PHYSICS

USEFUL DATA AND FORMULAE

JP MP VBk version

Assembled by Dr Andrew French

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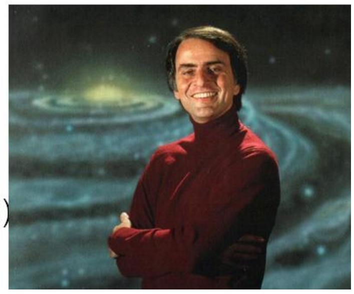
The Cosmos is all that is or ever was or ever will be.

In the last few millennia we have made the most astonishing and unexpected discoveries about the Cosmos and our place within it, explorations that are exhilarating to consider. They remind us that humans have evolved to wonder, that understanding is a joy, that knowledge is prerequisite to survival.

I believe our future depends on how well we know this Cosmos in which we float like a mote of dust in the morning sky.



Carl Sagan (1934-1996)
Cosmos pp20

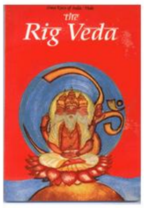
A human history of cosmology

20,000 BC — 5,000 BC — 3000 BC — 2000 BC — 3000 - 300 BC

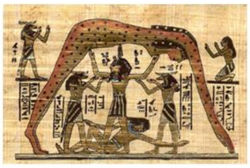
Ishango Bone
Africa

Megalithic structures

Babylonian



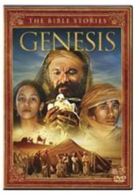
Hindu Rig Veda



Ancient Egypt

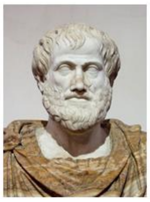
520BC — 500BC — 384-322BC — 200AD — 1564-1642

Jainism



Old Testament

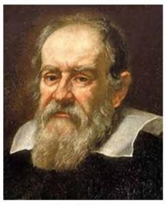
Parmenides
Anaxagoras



Aristotle

Aristarchus

Ptolemy



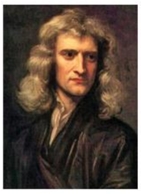
Galileo

500AD — 1473-1543 — 1571-1630 — 1642-1727 — 1879-1955 — 1894-1966

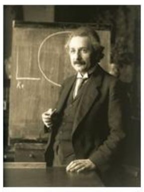
Aryabhata

Copernicus

Kepler



Newton



Einstein

Lemaitre

Hawking

CONTENTS

1. Physical quantities and units

$\pi \approx 3.142$ $e \approx 2.718$ $\sqrt{2} \approx 1.414$ **SI** means '*Système International d'Unités*' (International System of Units)

Quantity	Symbol	Vector or scalar?	Unit	Unit abbreviation	Notes
Mole **SI**	n	scalar	mole	mol	**Mol is a base SI unit** Avogadro's constant $N_A = 6.022 \times 10^{23}$ molecules per mole
Mass **SI**	m, M	scalar	kilogram tonne gram atomic mass unit	kg 10^3 kg 10^{-3}kg u	**kg is a base SI unit** Electron mass $m_e = 9.109 \times 10^{-31}$ kg Proton mass $m_p = 1.673 \times 10^{-27}$ kg Neutron mass $m_n = 1.675 \times 10^{-27}$ kg $u = 1.660 \times 10^{-27}$ kg Earth mass $M_\oplus = 5.974 \times 10^{24}$ kg Solar mass $M_\odot = 1.989 \times 10^{30}$ kg
Length **SI**	$l, L,$ $a, b, c, ..$ $...x, y, z$	scalar	angstrom nanometre micron millimetre centimetre metre kilometre mile Astronomical Unit parsec light-year	Å = 10^{-10}m nm = 10^{-9}m μm = 10^{-6}m mm = 10^{-3}m cm = 10^{-2}m m km = 10^3m mile AU parsec lyr	**m is a base SI unit** mile = 1,609m AU = 1.496 x10^{11}m parsec = 3.086 x10^{16}m lyr = 9.461 x10^{15}m
Angle	θ, ϕ $a, b, c ... \alpha, \beta$	scalar	degrees radians arc-minute arc-second	^{o}, deg rad arcmin = (1/60) deg arcsec = (1/3600) deg	π radians = 180^o
Area	A	scalar	square mm square centimetres square metres square kilometre hectares acre	mm^2 cm^2 m^2 km^2 ha acre	$mm^2 = 10^{-6}\ m^2$ $cm^2 = 10^{-4}\ m^2$ $km^2 = 10^6\ m^2$ ha = $10^4\ m^2$ acre = 4.047 x $10^3\ m^2$
Volume	V	scalar	cubic centimetre cubic metre cubic kilometre millilitre litre gallon	cm^3 m^3 km^3 ml l gal	$cm^3 = 10^{-6}\ m^3$ $km^3 = 10^9\ m^3$ ml = 1cm^3 (pure water at STP) l = $10^3 cm^3 = 10^{-3} m^3$ gal = 4.546 x$10^{-3}\ m^3$

Quantity	Symbol	Vector or scalar?	Unit	Unit abbreviation	Notes
Time **SI**	t, τ	scalar	picosecond nanosecond microsecond millisecond second minute hour day year	ps ns μs ms s min hr d = 24hr yr	**s is a base SI unit** min = 60s, hr = 60min = 3600s, $\text{yr} \approx 365 \times 24 \times 3600\text{s}$ $\text{yr} \approx 3.154 \times 10^7\text{s}$ $\text{yr} \approx \pi \times 10^7\text{s}$ Age of the Earth = 4.5 x 10^9 yr Age of the Universe = 13.8 x 10^9 yr
Speed	s u, v	scalar	metre per second kilometre per hour mile per hour	ms^{-1} kmh^{-1} mph	Speed of light in a vacuum c = 2.998 x 10^8 ms^{-1} Speed of sound in air (20^oC): 344 ms^{-1} Speed of sound in water: 1482 ms^{-1} $1\text{ms}^{-1} = 3.6\text{kmh}^{-1}$ 1ms^{-1} = 2.24 mph 1 min per mile at 60mph 3 mins per mile at 20mph 6 mins per mile at 10mph
Frequency	f	scalar	Hertz Kilohertz Megahertz Gigahertz Terahertz	Hz = s^{-1} kHz = 10^3 Hz MHz = 10^6 Hz GHz = 10^9 Hz THz = 10^{12} Hz	human voice sound waves 0 - 2kHz Radio waves 3kHz - 300MHz Microwaves 3MHz - 100GHz Infrared 100GHz - 300THz Visible light 10^{14} - 10^{15} Hz Ultraviolet 10^{15} Hz - 10^{16} Hz X-rays 10^{16} Hz - 10^{20} Hz Gamma rays > 10^{20} Hz
Period	T	scalar	Same as time	Same as time	Time to complete a single oscillation. $T = \frac{1}{f}$ e.g. period of Earth's rotation is 24 hours, period of Earth's orbit about the sun is 1 year.
Displacement	$\mathbf{x}$ x, y, z	vector	Same as length	Same as length	*Magnitude* as well as *direction*. Often we describe $\mathbf{x}$ in terms of a *coordinate system* e.g. *x,y,z Cartesians*. In this case a negative value of *x* means 'going backwards'. In one direction, displacement is the *area* under a (time,velocity) graph, where area below the time axis is negative.
Velocity	$\mathbf{v}$ u, v	vector	Same as speed	Same as speed	*Magnitude* as well as *direction*. Often we describe in terms of a *coordinate system* e.g. *x,y,z Cartesians*. In this case a negative value of *v* means 'going backwards'. In one direction, velocity is the *gradient* of a (time,displacement) graph. It is also the *area* under a (time, acceleration) graph, where area below the time axis is negative. u is typically a symbol for initial velocity v for final or 'current' velocity
Acceleration	$\mathbf{a}$ a	vector	metre per second squared	ms^{-2}	*Magnitude* as well as *direction*. Often we describe in terms of a *coordinate system* e.g. *x,y,z Cartesians*. In one direction, acceleration is the *gradient* of a (time,velocity) graph. 'Free-fall' acceleration under gravity: $g_{earth} = 9.81\text{ms}^{-2}$ $g_{moon} = 1.63\text{ms}^{-2}$

