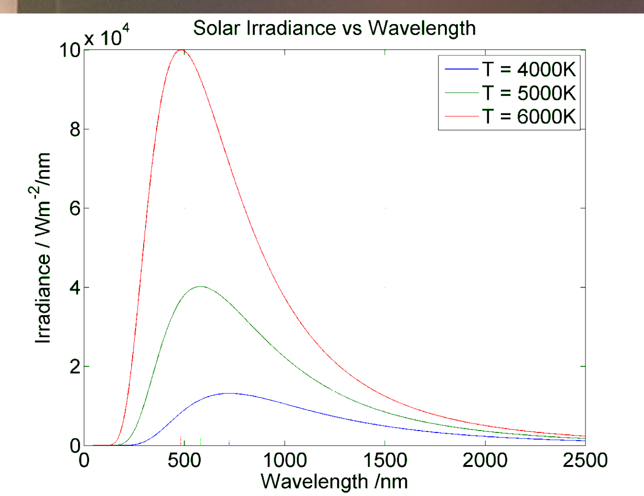
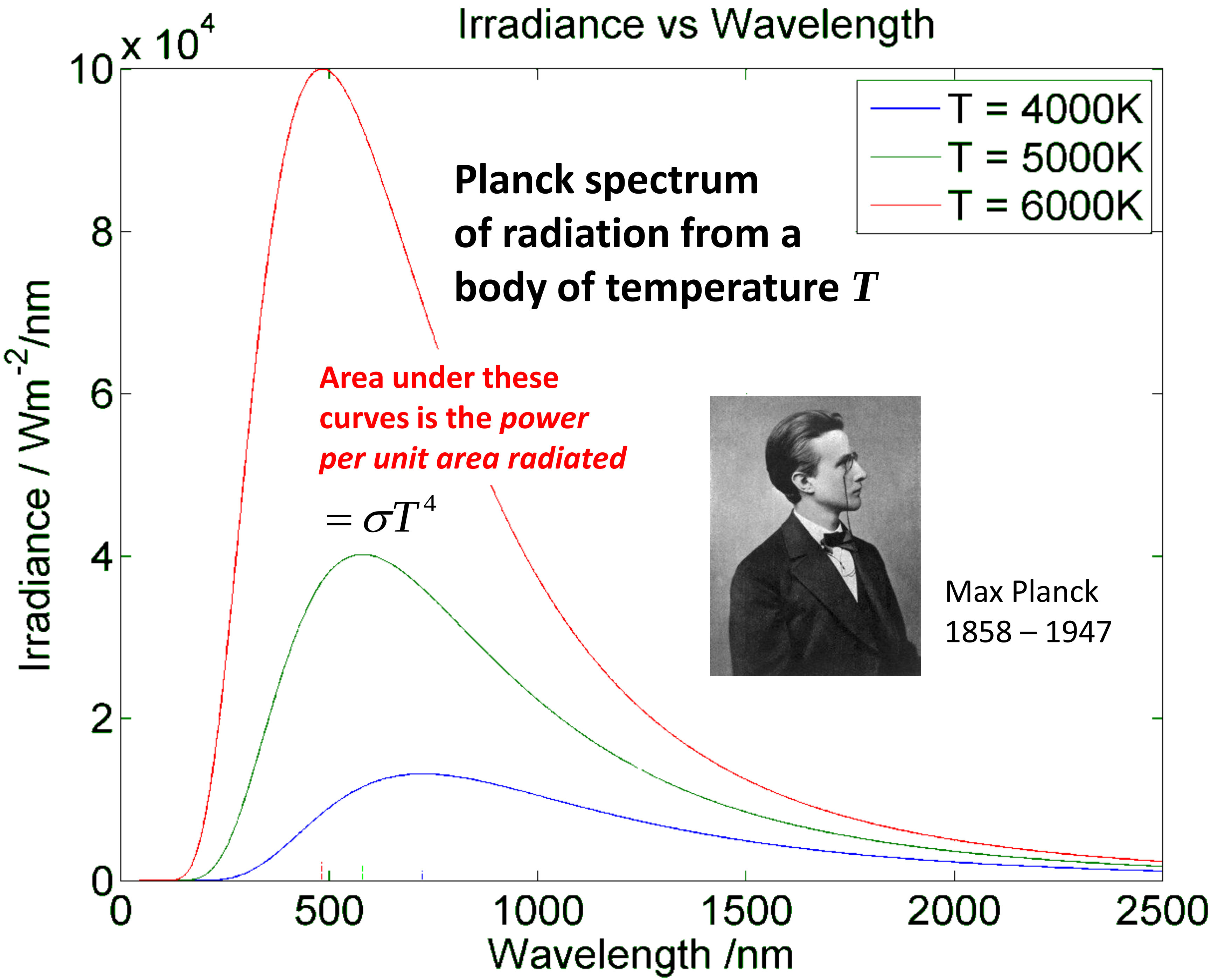
Planck spectrum from filament bulbs



Tony Ayres & Andrew French. June 2021


Irradiance vs Wavelength
x 10^4
Irradiance / Wm^-2/nm
Wavelength /nm
0
2
4
6
8
10
0
500
1000
1500
2000
2500
T = 4000K
T = 5000K
T = 6000K
Planck spectrum of radiation from a body of temperature T
Area under these curves is the power per unit area radiated
= σT^4
Max Planck
1858 – 1947

$$I = \int_0^\infty B(\lambda, T) d\lambda = \sigma T^4$$

$$\sigma = \frac{2\pi^5 k_B^4}{15 c^2 h^3}$$

$$B(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

This formula for irradiance is the Planck law

$k_B = 1.381 \times 10^{-23} \text{m}^2\text{kgs}^{-2}\text{K}^{-1}$	Boltzmann's constant
$h = 6.626 \times 10^{-34} \text{m}^2\text{kgs}^{-1}$	Planck's constant
$c = 2.998 \times 10^8 \text{ms}^{-1}$	Speed of light

