PHYSICS

USEFUL DATA AND FORMULAE

Assembled by Dr Andrew French

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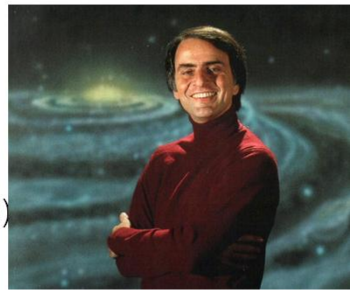
The Cosmos is all that is or ever was or ever will be.

In the last few millennia we have made the most astonishing and unexpected discoveries about the Cosmos and our place within it, explorations that are exhilarating to consider. They remind us that humans have evolved to wonder, that understanding is a joy, that knowledge is prerequisite to survival.

I believe our future depends on how well we know this Cosmos in which we float like a mote of dust in the morning sky.

Carl Sagan (1934-1996)
Cosmos pp20

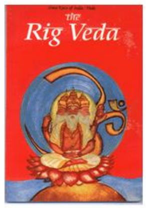


20,000 BC | 5,000 BC | 3000 BC | 2000 BC | 3000 - 300 BC

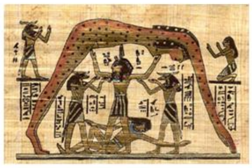
Ishango Bone
Africa

Megalithic structures

Babylonian



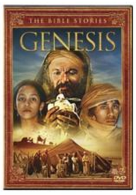
Hindu Rig Veda



Ancient Egypt

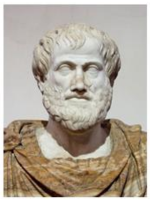
520BC | 500BC | 384-322BC | 200AD | 1564-1642

Jainism



Old Testament

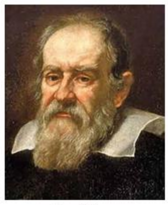
Parmenides
Anaxagoras



Aristotle

Aristarchus

Ptolemy



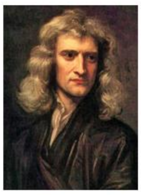
Galileo

500AD | 1473-1543 | 1571-1630 | 1642-1727 | 1879-1955 | 1894-1966

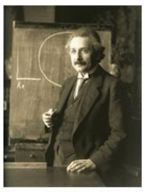
Aryabhata

Copernicus

Kepler



Newton



Einstein

Lemaitre

Hawking

CONTENTS

1. Physical quantities and units

$\pi \approx 3.142$ $e \approx 2.718$ $\sqrt{2} \approx 1.414$ **SI** means '*Système International d'Unités*' (International System of Units)

Quantity	**Symbol**	**Vector or scalar?**	**Unit**	**Unit abbreviation**	**Notes**
Mole **SI**	n	scalar	mole	mol	**Mol is a base SI unit** Avogadro's constant $N_A = 6.022\times10^{23}$ molecules per mole
Mass **SI**	m, M	scalar	kilogram tonne gram atomic mass unit	kg 10^3 kg 10^{-3}kg u	**kg is a base SI unit** Electron mass $m_e = 9.109\times10^{-31}$ kg Proton mass $m_p = 1.673\times10^{-27}$ kg Neutron mass $m_n = 1.675\times10^{-27}$ kg $u = 1.660\times10^{-27}$ kg Earth mass $M_\oplus = 5.974\times10^{24}$ kg Solar mass $M_\odot = 1.989\times10^{30}$ kg
Length **SI**	$l, L,$ $a, b, c, ..$ $...x, y, z$	scalar	angstrom nanometre micron millimetre centimetre metre kilometre mile Astronomical Unit parsec light-year	Å = 10^{-10}m nm = 10^{-9}m μm = 10^{-6}m mm = 10^{-3}m cm = 10^{-2}m m km = 10^3m mile AU parsec lyr	**m is a base SI unit** mile = 1,609m AU = 1.496 x10^{11}m parsec = 3.086 x10^{16}m lyr = 9.461 x10^{15}m
Angle	θ, ϕ $a, b, c...\alpha, \beta$	scalar	degrees radians arc-minute arc-second	^{o} , deg rad arcmin = (1/60) deg arcsec = (1/3600) deg	π radians = 180^o
Area	A	scalar	square mm square centimetres square metres square kilometre hectares acre	mm^2 cm^2 m^2 km^2 ha acre	$mm^2 = 10^{-6}\ m^2$ $cm^2 = 10^{-4}\ m^2$ $km^2 = 10^6\ m^2$ ha = $10^4\ m^2$ acre = 4.047 x $10^3\ m^2$
Volume	V	scalar	cubic centimetre cubic metre cubic kilometre millilitre litre gallon	cm^3 m^3 km^3 ml l gal	$cm^3 = 10^{-6}\ m^3$ $km^3 = 10^9\ m^3$ ml = 1cm^3 (pure water at STP) l = $10^3 cm^3 = 10^{-3} m^3$ gal = 4.546 x$10^{-3}\ m^3$

Quantity	Symbol	Vector or scalar?	Unit	Unit abbreviation	Notes
Time **SI**	t, τ	scalar	picosecond nanosecond microsecond millisecond second minute hour day year	ps ns μs ms s min hr d = 24hr yr	**s is a base SI unit** min = 60s, hr = 60min = 3600s, $\text{yr} \approx 365 \times 24 \times 3600\text{s}$ $\text{yr} \approx 3.154 \times 10^7\text{s}$ $\text{yr} \approx \pi \times 10^7\text{s}$ Age of the Earth = 4.5 x 10^9 yr Age of the Universe = 13.8 x 10^9 yr
Speed	s u, v	scalar	metre per second kilometre per hour mile per hour	ms^{-1} kmh^{-1} mph	Speed of light in a vacuum c = 2.998 x 10^8 ms^{-1} Speed of sound in air (20°C): 344 ms^{-1} Speed of sound in water: 1482 ms^{-1} $1\text{ms}^{-1} = 3.6\text{kmh}^{-1}$ 1ms^{-1} = 2.24 mph 1 min per mile at 60mph 3 mins per mile at 20mph 6 mins per mile at 10mph
Frequency	f	scalar	Hertz Kilohertz Megahertz Gigahertz Terahertz	Hz = s^{-1} kHz = 10^3 Hz MHz = 10^6 Hz GHz = 10^9 Hz THz = 10^{12} Hz	Voice sound waves 0 - 2kHz Radio waves 3kHz - 300MHz Microwaves 3MHz - 100GHz Infrared 100GHz - 300THz Visible light 10^{14} - 10^{15} Hz Ultraviolet 10^{15} Hz - 10^{16} Hz X-rays 10^{16} Hz - 10^{20} Hz Gamma rays > 10^{20} Hz
Period	T	scalar	Same as time	Same as time	Time to complete a single oscillation. $T = \frac{1}{f}$ e.g. period of Earth's rotation is 24 hours, period of Earth's orbit about the sun is 1 year.
Displacement	$\mathbf{x}$ x, y, z	vector	Same as length	Same as length	*Magnitude* as well as *direction*. Often we describe in terms of a *coordinate system* e.g. *x,y,z Cartesians*. In this case a negative value of *x* means 'going backwards'. In one direction, displacement is the *area* under a (time,velocity) graph, where area below the time axis is negative.
Velocity	$\mathbf{v}$ u, v	vector	Same as speed	Same as speed	*Magnitude* as well as *direction*. Often we describe in terms of a *coordinate system* e.g. *x,y,z Cartesians*. In this case a negative value of *v* means 'going backwards'. In one direction, velocity is the *gradient* of a (time,displacement) graph. It is also the *area* under a (time, acceleration) graph, where area below the time axis is negative. u is typically a symbol for initial velocity v for final or 'current' velocity
Acceleration	$\mathbf{a}$ a	vector	metre per second squared	ms^{-2}	*Magnitude* as well as *direction*. Often we describe in terms of a *coordinate system* e.g. *x,y,z Cartesians*. In one direction, acceleration is the *gradient* of a (time,velocity) graph. 'Free-fall' acceleration under gravity: $g_{earth} = 9.81\text{ms}^{-2}$ $g_{moon} = 1.63\text{ms}^{-2}$

Quantity	Symbol	Vector or scalar?	Unit	Unit abbreviation	Notes
Energy	E	scalar	Joules kilojoules megajoules calories kilo-calories kilowatt-hour electron-volts	J kJ = 10^3J MJ = 10^6J cal = 4.184J kcal = 10^3 cal kWh eV keV = 10^3eV MeV = 10^6eV	Energy is *conserved,* i.e. in a closed system has the *same numerical value*. It can be converted into different forms e.g. *kinetic* and *potential* energy. Calories measure energy in food 1kWh is a standard measure of domestic electricity consumption. *Total* UK energy consumption is about 125 kWh per person per day. eV = kinetic energy of an electron accelerated by a voltage V $eV = 1.602 \times 10^{-19}\,\mathrm{J}$
Power	P	scalar	Watts kilowatts megawatts gigawatts terawatts horsepower	W = Js^{-1} kW = 10^3 W MW = 10^6 W GW = 10^9 W TW = 10^{12} W hp = 746W	*Power is the rate of energy changed from one form into another* A light bulb uses about 20W Dr French's computers use about 250W A kettle uses about 2kW A Tour-de-France cyclist expends 250-500W A wind turbine generates 1-10MW A power station generates up to 5GW About 1.36 kWm^{-2} or solar radiation shine on the Earth.
Force	$\mathbf{f}$ f, F	vector	Newtons kilonewtons	N kN	*Newton's Second Law:* mass x acceleration = vector sum of forces
Weight	W	vector	Newtons kilonewtons	N kN	The gravitational force F acting upon a mass m is $F = mg$.
Tension	T	vector	Newtons kilonewtons	N kN	Force in a cable or string. Often these are modelled as *light* and *inextensible*. i.e. ignore the effect of their mass and assume they don't stretch.
Momentum	$\mathbf{p} = m\mathbf{v}$ $p = mv$	vector	kilogram-metres per second	$kgms^{-1}$	Note *impulse* is a change in momentum, e.g. due to a collision or from the action of some external force over a period of time.
Moment	$m = Fd$	scalar	Newton-metre	Nm	Force x perpendicular distance from a pivot
Torque	$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$	vector	Newton-metre	Nm	Vector quantity whose magnitude is the *moment* of force $\mathbf{F}$ about pivot. Force $\mathbf{F}$ acts from displacement $\mathbf{r}$ about pivot. Magnitude of torque is $\lvert\mathbf{F}\rvert$ multiplied by perpendicular distance of line of action of $\mathbf{F}$ from pivot.
Angular velocity	$\boldsymbol{\omega}$ ω	vector	radians per second	$rads^{-1}$	Used in circular and oscillatory motion. π radians = 180^o
Moment of inertia		scalar (or in general a matrix)	kilogram metre-squared	kgm^2	Sum of masses x square of perpendicular distance r from rotation axis of a rigid body. $I = \int r^2 dm$
Angular momentum	$\mathbf{J} = \mathbf{r} \times m\mathbf{v}$ $\mathbf{J} = \mathbf{I}\boldsymbol{\omega}$ $J = mvr$ $J = I\omega$	vector	kilogram-metres-squared per second	kgm^2s^{-1}	Angular momentum is conserved in a rigid body, in the absence of any torque. The inertia tensor $\mathbf{I} = \begin{pmatrix} I_x & I_{xy} & I_{xz} \\ I_{yx} & I_y & I_{yz} \\ I_{zx} & I_{zy} & I_z \end{pmatrix}$ is a matrix containing moments of inertia about the body x,y,z axis of a rigid body. If the body has symmetry, the off diagonal elements are typically zero

Quantity	Symbol	Vector or scalar?	Unit	Unit abbreviation	Notes
Elasticity	$k=\frac{\lambda}{l}$	scalar	Newtons per metre	Nm^{-1}	If an elastic body (such as spring or rubber band) is Hookean, the restoring force following extension by x is $F=\frac{\lambda}{l}x$ where l is the natural length and λ is the elastic modulus.
Stress	σ	scalar	Newtons per square metre	Nm^{-2}	Force per unit area - same as pressure $\sigma=\frac{F}{A}$
Strain	ε	scalar	Just a number	-	Ratio of extension to natural length $\varepsilon=\frac{x}{l}$ Young's Modulus is $Y=\frac{\sigma}{\varepsilon}=\frac{\lambda x}{lA}\times\frac{l}{x}=\frac{\lambda}{A}$ $\therefore \lambda = AY$ Young's modulus for different materials: (GPa = 10^9 Nm^{-2}) rubber 0.01-0.1 GPa Nylon 2-4 GPa Wood 11 GPa Bone 14 GPa Concrete 30 GPa Glass 50-90 GPa Aluminium 69 Copper 117 GPa Diamond 1,050-1,210 GPa
Viscosity	η	scalar	poiseuille	Pl $=Nm^{-2}s$ =Pa s $=kgm^{-1}s^{-1}$	$\eta=\frac{\text{shear stress}}{\text{velocity gradient}}$ Stress is force per unit area. Velocity gradient is change in velocity per metre of fluid. warm blood 3-4 x 10^{-3} $kgm^{-1}s^{-1}$ honey 2-10 $kgm^{-1}s^{-1}$ molten chocolate 10-25 $kgm^{-1}s^{-1}$ ketchup 50-100 $kgm^{-1}s^{-1}$ peanut butter 250 $kgm^{-1}s^{-1}$ corn syrup 1.4 $kgm^{-1}s^{-1}$ olive oil 8.1 x 10^{-2} $kgm^{-1}s^{-1}$ water 8.9 x 10^{-4} $kgm^{-1}s^{-1}$ mantle of Earth 10^{21} $kgm^{-1}s^{-1}$
Reynolds number	Re	scalar	just a number	-	$\mathrm{Re}=\frac{\rho v L}{\eta}=\frac{\text{density}\times\text{velocity}\times\text{length}}{\text{viscosity}}$ *Turbulent* flow if Re > few thousand Re = $\frac{\text{inertial force}}{\text{viscous force}}$
Mach number	Ma	scalar	just a number	-	$\mathrm{Ma}=\frac{v}{\text{speed of sound}}$