Quantity	Symbol	Vector or scalar?	Unit	Unit abbreviation	Notes
Energy	E	scalar	Joules kilojoules megajoules calories kilo-calories kilowatt-hour electron-volts	J kJ = 10^3J MJ = 10^6J cal = 4.184J kcal = 10^3 cal kWh eV keV = 10^3eV MeV = 10^6eV	Energy is *conserved,* i.e. in a closed system has the *same numerical value*. It can be converted into different forms e.g. *kinetic* and *potential* energy. Calories measure energy in food 1kWh is a standard measure of domestic electricity consumption. *Total* UK energy consumption is about 125 kWh per person per day. eV = kinetic energy of an electron accelerated by a voltage V $eV = 1.602 \times 10^{-19}\,\mathrm{J}$
Power	P	scalar	Watts kilowatts megawatts gigawatts terawatts horsepower	W = Js^{-1} kW = 10^3 W MW = 10^6 W GW = 10^9 W TW = 10^{12} W hp = 746W	*Power is the rate of energy changed from one form into another* A light bulb uses about 20W Dr French's computers use about 250W A kettle uses about 2kW A Tour-de-France cyclist expends 250-500W A wind turbine generates 1-10MW A power station generates up to 5GW About 1.36 kWm^{-2} of solar radiation shine on the Earth.
Force	$\mathbf{f}$ f, F	vector	Newtons kilonewtons	N kN	*Newton's Second Law:* mass x acceleration = vector sum of forces
Weight	W	vector	Newtons kilonewtons	N kN	The gravitational force F acting upon a mass m is $F = mg$.
Tension	T	vector	Newtons kilonewtons	N kN	Force applied by a taught cable or string. Often these are modelled as *light* and *inextensible*. i.e. ignore the effect of their mass and assume they don't stretch.
Momentum	$\mathbf{p} = m\mathbf{v}$ $p = mv$	vector	kilogram-metres per second	$kgms^{-1}$	Note *impulse* is a change in momentum, e.g. due to a collision or from the action of some external force over a period of time.
Moment	$m = Fd$	scalar	Newton-metre	Nm	Force x *perpendicular* distance from a pivot
Elasticity	$k = \frac{\lambda}{l}$	scalar	Newtons per metre	Nm^{-1}	If an elastic body (such as spring or rubber band) is *Hookean*, the restoring force following extension by x is $F = \frac{\lambda}{l}x$ where l is the natural length and λ is the elastic modulus.

Quantity	Symbol	Vector or scalar?	Unit	Unit abbreviation	Notes
Density	ρ	scalar	mass per unit volume	kgm^{-3} gcm^{-3}	Air is about 1.2 kgm^{-3} Wood is about 0.5 gcm^{-3} Water is about 1 gcm^{-3} Aluminium is 2.7 gcm^{-3} Iron is 7.8 gcm^{-3} Copper is 8.9 gcm^{-3} Mercury is 13.5 gcm^{-3} Gold is 19.3 gcm^{-3} Uranium is 19.1 gcm^{-3}
Pressure	p	scalar	Pascal kilopascal megapascal millibar Atmosphere	Pa kPa = 10^3 Pa MPa = 10^6 Pa mbar = 100Pa atm	*Force per unit area* Pa = $1Nm^{-2}$ atm = 101,325 Pa is essentially a 'reference' atmospheric pressure at sea level. atm = 1013.25 mbar. Millibars are used to measure air pressure in *meteorology,* i.e. climate science and weather forecasting.
Temperature **SI**	T	scalar	degrees celcius degrees fahrenheit degrees kelvin	^oC ^oF K	**K is a base SI unit** $T_K = T_C + 273.15$ $T_F = \frac{9}{5}T_C + 32$ Temperature in K is proportional to the mean kinetic energy of molecules. Hence nothing can be colder than 0K "absolute zero"
Solid or liquid specific heat capacity	c	scalar	joules per kilogram per Kelvin	$Jkg^{-1}K^{-1}$	water 4200 $Jkg^{-1}K^{-1}$ alcohol 2500 $Jkg^{-1}K^{-1}$ ice 2100 $Jkg^{-1}K^{-1}$ aluminium 900 $Jkg^{-1}K^{-1}$ concrete 800 $Jkg^{-1}K^{-1}$ glass 700 $Jkg^{-1}K^{-1}$ steel 500 $Jkg^{-1}K^{-1}$ copper 400 $Jkg^{-1}K^{-1}$
Gas specific heat capacity	c_p c_v	scalar	joules per kilogram per Kelvin	$Jkg^{-1}K^{-1}$	c_P is at constant pressure, c_V at constant volume. c_P for dry air is about 1000$Jkg^{-1}K^{-1}$
Specific latent heat of fusion	L ΔH	scalar	joules per kilogram	Jkg^{-1}	water 336,000 Jkg^{-1} alcohol 108,000 Jkg^{-1}
Specific latent heat of vaporisation	L ΔH	scalar	joules per kilogram	Jkg^{-1}	water 2,260,000 Jkg^{-1} alcohol 855,000 Jkg^{-1}