$$\lambda_{\max} = 590 \times 10^{-9}\text{m}$$

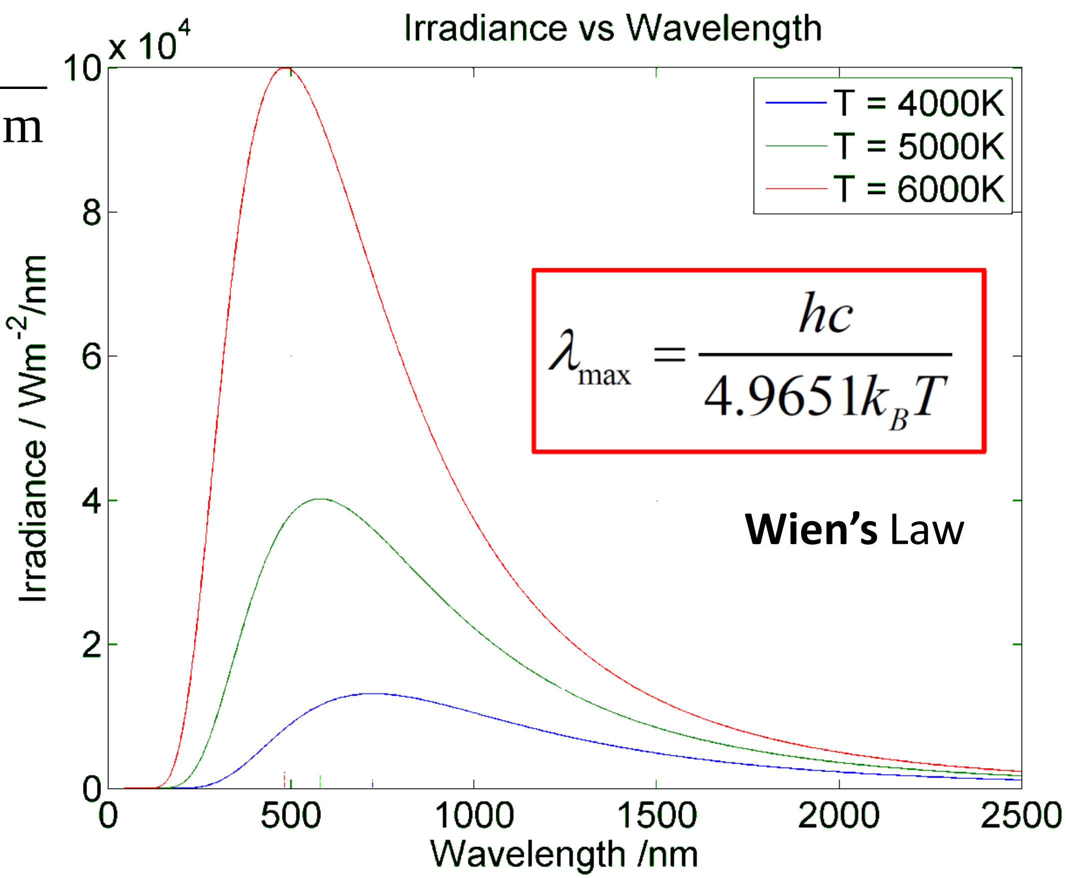
$$T = \frac{6.626 \times 10^{-34} \times 2.998 \times 10^8}{4.965 \times 1.381 \times 10^{-23} \times 590 \times 10^{-9}\text{m}}$$

$$T = 4{,}910\text{K}$$

So *peak* of radiation spectrum from a Tungsten filament must be for a larger wavelength than yellow, otherwise the filament would melt.

Red	620-750nm
Yellow	570-590nm
Green	495-570nm
Blue	450-495nm

The melting point of tungsten is **3,695K**.



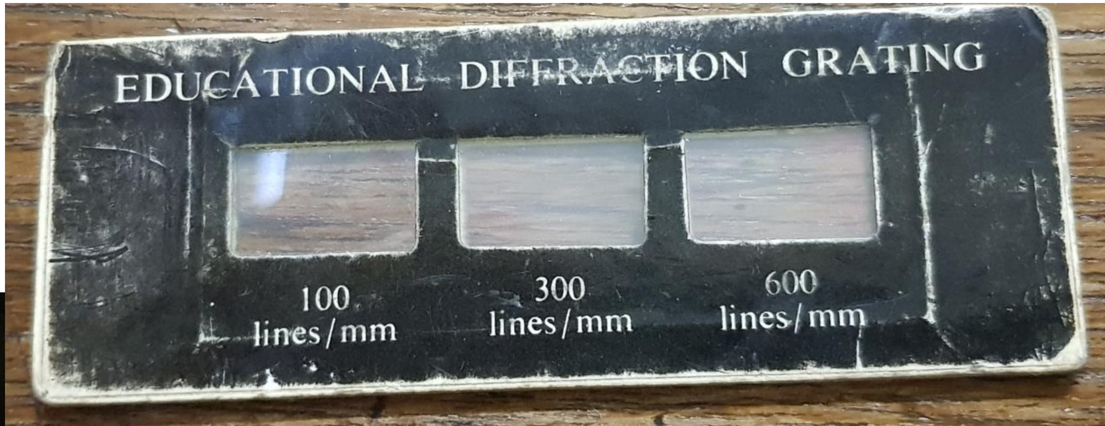
$$\lambda_{\max} = \frac{hc}{4.9651 k_B T}$$