Quantity	Symbol	Vector or scalar?	Unit	Unit abbreviation	Notes
Density	ρ	scalar	mass per unit volume	kgm^{-3} gcm^{-3}	Air is about 1.2 kgm^{-3} Wood is about 0.5 gcm^{-3} Water is about 1 gcm^{-3} Aluminium is 2.7 gcm^{-3} Iron is 7.8 gcm^{-3} Copper is 8.9 gcm^{-3} Mercury is 13.5 gcm^{-3} Gold is 19.3 gcm^{-3} Uranium is 19.1 gcm^{-3}
Pressure	p	scalar	Pascal kilopascal megapascal millibar Atmosphere	Pa kPa = 10^3 Pa MPa = 10^6 Pa mbar = 100Pa atm	*Force per unit area* Pa = $1Nm^{-2}$ atm = 101,325 Pa is essentially a 'reference' atmospheric pressure at sea level. atm = 1013.25 mbar. Millibars are used in meteorology i.e. climate science and weather forecasting to measure air pressure.
Temperature **SI**	T	scalar	degrees celcius degrees fahrenheit degrees kelvin	^oC ^oF K	**K is a base SI unit** $T_K = T_C + 273.15$ $T_F = \frac{9}{5} T_C + 32$ Temperature in K is proportional to the mean kinetic energy of molecules. Hence nothing can be colder than 0K "absolute zero"
Solid or liquid specific heat capacity	c	scalar	joules per kilogram per Kelvin	$Jkg^{-1}K^{-1}$	water 4,200 $Jkg^{-1}K^{-1}$ alcohol 2,500 $Jkg^{-1}K^{-1}$ ice 2,100 $Jkg^{-1}K^{-1}$ aluminium 900 $Jkg^{-1}K^{-1}$ concrete 800 $Jkg^{-1}K^{-1}$ glass 700 $Jkg^{-1}K^{-1}$ steel 500 $Jkg^{-1}K^{-1}$ copper 400 $Jkg^{-1}K^{-1}$
Gas specific heat capacity	c_p c_V	scalar	joules per kilogram per Kelvin	$Jkg^{-1}K^{-1}$	c_p is at constant pressure, c_V is at constant volume. c_p for dry air is about 1,000$Jkg^{-1}K^{-1}$. *Molar* heat capacities are: $c_V = \frac{1}{2}\alpha R$ and $c_p = c_V + R$ (Mayer Relation). Molar gas constant $R = 8.314 \text{Jmol}^{-1}\text{K}^{-1}$ and for air, molar volume is about $29gmol^{-1}$.
Specific latent heat of fusion	L ΔH	scalar	joules per kilogram	Jkg^{-1}	water 336,000 Jkg^{-1} alcohol 108,000 Jkg^{-1}
Specific latent heat of vaporisation	L ΔH	scalar	joules per kilogram	Jkg^{-1}	water 2,260,000 Jkg^{-1} alcohol 855,000 Jkg^{-1}

Quantity	Symbol	Vector or scalar?	Unit	Unit abbreviation	Notes
Charge	q, Q e	scalar	Coulombs	C	charge on electron $e = 1.602 \times 10^{-19}\,\mathrm{C}$
Voltage	V	scalar	Volts millivolts kilovolts Megavolts	V mV = 10^{-3}V kV = 10^{3}V MV = 10^{6}V	Potential energy per coulomb of charge Energy change per coulomb of charge *across* a resistor. 'Electromotive force' (EMF).
Current **SI**	I	scalar	Amps milliamps	A mA	**A is a base SI unit** Rate of charge flowing in an electrical circuit (coulombs per second).
Resistance	R	scalar	Ohms kilo-ohms mega-ohms	Ω	Ohm's Law: $V = IR$ Voltage drop across a resistor is proportional to resistance, and current flowing through it.
Resistivity	ρ	scalar	ohm-metre	$\Omega\mathrm{m}$	Resistance of a cylindrical wire of length l and cross sectional area A is $R = \rho \dfrac{l}{A}$ Copper ρ = 1.68 x 10^{-8} $\Omega\mathrm{m}$ Aluminium ρ = 2.82 x 10^{-8} $\Omega\mathrm{m}$ Gold ρ = 2.44 x 10^{-8} $\Omega\mathrm{m}$ Iron ρ = 1.00 x 10^{-7} $\Omega\mathrm{m}$ Sea water ρ = 2.00 x 10^{-1} $\Omega\mathrm{m}$ Glass ρ = 10^{11} - 10^{15} $\Omega\mathrm{m}$ Hard rubber ρ = 10^{13} $\Omega\mathrm{m}$ Dry wood ρ = 10^{14} - 10^{16} $\Omega\mathrm{m}$ Air ρ = 1.3 - 3.3 x 10^{16} $\Omega\mathrm{m}$
Capacitance	C	scalar	Farads picofarads nanofarads microfarads	F pF nF μF	Typical capacitances of electronic components are < a few μF. pF or nF are common. $\tau = RC$ is an approximate time period associated with a resistor, capacitor circuit. $\tau = \sqrt{LC}$ is an approximate time period associated with an inductor, capacitor circuit. These often exhibit resonance phenomenon, so can be used to amplify signals at a particular frequency.
Inductance	L	scalar	Henry	H	An inductor with an inductance of 1 Henry produces an 'electromotive force' (EMF) of 1 volt when the current through the inductor changes at the rate of 1 ampere per second.
Electric field strength	$\mathbf{E}$ E	vector	Volts per metre	Vm^{-1}	Force on a charge q coulombs in a electric field of strength E is $F = qE$ A dielectric will conduct electricity ('breakdown') when E exceeds a critical value. **Note: 1MVm^{-1} = 10^{6}Vm^{-1}** Air $E > 3\mathrm{MVm}^{-1}$ Glass $E > 10\mathrm{MVm}^{-1}$ Oil $E > 10\mathrm{MVm}^{-1}$ Rubber $E > 15\mathrm{MVm}^{-1}$ pure water $E > 65\mathrm{MVm}^{-1}$ Mica $E > 118\mathrm{MVm}^{-1}$ Diamond $E > 2{,}000 \times 10^{6}\,\mathrm{Vm}^{-1}$

Quantity	Symbol	Vector or scalar?	Unit	Unit abbreviation	Notes
Magnetic field strength	$\mathbf{B}$ B	vector	Tesla	T	Force on a wire of length l carrying current I in magnetic field B is $F = BIl$ Note force, current and field are *mutually perpendicular* Magnetic field inside a solenoid (a coil of wire carrying current I of n turns per unit length) is $B = \mu_0 nI$ 'Permeability of free space' $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{Hm}^{-1}$ Earth's magnetic field $\approx 25\mu\mathrm{T} - 65\mu\mathrm{T}$ (μT = 10^{-6}T) Fridge magnet = 5 x 10^{-3}T 1.5-3T field strength of a Magnetic Resonance Imaging (MRI) system 10^{6}T-10^{8}T field strength of a *neutron star* 10^{8}T-10^{11}T field strength of a *magnetar*
Gravitational field strength	$\mathbf{g}$ g	vector	metres per second squared	ms^{-2}	$g_{earth} = 9.81\mathrm{ms}^{-2}$ $g_{moon} = 1.63\mathrm{ms}^{-2}$
Radioactive activity	A	scalar	Becquerel Curies	Bq Ci	Bq = radioactive decays per second Ci = 3.7 10^{10}Bq (activity of one gram of ${}^{226}_{88}\mathrm{Ra}$)
Half life	$T_{\frac{1}{2}}$	scalar	Seconds, days, years	s, d, yr	Time for half of radioactive isotopes to have decayed ${}^{235}_{92}\mathrm{U}$ 7 x 10^{8} yr ${}^{14}_{12}\mathrm{C}$ 5,730 yr ${}^{123}_{53}\mathrm{I}$ 13 hrs
Refractive index	n	scalar	just a number	-	$n = \frac{c_{\mathrm{vacuum}}}{c_{\mathrm{material}}}$ i.e. ratio of speed of light in a vacuum (2.998 x $10^{8}\mathrm{ms}^{-1}$) to speed of light in a material vacuum 1 air 1.00 ice 1.31 water 1.33 human cornea 1.37-1.40 human lens 1.39-1.41 plexiglas 1.49 crown glass 1.52 sapphire 1.76-1.78 diamond 2.42

2. Mechanics

Name	Equation	Description of variables	Notes / diagram
Kinematics	$v = \frac{dx}{dt} \quad x = \int v dt$ $a = \frac{dv}{dt} \quad v = \int a dt$	x displacement v velocity a acceleration t time	***Velocity*** is the gradient of a (time,displacement) graph. ***Displacement*** is the area under a (time, velocity). graph. Note areas below the time axis are negative. ***Acceleration*** is the gradient of a (time,velocity) graph velocity is the area under a (time, velocity). graph. Note areas below the time axis are negative.
Constant acceleration motion	$v = u + at$ $x = x_0 + \frac{1}{2}(u+v)t$ $x = x_0 + ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2ax$	u initial velocity /ms^{-1} a acceleration /ms^{-2} t time /s v final velocity /ms^{-1} x displacement /m x_0 initial displacement /m	Only valid for motion when acceleration a is constant. Easily derived from linear velocity, time graph. a is the gradient $a = \frac{v-u}{t}$ and $x - x_0$ is the area under the graph, which is a trapezium hence $x - x_0 = \frac{1}{2}(u+v)t$
Newton's First Law	$\mathbf{a} = 0$ $\Rightarrow \mathbf{v} = \text{ constant}$	$\mathbf{a}$ acceleration $\mathbf{v}$ velocity	A object will move at *constant velocity* if it is *not accelerating*, and therefore the vector sum of forces is zero. It is in ***equilibrium***.
Newton's Second Law	$m\mathbf{a} = \sum_i \mathbf{f_i}$	mass x acceleration = vector sum of forces	Most mechanics problems are often solved by firstly writing down Newton II for each direction of a coordinate system (typically Cartesian *x,y*) appropriate for the problem.
Newton's Third Law	*"For every action there is an equal and opposite reaction"*		If body A imposes a contact force $\mathbf{F}$ upon body B, body B will in turn impose a contact force $-\mathbf{F}$ upon body A.
Conservation of momentum	$\underbrace{m_1\mathbf{u}_1 + m_2\mathbf{u}_2 + \ldots}_{\text{BEFORE COLLISION}} = \underbrace{m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + \ldots}_{\text{AFTER COLLISION}}$	momentum = mass x velocity	The vector sum of momenta is the same before and after a collision
Impulse	"Force x time = change in momentum" $\int_0^t \mathbf{f}(t)dt = m\mathbf{v} - m\mathbf{u}$	$\mathbf{f}(t)$ force (as a function of time t), m mass $\mathbf{v}$ final velocity $\mathbf{u}$ initial velocity 'impulse' means momentum change	In each direction of a coordinate system, the integral of the (time,force) graph is the change in momentum. If force is a constant force x time = change in momentum

Conservation of energy	$\frac{1}{2}mu^2 + \mathrm{GPE}_0 + \mathrm{EPE}_0 + ... = \frac{1}{2}mv^2 + \mathrm{GPE}_1 + \mathrm{EPE}_1 + ...$ $\mathrm{GPE} = mgh$ $\mathrm{GPE} = -\frac{GMm}{r}$ $\mathrm{EPE} = \frac{1}{2}kx^2 = \frac{1}{2}\frac{\lambda}{l}x^2$	m mass. u, v initial and final speeds. h change in vertical height. g gravitational field strength. M, m masses. r distance between masses. G gravitational force constant. x spring extension. k spring constant. λ modulus of elasticity. l original length of spring.	$g_{earth} = 9.81\mathrm{ms}^{-2}$ $g_{moon} = 1.63\mathrm{ms}^{-2}$ $G = 6.67 \times 10^{-11}\,\mathrm{Nm^2kg^{-2}}$
Coefficient of restitution	$C = \frac{\text{speed of separation}}{\text{speed of approach}}$		$C = 1$ *elastic* collision (kinetic energy conserved). $C = 0$ *inelastic* collision (objects remain together, some kinetic energy is lost)
Work done	'WORK DONE = FORCE x DISTANCE' $W = \int \mathbf{f} \cdot d\mathbf{r} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$	$\mathbf{f}$ force. $\mathbf{r}$ displacement. m mass. u, v initial and final speeds.	Work done is "the area under a (displacement, force) graph", noting that areas below the *x* axis are *negative*.
Moments	'MOMENT = FORCE x PERPENDICULAR DISTANCE FROM ROTATION AXIS' $M = Fd$	F force d distance from axis of rotation	In **equilibrium**, the sum of moments (clockwise or anticlockwise) is **zero**, regardless of the axis position chosen!
Moment of inertia	$I = \int r^2 dm$	I moment of inertia. r distance from rotation axis. m mass	thin rod, about centre $\frac{1}{12}ml^2$ solid sphere $\frac{2}{5}mr^2$ solid cone $\frac{3}{10}mr^2$
Perpendicular axis theorem	$I_z = I_x + I_y$		Only works for laminae defined in the x,y plane.
Parallel axis theorem	$I_{z'} = I_z + Md^2$	M mass d distance of new axis from 'z' rotation axis	
Angular momentum	$\mathbf{L} = \mathbf{r} \times m\mathbf{v}$	$\mathbf{r}$ displacement. m mass. $\mathbf{v}$ velocity.	For a rigid body, can decompose into angular momenta of centre of mass + angular momenta about centre of mass. i.e. *particle motion* of centre of mass plus *rotation* about centre of mass.