Quantity	Symbol	Vector or scalar?	Unit	Unit abbreviation	Notes
Charge	q, Q e	scalar	Coulombs	C	charge on the electron $e = 1.602 \times 10^{-19}\,\mathrm{C}$
Voltage	V	scalar	Volts millivolts kilovolts Megavolts	V mV = 10^{-3}V kV = 10^{3}V MV = 10^{6}V	Potential energy per coulomb of charge Energy change per coulomb of charge *across* a resistor. 'Electromotive force' (EMF).
Current **SI**	I	scalar	Amps milliamps	A mA	**A is a base SI unit** Rate of charge flowing in an electrical circuit (coulombs per second).
Resistance	R	scalar	Ohms kilo-ohms mega-ohms	Ω	Ohm's Law: $V = IR$ Voltage drop across a resistor is proportional to resistance, and current flowing through it.
Resistivity	ρ	scalar	ohm-metre	$\Omega\mathrm{m}$	Resistance of a cylindrical wire of length l and cross sectional area A is $R = \rho \frac{l}{A}$ Copper ρ = 1.68 x $10^{-8}\,\Omega\mathrm{m}$ Aluminium ρ = 2.82 x $10^{-8}\,\Omega\mathrm{m}$ Gold ρ = 2.44 x $10^{-8}\,\Omega\mathrm{m}$ Iron ρ = 1.00 x $10^{-7}\,\Omega\mathrm{m}$ Sea water ρ = 2.00 x $10^{-1}\,\Omega\mathrm{m}$ Glass ρ = 10^{11} - $10^{15}\,\Omega\mathrm{m}$ Hard rubber ρ = $10^{13}\,\Omega\mathrm{m}$ Dry wood ρ = 10^{14} - $10^{16}\,\Omega\mathrm{m}$ Air ρ = 1.3 - 3.3 x $10^{16}\,\Omega\mathrm{m}$
Electric field strength	$\mathbf{E}$ E	vector	Volts per metre	$\mathrm{Vm^{-1}}$	Force on a charge q coulombs in a electric field of strength E is $F = qE$ A dielectric will conduct electricity ('breakdown') when E exceeds a critical value. **Note: 1MVm^{-1} = 10^{6}Vm^{-1}** Air $E > 3\mathrm{MVm^{-1}}$ Glass $E > 10\mathrm{MVm^{-1}}$ Oil $E > 10\mathrm{MVm^{-1}}$ Rubber $E > 15\mathrm{MVm^{-1}}$ pure water $E > 65\mathrm{MVm^{-1}}$ Mica $E > 118\mathrm{MVm^{-1}}$ Diamond $E > 2{,}000 \times 10^{6}\,\mathrm{Vm^{-1}}$

Quantity	Symbol	Vector or scalar?	Unit	Unit abbreviation	Notes
Magnetic field strength	$\mathbf{B}$ B	vector	Tesla	T	Force on a wire of length l carrying current I in magnetic field B is $F = BIl$ Note force, current and field are *mutually perpendicular.* Magnetic field inside a solenoid (a coil of wire carrying current I of n turns per unit length) is $B = \mu_0 nI$ 'Permeability of free space' $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{Hm}^{-1}$ Earth's magnetic field $\approx 25\mu\mathrm{T} - 65\mu\mathrm{T}$ (μT = 10^{-6}T) 5 x 10^{-3}T strength of fridge magnet 1.5-3T field strength of a Magnetic Resonance Imaging (MRI) system 10^6T-10^8T field strength of a *neutron star* 10^8T-10^{11}T field strength of a *magnetar*
Gravitational field strength	$\mathbf{g}$ g	vector	metres per second squared	ms^{-2}	$g_{earth} = 9.81\mathrm{ms}^{-2}$ $g_{moon} = 1.63\mathrm{ms}^{-2}$
Radioactive activity	A	scalar	Becquerel Curies	Bq Ci	Bq = radioactive decays per second Ci = 3.7 10^{10}Bq (activity of one gram of $^{226}_{88}\mathrm{Ra}$)
Half life	$T_{\frac{1}{2}}$	scalar	Seconds, days, years	s, d, yr	Time for half of a sample of radioactive isotopes to have decayed $^{235}_{92}\mathrm{U}$ 7 x 10^8 yr $^{14}_{12}\mathrm{C}$ 5,730 yr $^{123}_{53}\mathrm{I}$ 13 hrs
Refractive index	n	scalar	just a number	-	$n = \frac{c_{\mathrm{vacuum}}}{c_{\mathrm{material}}}$ i.e. ratio of speed of light in a vacuum (2.998 x $10^8\mathrm{ms}^{-1}$) to speed of light in a material. vacuum 1 air 1.00 ice 1.31 water 1.33 human cornea 1.37-1.40 human lens 1.39-1.41 plexiglas 1.49 crown glass 1.52 sapphire 1.76-1.78 diamond 2.42

2. Mechanics

Name	Equation	Description of variables	Notes / diagram
Kinematics	$v = \frac{dx}{dt} \quad x = \int v dt$ $a = \frac{dv}{dt} \quad v = \int a dt$	x displacement v velocity a acceleration t time	***Velocity*** is the gradient of a (time,displacement) graph. ***Displacement*** is the area under a (time, velocity). graph. Note areas below the time axis are negative. ***Acceleration*** is the gradient of a (time,velocity) graph velocity is the area under a (time, velocity). graph. Note areas below the time axis are negative.
Constant acceleration motion	$v = u + at$ $x = x_0 + \frac{1}{2}(u+v)t$ $x = x_0 + ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2ax$	u initial velocity /ms^{-1} a acceleration /ms^{-2} t time /s v final velocity /ms^{-1} x displacement /m x_0 initial displacement /m	Only valid for motion when acceleration a is *constant*. Easily derived from linear velocity, time graph. a is the *gradient* $a = \frac{v-u}{t}$ and $x - x_0$ is the *area under the graph*, which is a *trapezium* hence $x - x_0 = \frac{1}{2}(u+v)t$
Newton's First Law	$\mathbf{a} = 0$ $\Rightarrow \mathbf{v} =$ constant	$\mathbf{a}$ acceleration $\mathbf{v}$ velocity	A object will move at *constant velocity* if it is *not accelerating*, and therefore the vector sum of forces is zero. It is in ***equilibrium***.
Newton's Second Law	$m\mathbf{a} = \sum_i \mathbf{f_i}$	mass x acceleration = vector sum of forces	Most mechanics problems are often solved by firstly writing down Newton II for each direction of a coordinate system (typically Cartesian *x,y*) appropriate for the problem.
Newton's Third Law	*"For every action there is an equal and opposite reaction"*		If body A imposes a contact force $\mathbf{F}$ upon body B, body B will in turn impose a contact force $-\mathbf{F}$ upon body A.
Conservation of momentum	$\underbrace{m_1\mathbf{u}_1 + m_2\mathbf{u}_2 + ...}_{\text{BEFORE COLLISION}} = \underbrace{m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + ...}_{\text{AFTER COLLISION}}$	momentum = mass x velocity	The vector sum of momenta is the same before and after a collision
Impulse	"Force x time = change in momentum" $\int_0^t \mathbf{f}(t)dt = m\mathbf{v} - m\mathbf{u}$	$\mathbf{f}(t)$ force (as a function of time t), m mass $\mathbf{v}$ final velocity $\mathbf{u}$ initial velocity 'impulse' means momentum change	In each direction of a coordinate system, the area under the (time,force) graph is the *change in momentum*. If force is a constant force x time = change in momentum