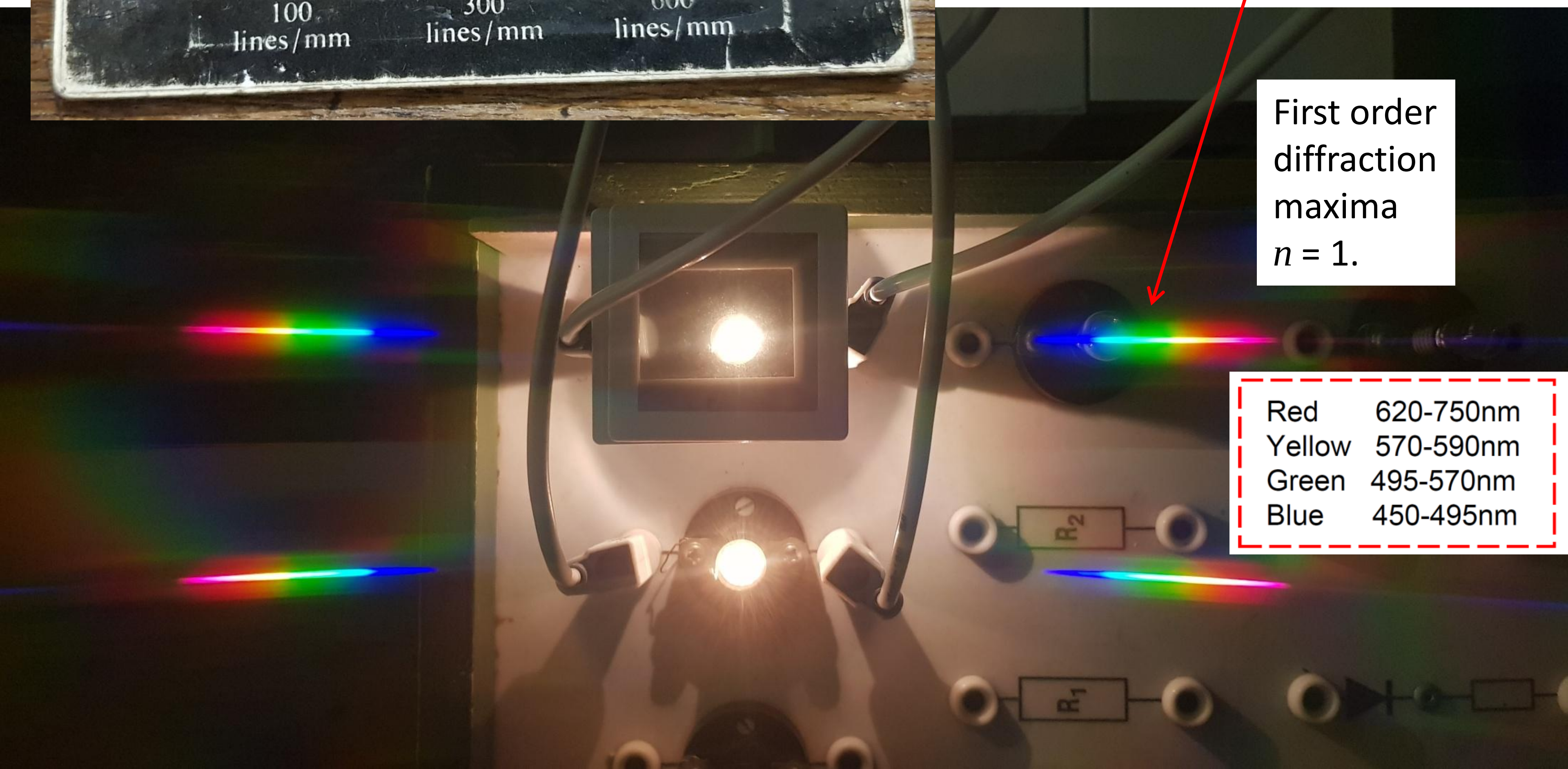
Wien's Law

We can (crudely) measure the visible part of the spectra of light from a filament bulb by fixing a **diffraction grating** over the camera of a smartphone. (I used a Samsung Galaxy S8). For the diffraction grating, I used 600 lines /mm.

$$s = \frac{10^{-3}}{600}\,\text{m}$$

$$\theta_n = \sin^{-1}\left(\frac{n\lambda}{s}\right)$$

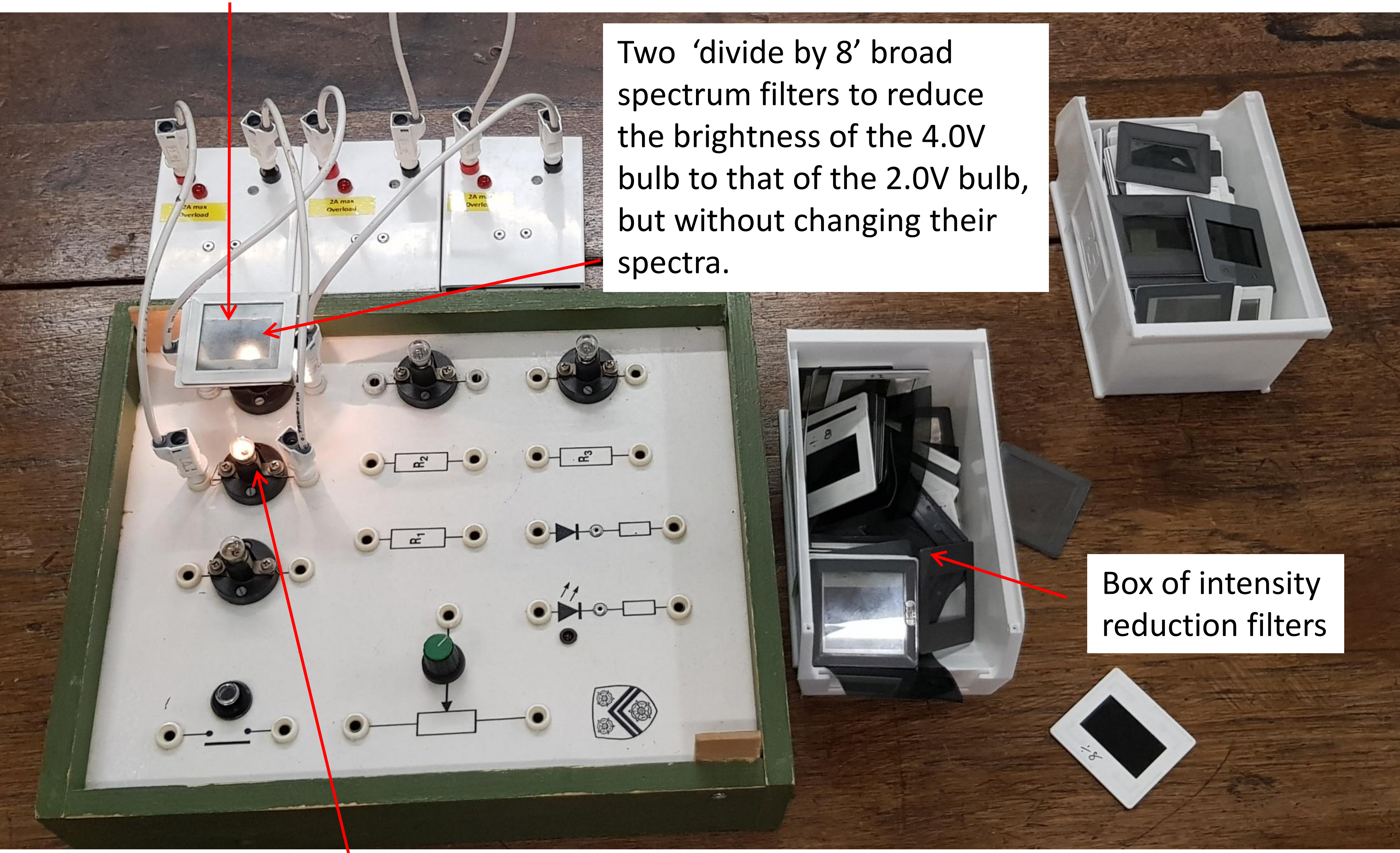




First order diffraction maxima $n = 1$.