Torque	$\boldsymbol{\tau}=\mathbf{r}\times\mathbf{f}$ $\boldsymbol{\tau}=\frac{d\mathbf{L}}{dt}$ $\boldsymbol{\tau}=\mathbf{I}\dot{\boldsymbol{\omega}}=\begin{pmatrix} I_x & I_{xy} & I_{xz} \\ I_{yx} & I_y & I_{yz} \\ I_{zx} & I_{zy} & I_z \end{pmatrix}\begin{pmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{pmatrix}$ $Fd=I\ddot{\theta}$	$\mathbf{r}$ displacement $\mathbf{f}$ force t time $\mathbf{I}$ inertia tensor F force d distance from axis of rotation $\ddot{\theta}$ angular acceleration	If torque is zero this means angular momentum is a constant. Rotational equivalent of Newton's Second Law. angular acceleration in radians per second^2
Projectile motion	$v_x=u\cos\theta \quad v_y=u\sin\theta-gt$ $v=\sqrt{v_x^2+v_y^2}=\sqrt{u^2-2g(y-y_0)}$ $x=ut\cos\theta$ $y=y_0+x\tan\theta-\frac{g}{2u^2}(1+\tan^2\theta)x^2$ $t_a=\frac{u\sin\theta}{g} \quad y_a=y_0+\frac{u^2\sin^2\theta}{2g} \quad x_a=\frac{u^2\sin\theta\cos\theta}{g}$ $R=\frac{u^2\sin 2\theta}{g}$	v_x horizontal velocity v_y vertical velocity v speed u launch speed θ launch elevation g gravitational acceleration t time since launch x horizontal displacement y vertical displacement y_0 initial vertical displacement t_a apogee time x_a, y_a apogee coordinates R horizontal range if $y_0=0$	Projectile motion is essentially constant acceleration motion in both x and y directions. Air resistance is ignored. In the x direction acceleration is zero, hence a *constant* velocity $v_x=u\cos\theta$. The x,y curve traced out by particle is an inverted parabola. Typically for a given range there are two possible trajectories for a given launch velocity u corresponding to 'steep' and 'shallow' solutions for elevation θ.
Motion in a circle	$\omega=\dot{\theta}=\frac{d\theta}{dt} \qquad \dot{\omega}=\ddot{\theta}=\frac{d^2\theta}{dt^2}$ $\mathbf{v}=r\omega\hat{\boldsymbol{\theta}}$ $\mathbf{a}=-\frac{v^2}{r}\hat{\mathbf{r}}+r\ddot{\theta}\hat{\boldsymbol{\theta}} \; =-r\omega^2\hat{\mathbf{r}}+r\ddot{\theta}\hat{\boldsymbol{\theta}}$ $\frac{d\hat{\mathbf{r}}}{dt}=\dot{\theta}\hat{\boldsymbol{\theta}} \qquad \frac{d\hat{\boldsymbol{\theta}}}{dt}=-\dot{\theta}\hat{\mathbf{r}}$ $\dot{\mathbf{r}}=\frac{d}{dt}(r\hat{\mathbf{r}})=\dot{r}\hat{\mathbf{r}}+r\dot{\theta}\hat{\boldsymbol{\theta}}$ $\ddot{\mathbf{r}}=\frac{d^2}{dt^2}(r\hat{\mathbf{r}})=\left(\ddot{r}-r\dot{\theta}^2\right)\hat{\mathbf{r}}+\left(2\dot{r}\dot{\theta}+r\ddot{\theta}\right)\hat{\boldsymbol{\theta}}$ $\hat{\mathbf{r}}=\cos\theta\hat{\mathbf{x}}+\sin\theta\hat{\mathbf{y}}$ $\hat{\boldsymbol{\theta}}=-\sin\theta\hat{\mathbf{x}}+\cos\theta\hat{\mathbf{y}}$ $x=r\cos\theta$ $y=r\sin\theta$	θ anticlockwise angle /radians r circle radius /m ω angular velocity /rads^{-1} $\mathbf{v}, v$ velocity, speed $\mathbf{a}$ acceleration $\hat{\mathbf{r}}$ radial unit vector $\hat{\boldsymbol{\theta}}$ polar angle unit vector $\hat{\mathbf{x}}$ x direction unit vector $\hat{\mathbf{y}}$ y direction unit vector	Assumes the circle radius r is a constant In general we can incorporate an $\dot{r}$ term into both velocity and acceleration expressions. Note if one chooses a reference frame which rotates at ω then one will experience a 'centrifugal force' of magnitude $F_r=mr\dot{\theta}^2$ *away from* the axis of rotation.

Lift and drag	$F_L = \frac{1}{2} c_L \rho A v^2$ $F_D = \frac{1}{2} c_D \rho A v^2$ $F = 6\pi a \eta v$	c_L lift coefficient c_D drag coefficient ρ density of air/fluid A area of object in fluid stream v speed a radius of sphere η viscosity	The linear Stokes Drag equation $F = 6\pi a \eta v$ is typically applicable in low Reynolds number scenarios when viscous forces dominate. Air resistance models for bikes, cars, planes, skydivers are typically better served by the v^2 models.
Force of gravity & Kepler's Laws of orbital motion	$\mathbf{W} = m\mathbf{g}$ $\mathbf{F} = -\frac{GMm}{r^2}\hat{\mathbf{r}}$ $r = \frac{a\left(1-\varepsilon^2\right)}{1+\varepsilon\cos\theta}$ $\varepsilon = \sqrt{1-\frac{b^2}{a^2}}$ $P^2 = \frac{4\pi^2}{G(M+m)}a^3$ $\frac{dA}{dt} = \frac{1}{2}\sqrt{G(M+m)\left(1-\varepsilon^2\right)a}$	$\mathbf{W}$ weight g gravitational field strength. M, m masses. r distance between masses. G gravitational force constant. ε eccentricity of elliptical orbit a semi-major axis of the ellipse. b semi-minor axis of the ellipse. M, m star and planet masses θ polar angle (anticlockwise from semi-major axis) P orbital period	$g_{earth} = 9.81\text{ms}^{-2}$ $g_{moon} = 1.63\text{ms}^{-2}$ $G = 6.67\times10^{-11}\,\text{Nm}^2\text{kg}^{-2}$ **Kepler's First Law:** *Bound gravitational orbits of two masses are ellipses, with foci about the common centre of mass.* **Kepler's Second Law:** *The rate of ellipse area swept out (radially from the focus of the ellipse) is a constant* **Kepler's Third Law:** *The square of orbital period is proportional to the cube of the ellipse semi-major axis*
Elasticity (or elastic strings)	$F = kx = \frac{\lambda}{l}x$ $E = \frac{1}{2}kx^2$	F force k elastic constant λ elastic modulus l original length of elastic string x extension E elastic potential energy	Most elasticity models are *Hookean* and assume a constant modulus of elasticity. In reality for large extensions there will be plastic deformation and ultimately breakage.
Friction	$F \le \mu R$ $F = \mu R$	F frictional force μ coefficient of friction R normal contact force	A system is said to be in 'limiting' equilibrium' if $F = \mu R$ i.e. 'on the point of sliding'. Once an object is sliding along a surface, the friction force 'maxes out' out $F = \mu R$. Note μ may change in this dynamic case. $F \le \mu R$ can be used to determine conditions (e.g. tilt angle of a slope) for sliding to occur.

Power & driving force	$P = Fv$ $P = \mathbf{F} \cdot \mathbf{v}$ $D = \frac{P}{v}$	$\mathbf{F}, F$ force P power $\mathbf{v}, v$ velocity D driving force	One *Horsepower* (hp) = 746W. This equation is useful in relating energy conversion rates in engines to resulting motion.
Bernoulli's equation for a fluid stream	$\frac{1}{2}\rho v^2 + p + \rho g z = \text{constant}$ incompressible $\frac{1}{2}v^2 + \frac{\gamma p}{(\gamma - 1)\rho} + gz = \text{constant}$ compressible $\gamma = \frac{c_p}{c_V}$	ρ fluid density p pressure z height v velocity c_p, c_V specific heat capacities of fluid and constant pressure, volume	This explains why lift occurs across an aircraft wing. The fluid has to travel further over the upper edge. To preserve continuity of air, it therefore travels faster. Hence pressure is lower above the wind than below. The pressure difference causes a lift force.
Young's modulus	$E = \frac{\sigma}{\varepsilon} = \frac{F/A}{\delta l / l}$	σ stress ε strain F force A area δl extension l original length	Rubber $E = 0.01\text{GPa}$ Wood $E = 11\text{GPa}$ Concrete $E = 30\text{GPa}$ Glass $E = 70\text{GPa}$ Steel $E = 200\text{GPa}$ Diamond $E = 1{,}100\text{GPa}$
Bending beam	$EI\frac{d^4 y}{dx^4} = w(x)$ $I = \iint y^2 dx dy$	$y(x)$ beam vertical deflection vs horizontal displacement E Young's modulus I moment of *area* $w(x)$ beam weight per unit length	
Simple Harmonic Motion (SHM)	$\frac{d^2 x}{dt^2} + 2\gamma \frac{dx}{dt} + \omega_0^2 x = A_0 \omega_0^2 \sin \omega t$ $x(t) = \frac{A_0 \omega_0^2}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 4\gamma^2 \omega^2}} \sin\left(\omega t - \phi\right)$ $\phi = \tan^{-1}\left(\frac{2\gamma\omega}{\omega_0^2 - \omega^2}\right)$ $\omega_{res} = \sqrt{\omega_0^2 - 2\gamma^2}$, $x_{\max} = \frac{A_0 \omega_0^2}{2\gamma\sqrt{\omega_0^2 - \gamma^2}}$ $Q = \frac{\omega_0}{2\gamma}$	$\omega = 2\pi f = \frac{2\pi}{T}$ f frequency T period Steady-state solution phase /radians Resonance frequency Quality factor	e.g. a damped, driven mechanical or electrical *oscillator*. x could be displacement, electrical current, angle .... For a mass, spring system $\omega_0 = \sqrt{\frac{\lambda}{ml}}$ For a series L,C,R circuit $\omega_0 = \sqrt{\frac{1}{LC}}$ Higher Q values mean sharper peaks in the (f, x) graph

3. Electricity & Magnetism

Name	Equation	Description of variables	Notes / diagram
Charge on a capacitor	$Q = CV$	Q charge /coulombs C capacitance /Farads V voltage /volts	Voltage across two capacitor plates separated by an insulating *dielectric.*
Ohm's law	$V = IR$	V voltage /volts I current /amps R resistance /ohms	Voltage or 'potential difference' *across* a resistive element.
Electrical power	$P = VI$	P power /watts V voltage /volts I current /amps	
Resistive power loss	$P = I^2 R$	P power /watts I current /amps R resistance /ohms	
Electric field strength	$E = \frac{V}{d}$ $E_x = -\frac{\partial V}{\partial x}$ $\mathbf{E} = -\nabla V$	V voltage /volts d distance between charged parallel plates x displacement $\mathbf{E}$ electric field $\nabla V = \hat{\mathbf{x}}\frac{\partial V}{\partial x} + \hat{\mathbf{y}}\frac{\partial V}{\partial y} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$	
(Lorentz) force on a charge in an electric and magnetic field	$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$	$\mathbf{E}$ electric field $\mathbf{B}$ magnetic field $\mathbf{v}$ velocity of charge q	
Force between two static charges	$\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$	q_1, q_2 charges r charge separation $\hat{\mathbf{r}}$ charge separation unit vector ε_0 permittivity of free space = 8.85 × 10^{-12} m^{-3} kg^{-1} s^4A^2	Coulomb's law of electrostatics.
Resistance of a wire	$R = \frac{\rho l}{A}$	R resistance l length A cross sectional area ρ resistivity	Assume uniform resistivity and cross sectional area along length of wire Copper ρ = 1.68 x 10^{-8} Ωm Aluminium ρ = 2.82 x 10^{-8} Ωm Air ρ = 1.3 - 3.3 x 10^{16} Ωm
Energy stored in a capacitor	$E = \frac{1}{2}CV^2$	C capacitance V voltage between capacitor plates E energy	
Energy stored in an inductor	$E = \frac{1}{2}LI^2$	L inductance I current E energy	
Addition of series resistors	$R = R_1 + R_2 + ...$	R resistance	
Addition of parallel resistors	$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + ...$	R resistance	
Addition of series capacitors	$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + ...$	C capacitance	
Addition of parallel capacitors	$C = C_1 + C_2 + ...$	C capacitance	

Addition of series inductors	$L = L_1 + L_2 + ...$	L inductance	
Addition of parallel inductors	$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + ...$	L inductance	
Magnetic field inside an infinite solenoid	$B = \mu\mu_0 \frac{NI}{l}$	B magnetic field strength μ relative permeability μ_0 permeability of free space = $4\pi \times 10^{-7}\,\mathrm{Hm^{-1}}$ N turns in length l I current	A soft magnetic material inside the coil will enhance the magnetic field. For ferrite $\mu > 640$
Characteristic frequency of an inductor-capacitor 'tuned' circuit	$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$	f_0 frequency C capacitance L inductance	
Voltage vs time curves for charging and discharging of a capacitor	$\frac{V}{V_0} = e^{-\frac{t}{RC}}$ discharging $\frac{V}{V_0} = 1 - e^{-\frac{t}{RC}}$ charging	V voltage at time t V_0 maximum voltage R resistance C capacitance	Note *RC* is a **time constant** for a capacitor charging/discharging through a resistor.
Capacitance of a spherical conductor	$C = 4\pi\varepsilon_0 a$	ε_0 permittivity of free space = 8.85 × 10^{-12} m^{-3} kg^{-1} s^4A^2 a radius of sphere	
Capacitance of a parallel plate capacitor	$C = \frac{\varepsilon\varepsilon_0 A}{d}$	ε relative permittivity (of dielectric) A area of capacitor plates d plate separation C capacitance	ε vacuum 1 paper 3.9 silicon 11.7 calcium copper titanate > 250,000
Voltage induced by an inductor	$V = -L\frac{dI}{dt}$	V voltage induced I current L inductance t time	- sign due to *Lenz's law* i.e. an inductor will resist changes in electrical current passing through it
Energy density of electric and magnetic fields	$u = \frac{1}{2}\varepsilon\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu\mu_0}$	u potential energy per unit volume E electric field strength B magnetic field strength	
Inductance of a coil	$L = \frac{\mu\mu_0 K N^2 A}{l}$	N turns in length l with cross sectional area A If $l \gg$ coil radius ("infinite solenoid") then *Nagaoka coefficient* $K \approx 1$	
Inductance of a toroidal coil	$L \approx 0.007975\frac{d^2N^2}{D}, \quad d < 0.1D$	d diameter of coil windings N number of windings D diameter of torus	Semi-empirical formula
Biot-Savart law for calculating magnetic fields due to current elements	$d\mathbf{B} = \frac{\mu_0 I}{4\pi}\frac{d\mathbf{l} \times \mathbf{r}}{r^3}$	$\mathbf{B}$ magnetic field I current $d\mathbf{l}$ vector line element $\mathbf{r}$ position vector at which $\mathbf{B}$ is calculated $r = \lvert\mathbf{r}\rvert$	