Conservation of energy	$\frac{1}{2}mu^2 + \mathrm{GPE}_0 + \mathrm{EPE}_0 + ... = \frac{1}{2}mv^2 + \mathrm{GPE}_1 + \mathrm{EPE}_1 + ...$ $\mathrm{GPE} = mgh$ $\mathrm{GPE} = -\frac{GMm}{r}$ $\mathrm{EPE} = \frac{1}{2}kx^2 = \frac{1}{2}\frac{\lambda}{l}x^2$	u,v initial and final speeds. h change in vertical height. g gravitational field strength. M,m masses. r distance between masses. G gravitational force constant. x spring extension. k spring constant. λ modulus of elasticity. l original length of spring.	$g_{earth} = 9.81\mathrm{ms}^{-2}$ $g_{moon} = 1.63\mathrm{ms}^{-2}$ $G = 6.67 \times 10^{-11}\,\mathrm{Nm}^2\mathrm{kg}^{-2}$
Coefficient of restitution	$C = \frac{\text{speed of separation}}{\text{speed of approach}}$	$C = 1$ *elastic* collision (kinetic energy conserved). $C = 0$ *inelastic* collision (objects remain together, some kinetic energy is lost)	
Work done	'WORK DONE = FORCE x DISTANCE' $W = \int \mathbf{f} \cdot d\mathbf{r} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$	$\mathbf{f}$ force. $\mathbf{r}$ displacement. m mass. u,v initial and final speeds.	Work done is "the area under a (displacement, force) graph", noting that areas below the *x* axis are *negative*.
Moments	'MOMENT = FORCE x PERPENDICULAR DISTANCE FROM ROTATION AXIS' $M = Fd$	F force d distance from axis of rotation	In **equilibrium**, the sum of moments (clockwise or anticlockwise) is **zero**, regardless of the axis position chosen!
Projectile motion	$v_x = u\cos\theta \quad v_y = u\sin\theta - gt$ $v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 - 2g(y - y_0)}$ $x = ut\cos\theta$ $y = y_0 + x\tan\theta - \frac{g}{2u^2}(1 + \tan^2\theta)x^2$ $t_a = \frac{u\sin\theta}{g}$ $y_a = y_0 + \frac{u^2\sin^2\theta}{2g}$ $x_a = \frac{u^2\sin\theta\cos\theta}{g}$ $R = \frac{u^2\sin 2\theta}{g}$	v_x horizontal velocity v_y vertical velocity v speed u launch speed θ launch elevation g gravitational acceleration t time since launch x horizontal displacement y vertical displacement y_0 initial vertical displacement t_a apogee time x_a, y_a apogee coordinates R horizontal range if $y_0 = 0$	Projectile motion is essentially *constant acceleration motion in both x and y directions*, if air resistance is ignored. In the x direction acceleration is zero, hence a *constant* velocity $v_x = u\cos\theta$. The x,y curve traced out by particle is an *inverted parabola.* Typically for a given range there are two possible trajectories for a given launch velocity u corresponding to 'steep' and 'shallow' solutions for elevation θ.

Lift and drag	$F_L = \frac{1}{2} c_L \rho A v^2$ $F_D = \frac{1}{2} c_D \rho A v^2$ $F = 6\pi a \eta v$	c_L lift coefficient. c_D drag coefficient. ρ density of air/fluid. A area of object in fluid stream. v speed. a radius of sphere. η viscosity	The linear *Stokes drag* equation $F = 6\pi a \eta v$ is typically applicable in *low Reynolds number* scenarios when *viscous forces dominate.* Air resistance models for bikes, cars, planes, skydivers are typically better served by the v^2 models.
Force of gravity & Kepler's Laws of orbital motion	$\mathbf{W} = m\mathbf{g}$ $\mathbf{F} = -\frac{GMm}{r^2}\hat{\mathbf{r}}$ $r = \frac{a\left(1-\varepsilon^2\right)}{1+\varepsilon\cos\theta}$ $\varepsilon = \sqrt{1-\frac{b^2}{a^2}}$ $P^2 = \frac{4\pi^2}{G(M+M_\odot)}a^3$ $\frac{dA}{dt} = \frac{1}{2}\sqrt{G(M+M_\odot)\left(1-\varepsilon^2\right)a}$	$\mathbf{W}$ weight g gravitational field strength. M, m masses. r distance between masses. G gravitational force constant. ε eccentricity of elliptical orbit. a semi-major axis of the ellipse. b semi-minor axis of the ellipse. $M_\odot$ stellar mass. θ polar angle (anticlockwise from semi-major axis). P orbital period.	$g_{earth} = 9.81\text{ms}^{-2}$ $g_{moon} = 1.63\text{ms}^{-2}$ $G = 6.67\times10^{-11}\text{Nm}^2\text{kg}^{-2}$ **Kepler's First Law:** *Bound gravitational orbits of two masses are ellipses, with foci about the common centre of mass.* **Kepler's Second Law:** *The rate of ellipse area swept out (radially from the focus of the ellipse) is a constant* **Kepler's Third Law:** *The square of orbital period is proportional to the cube of the ellipse semi-major axis*
Elasticity (or elastic strings)	$F = kx = \frac{\lambda}{l}x$ $E = \frac{1}{2}kx^2$	F force. k elastic constant. λ elastic modulus. l original length of elastic string. x extension. E elastic potential energy	Most elasticity models are *Hookean* and assume a constant modulus of elasticity. In reality for large extensions there will be plastic deformation and ultimately breakage.
Friction	$F \le \mu R$ $F = \mu R$	F frictional force. μ coefficient of friction. R normal contact force	A system is said to be in 'limiting' equilibrium' if $F = \mu R$, i.e. 'on the point of sliding'. Once an object is sliding along a surface, the friction force 'maxes out' out $F = \mu R$. Note μ may change in this dynamic case. $F \le \mu R$ can be used to determine conditions (e.g. tilt angle of a slope) for sliding to occur.

Power & driving force	$P = Fv$ $P = \mathbf{F} \cdot \mathbf{v}$ $D = \frac{P}{v}$	$\mathbf{F}, F$ force P power $\mathbf{v}, v$ velocity D driving force	One *Horsepower* (hp) = 746W. This equation is useful in relating energy conversion rates in engines to resulting motion.