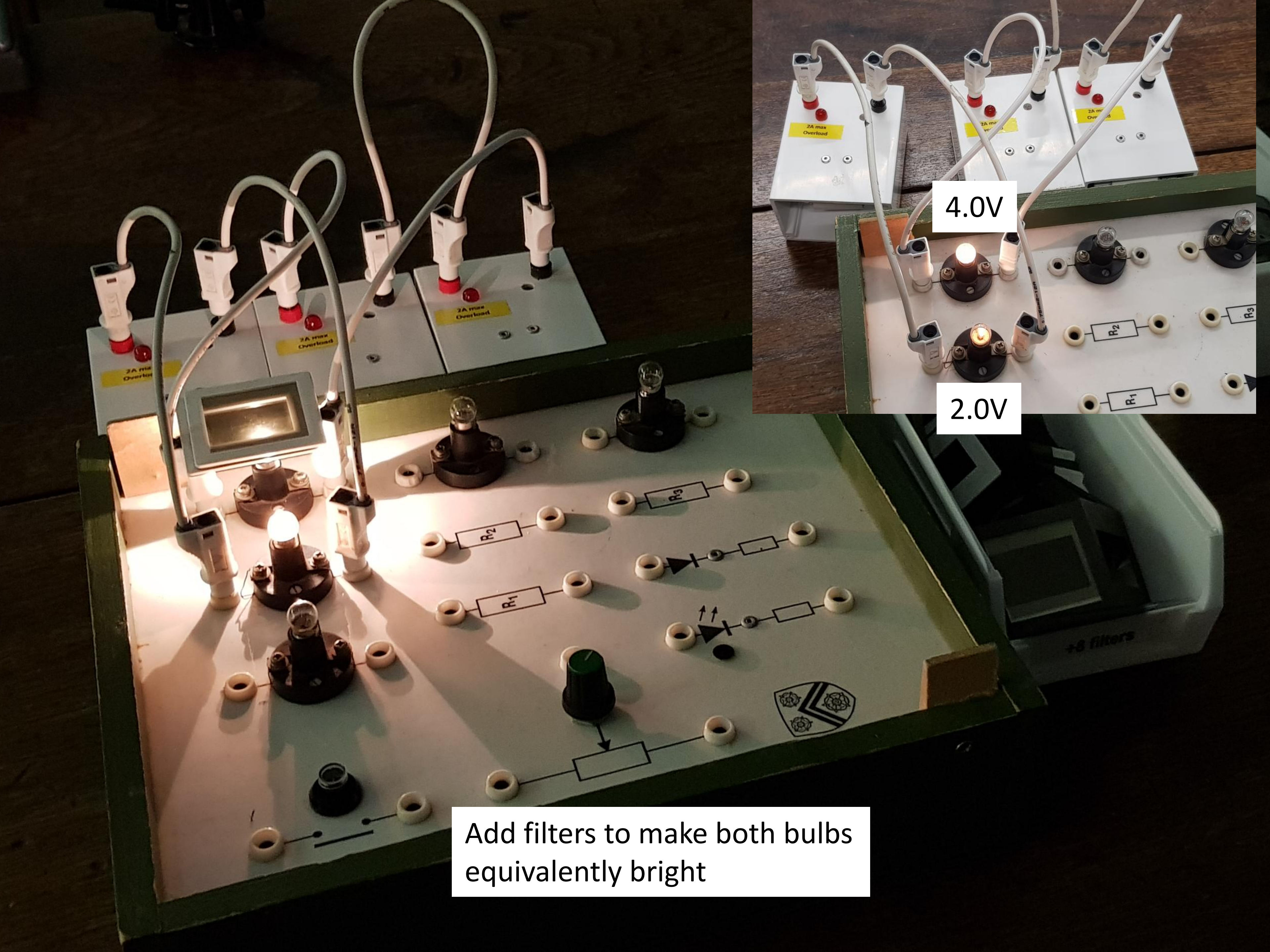
Red	620-750nm
Yellow	570-590nm
Green	495-570nm
Blue	450-495nm