Maxwell's equations for electric and magnetic fields	$\int_S \mathbf{E}\cdot d\mathbf{S} = \frac{1}{\varepsilon_0}Q$ $\int_S \mathbf{B}\cdot d\mathbf{S} = 0$ $\oint \mathbf{E}\cdot d\mathbf{l} = -\frac{\partial}{\partial t}\int_S \mathbf{B}\cdot d\mathbf{S}$ $\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 I + \mu_0\varepsilon_0 \frac{\partial}{\partial t}\int_S \mathbf{E}\cdot d\mathbf{S}$	S surface enclosing charge Q $\oint ...$ means integrate around a closed loop, usig lie elements $d\mathbf{l}$	#1 is **Gauss's Law of electrostatics** #2 means **'no magnetic magnetic monopoles'** (although dipoles can be though as a 'source' of magnetic fields) #3 is **Faraday's/Lenz's law of electromagnetic induction** #4 is **Coulombs law + Maxwell's 'displacement current' term** $\mu_0\varepsilon_0 \frac{\partial}{\partial t}\int_S \mathbf{E}\cdot d\mathbf{S}$
Generalized Ohm's law for Alternating Current (AC) circuits Impedance of resistors, capacitors and inductors	$V = IZ$ $V = V_0 e^{i\omega t}$ $I = I_0 e^{i(\omega t - \phi)}$ $Z_R = R$ $Z_L = i\omega L$ $Z_C = \frac{1}{i\omega C}$ $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 376.7\Omega$	V voltage I current Z impedance t time $\omega = 2\pi f$ f frequency of AC input voltage L inductance C capacitance Z_0 impedance of free space (i.e. vacuum, and to a very good approximation, air)	Complex impedance is a useful rick for finding out voltages across circuit elements e.g. in an LCR series circuit $\frac{V_C}{V_0} = \frac{Z_C}{Z_C + Z_L + Z_R}$ i.e. *'potential divider'* concept. $\left\lvert\frac{V_C}{V_0}\right\rvert$ is the voltage amplitude response and $\phi = \arg\left(\frac{V}{V_0}\right)$ is the phase.
Skin depth i.e. penetration depth of electromagnetic fields within a conductor transmitting AC	$\delta = \sqrt{\frac{\rho}{\mu_0 \pi f}}$	δ field penetration depth ρ resistivity f frequency μ_0 permeability of free space = $4\pi \times 10^{-7}\,\mathrm{Hm}^{-1}$	
Transformers	$\frac{V_2}{V_1} = \frac{N_2}{N_1}$ $\frac{I_2}{I_1} \approx \frac{N_1}{N_2}$	V_1, I_1 voltage, current in primary coil. V_2, I_2 voltage, current in secondary coil. N_1, N_2 are number of turns in (respectively) primary and secondary coils.	This assumes no power is lost in the transfer of electrical energy from coil 1 to coil 2 i.e. $I_1 V_1 = I_2 V_2$.

4. Waves & optics

Name	Equation	Description of variables	Notes / diagram
Wave speed equation	$c = f\lambda$	c wave speed f frequency λ wavelength	Speed of light in a vacuum c = 2.998 x 10^8 ms^{-1} Speed of sound in air (20^oC): 344 ms^{-1} Speed of sound in water: 1482 ms^{-1}
Wavenumber	$k = \frac{2\pi}{\lambda}$	λ wavelength	Phase of a wave is $\phi = kx - \omega t$
Frequency and period	$f = \frac{1}{T}$	f frequency T period	
Speed of waves in elastic media	$c = \sqrt{\frac{T}{\mu}}$ string under tension $c = \sqrt{\frac{E}{\rho}}$ elastic solid	c wave speed T string tension μ mass per unit length E Elastic modulus ρ Density	e.g. guitar string low E μ = 0.0059 kgm^{-1} f = 82.41Hz (fundamental mode, so string length = $\frac{1}{2}\lambda$) $T =$ 67.8N $\lambda = 1.30$ m
Antenna beamwidth / optical resolving power	$\delta\theta \approx \frac{\lambda}{d}$	$\delta\theta$ minimum resolvable angular separation (radians) between objects λ wavelength d diameter of aperture of optical device (e.g. lens, dish antenna etc)	
Spherical adiabatic shock wave	$R \approx \left(\frac{Et^2}{\rho_0}\right)^{\frac{1}{5}}$	R shock radius E energy release t time ρ_0 density of undisturbed medium, e.g. air in front ('upstream') of the shock front	*Sedov-Taylor* relation. Describes the shock wave resulting from a localized explosion! An *adiabatic* process is one that occurs *without* transfer of heat or matter between a system and its surroundings.
Snell's law of refraction	$n_1 \sin\theta_1 = n_2 \sin\theta_2$ $n = \frac{\text{speed of light in vaccum}}{\text{speed of light in medium}}$ $\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2}$ $0 \le \theta \le \frac{1}{2}\pi \;\; \therefore 0 \le \sin\theta_2 \le 1 \;\; \therefore 0 \le \frac{n_1 \sin\theta_1}{n_2} \le 1$ $\theta_1 \le \sin^{-1}\left(\frac{n_2}{n_1}\right)$	n_1 refractive index of medium 1 θ_1 angle of incidence to boundary of medium 1 to 2 n_2 refractive index of medium 2 θ_2 angle of refraction in medium 2	Angles measured from normal to the reflecting surface Total internal reflection at glass : air interface i.e. no refraction if $\theta_i > \sin^{-1}\left(\frac{n_{air}}{n_{glass}}\right)$ $\theta_i > \sin^{-1}\left(\frac{1}{1.52}\right)$ $\theta_i > 41.1^o$ This is the *critical angle*
Law of reflection	$\theta_i = \theta_r$	θ_i angle of incidence	Angles measured from normal to the

		θ_r angle of reflection	reflecting surface.
Fraunhofer diffraction limit	$L \gg \frac{(\Delta x)^2}{\lambda}$	Δx aperture size λ wavelength L distance of aperture from observer	Beyond this range we can assume waves are planar and not spherical.
Diffraction pattern from a slit of width a	$I(\theta) = \frac{I_0 \sin^2\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\left(\frac{\pi a \sin\theta}{\lambda}\right)^2}$	θ diffraction angle λ wavelength I_0 peak intensity a slit width	Broad '1D' slit Assume uniform illumination normal to slits. Ignore effect of 'height' only width. i.e. assume slit is 'long and thin'. *A rectangular slit is a product of these functions.*
Diffraction pattern from two thin slits of separation D	$I(\theta) = I_0 \cos^2\left(\frac{\pi D \sin\theta}{\lambda}\right)$	D slit spacing θ diffraction angle λ wavelength I_0 peak intensity	Young's double slits Assume uniform illumination normal to slits.
Diffraction pattern due to a grating of N slits of width a with separation d	$I(\theta) = I_0 \left(\frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\frac{\pi a \sin\theta}{\lambda}} \times \frac{\sin\left(\frac{N\pi d \sin\theta}{\lambda}\right)}{N \sin\left(\frac{\pi d \sin\theta}{\lambda}\right)} \right)$	N number of slits θ diffraction angle λ wavelength d slit spacing a slit width I_0 peak intensity	Assume uniform illumination normal to slits.
Gauss' Lens Formula	$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$	u object distance v image distance f focal length of a lens	$1/f$ *= Dioptre number.* *f-number* is: $\frac{f}{\text{aperture diameter}}$ An f-number of 2 would conventionally be written as *f/2,* which gives the aperture diameter given the lens focal length.
Lensmakers' formula	$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$	f focal length of a lens n refractive index of lens R_1, R_2 radii of curvature of lens surface	
Mach cone shockwave angle	$\sin\theta = \frac{c}{v}$	θ shockwave angle c speed of sound in external medium v velocity of source of wave disturbances	Shock front is only formed when $v > c$. When $v = c$ a broad shockwave is formed, which is the 'sonic boom'.

Kelvin Wedge wave pattern for a vessel moving over deep water	$\theta = \sin^{-1}\tfrac{1}{3} \approx 19.5^{\circ}$ $\alpha = \tan^{-1}\sqrt{2} \approx 54.7^{\circ}$ $v = \sqrt{\dfrac{2g\lambda}{4\pi}}$ $\omega^2 = gk$ $c_p = f\lambda = \dfrac{\omega}{k}$ $c_g = \dfrac{d\omega}{dk} = \tfrac{1}{2}c_p$	θ angle of bow waves α angle of wave-fronts relative to direction of motion v vessel velocity g gravitational field strength λ wavelength f frequency $k = \dfrac{2\pi}{\lambda}$ wavenumber c_p phase velocity c_g group velocity	$\omega^2 = gk$ is the dispersion relationship for deep water waves Wave packets travel at the group velocity. *Information carried by waves can only travel at the group velocity.*
Generalized dispersion relationship for interfacial waves between two fluids	$\omega^2 = \dfrac{\sigma k^3 + g(\rho_1 - \rho_2)k}{\rho_2 + \rho_1 \text{cotanh}(kD)}$ $\omega^2 = \begin{cases} \dfrac{\sigma k^4 D}{\rho_1} + gk^2 D & \tanh(kD) \approx kD \\ \dfrac{\sigma k^3}{\rho_1} + gk & \tanh(kD) \approx 1 \end{cases}$ *Ripples* are when $\tanh(kD) \approx 1$ and $\rho_1 \gg \rho_2$ $c_p = \dfrac{\omega}{k} = \sqrt{\dfrac{\sigma k}{\rho_1} + \dfrac{g}{k}} = \sqrt{\dfrac{2\pi\sigma}{\lambda\rho_1} + \dfrac{g\lambda}{2\pi}}$ $c_p = \sqrt[4]{\dfrac{4g\sigma}{\rho_1}}$	σ surface tension ρ_1 , ρ_2 densities D depth of fluid 1 $k = \dfrac{2\pi}{\lambda}$ wavenumber g gravitational field strength λ wavelength Minimum phase velocity of ripples	
Wave transmission and reflection coefficients	$Z_n = \rho_n c_n$ $t = \dfrac{2Z_1}{Z_1 + Z_2}$ $r = \dfrac{Z_1 - Z_2}{Z_1 + Z_2}$	Z wave impedance ρ density of medium c wave speed in medium t transmission coefficient r reflection coefficient	If $\psi = \psi_0 e^{i(kx-\omega t)}$ is an incident plane wave $r\psi$ is the reflected wave (moving in the the $-x$ direction) and $t\psi$ is the transmitted wave from the interface of two media of differing wave impedances.
Free-space wave equations for electrical and magnetic fields	$\nabla^2 \mathbf{E} = \dfrac{1}{c^2}\dfrac{\partial^2 \mathbf{E}}{\partial t^2}$ $\nabla^2 \mathbf{B} = \dfrac{1}{c^2}\dfrac{\partial^2 \mathbf{B}}{\partial t^2}$ $c = \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}}$	$\mathbf{E}$ electrical field $\mathbf{B}$ magnetic field c speed of light t time ε_0 permittivity of free space = 8.85×10^{-12} $m^{-3}\,kg^{-1}\,s^4A^2$ μ_0 permeability of free space = $4\pi \times 10^{-7}\,\text{Hm}^{-1}$	$c = 2.998 \times 10^8$ ms^{-1}
Wave equation	$\dfrac{\partial^2 \psi}{\partial x^2} = \dfrac{1}{c^2}\dfrac{\partial^2 \psi}{\partial t^2}$	ψ wave amplitude x displacement t time c wave speed	

Doppler effect	$\Delta f = -\dfrac{\cos\theta}{\dfrac{c}{u}+\cos\theta} f$ $\lambda_e = \dfrac{c}{f} \quad \lambda_o = \dfrac{c}{f+\Delta f}$ $u\cos\theta = \dfrac{\Delta\lambda}{\lambda_e}$	v velocity *away from* observer c wave speed f emitted wave frequency Δf frequency shift (from f) of waves arriving at observer. Wavelength Doppler shift formula.	If $\theta = 0^{\circ}$ $\Delta f \approx -\dfrac{u}{c} f$.
Rainbows	$\left(n^2-1\right)^{-2} = \beta + \alpha\left(\dfrac{f}{10^{15}\,\mathrm{Hz}}\right)^2$ $\alpha = -0.3612$ $\beta = 1.7587$ $\varepsilon = 4\sin^{-1}\left(\sqrt{\dfrac{4-n^2}{3n^2}}\right) - 2\sin^{-1}\left(\sqrt{\dfrac{4-n^2}{3}}\right)$	n refractive index f frequency of light ε elevation of rainbow element of colour corresponding to frequency f	Colour Wavelength in vacuo /nm Red 780-622 Orange 622-597 Yellow 597-577 Green 577-492 Blue 492-455 Violet 455-390 A rainbow is observed at a mean angle of about 41.7° with an angular width of about 1.6°.