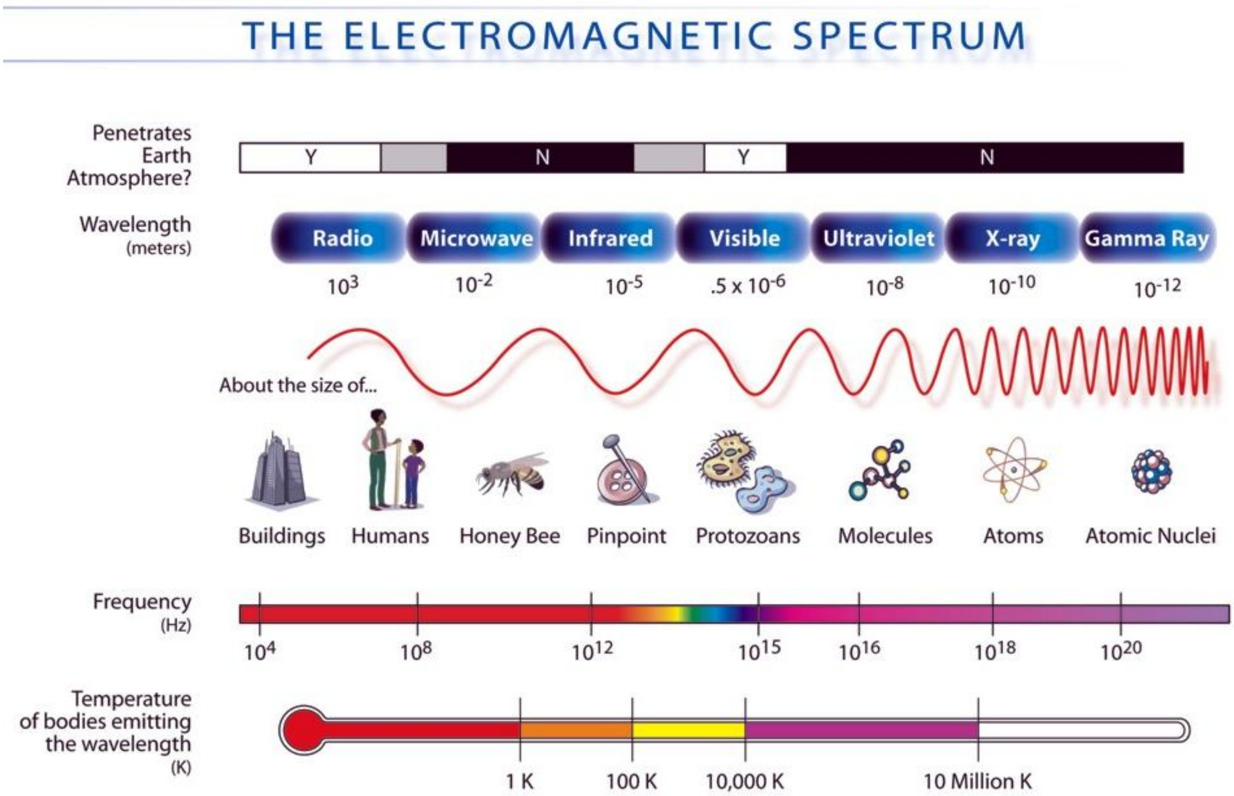
3. Electricity & Magnetism

Name	Equation	Description of variables	Notes / diagram
Charge on a capacitor	$Q = CV$	Q charge /coulombs C capacitance /Farads V voltage /volts	Voltage across two capacitor plates separated by an insulating *dielectric*
Ohm's law	$V = IR$	V voltage /volts I current /amps R resistance /ohms	Voltage or 'potential difference' *across* a resistive element
Electrical power	$P = VI$	P power /watts V voltage /volts I current /amps	
Resistive power loss	$P = I^2 R$	P power /watts I current /amps R resistance /ohms	
Resistance of a wire	$R = \frac{\rho l}{A}$	R resistance l length A cross sectional area ρ resistivity	Assume uniform resistivity and cross sectional area. Copper $\rho = 1.68 \times 10^{-8}\ \Omega\text{m}$ Aluminium $\rho = 2.82 \times 10^{-8}\ \Omega\text{m}$ Air $\rho = 1.3 - 3.3 \times 10^{16}\ \Omega\text{m}$
Addition of series resistors	$R = R_1 + R_2 + ...$	R resistance	
Addition of parallel resistors	$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + ...$	R resistance	
Magnetic field inside an infinite solenoid	$B = \mu\mu_0 \frac{NI}{l}$	B magnetic field strength μ relative permeability μ_0 permeability of free space = $4\pi \times 10^{-7}\ \text{Hm}^{-1}$ N turns in length l I current	A soft magnetic material inside the coil will enhance the magnetic field. For ferrite $\mu > 640$
Transformers	$\frac{V_2}{V_1} = \frac{N_2}{N_1}$ $\frac{I_2}{I_1} = \frac{N_1}{N_2}$	V_1, I_1 Voltage, current in primary coil, V_2, I_2 voltage, current in secondary coil, N_1, N_2 number of turns in primary, secondary coils	This assumes no power is lost i.e. $V_1 I_1 = V_2 I_2$

4. Waves & optics

Name	Equation	Description of variables	Notes / diagram
Wave speed equation	$c = f\lambda$	c wave speed f frequency λ wavelength	Speed of light in a vacuum c = 2.998 x 10^8 ms^{-1} Speed of sound in air (20^oC): 344 ms^{-1} Speed of sound in water: 1482 ms^{-1}
Wavenumber	$k = \frac{2\pi}{\lambda}$	λ wavelength	Phase of a wave is $\phi = kx - \omega t$
Frequency and period	$f = \frac{1}{T}$	f frequency T period	
Speed of waves in elastic media	$c = \sqrt{\frac{T}{\mu}}$ string under tension $c = \sqrt{\frac{E}{\rho}}$ elastic solid	c wave speed T string tension μ mass per unit length E Elastic modulus ρ Density	e.g. guitar string low E μ = 0.0059 kgm^{-1} f = 82.41Hz (fundamental mode, so string length = $\frac{1}{2}\lambda$) $T =$ 67.8N $\lambda = 1.30$ m
Law of reflection	$\theta_i = \theta_r$	θ_i angle of incidence θ_r angle of reflection	Angles measured from normal to the reflecting surface
Snell's law of refraction	$n_1 \sin\theta_1 = n_2 \sin\theta_2$ $n = \frac{\text{speed of light in vaccum}}{\text{speed of light in medium}}$ $\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2}$ $0 \le \theta \le \frac{1}{2}\pi$ $\therefore 0 \le \sin\theta_2 \le 1$ $\therefore 0 \le \frac{n_1 \sin\theta_1}{n_2} \le 1$ $\theta_1 \le \sin^{-1}\left(\frac{n_2}{n_1}\right)$	n_1 refractive index of medium 1 θ_1 angle of incidence to boundary of medium 1 to 2 n_2 refractive index of medium 2 θ_2 angle of refraction in medium 2	Angles measured from normal to the reflecting surface Total internal reflection at glass : air interface i.e. no refraction if $\theta_i > \sin^{-1}\left(\frac{n_{air}}{n_{glass}}\right)$ $\theta_i > \sin^{-1}\left(\frac{1}{1.52}\right)$ $\theta_i > 41.1^o$ *critical angle*
Gauss' Lens Formula	$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$	u object distance v image distance f focal length of a lens	$1/f$ = **Dioptre number.** **f-number** is $\frac{f}{\text{aperture diameter}}$ An f-number of 2 would conventionally be written as **f/2**, which gives the aperture diameter given the lens focal length.
Lensmakers' formula	$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$	f focal length of a lens n refractive index of lens R_1, R_2 radii of curvature of lens surface	

Doppler effect	$\Delta f = \frac{v}{c} f$	v velocity towards observer c wave speed f emitted wave frequency Δf frequency shift (from f) of waves arriving at observer.	



5. Thermal physics

Name	Equation	Description of variables	Notes / diagram
Ideal gas laws	$pV = nRT$ $V \propto T$ Charles' Law $p \propto \frac{1}{V}$ Boyle's Law	p pressure V volume n number of moles of gas R molar gas constant T absolute temperature (in kelvin)	R = 8.314 $Jmol^{-1}K^{-1}$
Heat capacity and energy change	$\Delta E = mc\Delta T$	ΔE energy required to raise the temperature of a mass m by ΔT c is the specific heat capacity	
Kelvin, Celcius and Fahrenheit temperature scales	$T_K = T_C + 273.15$ $T_F = \frac{9}{5}T_C + 32$		
Fluid pressure	$p = \rho gh$	p pressure ρ fluid density g gravitational field strength h height of fluid column	