About 4.0V (from two cells) powering a light bulb

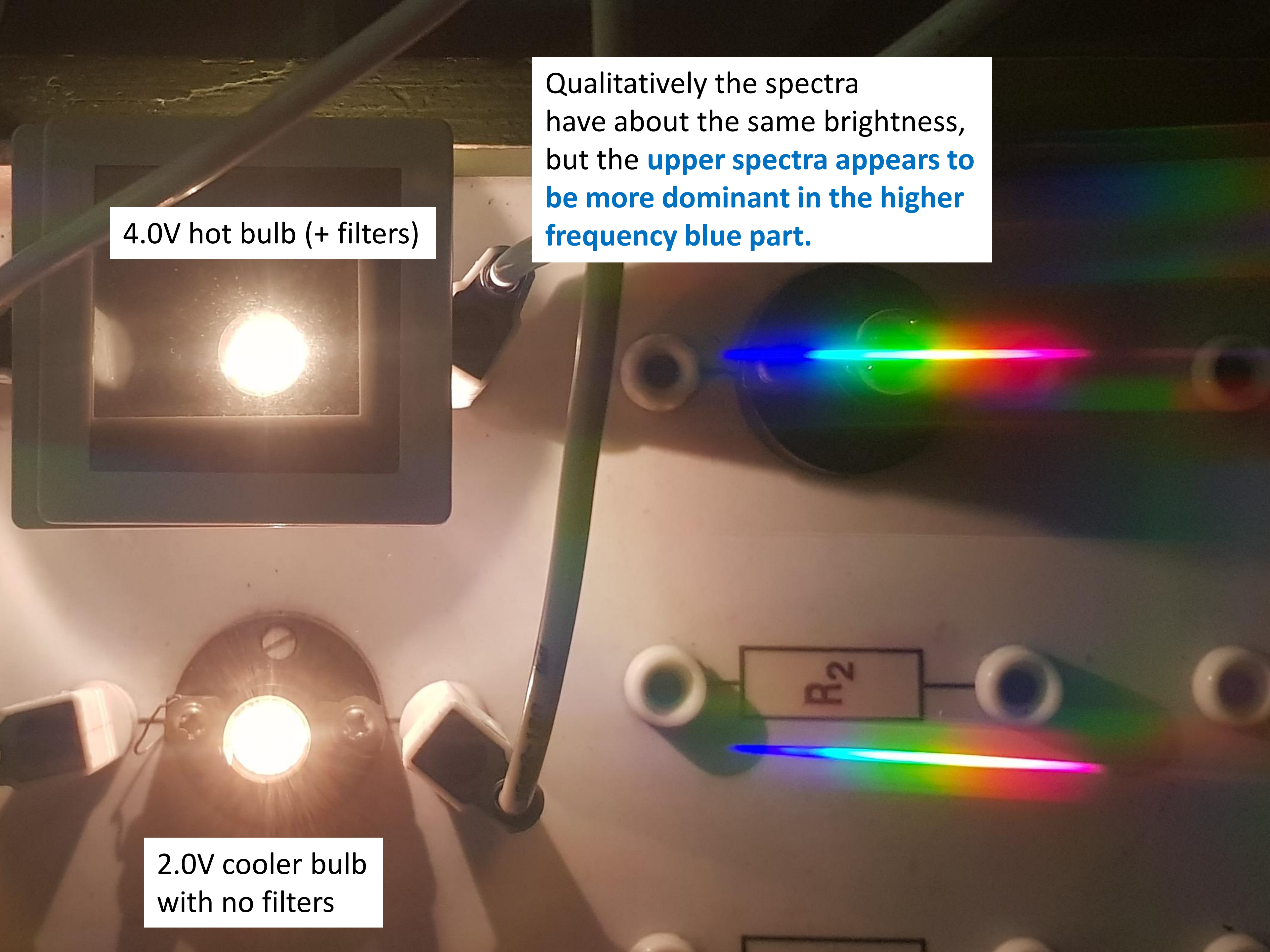


Just one (2.0V) cell powering a light bulb

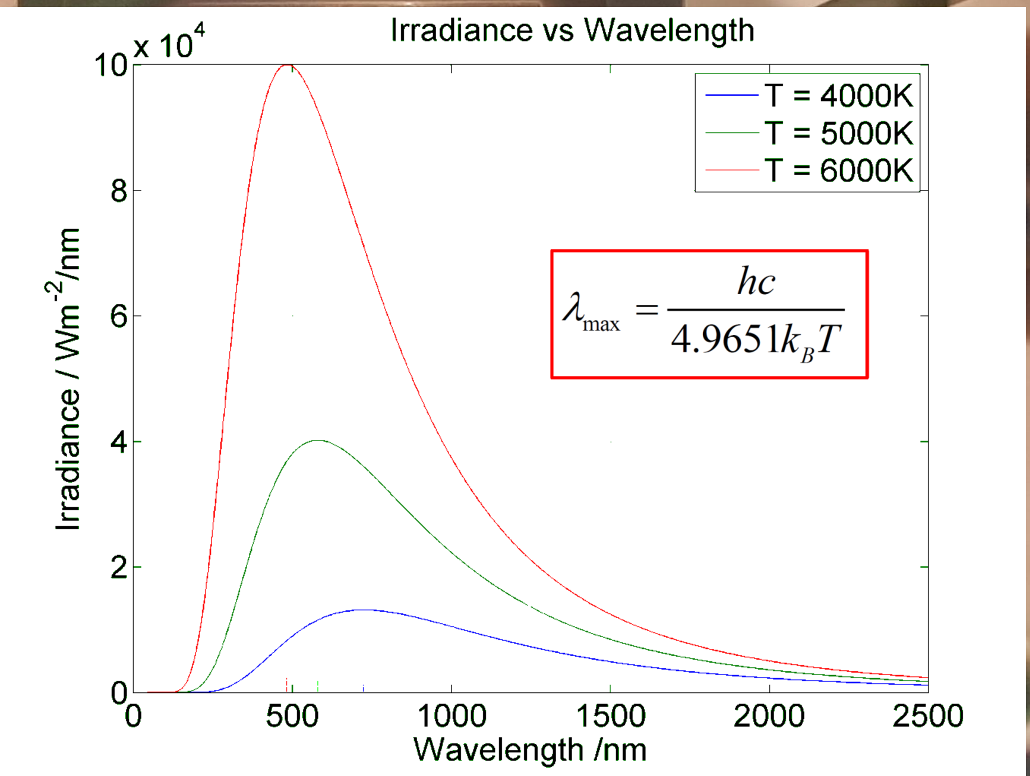
Equipment

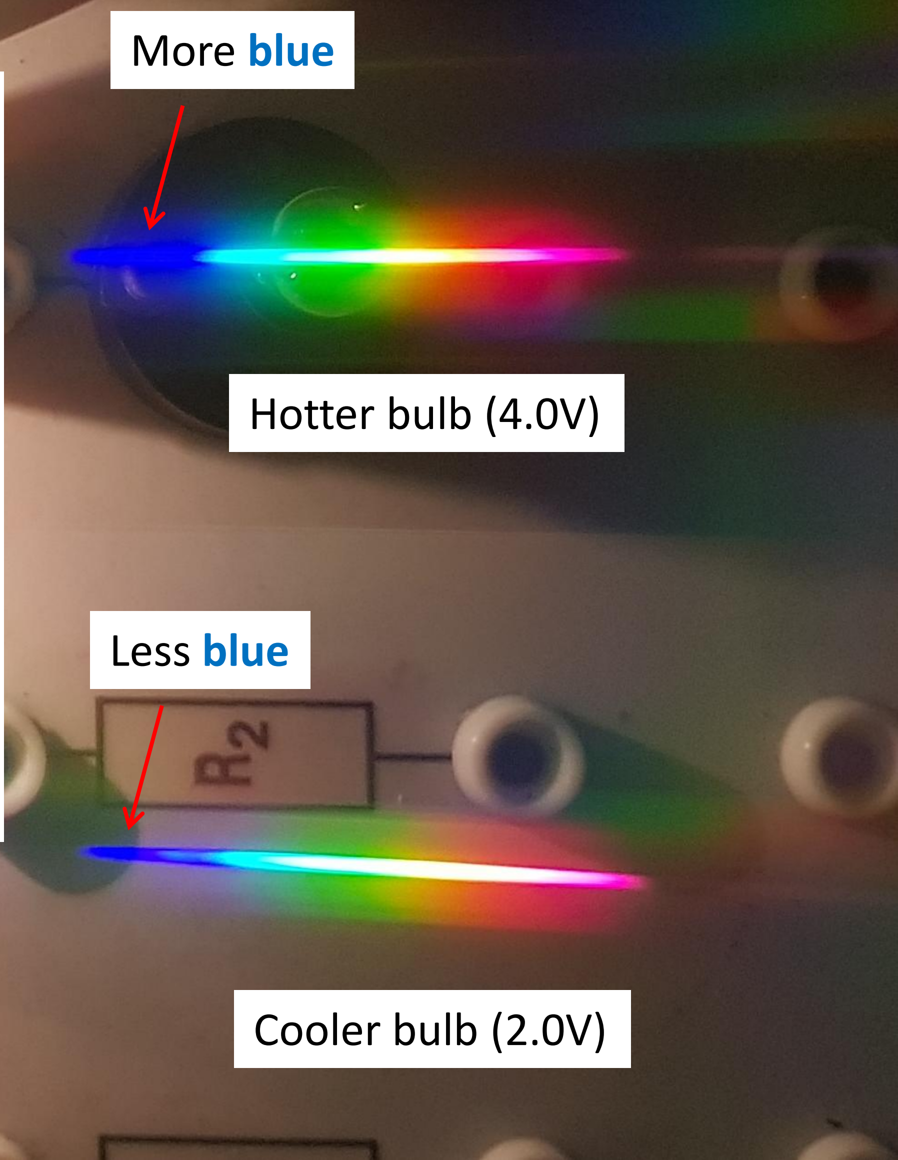


Add filters to make both bulbs equivalently bright

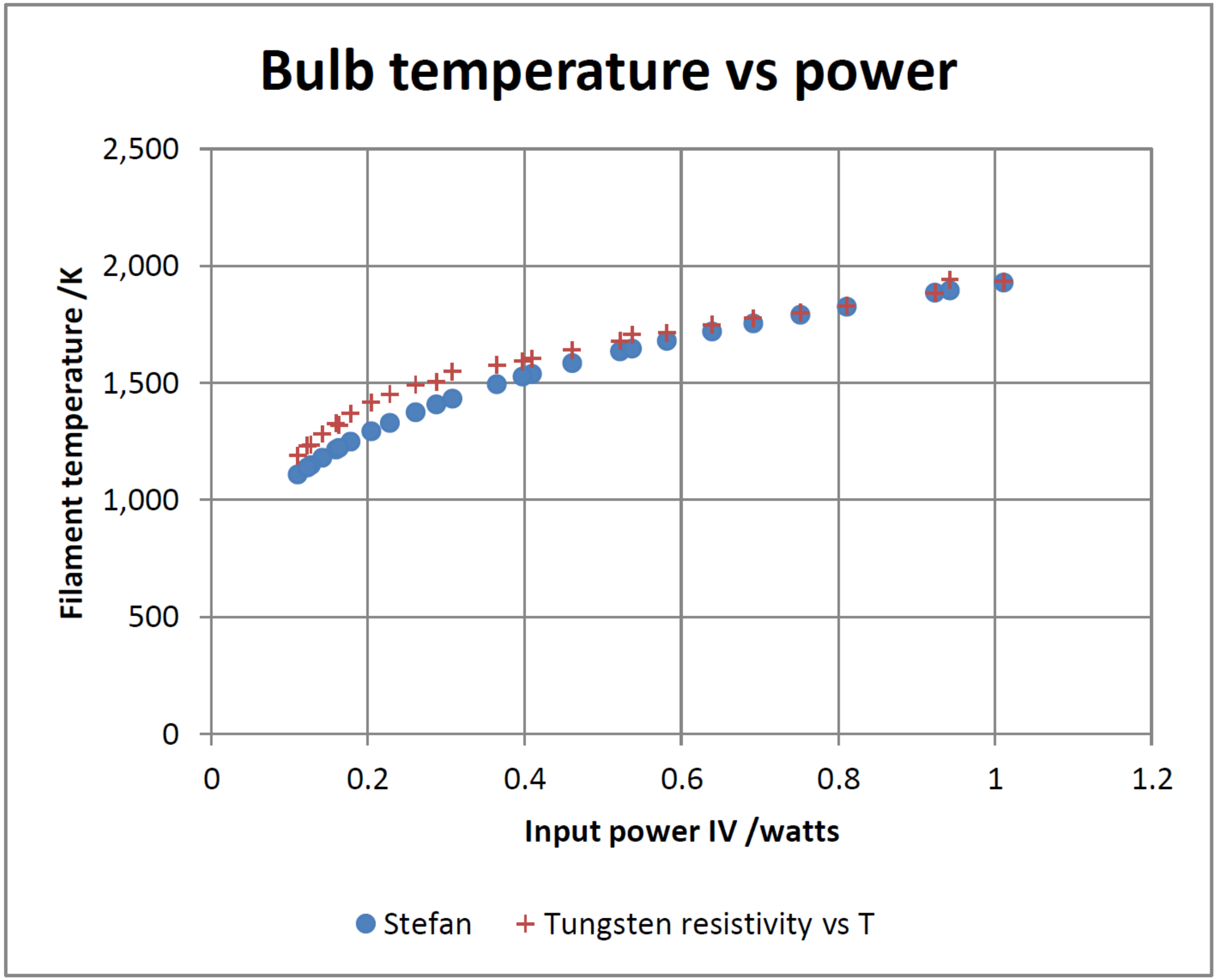

Qualitatively the spectra have about the same brightness, but the **upper spectra appears to be more dominant in the higher frequency blue part.**
4.0V hot bulb (+ filters)
R2
2.0V cooler bulb with no filters

Hotter lightbulb filaments should have a slightly narrower spectrum, *shifted to lower wavelengths.*





Red	620-750nm
Yellow	570-590nm
Green	495-570nm
Blue	450-495nm



From Tungsten resistivity vs T

$$\left(\frac{\pi r^2}{La}\right)^{\frac{1}{1.205}} \approx 220$$

$$T = \left(\frac{V}{I}\right)^{\frac{1}{1.205}} \left(\frac{\pi r^2}{La}\right)^{\frac{1}{1.205}}$$

$$T = \left(\frac{IV}{A\sigma}\right)^{\frac{1}{4}}$$

From Stefan's law

Predict temperature well below tungsten melting point of 3,695K.