5. Thermal physics

Name	Equation	Description of variables	Notes / diagram
Ideal gas laws	$pV = nRT$ $V \propto T$ Charles' Law $p \propto \frac{1}{V}$ Boyle's Law	p pressure V volume n number of moles of gas R molar gas constant T absolute temperature (in kelvin)	R = 8.314 $\text{Jmol}^{-1}\text{K}^{-1}$
Equipartition	$U = \sum_{d.o.f} \frac{1}{2} k_B T$ $k_B = \frac{R}{N_A}$	k_B Boltzmann's constant = 1.38 x 10^{-23} $\text{m}^2\text{kgs}^{-2}\text{K}^{-1}$ T absolute temperature (in Kelvin) R molar gas constant N_A Avogadro's number = 6.02 x 10^{23} molecules per mole	$\frac{1}{2} k_B T$ is the energy per 'degree of freedom' of molecular motion. If a molecule can translate in three dimensions, vibrate in two modes and rotate in two orientations, this means the mean thermal energy per molecule is $\frac{7}{2} k_B T$
Maxwell-Boltzmann molecular speed distribution	$p(v) = 4\pi v^2 \left(\frac{m}{2\pi k_B T} \right)^{\frac{3}{2}} e^{-\frac{mv^2}{2k_B T}}$ $p(E) = \frac{2E^{\frac{1}{2}}}{\sqrt{\pi}\left(k_B T\right)^{\frac{3}{2}}} e^{-\frac{E}{k_B T}}$	v molecular speed k_B Boltzmann's constant = 1.38 x 10^{-23} $\text{m}^2\text{kgs}^{-2}\text{K}^{-1}$ T absolute temperature (in Kelvin) m molecular mass $p(...)$ probability density E kinetic energy	The probability of a speed being in the range v to $v + dv$ is defined as $p(v)dv$. Hence $1 = \int_0^\infty p(v)dv$
Boltzmann entropy	$S = k_B \ln W$ $W = \frac{N!}{(N-n)!n!}$	S entropy k_B Boltzmann's constant = 1.38 x 10^{-23} $\text{m}^2\text{kgs}^{-2}\text{K}^{-1}$ W number of ways of arranging N two state systems, with n in the 'excited' state.	
Second Law of Thermodynamics	$\Delta S_{total} = \Delta S_{system} + \Delta S_{surroundings}$ $\Delta S_{surroundings} = \frac{\Delta H}{T}$ $\Delta S_{total} > 0$	ΔS entropy change ΔH enthalpy i.e. heat exchanged with surroundings T absolute temperature (in Kelvin)	For any chemical change, the total amount of entropy must increase
Ratio of specific heat capacities	$\gamma = \frac{c_p}{c_V}$ $c_p = c_V + \frac{1}{2} D \frac{RT}{m}$	c_p constant pressure specific heat capacity c_V constant volume specific heat capacity D degrees of freedom of molecular motion R molar gas constant m molecular mass T absolute temperature (in Kelvin)	R = 8.314 $\text{Jmol}^{-1}\text{K}^{-1}$
Adiabatic changes	$pV^\gamma = \text{constant}$ $\Delta W = \frac{1}{\gamma - 1}\left(p_1 V_2 - p_1 V_1 \right)$	p pressure V volume	i.e. no mass or energy is exchanged between system and surroundings

Conservation of energy (First Law of Thermodynamics)	$dU = dQ + dW$ $dW = -pdV$ $dW = \sigma dA$	U internal energy Q heat energy W work done by gas p pressure V volume σ surface tension A area	
Constant volume heat capacity	$C_V = \left.\frac{\partial Q}{\partial T}\right\|_V$ $c_V = C_V / m$	Q heat energy T absolute temperature (in Kelvin) m molecular mass	Change in gas state occurs at constant volume
Constant volume heat capacity	$C_P = \left.\frac{\partial Q}{\partial T}\right\|_P$ $c_P = C_P / m$	Q heat energy T absolute temperature (in Kelvin) m molecular mass	Change in gas state occurs at constant pressure
Clausius-Clapeyron equation for the (solid,liquid,gas) phase boundary in a p,V,T space	$\frac{dp}{dT} = \frac{L}{T\Delta V}$	p pressure ΔV volume change during phase change L latent heat absorbed T absolute temperature (in Kelvin)	
Planck radiation distribution and Stefan's Law for black body radiation	$B(\lambda,T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$ $I = \int_0^\infty B(\lambda,T)d\lambda = \sigma T^4 = \frac{1}{4}uc$ $\sigma = \frac{2\pi^5 k_B^4}{15c^2h^3}$	I 'irradiance' (measure of radiation intensity per wavelength) c speed of light T absolute temperature (in Kelvin) λ frequency h Planck's constant = 6.63 x 10^{-34} m^2kgs^{-1} σ Stefan-Boltzmann constant = 5.67 x 10^{-8} $Wm^{-2}K^{-4}$ k_B Boltzmann's constant = 1.38 x 10^{-23} $m^2kgs^{-2}K^{-1}$ u radiation energy density.	Emmisitivies: $I = \varepsilon\sigma T^4$ $0 \le \varepsilon \le 1$ Black-Body $\varepsilon = 1$ *Albedo* $A = 1 - \varepsilon$
Isothermal atmospheric pressure model	$p = p_0 \exp\left(-\frac{mg}{RT_0}(h - h_0)\right)$ $T = T_0$ $L = 0$	p pressure at altitude h p_0 pressure at altitude h_0 T temperature (Kelvin) at altitude h T_0 temperature (Kelvin) at altitude h_0 R molar gas constant m molar mass of air g gravitational field strength	R = 8.314 $Jmol^{-1}K^{-1}$ for dry air $m = 0.02896\,kgmol^{-1}$ $g = 9.81ms^{-2}$
Standard Atmospheric pressure model incorporating *lapse rate* (i.e. change of temperature with altitude)	$p = p_0\left(1 - \frac{L(h-h_0)}{T_0}\right)^{\frac{mg}{LR}}$ $T = T_0 - L(h - h_0)$ $L \ne 0$ $p_0 = 1013.25\text{mbar}$ $T_0 = 288.15^\circ\text{K}$	p pressure at altitude h p_0 pressure at altitude h_0 T_0 temperature (Kelvin) at altitude h_0 L lapse rate	h_0 = 0 km h_1 = 11km h_2 = 20 km h_3 = 32 km h_4 = 47 km h_5 = 51 km h_6 = 71 km h_7 = 85 km L_0 = 6.5^o/km L_1 = 0^o/km L_2 = -1^o/km L_3 = -2.8^o/km L_4 = 0^o/km L_5 = 2.8^o/km L_6 = 2^o/k

Vapour pressure	$p_{vap} = UE_s$ $E_s = a\exp\left\{\left(b-\frac{T}{234.5}\right)\left(\frac{T}{T+257.14}\right)\right\}$ $a = 6.1121$ $b = 18.678$ $\rho = \frac{m_d}{RT}\left(p - U\left(1-\frac{m_{vap}}{m_d}\right)E_s(T)\right)$ $dp = -\rho g dh$	p_{vap} vapour pressure U relative humidity E_s Saturation vapour pressure T temperature (kelvin) m_d molar mass of dry air m_{vap} molar mass of vapour R molar gas constant ρ overall vapour plus dry air density g gravitational field strength p pressure h altitude	$m_d = 0.02896\text{kgmol}^{-1}$ $m_{vap} = 0.01802\text{kgmol}^{-1}$ $g = 9.81\text{ms}^{-2}$ $0 \le U \le 1$
Dew point	$T_d = \frac{b\left(\ln U + \frac{aT}{b+T}\right)}{a - \ln U - \frac{aT}{b+T}}$ $a = 17.625$ $b = 243.04$	U relative humidity T temperature in degrees Celcius	i.e. temperature at which vapour will be saturated and condensation occurs
Boiling point	$T = \left(\frac{1}{T_*} - \frac{R}{\Delta H}\ln\left(\frac{p}{p_*}\right)\right)^{-1}$	T boiling point at pressure p given known boiling point T_* at pressure p_* R molar gas constant ΔH latent heat of vaporization	ΔH = 40.7 kJ mol^{-1} at 100 ^{o}C and 1013.25 mbar ambient air pressure.
Heat capacity and energy change	$\Delta E = mc\Delta T$	ΔE energy required to raise the temperature of a mass m by ΔT c is the specific heat capacity	
Kelvin, Celsius and Fahrenheit temperature scales	$T_K = T_C + 273.15$ $T_F = \frac{9}{5}T_C + 32$		
Fluid pressure	$p = \rho g h$	p pressure ρ fluid density g gravitational field strength h height of fluid column	
Fourier's law of heat transfer via conduction	$q_x = -k\frac{\partial T}{\partial x}$	q_x heat flux (Wm^{-2}) in x direction, k thermal	$\frac{\partial T}{\partial x}$ temperature

		conductivity	gradient
Heat diffusion equation	$\frac{\partial^2 T}{\partial x^2} = \frac{1}{D}\frac{\partial T}{\partial t}$ $l = \frac{1}{\sqrt{2\pi}d^2 n}$ $D \approx \frac{2}{3} l \langle c \rangle$ $\eta \approx \frac{1}{2}\rho l \langle c \rangle$	T temperature x direction t time D diffusion coefficient l mean free molecular path d molecular diameter $\langle c \rangle$ mean molecular speed ρ density η viscosity	These transport properties assume the kinetic theory i.e. matter is comprised of molecules in constant, largely random, motion. Heat, temperature is a measure of the energy of these random movements.
Newton's law of cooling	$\frac{dQ}{dt} = hA(T - T_a)$	Q thermal energy /J h heat transfer coefficient, A surface area, T temperature of body, T_a ambient temperature of environment	If $dQ = -mcdT$ $m =$ thermal mass, c = specific heat capacity $\frac{dT}{dt} = -\frac{hA}{mc}(T - T_a)$ $T = T_a + (T_0 - T_a)e^{-\frac{hAt}{mc}}$

6. Nuclear & Quantum physics

Name	Equation	Description of variables	Notes / diagram
Photon energy	$E = hf$	h Planck's constant = 6.63 x 10^{-34} m^2kgs^{-1} f frequency	
Mass-energy relation	$\Delta E = \Delta mc^2$	ΔE energy change Δm mass change c speed of light	Mass change in a nuclear reaction equates to an energy change - essentially due to the changes in nucleon binding energies $c = 2.998 \times 10^8 \text{ ms}^{-1}$
Photon momentum	$p = \frac{h}{\lambda}$ $p = \hbar k$ $k = \frac{2\pi}{\lambda}$	p momentum h Planck's constant = 6.63 x 10^{-34} m^2kgs^{-1} $\hbar = \frac{h}{2\pi}$ k wavenumber	Note although photons have momentum, they don't have mass!
Heisenberg's uncertainly principle	$\Delta p \Delta x \geq \frac{1}{2}\hbar$ $\Delta E \Delta t \geq \frac{1}{2}\hbar$	Δp momentum uncertainty Δx positional uncertainty ΔE energy uncertainty Δt time uncertainty h Planck's constant = 6.63 x 10^{-34} m^2kgs^{-1} $\hbar = \frac{h}{2\pi}$	
Schrödinger's wave equation	$-\frac{\hbar}{2m}\frac{\partial^2 \psi}{\partial t^2} + V\psi = E\psi$ time independent $-\frac{\hbar}{2m}\frac{\partial^2 \psi}{\partial t^2} + V\psi = i\hbar\frac{\partial \psi}{\partial t}$ time dependent	$\left\|\psi\right\|^2 dx$ is the probability of a particle 'existing' within x position x to $x+dx$ h Planck's constant = 6.63 x 10^{-34} m^2kgs^{-1} $\hbar = \frac{h}{2\pi}$ t time, V potential energy E total energy, m mass 'Classic' (!) textbook solutions are - Free particle - Particle in a box - Particle in a potential well - Particle 'tunnelling' through a barrier - Harmonic oscillator - Hydrogenic spherical atom	
Particle in a box	$\psi(x,y,z) = \left(\frac{8}{abc}\right)^{\frac{1}{2}} \sin\frac{l\pi x}{a} \sin\frac{m\pi y}{b} \sin\frac{n\pi z}{c}$ $E_{nlm} = \frac{h^2}{8m}\left(\frac{l^2}{a^2} + \frac{m^2}{b^2} + \frac{n^2}{c^2}\right)$	Solutions to time independent Schrödinger equation for a particle of mass m in a box of dimensions $a \times b \times c$ n,l,m are integers ≥ 1 E_{nlm} are particle energies characterized by *energy state* n,l,m	

Bohr model of a Hydrogenic atom 'Orbital angular momentum is quantized' Model is a quasi-classical analogy. Electrons follow circular orbits, but at fixed (quantized) angular momentum. This must be an *analogy*, since an accelerating point-like electron would *radiate*, and hence rapidly lose energy.	$L_n = \mu r_n v_n = n\hbar$ $\mu = \frac{m_e m_{nuc}}{m_e + m_{nuc}} \approx m_e$ $\alpha \approx \frac{1}{137} = \frac{e^2}{4\pi\varepsilon_0 \hbar c}$ $a_0 = \frac{\alpha}{4\pi R_\infty}$ $r_n = \frac{n^2 m_e}{Z\mu} a_0$ $E_n = -\frac{R_\infty hc\mu Z^2}{m_e n^2}$ $R_\infty = \frac{m_e e^4}{8h^3\varepsilon_0^2 c}$ $\frac{1}{\lambda_{mn}} = \frac{R_\infty \mu Z^2}{m_e}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$	L_n orbital angular momentum of electron, nucleus two-body system n, m orbital numbers (positive integers) m_e electron mass 9.11 x 10^{-31}kg m_{nuc} nucleus mass = Au, u = 1.66 x 10-^{27}kg, A is the atomic mass number μ reduced mass r_n radius of nth electron 'orbit' v_n velocity of electron in nth 'circular orbit' c speed of light 2.998 x 10^8ms^{-1} e charge on electron 1.60 x 10^{-19} coulombs h Planck's constant = 6.63 x 10^{-34} m^{2}kgs^{-1} $\hbar = \frac{h}{2\pi}$ α Fine structure constant R_∞ Rydberg constant Z Atomic number (number of protons in nucleus) ε_0 permittivity of free space = 8.85 × 10^{-12} m^{-3} kg^{-1} s^{4}A^2 λ_{mn} wavelength of photon produced/absorbed resulting from an electron energy state change between 'orbits' m to n
'Liquid drop' nuclear binding energy model	$B = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z^2}{A^{\frac{2}{3}}} - a_a \frac{(N-Z)^2}{A} + \delta A^{-\frac{3}{4}}$ $\delta = \begin{cases} a_p & Z,N \text{ both even} \\ -a_p & Z,N \text{ both odd} \\ 0 & \text{otherwise} \end{cases}$ $m = Zm_H + Nm_n - B$	B nuclear binding energy A Atomic mass number Z Atomic number (number of protons) N Number of neutrons $M(Z,A)$ atomic mass c speed of light 2.998 x 10^8ms^{-1} m_H = 1.673 x 10^{-27}kg m_n = 1.675 x 10^{-27} kg $a_V \approx 15.8\text{MeV}$, $a_S \approx 17.8\text{MeV}$, $a_C \approx 0.71\text{MeV}$, $a_A \approx 23.7\text{MeV}$, $a_P \approx 34\text{MeV}$ 1eV **=** 1.60 x 10^{-19} J 1MeV = 1.60 x 10^{-13} J
Radioactive decay	$\frac{dN}{dt} = -\lambda N$ $\lambda = \frac{\ln 2}{T_{\frac{1}{2}}}$ $N = N_0 \exp\left(-\frac{t}{T_{\frac{1}{2}}}\ln 2\right)$	N Number of radioactive atoms at time t that have not yet decayed N_0 Number of radioactive atoms at $t = 0$ λ decay constant $T_{\frac{1}{2}}$ half life. The time taken for $N = \frac{1}{2}N_0$ t time
Geiger-Nuttall rule	$\log\lambda = A + B\log x$	λ decay constant A, B empirical parameters or radioactive sample, and medium in which they are decaying into (e.g. air, paper, metal, lead ...) x distance from source
Alpha decay	${}^{Z+N}_{Z}\text{X} \rightarrow {}^{Z+N-4}_{Z-2}\text{Y} + \alpha$ ${}^{229}_{90}\text{Th} \rightarrow {}^{225}_{88}\text{Ra} + \alpha$	**Alpha decay.** Atomic number (Z) *reduces* by 2. Mass number *reduces* by 4 Kinetic energy of alpha particle approximately 5MeV. (100,000 x ionization energy for an air molecule)