6. Nuclear & Quantum physics

Name	**Equation**	**Description of variables**	**Notes / diagram**
Photon energy	$E = hf$	h Planck's constant = 6.63 x 10^{-34} m^2kgs^{-1} f frequency	
Mass-energy relation	$\Delta E = \Delta mc^2$	ΔE energy change Δm mass change c speed of light	Mass change in a nuclear reaction equates to an energy change, essentially due to the changes in nucleon binding energies. $c = 2.998 \times 10^8\,\mathrm{ms}^{-1}$
Radioactive decay	$\frac{dN}{dt} = -\lambda N$ $\lambda = \frac{\ln 2}{T_{\frac{1}{2}}}$ $N = N_0 \exp\left(-\frac{t}{T_{\frac{1}{2}}}\ln 2\right)$	N Number of radioactive atoms at time t that have not yet decayed N_0 Number of radioactive atoms at $t = 0$ λ decay constant $T_{\frac{1}{2}}$ half life. The time taken for $N = \frac{1}{2}N_0$ t time	
Geiger-Nuttall rule	$\log\lambda = A + B\log x$ $I = I_0 e^{-\mu x}$ gamma rays $I = \frac{dN}{dt}$ 'activity'	λ decay constant μ absorption coefficient (Gamma rays) A, B empirical parameters or radioactive sample, and medium in which they are decaying into (e.g. air, paper, metal, lead ...) x distance from source	
Alpha decay (alpha particles are Helium nuclei)	${}^{Z+N}_{Z}\mathrm{X} \rightarrow {}^{Z+N-4}_{Z-2}\mathrm{Y} + \alpha$ ${}^{229}_{90}\mathrm{Th} \rightarrow {}^{225}_{88}\mathrm{Ra} + \alpha$	Alpha decay. Atomic number (Z) *reduces* by 2. Mass number *reduces* by 4 Kinetic energy of alpha particle approximately 5MeV. (100,000 x ionization energy for an air molecule)	
Beta decay (beta particles are high energy electrons)	${}^{Z+N}_{Z}\mathrm{X} \rightarrow {}^{Z+N}_{Z+1}\mathrm{Y} + \beta$ ${}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + \beta$	Beta decay. Atomic number (Z) increases by 1 Mass number stays the same Kinetic energy of beta particles 0.01 to 10MeV i.e. a spectrum of energies [1MeV = 1.60 x 10^{-13} J]	
Nuclear fission	${}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3{}^{1}_{0}\mathrm{n}$	174 MeV per reaction 71.5 million MJ /kg of fuel coal 24 MJ per kg gas 46 MJ per kg sandwich 10 MJ per kg 1MeV = 1.60 x 10^{-13} J	
Nuclear fusion	${}^{2}_{1}D + {}^{3}_{0}\mathrm{T} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n}$	17.6 MeV per reaction 338 million MJ /kg of fuel [1MeV = 1.60 x 10^{-13} J]	

Periodic Table of the Elements

Atomic Number
Symbol
Name
Atomic Mass

Atomic mass values reflect the IUPAC accepted values as of 09/2013.

Masses expressed in [a;b] format show the lower and upper limit of atomic mass depending on the physical and chemical history of the element.

Masses expressed in < > format are the mass numbers of the longest-lived isotope for elements with no stable nucleus.

1 IA 1A	2 IIA 2A	3 IIIB 3B	4 IVB 4B	5 VB 5B	6 VIB 6B	7 VIIB 7B	8 VIII 8	9 VIII 8	10 VIII 8	11 IB 1B	12 IIB 2B	13 IIIA 3A	14 IVA 4A	15 VA 5A	16 VIA 6A	17 VIIA 7A	18 VIIIA 8A
1 H Hydrogen [1.00784;1.00811]																	2 He Helium 4.002602(2)
3 Li Lithium [6.938;6.997]	4 Be Beryllium 9.0121831(5)											5 B Boron [10.806;10.821]	6 C Carbon [12.0096;12.0116]	7 N Nitrogen [14.00643;14.00728]	8 O Oxygen [15.99903;15.99977]	9 F Fluorine 18.998403163(6)	10 Ne Neon 20.1797(6)
11 Na Sodium 22.98976928(2)	12 Mg Magnesium [24.304,24.307]											13 Al Aluminum 26.9815386(8)	14 Si Silicon [28.084;28.086]	15 P Phosphorus 30.973761998(5)	16 S Sulfur [32.059;32.076]	17 Cl Chlorine [35.446;35.457]	18 Ar Argon 39.948(1)
19 K Potassium 39.0983(1)	20 Ca Calcium 40.078(4)	21 Sc Scandium 44.955908(5)	22 Ti Titanium 47.867(1)	23 V Vanadium 50.9415(1)	24 Cr Chromium 51.9961(6)	25 Mn Manganese 54.938045(5)	26 Fe Iron 55.845(2)	27 Co Cobalt 58.933194(4)	28 Ni Nickel 58.6934(4)	29 Cu Copper 63.546(3)	30 Zn Zinc 65.38(2)	31 Ga Gallium 69.723(1)	32 Ge Germanium 72.630(8)	33 As Arsenic 74.921595(6)	34 Se Selenium 78.971(8)	35 Br Bromine [79.901,79.907]	36 Kr Krypton 83.798(2)
37 Rb Rubidium 85.4678(3)	38 Sr Strontium 87.62(1)	39 Y Yttrium 88.90584(2)	40 Zr Zirconium 91.224(2)	41 Nb Niobium 92.90637(2)	42 Mo Molybdenum 95.95(1)	43 Tc Technetium <98>	44 Ru Ruthenium 101.07(2)	45 Rh Rhodium 102.90550(2)	46 Pd Palladium 106.42(1)	47 Ag Silver 107.8682(2)	48 Cd Cadmium 112.414(4)	49 In Indium 114.818(1)	50 Sn Tin 118.710(7)	51 Sb Antimony 121.760(1)	52 Te Tellurium 127.60(3)	53 I Iodine 126.90447(3)	54 Xe Xenon 131.293(6)
55 Cs Cesium 132.90545196(6)	56 Ba Barium 137.327(7)	57-71	72 Hf Hafnium 178.49(2)	73 Ta Tantalum 180.94788(2)	74 W Tungsten 183.84(1)	75 Re Rhenium 186.207(1)	76 Os Osmium 190.23(3)	77 Ir Iridium 192.217(3)	78 Pt Platinum 195.084(9)	79 Au Gold 196.966569(5)	80 Hg Mercury 200.592(3)	81 Tl Thallium [204.382;204.385]	82 Pb Lead 207.2(1)	83 Bi Bismuth 208.98040(1)	84 Po Polonium <209>	85 At Astatine <210>	86 Rn Radon <222>
87 Fr Francium <223>	88 Ra Radium <226>	89-103	104 Rf Rutherfordium <267>	105 Db Dubnium <268>	106 Sg Seaborgium <271>	107 Bh Bohrium <272>	108 Hs Hassium <270>	109 Mt Meitnerium <276>	110 Ds Darmstadtium <281>	111 Rg Roentgenium <280>	112 Cn Copernicium <285>	113 Uut Ununtrium unknown	114 Fl Flerovium <289>	115 Uup Ununpentium unknown	116 Lv Livermorium <293>	117 Uus Ununseptium unknown	118 Uuo Ununoctium unknown

Lanthanide Series	57 La Lanthanum 138.90547(7)	58 Ce Cerium 140.116(1)	59 Pr Praseodymium 140.90766(2)	60 Nd Neodymium 144.242(3)	61 Pm Promethium <145>	62 Sm Samarium 150.36(2)	63 Eu Europium 151.964(1)	64 Gd Gadolinium 157.25(3)	65 Tb Terbium 158.92535(2)	66 Dy Dysprosium 162.500(1)	67 Ho Holmium 164.93033(2)	68 Er Erbium 167.259(3)	69 Tm Thulium 168.93422(2)	70 Yb Ytterbium 173.054(5)	71 Lu Lutetium 174.9668(1)
Actinide Series	89 Ac Actinium <227>	90 Th Thorium 232.0377(4)	91 Pa Protactinium 231.03588(2)	92 U Uranium 238.02891(3)	93 Np Neptunium <237>	94 Pu Plutonium <244>	95 Am Americium <243>	96 Cm Curium <247>	97 Bk Berkelium <247>	98 Cf Californium <251>	99 Es Einsteinium <252>	100 Fm Fermium <257>	101 Md Mendelevium <258>	102 No Nobelium <259>	103 Lr Lawrencium <262>