$$\sigma = 5.67\times10^{-8}\,\mathrm{Wm^{-2}K^{-1}}$$

i.e. we ignore coiling for the radiating area

$$A = \pi dl$$

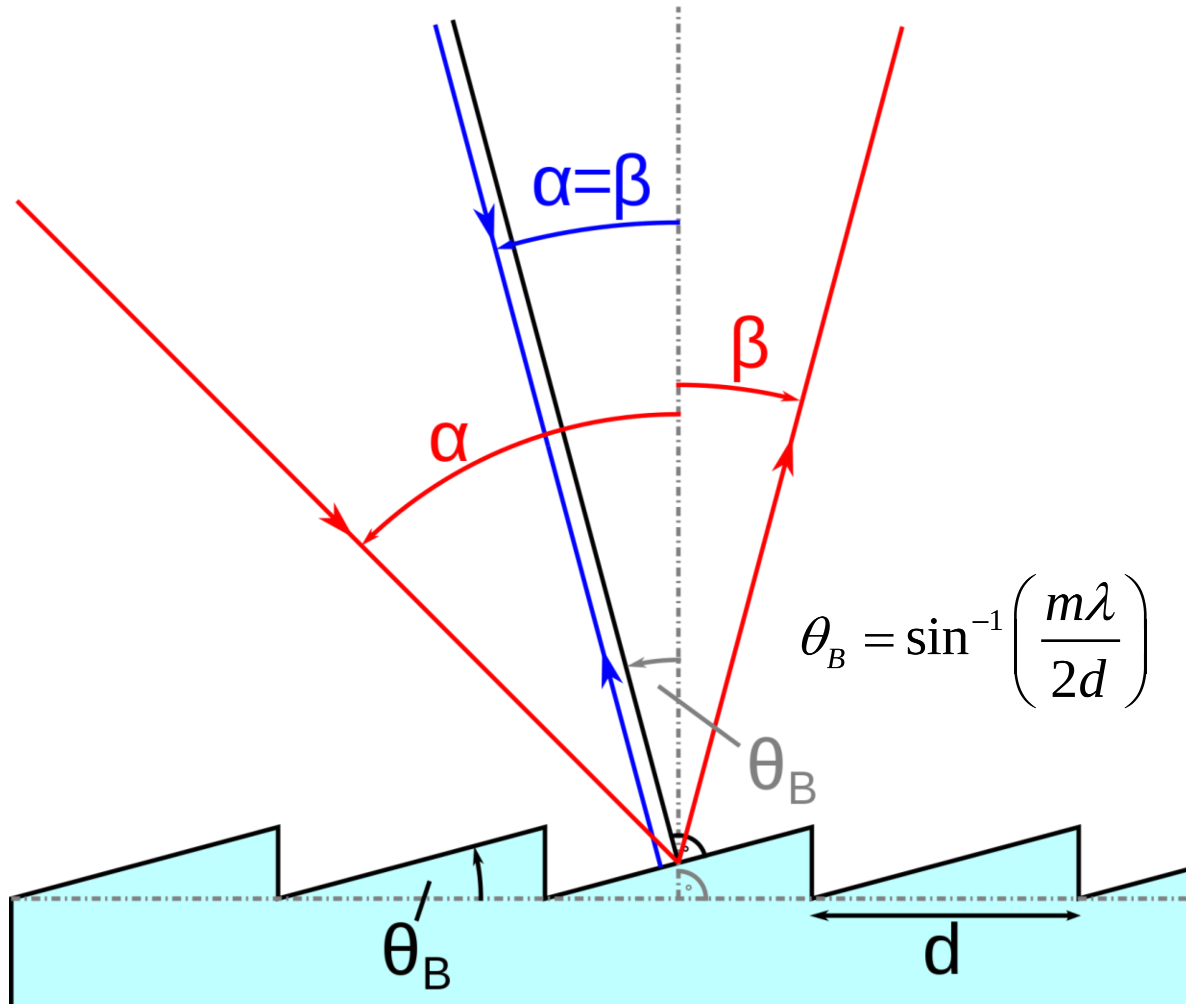
$$A = 1.29\times10^{-6}\,\mathrm{m^2}$$

Could you measure the Planck spectrum directly?

Rather than a diffraction grating, a **blazed reflection grating** is preferable.
https://en.wikipedia.org/wiki/Blazed_grating

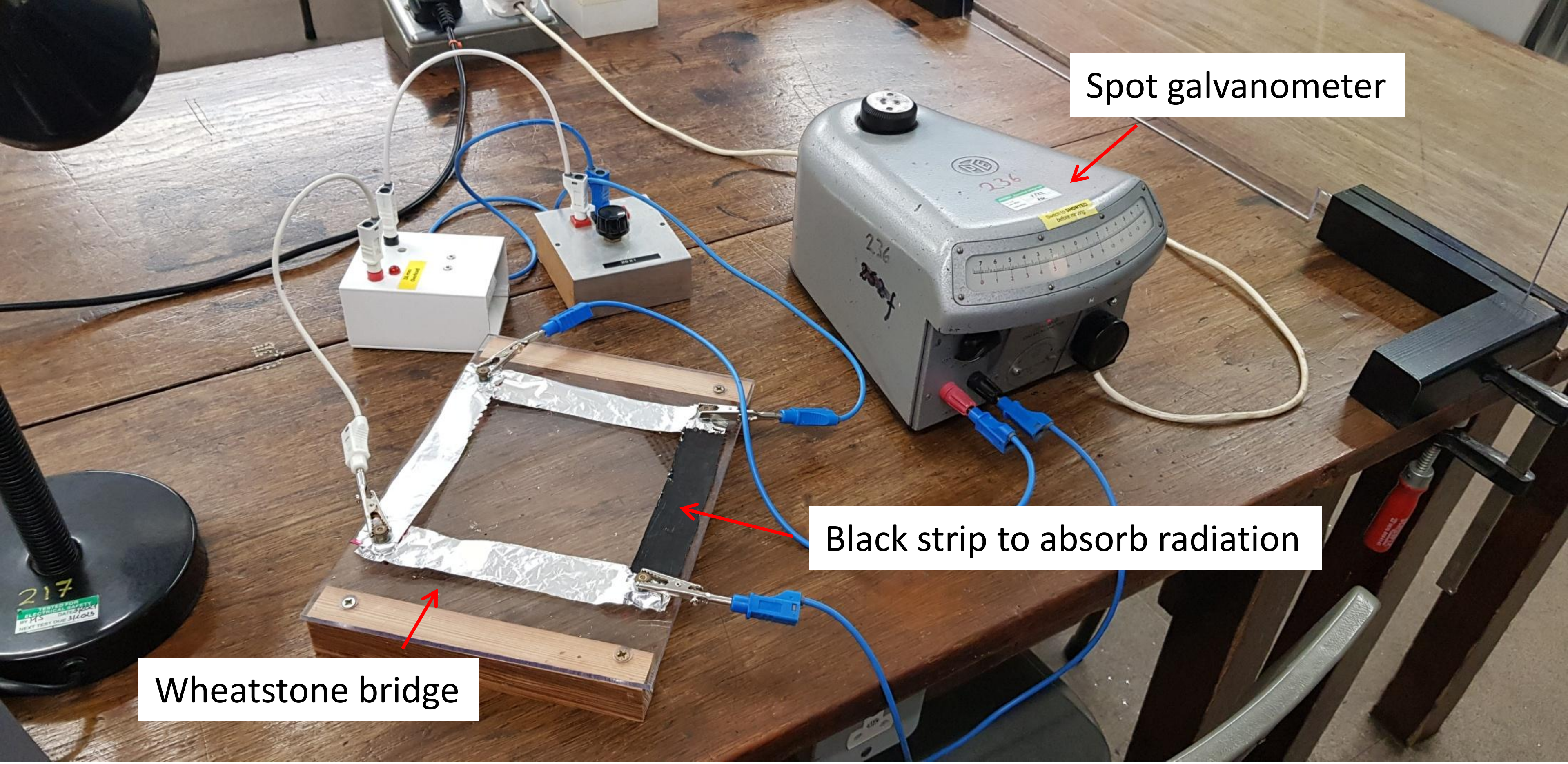
A photo-detector could move around in a circular arc and record the intensity corresponding to each colour.