Beta decay	${}^{Z+N}_{Z}\mathrm{X} \rightarrow {}^{Z+N}_{Z+1}\mathrm{Y} + \beta$ ${}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + \beta$	**Beta decay.** Atomic number (Z) increases by 1 Mass number stays the same Kinetic energy of beta particles 0.01 to 10MeV i.e. a spectrum of energies [1MeV = 1.60 x 10^{-13} J]
Nuclear fission	${}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3{}^{1}_{0}\mathrm{n}$	174 MeV per reaction 71.5 million MJ /kg of fuel coal 24 MJ per kg gas 46 MJ per kg sandwich 10 MJ per kg 1MeV = 1.60 x 10^{-13} J
Nuclear fusion	${}^{2}_{1}D + {}^{3}_{0}\mathrm{T} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n}$	17.6 MeV per reaction 338 million MJ /kg of fuel [1MeV = 1.60 x 10^{-13} J]

Periodic Table of the Elements

Atomic Number
Symbol
Name
Atomic Mass

Atomic mass values reflect the IUPAC accepted values as of 09/2013.

Masses expressed in [a;b] format show the lower and upper limit of atomic mass depending on the physical and chemical history of the element.

Masses expressed in < > format are the mass numbers of the longest-lived isotope for elements with no stable nucleus.

1 IA 1A	2 IIA 2A	3 IIIB 3B	4 IVB 4B	5 VB 5B	6 VIB 6B	7 VIIB 7B	8 VIII 8	9 VIII 8	10 VIII 8	11 IB 1B	12 IIB 2B	13 IIIA 3A	14 IVA 4A	15 VA 5A	16 VIA 6A	17 VIIA 7A	18 VIIIA 8A
1 H Hydrogen [1.00784;1.00811]																	2 He Helium 4.002602(2)
3 Li Lithium [6.938;6.997]	4 Be Beryllium 9.0121831(5)											5 B Boron [10.806;10.821]	6 C Carbon [12.0096;12.0116]	7 N Nitrogen [14.00643;14.00728]	8 O Oxygen [15.99903;15.99977]	9 F Fluorine 18.998403163(6)	10 Ne Neon 20.1797(6)
11 Na Sodium 22.98976928(2)	12 Mg Magnesium [24.304,24.307]											13 Al Aluminum 26.9815386(8)	14 Si Silicon [28.084;28.086]	15 P Phosphorus 30.973761998(5)	16 S Sulfur [32.059;32.076]	17 Cl Chlorine [35.446;35.457]	18 Ar Argon 39.948(1)
19 K Potassium 39.0983(1)	20 Ca Calcium 40.078(4)	21 Sc Scandium 44.955908(5)	22 Ti Titanium 47.867(1)	23 V Vanadium 50.9415(1)	24 Cr Chromium 51.9961(6)	25 Mn Manganese 54.938045(5)	26 Fe Iron 55.845(2)	27 Co Cobalt 58.933194(4)	28 Ni Nickel 58.6934(4)	29 Cu Copper 63.546(3)	30 Zn Zinc 65.38(2)	31 Ga Gallium 69.723(1)	32 Ge Germanium 72.630(8)	33 As Arsenic 74.921595(6)	34 Se Selenium 78.971(8)	35 Br Bromine [79.901,79.907]	36 Kr Krypton 83.798(2)
37 Rb Rubidium 85.4678(3)	38 Sr Strontium 87.62(1)	39 Y Yttrium 88.90584(2)	40 Zr Zirconium 91.224(2)	41 Nb Niobium 92.90637(2)	42 Mo Molybdenum 95.95(1)	43 Tc Technetium <98>	44 Ru Ruthenium 101.07(2)	45 Rh Rhodium 102.90550(2)	46 Pd Palladium 106.42(1)	47 Ag Silver 107.8682(2)	48 Cd Cadmium 112.414(4)	49 In Indium 114.818(1)	50 Sn Tin 118.710(7)	51 Sb Antimony 121.760(1)	52 Te Tellurium 127.60(3)	53 I Iodine 126.90447(3)	54 Xe Xenon 131.293(6)
55 Cs Cesium 132.90545196(6)	56 Ba Barium 137.327(7)	57-71	72 Hf Hafnium 178.49(2)	73 Ta Tantalum 180.94788(2)	74 W Tungsten 183.84(1)	75 Re Rhenium 186.207(1)	76 Os Osmium 190.23(3)	77 Ir Iridium 192.217(3)	78 Pt Platinum 195.084(9)	79 Au Gold 196.966569(5)	80 Hg Mercury 200.592(3)	81 Tl Thallium [204.382;204.385]	82 Pb Lead 207.2(1)	83 Bi Bismuth 208.98040(1)	84 Po Polonium <209>	85 At Astatine <210>	86 Rn Radon <222>
87 Fr Francium <223>	88 Ra Radium <226>	89-103	104 Rf Rutherfordium <267>	105 Db Dubnium <268>	106 Sg Seaborgium <271>	107 Bh Bohrium <272>	108 Hs Hassium <270>	109 Mt Meitnerium <276>	110 Ds Darmstadtium <281>	111 Rg Roentgenium <280>	112 Cn Copernicium <285>	113 Uut Ununtrium unknown	114 Fl Flerovium <289>	115 Uup Ununpentium unknown	116 Lv Livermorium <293>	117 Uus Ununseptium unknown	118 Uuo Ununoctium unknown

Lanthanide Series	57 La Lanthanum 138.90547(7)	58 Ce Cerium 140.116(1)	59 Pr Praseodymium 140.90766(2)	60 Nd Neodymium 144.242(3)	61 Pm Promethium <145>	62 Sm Samarium 150.36(2)	63 Eu Europium 151.964(1)	64 Gd Gadolinium 157.25(3)	65 Tb Terbium 158.92535(2)	66 Dy Dysprosium 162.500(1)	67 Ho Holmium 164.93033(2)	68 Er Erbium 167.259(3)	69 Tm Thulium 168.93422(2)	70 Yb Ytterbium 173.054(5)	71 Lu Lutetium 174.9668(1)
Actinide Series	89 Ac Actinium <227>	90 Th Thorium 232.0377(4)	91 Pa Protactinium 231.03588(2)	92 U Uranium 238.02891(3)	93 Np Neptunium <237>	94 Pu Plutonium <244>	95 Am Americium <243>	96 Cm Curium <247>	97 Bk Berkelium <247>	98 Cf Californium <251>	99 Es Einsteinium <252>	100 Fm Fermium <257>	101 Md Mendelevium <258>	102 No Nobelium <259>	103 Lr Lawrencium <262>

7. Relativity, cosmology

Name	Equation	Description of variables	Notes / diagram
Bode's law for the solar system	$D_{AU} = \frac{4+3\times 2^n}{10}$	D_{AU} planetary orbital radius /AU $n=0$ Venus $n=1$ Earth $n=2$ Mars $n=3$ Ceres $n=4$ Jupiter $n=5$ Saturn $n=6$ Uranus $n=7$ Neptune	1AU (Astronomical unit) = mean Earth-Sun separation = 1.496 x10^{11}m
Cepheid variable luminosity relationship	$\log_{10}\frac{\bar{L}}{L_\odot} \approx 1.15\log_{10} T_d + 2.47$	$\bar{L}$ mean Cepheid luminosity $L_\odot$ Solar luminosity T_d pulsation period /days	$L_\odot \approx 3.85\times 10^{26}\,\mathrm{W}$
Hubble's law	$v = H_0 d$	v cosmological recession velocity H_0 Hubble constant = 67.8 kms^{-1}/Mpc d distance of galaxy	The entire universe is expanding, so d can be measured from *any* observation point. 1Mpc = 3.09 x 10^{22}m.
Gravitational lensing (Einstein rings)	$\theta^2 = \frac{4GM}{c^2}\left(\frac{d_{OS}-d_{OL}}{d_{OS}d_{OL}}\right)$	$G = 6.67\times 10^{-11}\,\mathrm{Nm^2kg^{-2}}$ M lens mass c speed of light 2.998 x 10^8ms^{-1} d_{OS} distance from observer to source of light d_{OL} distance from observer to lensing mass M θ ring angular half-width	Light is bent by the presence of massive objects such as black holes. Objects (stars, galaxies..) behind the 'lens mass' will appear to be distorted into a ring formation.
Schwarzschild radius of a Black Hole	$R_s = \frac{2GM}{c^2} \approx 3\frac{M}{M_\odot}\,\mathrm{km}$	$G = 6.67\times 10^{-11}\,\mathrm{Nm^2kg^{-2}}$ M black hole mass c speed of light 2.998 x 10^8ms^{-1}	$M_\odot$ solar mass = 1.99 x 10^{30}kg. The Schwarzschild radius is the radius of a spherical mass whose *gravitational escape velocity* equals the speed of light.
Escape velocity	$E = \frac{1}{2}mu^2 - \frac{GMm}{r}$ $E_R = \frac{1}{2}mu^2 - \frac{GMm}{R}$ $E_\infty = \frac{1}{2}mv^2$ $v > 0 \Rightarrow E_R > 0$ $\therefore\ u > \sqrt{\frac{2GM}{R}}$	$G = 6.67\times 10^{-11}\,\mathrm{Nm^2kg^{-2}}$ M mass of (spherical) object R radius of object u launch velocity r radius from object centre m mass of object escaping E total energy of escaping object	For Earth, the escape velocity is $u > \sqrt{\frac{2\times 6.67\times 10^{-11}\times 5.97\times 10^{24}}{6.38\times 10^{6}}}$ $u > 11.2\,\mathrm{kms^{-1}}$

Equations of static (i.e. time independent) structure of stars	$\frac{dM}{dr}=4\pi\rho r^2$ $\frac{dp}{dr}=-\frac{G\rho M}{r^2}$ $\frac{dL}{dr}=4\pi\rho r^2\varepsilon$ $\frac{dT}{dr}=\frac{-3\kappa\rho}{16\sigma T^3}\frac{L}{4\pi r^2}$ $\frac{dT}{dr}=\frac{\gamma-1}{\gamma}\frac{T}{p}\frac{dp}{dr}$	M mass within radius r ρ density $G=6.67\times10^{-11}\,\mathrm{Nm^2kg^{-2}}$ L luminosity ε power generated per unit mass T temperature /K κ mean opacity p pressure σ Stefan-Boltzmann constant = 5.67×10^{-8} $Wm^{-2}K^{-4}$ $\gamma=\frac{c_p}{c_V}$ ratio of constant pressure and constant volume heat capacities	
Gravitational Redshift	$\frac{f_\infty}{f_r}=\sqrt{1-\frac{2GM}{rc^2}}$	f_∞ frequency at range $r=\infty$ from mass centre f_r frequency at range r $G=6.67\times10^{-11}\,\mathrm{Nm^2kg^{-2}}$ M mass c speed of light 2.998 x $10^8 ms^{-1}$	
Redshift	$z=\frac{\lambda_o-\lambda_e}{\lambda_e}$	λ_o observed wavelength λ_e emitted wavelength	
Black hole temperature	$T=\frac{\hbar c^3}{8\pi GMk_B}\approx10^{-7}\frac{M_\odot}{M}$	h Planck's constant = 6.63 x 10^{-34} m^2kgs^{-1} $\hbar=\frac{h}{2\pi}$ k_B Boltzmann's constant = 1.38 x 10^{-23} $m^2kgs^{-2}K^{-1}$ $G=6.67\times10^{-11}\,\mathrm{Nm^2kg^{-2}}$ M black hole mass $M_\odot$ solar mass c speed of light 2.998 x $10^8 ms^{-1}$ T temperature /Kelvin	$M_\odot=1.99\times10^{30}\,\mathrm{kg}$