7. Cosmology

Name	Equation	Description of variables	Notes / diagram
Bode's law for the solar system	$D_{AU} = \frac{4+3\times 2^n}{10}$	D_{AU} planetary orbital radius /AU $n=0$ Venus $n=1$ Earth $n=2$ Mars $n=3$ Ceres $n=4$ Jupiter $n=5$ Saturn $n=6$ Uranus $n=7$ Neptune	1AU (Astronomical unit) = mean Earth-Sun separation = 1.496 x10^{11}m
Hubble's law	$v = H_0 d$	v cosmological recession velocity H_0 Hubble constant = 67.8 kms^{-1}/Mpc d distance of galaxy	The entire universe is expanding, so d can be measured from *any* observation point. 1Mpc = 3.09 x 10^{22}m
Schwarzschild radius of a Black Hole	$R_s = \frac{2GM}{c^2} \approx 3\frac{M}{M_\odot}\text{km}$	$G = 6.67\times 10^{-11}\,\text{Nm}^2\text{kg}^{-2}$ M black hole mass c speed of light 2.998 x 10^8ms^{-1}	$M_\odot$ solar mass =1.99 x 10^{30}kg The Schwarzschild radius is the radius of a spherical mass whose *gravitational escape velocity* equals the speed of light.
Escape velocity	$E = \frac{1}{2}mu^2 - \frac{GMm}{r}$ $E_R = \frac{1}{2}mu^2 - \frac{GMm}{R}$ $E_\infty = \frac{1}{2}mv^2$ $v > 0 \Rightarrow E_R > 0$ $\therefore\ u > \sqrt{\frac{2GM}{R}}$	$G = 6.67\times 10^{-11}\,\text{Nm}^2\text{kg}^{-2}$ M mass of (spherical) object R radius of object u launch velocity r radius from object centre m mass of object escaping E total energy of escaping object	For Earth, the escape velocity is $u > \sqrt{\frac{2\times 6.67\times 10^{-11}\times 5.97\times 10^{24}}{6.38\times 10^6}}$ $u > 11.2\,\text{kms}^{-1}$
Redshift	$z = \frac{\lambda - \lambda_0}{\lambda_0}$	λ observed wavelength λ_0 emitted wavelength	

The Solar System has the following parameters. (Woan, 2000 pp176). All orbits are assumed to be elliptical about the sun. Note

$$\frac{M_\odot}{M_\oplus} \approx 332{,}948$$

and

$$R_\oplus \approx \frac{\text{AU}}{23,455}$$

In SI units:

$$\begin{aligned} M_\odot &= 1.9891 \times 10^{30}\ \text{kg} \\ R_\odot &= 6.960 \times 10^{8}\ \text{m} \\ M_\oplus &= 5.9742 \times 10^{24}\ \text{kg} \\ R_\oplus &= 6.37814 \times 10^{6}\ \text{m} \\ 1\text{AU} &= 1.495979 \times 10^{11}\ \text{m} \end{aligned}$$

Object	$M/M_\oplus$	a /AU	ε[4]	θ_0	β	α	$R/R_\oplus$	T_{rot} / days	P /Yr
Sun	332,837	-	-	-	-	-	109.123	-	-
Mercury	0.055	0.387	0.21	*	7.00	0	0.383	58.646	0.241
Venus†	0.815	0.723	0.01	*	3.39	0	0.949	243.018	0.615
Earth	1.000	1.000	0.02	*	0.00	0	1.000	0.997	1.000
Mars	0.107	1.523	0.09	*	1.85	0	0.533	1.026	1.881
Jupiter	317.85	5.202	0.05	*	1.31	0	11.209	0.413	11.861
Saturn	95.159	9.576	0.06	*	2.49	0	9.449	0.444	29.628
Uranus†	14.500	19.293	0.05	*	0.77	0	4.007	0.718	84.747
Neptune	17.204	30.246	0.01	*	1.77	0	3.883	0.671	166.344
Pluto†	0.003	39.509	0.25	*	17.5	0	0.187	6.387	248.348

where β is the orbital inclination /degrees. In all cases the semi-major axis pointing direction is

$$\mathbf{d} = d_x\hat{\mathbf{x}} + d_y\hat{\mathbf{y}} + d_{\mathbf{y}}\hat{\mathbf{z}} = \cos\beta\hat{\mathbf{x}} + \sin\beta\hat{\mathbf{z}}$$

* For the current orbital polar angle θ_0 (and indeed more accurate values for solar system parameters) see the website of the Jet Propulsion Laboratory (JPL) `http://ssd.jpl.nasa.gov/`

†These planets rotate clockwise about their own internal polar axis. ("Retrograde"). All the other planets rotate anti-clockwise about their own internal axis. All the planets orbit the sun in an anticlockwise direction.

[4] http://nineplanets.org/data.html

8. Mathematics

Name	Equation	Notes / diagram
Trigonometry & Pythagoras' theorem	$x = r\cos\theta$ $y = r\sin\theta$ $r = \sqrt{x^2 + y^2}$ $\sin^2\theta + \cos^2\theta = 1$ $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	$\cos\theta$ is the x coordinate of the unit circle $\sin\theta$ is the y coordinate. θ is measured anticlockwise from the x axis.
Special triangles	$\sin 30^o = \frac{1}{2}$ $\sin 60^o = \frac{\sqrt{3}}{2}$ $\cos 60^o = \frac{1}{2}$ $\cos 30^o = \frac{\sqrt{3}}{2}$ $\tan 30^o = \frac{1}{\sqrt{3}}$ $\tan 60^o = \sqrt{3}$ $\sin 45^o = \frac{1}{\sqrt{2}}$ $\cos 45^o = \frac{1}{\sqrt{2}}$ $\tan 45^o = 1$	
Laws of indices	$x^a x^b = x^{ab}$ $\left(x^a\right)^b = x^{ab}$ $x^{-a} = \dfrac{1}{x^a}$ $\sqrt[n]{x} = x^{\frac{1}{n}}$	
Laws of logarithms	$y = \log_b x \Rightarrow x = b^y$ $\log_b x + \log_b y = \log_b xy$ $\log_b x - \log_b y = \log_b \frac{x}{y}$ $\log_b x^n = n\log_b x$ $x = b^{\log_b x}$ $\log_b x = \dfrac{\log_{10} x}{\log_{10} b} = \dfrac{\log_c x}{\log_c b}$	Base $b > 0$
Binomial expansion	$\left(a+b\right)^n = \binom{n}{0}a^0b^n + \binom{n}{1}a^1b^{n-1} + \binom{n}{2}a^2b^{n-2} + ... + \binom{n}{n}a^nb^0$ $\binom{n}{r} = \dfrac{n!}{(n-r)!r!}$ $\left(1+x\right)^n = 1 + nx + n(n-1)x + n(n-1)(n-2)\dfrac{x^2}{2!} + ...$ $+n(n-1)(n-2)(n-3)\dfrac{x^3}{3!} + ...$	Binomial expansion n integer, >0 Generalized binomial expansion $\lvert x\rvert < 1$
Arithmetic progression	$u_n = a + (n-1)d$ $u_1 = a$ $u_{n+1} - u_n = d$ $S_n = \sum_{i=1}^{n} u_i = \frac{1}{2}n\left(u_1 + u_n\right)$	
Geometric progression	$u_n = ar^{n-1}$ $u_1 = a$ $\dfrac{u_{n+1}}{u_n} = r$ $S_n = a + ar + ar^2 + ... + ar^{n-1} = \dfrac{a\left(1-r^n\right)}{1-r}$	If $\lvert r\rvert < 1$ $S_\infty \to \dfrac{a}{1-r}$