Perhaps one could make a crude approximation by summing the squares of **R**,**G**,**B** values of the images in the previous slide, and then use the diffraction angle formula to relate to wavelength. But for this to be accurate you will need to mount the camera to a stand so the angles could be measured precisely.



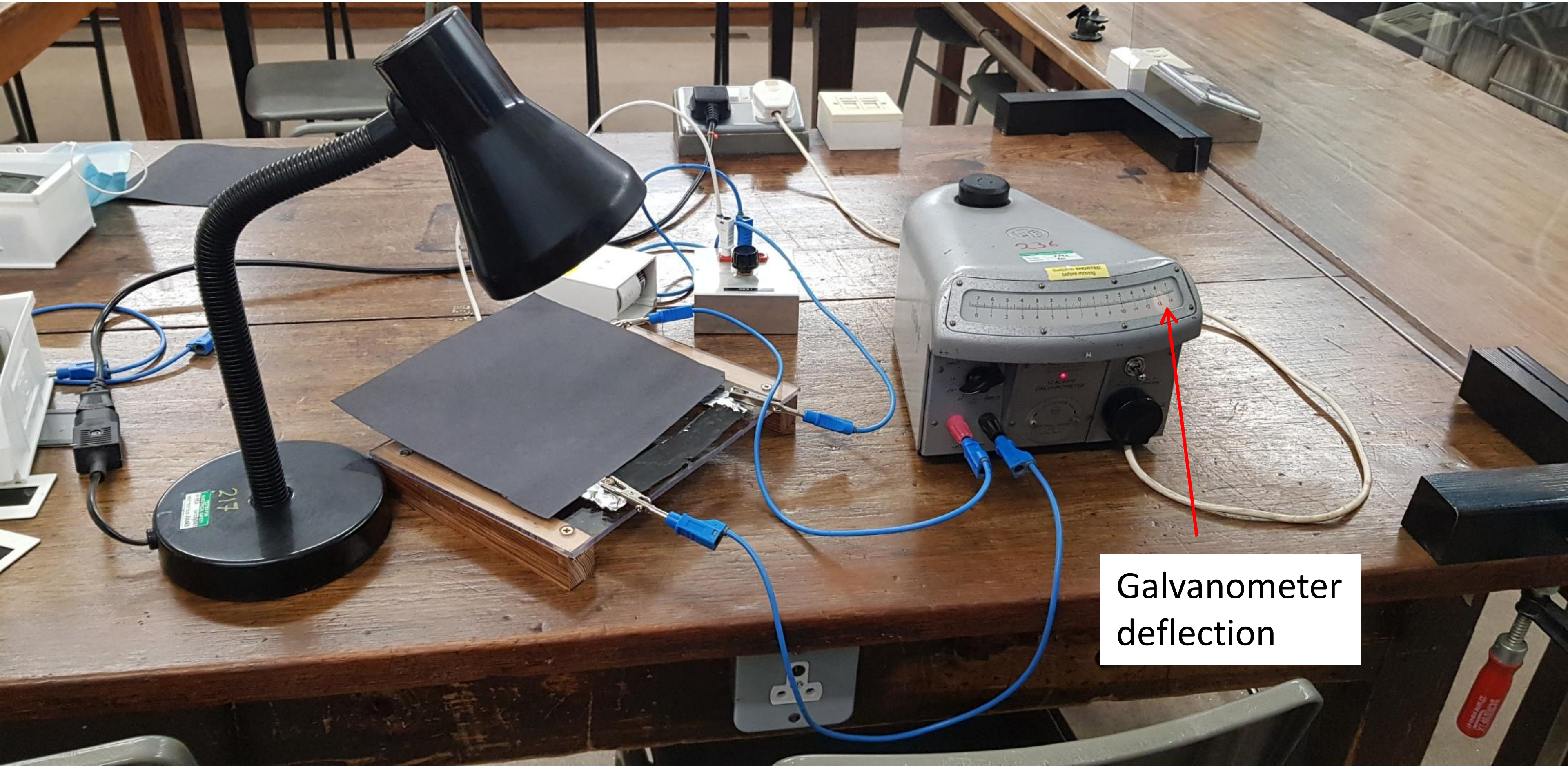
$$\theta_B = \sin^{-1}\left(\frac{m\lambda}{2d}\right)$$

Littrow configuration →



For a DIY photodetector, one can measure the power of electromagnetic radiation using a **Bolometer**. For the arrangement above, the black strip forms part of a Wheatstone Bridge. When it absorbs radiation it will heat up and its resistance will increase (perhaps linearly over a small range). Therefore one can measure a galvanometer voltage reading, resulting from an unbalanced bridge, and this should correlate with the radiation intensity.

https://en.wikipedia.org/wiki/Bolometer



Example bolometric measurement using a white light source.