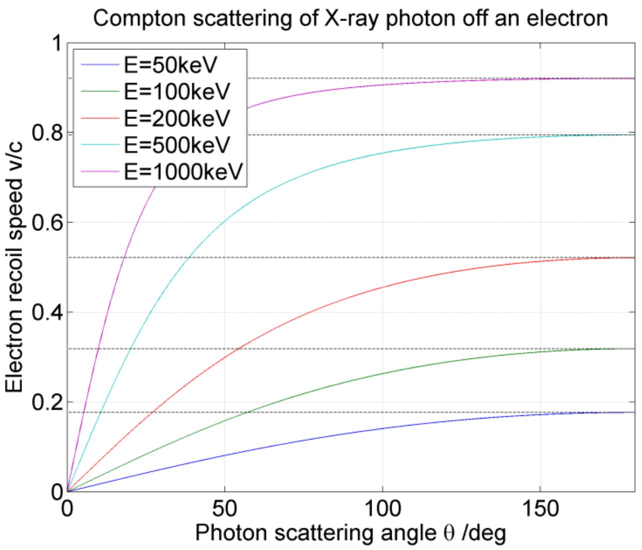
(vii) By considering the photo scattering angle situation which results in the largest wavelength shift, explain why this is also the scenario for the *maximum* electron recoil. Hence, from conservation of energy, show that the maximum electron recoil speed is:

$$v = c\sqrt{1-\left(\frac{m_e c^2}{m_e c^2 + hc/\lambda - hc/\lambda'}\right)^2}; \qquad \lambda' = \lambda + 2h/m_e c$$

(viii) Write a computer program to generate the following graphs: E is the energy of the incident X-ray photon.



Compton scattering of X-ray photon off an electron



Compton scattering of X-ray photon off an electron