Summation formulae	$\sum_{n=1}^{N} n = \frac{1}{2}n(n+1)$ $\sum_{n=1}^{N} n^2 = \frac{1}{6}n(n+1)(2n+1)$ $\sum_{n=1}^{N} n^3 = \frac{1}{4}n^2\left(n+1\right)^2$	
Triangle	$A = \frac{1}{2}bh = \frac{1}{2}ab\sin C$ Area of a triangle $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ Sine and Cosine rules $a^2 = b^2 + c^2 - 2bc\cos A$	b is base of triangle h perpendicular height a,b,c sides of triangle A,B,C opposite angles to sides
Circle	$\left(x-a\right)^2 + \left(x-b\right)^2 = r^2$ $C = 2\pi r$ $A = \pi r^2$ $s = r\theta$ $a = \frac{1}{2}r^2\theta$	Circle centre (a,b) and radius r Circumference C and area A Arc angle (radians) θ and area a
Ellipse	$\frac{\left(x-x_0\right)^2}{a^2} + \frac{\left(y-y_0\right)^2}{b^2} = 1$ $A = \pi ab$	Geometric centre (x_0, y_0) semi-major axis a and semi-minor axis b Area A
Cylinder	$A = 2\pi rh + 2\pi r^2$ $V = \pi r^2 h$	Area A and volume V h height or length of cylinder
Cone	$A = \pi rl$ $V = \frac{1}{3}\pi r^2 h$	l slant height r radius of base h perpendicular height
Frustum	$V = \frac{1}{3}h\left(A + \sqrt{aA} + a\right)$	Top and base areas a, A Perpendicular height h
Combinatorics	$P = \frac{n!}{p!q!r!\ldots}$ ${}^nC_r = \frac{n!}{(n-r)!r!}$ ${}^nP_r = \frac{n!}{(n-r)!}$	n objects, p repeats of type A, q repeats of type B etc. ${}^nC_r$ is umber of *combinations* of r distinct objects from a population of n distinct objects i.e. order of subset doesn't matter. ${}^nP_r$ is umber of *permutations* of r distinct objects from a population of n distinct objects i.e. order of subset *does* matter.
Quadratic equations	$y = ax^2 + bx + c$ $y = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic formula Discriminant $\Delta = b^2 - 4ac$
Matrices	$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix}$ $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	 Inverse matrix Identity matrix

Basic differentiation	$f'(x) = \frac{d}{dx} f(x)$ $\frac{d}{dx} x^n = nx^{n-1}$ $\frac{d}{dx} e^{ax} = ae^{ax}$	$\frac{dy}{dx}$ is the *gradient* at point (x, y) along a curve $y(x)$. Stationary point (x_s, y_s) when $\frac{dy}{dx} = 0$ maxima if $\left.\frac{d^2y}{dx^2}\right\|_{x=x_s} < 0$ minima if $\left.\frac{d^2y}{dx^2}\right\|_{x=x_s} > 0$
Basic integration	$\int f'(x)dx = f(x) + c$ $\int_a^b f'(x)dx = \left[f(x) \right]_a^b = \left(f(b) \right) - \left(f(a) \right)$ $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$ $\int e^{ax} dx = \frac{1}{a} e^{ax} + c$	$\int g(x)dx$ is the area between the curve $g(x)$ and the x axis, with the caveat that area beneath the axis counts as negative. $\int g(x)dx$ is also the *inverse* of differentiating $y = g(x)$ i.e. $\int \frac{dy}{dx} dx = y + c$ (which is true up to a *constant of integration*, which must be specified).

Linear regression	$\overline{x} = \frac{1}{N}\sum_{n=1}^{N} x_n \quad \overline{y} = \frac{1}{N}\sum_{n=1}^{N} y_n$ $\overline{x^2} = \frac{1}{N}\sum_{n=1}^{N} x_n^2 \quad \overline{y^2} = \frac{1}{N}\sum_{n=1}^{N} y_n^2$ $\overline{xy} = \frac{1}{N}\sum_{n=1}^{N} x_n y_n$ $\mathrm{cov}[x,y] = \overline{xy} - \overline{x} \times \overline{y}$ $V[x] = \overline{x^2} - \overline{x}^2 \quad V[\mathrm{y}] = \overline{y^2} - \overline{y}^2$ $p = \frac{\mathrm{cov}[x,y]}{\sqrt{V[x]V[y]}}$ $y = mx + c$ $m = \frac{\mathrm{cov}[x,y]}{V[x]} \quad c = \overline{y} - m\overline{x}$ vertical fit $m = \frac{V[y]}{\mathrm{cov}[x,y]} \quad c = \overline{y} - m\overline{x}$ horizontal fit	Formulae for calculating the line of best fit to a set of data $\{x_n, y_n\}$ $\mathrm{cov}[x,y]$ is the covariance $p = \frac{\mathrm{cov}[x,y]}{\sqrt{V[x]V[y]}}$ is the product moment correlation coefficient. $p = -1$ perfect negative correlation between x and y $p = +1$ perfect positive correlation between x and y $p = 0$ no correlation between x and y
Statistical analysis	$\overline{x} = E[x] = \frac{1}{N}\sum_{n=1}^{N} x_n$ $\sigma^2 = V[x] = \frac{1}{N-1}\sum_{n=1}^{N}\left(x_n - \overline{x}\right)^2$	$\overline{x}$ mean, or expectation σ^2 variance
Bayes' Theorem & statistical inference	$P(H \mid T)P(T) = P(T \mid H)P(H)$ $P(H \mid T) = \frac{P(T \mid H)P(H)}{P(T \mid H)P(H) + P(T \mid H')P(H')}$ $P(H \mid T') = \frac{P(T' \mid H)P(H)}{P(T' \mid H)P(H) + P(T' \mid H')P(H')}$	H hypothesis true H' hypothesis false T test for hypothesis pass T' test for hypothesis fail $P(H \mid T)$ is probability of hypothesis being true given a test has been passed. This is often what a patient wants to know following a test for a disease. Note in medical applications a pharmaceutical company will instead measure $P(T \mid H)$ e.g. probability that a test passes given a sample has the disease. If a disease is rare, $P(H) \ll 1$ which may mean $P(H \mid T)$ is low even if $P(T \mid H)$ is close to 100%. $P(H \mid T')$ is called a *false positive.*

9. Recommended books and resources

Online portal to Physics, Mathematics references (including this document)

www.eclecticon.info

General Physics

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