Lorentz space-time transformations	$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$ $x = \gamma(x' + vt') \quad y = y' \quad z = z'$ $t = \gamma\left(t' + \frac{v}{c^2}x'\right)$	v velocity of frame S' relative to x direction of frame S $c = 2.998 \times 10^8 \text{ms}^{-1}$ x, y, z coordinates in frame S x', y', z' coordinates in frame S t time in frame S t' time in frame S'	S,S' are Cartesian x, y, z frames of reference. Relatively speaking, S is the 'stationary frame' and S' is the 'moving frame'. (But obviously the converse is true from the perspective of S'). Assume S and S' are coincident at $t = 0$ (which may not be $t' = 0$)
Relativistic velocity transformations	$u_x = \frac{u'_x + v}{1 + u'_x v/c^2}$ $u_y = \frac{u'_y}{\gamma(1 + u'_x v/c^2)}$ $u_z = \frac{u'_z}{\gamma(1 + u'_x v/c^2)}$	v velocity of frame S' relative to x direction of frame S $c = 2.998 \times 10^8 \text{ms}^{-1}$ u_x, u_y, u_z velocities in S frame u'_x, u'_y, u'_z velocities in S' frame	S,S' are Cartesian x, y, z frames of reference. $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$
Relativistic momentum & energy transformations	$\mathbf{p} = \gamma m \mathbf{u}$ $E = \gamma mc^2$ $E^2 - p^2c^2 = \text{constant}$ $p_x = \gamma\left(p'_x + \frac{vE'}{c^2}\right)$ $p_y = p'_y \quad p_z = p'_z$ $E = \gamma(E' + vp'_x)$	$\mathbf{p}$, $p = \lvert\mathbf{p}\rvert$ momentum in S frame $p'_{x,y,z}$ momentum in S' frame m mass $\mathbf{u}$ velocity v velocity of frame S' $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$ $c = 2.998 \times 10^8 \text{ms}^{-1}$ E total energy	
Relativistic Doppler shift	$\frac{f'}{f} = \gamma\left(1 + \frac{v}{c}\cos\theta\right)$	θ photon arrival angle (anticlockwise from horizontal) in S frame v velocity of S' $c = 2.998 \times 10^8 \text{ms}^{-1}$ f' frequency emitted in S' frame f frequency received in S frame	
Relativistic aberration	$\cos\theta = \frac{\cos\theta' + v/c}{1 + (v/c)\cos\theta'}$	θ emission angle of light in S θ' emission angle of light in S' v velocity of S' relative to S $c = 2.998 \times 10^8 \text{ms}^{-1}$	

The Solar System has the following parameters. (Woan, 2000 pp176). All orbits are assumed to be elliptical about the sun. Note

$$\frac{M_\odot}{M_\oplus} \approx 332{,}948$$

and

$$R_\oplus \approx \frac{\text{AU}}{23,455}$$

In SI units:

$$\begin{aligned} M_\odot &= 1.9891 \times 10^{30} \text{ kg} \\ R_\odot &= 6.960 \times 10^{8} \text{ m} \\ M_\oplus &= 5.9742 \times 10^{24} \text{ kg} \\ R_\oplus &= 6.37814 \times 10^{6} \text{ m} \\ 1\text{AU} &= 1.495979 \times 10^{11} \text{ m} \end{aligned}$$

Object	$M/M_\oplus$	a /AU	ε[4]	θ_0	β	α	$R/R_\oplus$	T_{rot} / days	P /Yr
Sun	332,837	-	-	-	-	-	109.123	-	-
Mercury	0.055	0.387	0.21	*	7.00	0	0.383	58.646	0.241
Venus†	0.815	0.723	0.01	*	3.39	0	0.949	243.018	0.615
Earth	1.000	1.000	0.02	*	0.00	0	1.000	0.997	1.000
Mars	0.107	1.523	0.09	*	1.85	0	0.533	1.026	1.881
Jupiter	317.85	5.202	0.05	*	1.31	0	11.209	0.413	11.861
Saturn	95.159	9.576	0.06	*	2.49	0	9.449	0.444	29.628
Uranus†	14.500	19.293	0.05	*	0.77	0	4.007	0.718	84.747
Neptune	17.204	30.246	0.01	*	1.77	0	3.883	0.671	166.344
Pluto†	0.003	39.509	0.25	*	17.5	0	0.187	6.387	248.348

where β is the orbital inclination /degrees. In all cases the semi-major axis pointing direction is

$$\mathbf{d} = d_x\hat{\mathbf{x}} + d_y\hat{\mathbf{y}} + d_{\mathbf{y}}\hat{\mathbf{z}} = \cos\beta\hat{\mathbf{x}} + \sin\beta\hat{\mathbf{z}}$$

* For the current orbital polar angle θ_0 (and indeed more accurate values for solar system parameters) see the website of the Jet Propulsion Laboratory (JPL) `http://ssd.jpl.nasa.gov/`

†These planets rotate clockwise about their own internal polar axis. ("Retrograde"). All the other planets rotate anti-clockwise about their own internal axis. All the planets orbit the sun in an anticlockwise direction.

[4] http://nineplanets.org/data.html

8. Mathematics

Name	Equation	Notes / diagram
Trigonometry & Pythagoras' theorem	$x = r\cos\theta$ $y = r\sin\theta$ $r = \sqrt{x^2 + y^2}$ $\sin^2\theta + \cos^2\theta = 1$ $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$ $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$ $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$	$\cos\theta$ is the x coordinate of the unit circle $\sin\theta$ is the y coordinate. θ is measured anticlockwise from the x axis.
Special triangles	$\sin 30^\circ = \frac{1}{2}$ $\sin 60^\circ = \frac{\sqrt{3}}{2}$ $\cos 60^\circ = \frac{1}{2}$ $\cos 30^\circ = \frac{\sqrt{3}}{2}$ $\tan 30^\circ = \frac{1}{\sqrt{3}}$ $\tan 60^\circ = \sqrt{3}$ $\sin 45^\circ = \frac{1}{\sqrt{2}}$ $\cos 45^\circ = \frac{1}{\sqrt{2}}$ $\tan 45^\circ = 1$	
Laws of indices	$x^a x^b = x^{ab}$ $\left(x^a\right)^b = x^{ab}$ $x^{-a} = \dfrac{1}{x^a}$ $\sqrt[n]{x} = x^{\frac{1}{n}}$	
Laws of logarithms	$y = \log_b x \Rightarrow x = b^y$ $\log_b x + \log_b y = \log_b xy$ $\log_b x - \log_b y = \log_b \frac{x}{y}$ $\log_b x^n = n\log_b x$ $x = b^{\log_b x}$ $\log_b x = \dfrac{\log_{10} x}{\log_{10} b} = \dfrac{\log_c x}{\log_c b}$	Base $b > 0$
De-Moivre's Theorem	$e^{i\theta} = \cos\theta + i\sin\theta$	
Taylor & Maclaurin expansions	$f(x) = f(0) + f'(0)x + f''(0)\dfrac{x^2}{2!} + f^{(3)}(0)\dfrac{x^3}{3!} + ...$ $f(x+h) = f(h) + f'(h)\text{x} + f''(h)\dfrac{x^2}{2!} + ...$	
Binomial expansion	$(a+b)^n = \binom{n}{0}a^0b^n + \binom{n}{1}a^1b^{n-1} + \binom{n}{2}a^2b^{n-2} + ... + \binom{n}{n}$ $\binom{n}{r} = \dfrac{n!}{(n-r)!r!}$ $(1+x)^n = 1 + nx + n(n-1)x + n(n-1)(n-2)\dfrac{x^2}{2!} + ...$ $+n(n-1)(n-2)(n-3)\dfrac{x^3}{3!} + ...$	Binomial expansion n integer, >0 Generalized binomial expansion $\|x\| < 1$

Arithmetic progression	$u_n = a + (n-1)d$ $u_1 = a$ $u_{n+1} - u_n = d$ $S_n = \sum_{i=1}^{n} u_i = \frac{1}{2}n(u_1 + u_n)$	
Geometric progression	$u_n = ar^{n-1}$ $u_1 = a$ $\frac{u_{n+1}}{u_n} = r$ $S_n = a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1-r^n)}{1-r}$	If $\lvert r \rvert < 1$ $S_\infty \to \frac{a}{1-r}$
Summation formulae	$\sum_{n=1}^{N} n = \frac{1}{2}n(n+1)$ $\sum_{n=1}^{N} n^2 = \frac{1}{6}n(n+1)(2n+1)$ $\sum_{n=1}^{N} n^3 = \frac{1}{4}n^2(n+1)^2$	
Triangle	$A = \frac{1}{2}bh = \frac{1}{2}ab\sin C$ Area of a triangle $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ Sine and Cosine rules $a^2 = b^2 + c^2 - 2bc\cos A$	b is base of triangle h perpendicular height a,b,c sides of triangle A,B,C opposite angles to sides
Circle	$(x-a)^2 + (x-b)^2 = r^2$ $C = 2\pi r$ $A = \pi r^2$ $s = r\theta$ $a = \frac{1}{2}r^2\theta$	Circle centre (a,b) and radius r Circumference C and area A Arc angle (radians) θ and area a
Ellipse	$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$ $A = \pi ab$	Geometric centre (x_0, y_0) semi-major axis a and semi-minor axis b Area A
Cylinder	$A = 2\pi rh + 2\pi r^2$ $V = \pi r^2 h$	Area A and volume V h height or length of cylinder
Cone	$A = \pi rl$ $V = \frac{1}{3}\pi r^2 h$	l slant height r radius of base h perpendicular height
Frustum	$V = \frac{1}{3}h\left(A + \sqrt{aA} + a\right)$	Top and base areas a, A Perpendicular height h
Stirling's Formula	$n! \approx n^{n+\frac{1}{2}}e^{-n}\sqrt{2\pi}$ $\ln(n!) \approx n\ln n - n$	
Combinatorics	$P = \frac{n!}{p!q!r!\ldots}$ ${}^{n}C_r = \frac{n!}{(n-r)!r!}$ ${}^{n}P_r = \frac{n!}{(n-r)!}$	n objects, p repeats of type A, q repeats of type B etc. ${}^{n}C_r$ is umber of *combinations* of r distinct objects from a population of n distinct objects i.e. order of subset doesn't matter. ${}^{n}P_r$ is umber of *permutations* of r distinct objects from a population of n distinct objects i.e. order of subset *does* matter.

Quadratic equations	$y = ax^2 + bx + c$ $y = 0 \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic formula *Discriminant* $\Delta = b^2 - 4ac$
Vector scalar (dot) product	$\mathbf{a} \cdot \mathbf{b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert \cos\theta$ $\hat{\mathbf{x}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{x}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{z}} = 0$ $\hat{\mathbf{x}} \cdot \hat{\mathbf{x}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1$ $\mathbf{a} = a_x\hat{\mathbf{x}} + a_y\hat{\mathbf{y}} + a_z\hat{\mathbf{z}} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$ $\mathbf{b} = b_x\hat{\mathbf{x}} + b_y\hat{\mathbf{y}} + b_z\hat{\mathbf{z}} = \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix}$ $\mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y + a_z b_z$	Projection (shadow) of one vector on another is $\dfrac{\mathbf{a} \cdot \mathbf{b}}{\lvert\mathbf{b}\rvert}$
Vector (cross) product	$\lvert\mathbf{a} \times \mathbf{b}\rvert = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert \sin\theta$ $\mathbf{b} \times \mathbf{a} = -\mathbf{a} \times \mathbf{b}$ $\hat{\mathbf{x}} \times \hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}} \times \hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}} \times \hat{\mathbf{x}} = \hat{\mathbf{y}}$ $\mathbf{a} \times \mathbf{b} = \hat{\mathbf{x}}\left(a_y b_z - a_z b_y\right) + \hat{\mathbf{y}}\left(a_z b_x - a_x b_z\right) + \hat{\mathbf{z}}\left(a_x b_y - a_y b_x\right.$ $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) = (\mathbf{a} \cdot \mathbf{c})(\mathbf{b} \cdot \mathbf{d}) - (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c})$ $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{b} \cdot \mathbf{c})\mathbf{a}$ $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$	Evaluate using 'right hand screw rule'
Vector equations and planes, and distances between lines and planes.	$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ $d = \sqrt{\lvert\mathbf{b} - \mathbf{a}\rvert^2 - \dfrac{\left((\mathbf{b} - \mathbf{a}) \cdot \mathbf{c}\right)^2}{\lvert\mathbf{c}\rvert^2}} = \dfrac{\lvert(\mathbf{b} - \mathbf{a}) \times \mathbf{c}\rvert}{\lvert\mathbf{c}\rvert}$ $\hat{\mathbf{n}} = \dfrac{(\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{a})}{\lvert(\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{a})\rvert}$ $(\mathbf{r} - \mathbf{a}) \cdot \hat{\mathbf{n}} = 0$ $\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) + \mu(\mathbf{c} - \mathbf{a})$ $d = \lvert(\mathbf{a} - \mathbf{p}) \cdot \hat{\mathbf{n}}\rvert$ $\Delta = \dfrac{\lvert(\mathbf{b} - \mathbf{a}) \cdot (\mathbf{c} \times \mathbf{d})\rvert}{\lvert\mathbf{c} \times \mathbf{d}\rvert}$ $d = \lvert(\mathbf{c} - \mathbf{a}) \cdot \hat{\mathbf{n}}\rvert$	Vector equation of a straight line through point $\mathbf{a}$ and with direction vector $\mathbf{b}$ 'Foot of the perpendicular' i.e. closest distance from $\mathbf{b}$ to line passing through $\mathbf{a}$ with direction vector $\mathbf{c}$ Normal unit vector $\hat{\mathbf{n}}$ to plane containing non-parallel position vectors $\mathbf{a,b,c}$ Vector equation of a plane Distance from a point $\mathbf{p}$ to plane characterized by point $\mathbf{a}$ on the plane and unit normal $\hat{\mathbf{n}}$ Distance between line through $\mathbf{a}$ with direction vector $\mathbf{c}$ and a line through $\mathbf{b}$ with direction vector $\mathbf{d}$ If a line $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ is parallel to a plane, $\hat{\mathbf{n}} \cdot \mathbf{b} = 0$. Distance between line and plane is $d = \lvert(\mathbf{c} - \mathbf{a}) \cdot \hat{\mathbf{n}}\rvert$ Volume of a parallelepiped formed from vectors $\mathbf{a,b,c}$

	$V=[\mathbf{a},\mathbf{b},\mathbf{c}]=\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})$	
Matrices	$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix}=\begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix}$ $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1}=\dfrac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1}\begin{pmatrix} a & b \\ c & d \end{pmatrix}=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	Inverse matrix Identity matrix
Basic differentiation	$f'(x)=\dfrac{d}{dx}f(x)$ $\dfrac{d}{dx}x^n=nx^{n-1}$ $\dfrac{d}{dx}e^{ax}=ae^{ax}$ $\dfrac{d}{dx}\ln\lvert x\rvert=\frac{1}{x}$ $\dfrac{d}{dx}\ln\lvert f(x)\rvert=\dfrac{f'(x)}{f(x)}$ $\dfrac{d}{dx}\sin ax=a\cos ax$ $\dfrac{d}{dx}\cos ax=-a\sin ax$ $\dfrac{d}{dx}\tan ax=a\sec^2 ax$ $\dfrac{d}{dx}(uv)=u\dfrac{dv}{dx}+v\dfrac{du}{dx}$ $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)=\dfrac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ $\dfrac{dy}{dx}=\dfrac{dy}{dz}\times\dfrac{dz}{dx}$	For trigonometric functions, variables must be in radians π radians = 180° Note: if $\theta\ll 1$ $\theta\approx\sin\theta\approx\tan\theta$ Product Rule Quotient Rule Chain Rule
Basic integration	$\int f'(x)dx=f(x)+c$ $\int_a^b f'(x)dx=\left[f(x)\right]_a^b=(f(b))-(f(a))$ $\int e^{ax}dx=\frac{1}{a}e^{ax}+c$ $\int\dfrac{f'(x)}{f(x)}dx=\ln\lvert f(x)\rvert+c$ $\int\sin ax dx=-\frac{1}{a}\cos ax$ $\int\cos ax dx=\frac{1}{a}\sin ax$ $\int\tan ax dx=-\frac{1}{a}\ln\lvert\cos ax\rvert+c$ $\int(uv)dx=u\int vdx-\int\left(\dfrac{du}{dx}\times\int vdx\right)dx$	$\int g(x)dx$ is the area between the curve $g(x)$ and the x axis, with the caveat that the area beneath the axis counts a negative. $\int g(x)dx$ is also the *inverse* of differentiating $y=g(x)$. i.e. $\int\dfrac{dy}{dx}dx=y+c$ (which is true up to a constant of integration c, which must be specified. Integration by parts

Volumes of revolution	$V_x = \int_{x=a}^{b} \pi y^2 dx$	About x axis.
	$V_y = \int_{y=a}^{b} \pi x^2 dy$	About y axis.
	$A_x = 2\pi \int_a^b y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$	Surface area
	$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$	Length of a curve
	$R = \left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} \left(\frac{d^2y}{dx^2}\right)^{-1}$	Radius of curvatire

Linear regression	$\bar{x} = \frac{1}{N}\sum_{n=1}^{N} x_n \quad \bar{y} = \frac{1}{N}\sum_{n=1}^{N} y_n$ $\overline{x^2} = \frac{1}{N}\sum_{n=1}^{N} x_n^2 \quad \overline{y^2} = \frac{1}{N}\sum_{n=1}^{N} y_n^2$ $\overline{xy} = \frac{1}{N}\sum_{n=1}^{N} x_n y_n$ $\mathrm{cov}[x,y] = \overline{xy} - \bar{x}\times\bar{y}$ $V[x] = \overline{x^2} - \bar{x}^2 \quad V[\mathrm{y}] = \overline{y^2} - \bar{y}^2$ $p = \frac{\mathrm{cov}[x,y]}{\sqrt{V[x]V[y]}}$ $y = mx + c$ $m = \frac{\mathrm{cov}[x,y]}{V[x]} \quad c = \bar{y} - m\bar{x}$ vertical fit $m = \frac{V[y]}{\mathrm{cov}[x,y]} \quad c = \bar{y} - m\bar{x}$ horizontal fit	Formulae for calculating the line of best fit to a set of data $\{x_n, y_n\}$ $\mathrm{cov}[x,y]$ is the covariance $p = \frac{\mathrm{cov}[x,y]}{\sqrt{V[x]V[y]}}$ is the product moment correlation coefficient. $p = -1$ perfect negative correlation between x and y $p = +1$ perfect positive correlation between x and y $p = 0$ no correlation between x and y
Statistical analysis	$\bar{x} = E[x] = \frac{1}{N}\sum_{n=1}^{N} x_n$ $\sigma^2 = V[x] = \frac{1}{N-1}\sum_{n=1}^{N}\left(x_n - \bar{x}\right)^2$ $\mathrm{skew}[x] = \frac{N}{(N-1)(N-2)}\sum_{n=1}^{N}\left(\frac{x_n - \bar{x}}{\sigma}\right)^3$	$\bar{x}$ mean, or expectation σ^2 variance For continuous variables and probability distribution $p(x)$ $E[x] = \int xp(x)dx$ $V[x] = E[x^2] - \left(E[x]\right)^2$ $V[x] = \int x^2 p(x)dx - \left(\int xp(x)dx\right)^2$
Numeric integration: Trapezium Rule	$\int_{x_0}^{x_n} f(x)dx \approx \frac{1}{2}\Delta x\left(f(x_0) + 2\left(f(x_1) + f(x_2) + ... + f(x_{n-1})\right) + f(x_n)\right)$ $\Delta x = \frac{x_n - x_0}{N}$	Estimate integral by summing trapezia fitted to the curve. Each trapezia has a fixed base width Δx
Solving Ordinary Differential Equations - *Euler's Method*	$\frac{dy}{dx} = f(x,y)$ $x_{n+1} = x_n + \Delta x$ $y_{n+1} = y_n + f(x_n, y_n)\Delta x$	Errors of the order of Δx
Solving Ordinary Differential Equations - *Runge-Kutta Method*	$\frac{dy}{dx} = f(x,y)$ $x_{n+1} = x_n + \Delta x$ $k_1 = f(x_n, y_n)\Delta x$ $k_2 = f(x_n + \frac{1}{2}\Delta x, y_n + \frac{1}{2}k_1)\Delta x$ $k_3 = f(x_n + \frac{1}{2}\Delta x, y_n + \frac{1}{2}k_2)\Delta x$ $k_4 = f(x_n + \frac{1}{2}\Delta x, y_n + k_3)\Delta x$ $y_{n+1} = y_n + \frac{1}{6}k_1 + \frac{1}{3}k_2 + \frac{1}{3}k_3 + \frac{1}{6}k_4$	Errors of the order of Δx^4
Solving vector dynamics problems via Verlet Method	$\mathbf{a}_n = f\left(t_n, \mathbf{r}_n\right)$ $\mathbf{v}_{n+1} = \mathbf{v}_n + \frac{1}{2}\left(\mathbf{a}_n + \mathbf{a}_{n+1}\right)\Delta t$ $\mathbf{r}_{n+1} = \mathbf{r}_n + \mathbf{v}_n\Delta t + \frac{1}{2}\mathbf{a}_n\Delta t^2$	Errors of the order of Δx^2
Newton's Raphson method for root finding	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$	Requires initial guess of root x_0 Fails near to a stationary point $f'(x) = 0$

Normal distribution	$p(x\mid\mu,\sigma)=\dfrac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ $\bar{x}=\mu$ $M(x,t)=e^{\mu t+\frac{1}{2}\sigma^2t^2}$	*Gaussian distribution or 'bell curve'.* ***Central limit theorem:*** *"The distribution of the mean values of a set of independent random values tends towards a Gaussian distribution if the number of samples is large enough."*
Exponential distribution	$p(x\mid\lambda)=\lambda e^{-\lambda x}$ $\bar{x}=\dfrac{1}{\lambda}$ $\sigma^2=\dfrac{1}{\lambda^2}$	
Rayleigh distribution	$p(x\mid\sigma)=\dfrac{x}{\sigma^2}e^{-\frac{x^2}{2\sigma^2}}$ $\bar{x}=\sigma\sqrt{\pi/2}$ $V[x]=2\sigma^2(1-\frac{1}{4}\pi)$	
Chi-squared distribution	$p(x\mid r)=\dfrac{e^{-x/2}x^{(r/2)-1}}{2^{r/2}\Gamma(r/2)}$ $\bar{x}=r$ $\sigma^2=2r$ $\Gamma(x)=\int_0^\infty t^{z-1}e^{-t}dt$	$\Gamma(x)$ is the Gamma function.
Binomial distribution	$p(x\mid n,p)=\binom{n}{x}p^x(1-p)^{n-x}$ $p(x\mid n,p)\approx\dfrac{1}{\sqrt{2\pi np(1-p)}}\exp\left[-\frac{1}{2}\dfrac{(x-np)^2}{np(1-p)}\right]$ $\bar{x}=np$ $\sigma^2=np(1-p)$ $M(x,t)=(pe^t+1-p)^n$	Probability of x successes out of n independent trials, each with probability of success p
Geometric distribution	$p(x\mid p)=(1-p)^{x-1}p$ $\bar{x}=\dfrac{1}{p}$ $\sigma^2=\dfrac{1-p}{p^2}$	Probability of success in the x^{th} independent trial, following *x-1* failures. Each trial has probability of success p.
Poisson distribution	$p(x\mid\lambda)=\dfrac{\lambda^x e^{-\lambda}}{x!}\approx\dfrac{1}{\sqrt{2\pi\lambda}}e^{-\frac{(x-\lambda)^2}{2\lambda}}$ $\bar{x}=\lambda$ $\sigma^2=\lambda$ $M(x,t)=e^{\lambda(e^t-1)}$	Probability of success rate x, given mean success rate λ. For example, goals per football game, decays per second etc...
Moment generating functions	$M(x,t)=E\left[e^{tx}\right]$ $e^{tx}=1+tx+\frac{1}{2!}(tx)^2+\frac{1}{3!}(tx)^3+\ldots$ $M(x,t)=1+tE[x]+\frac{1}{2!}t^2E[x^2]+\ldots$ $E[x^n]=\left.\dfrac{\partial^n M(x,t)}{\partial t^n}\right\|_{t=0}$ $V[x]=E[x^2]-\left(E[x]\right)^2$	$E[x]$ expectation $V[x]$ variance

Bayes' Theorem & statistical inference	$P(H\mid T)P(T)=P(T\mid H)P(H)$ $P(H\mid T)=\dfrac{P(T\mid H)P(H)}{P(T\mid H)P(H)+P(T\mid H')P(H')}$ $P(H\mid T')=\dfrac{P(T'\mid H)P(H)}{P(T'\mid H)P(H)+P(T'\mid H')P(H')}$	H hypothesis true H' hypothesis false T test for hypothesis pass T' test for hypothesis fail $P(H\mid T)$ is probability of hypothesis being true given a test has been passed. This is often what a patient wants to know following a test for a disease. Note in medical applications a pharmaceutical company will instead measure $P(T\mid H)$ e.g. probability that a test passes given a sample has the disease. If a disease is rare, $P(H)\ll 1$ which may mean $P(H\mid T)$ is low even if $P(T\mid H)$ is close to 100%. $P(H\mid T')$ is called a *false positive.*

9. Recommended books and resources

Online portal to Physics, Mathematics references (including this document)

www.eclecticon.info

* Don't start a Physics University course without these! ** Standard texts

General Physics

French, A., *Science by Simulation**

Woan, G., *The Cambridge Handbook of Physics Formulas**

Cullerne, J.P., Machacek, A., *The Language of Physics**

Rees, W.G., *Physics by Example**

Kirk, T., *IB Study Guide: Physics 2nd Edition**

Chadha, G *et al., A Level Physics for OCR A**

*Feynman Lectures on Physics***

Mechanics

Kleppner, D., & Kolenkow, R., *An Introduction to Mechanics**

Morin, D., *Introduction to Classical mechanics: With problems and solutions**

Pain, H.J., *The Physics of Vibrations and Waves*

Hand, L.N., Finch, J.D. *Analytical Mechanics*

Strogatz, S.H., *Nonlinear Dynamics & Chaos***

Faber, T.E., *Fluid Dynamics for Physicists**

Goldstein, H., Poole, C.P., Safko, J.L. *Classical Mechanics***

Waves & Optics

Hecht, E., *Optics***

Thermal Physics

Mandl, F., *Statistical Physics***

Electricity & Magnetism

Bleaney,B., Bleaney, B., *Electricity & Magnetism (volumes 1 and 2)**

Jackson, J,D., *Classical Electrodynamics***

Quantum Physics / Solid State Physics / Nuclear Physics/ Particle Physics etc

McEvoy, J.P., Zarate, O., *Introducing Quantum Theory**

Martin, B., *Nuclear & Particle Physics: An Introduction***

McCaw, C.S., Orbitals With Applications in Atomic Spectra

Kittel, C., Introduction to Solid State Physics**

*Haken, H., Wolf, C. H., The Physics of Atoms and Quanta***

*Warner, M., Cheung, A., A Cavendish Quantum Mechanics Primer**

Cosmology

Basset, B., Edney, R., *Relativity: A Graphic Guide*

d'Inverno, R., *Introducing Einstein's Relativity***

Earth Sciences & Remote Sensing

Fowler, C.M.R., *The Solid Earth: An Introduction to Global Geophysics***

Shearer, P.M., *Introduction to Seismology***

Rees, W.G., *Physical Principles of Remote Sensing***

Mathematics

Riley, K.F., Hobson, M.P., Bence, S.J., *Mathematical Methods for Physics and Engineering**

Rayner, D., *Extended Mathematics for Cambridge IGCSE**

Quadling et al, *OCR (Cambridge Advanced Level Mathematics) Core1&2,Core3&4,Further Pure1,Further Pure 2&3,Statistics1,Statistics2&3, Mechanics1,Mechanics2,Mechanics3&4,Decision1,Decision2&3 ***

Misc

Gleick, J., *Chaos: Making a New Science*

MacKay, D.J.C., *Sustainable Energy - Without the Hot Air*

MacKay, D.J.C., *Information Theory, Inference and Learning Algorithms*

Feynman, R.P., Leighton, R., Hutchings, E., *Surely You're Joking Mr Feynman*

Bellos, A., *Alex's Adventures in Numberland*

Computer Programming & document processing

Hanselman, D.C., Littlefield, B.L., *Mastering MATLAB*

Press, W.H., Teukolsky, S.A., Vetterling, T., Flannery, B.P., *Numerical Recipes: The Art of Scientific Computing***

`www.mathworks.co.uk (MATLAB)`

`www.python.org`

`https://thonny.org/`

`www.irfanview.com/`

`http://www.lyx.org/` (LaTeX)

`www.pcspecialist.co.uk`