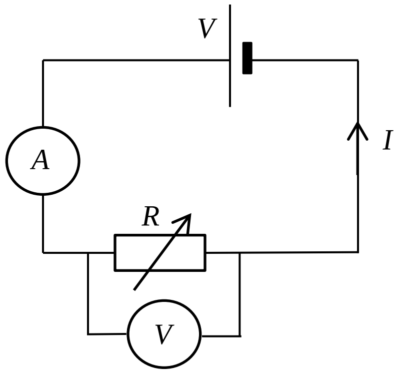
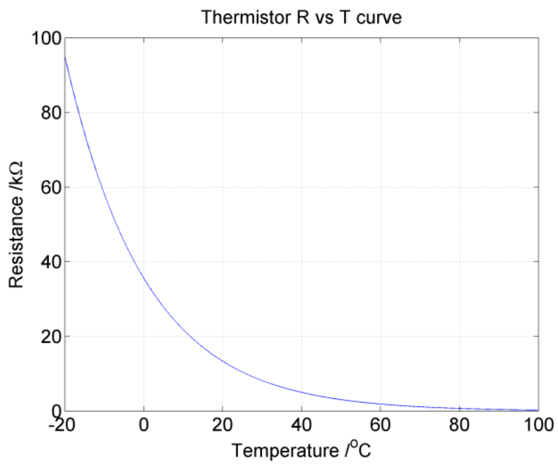
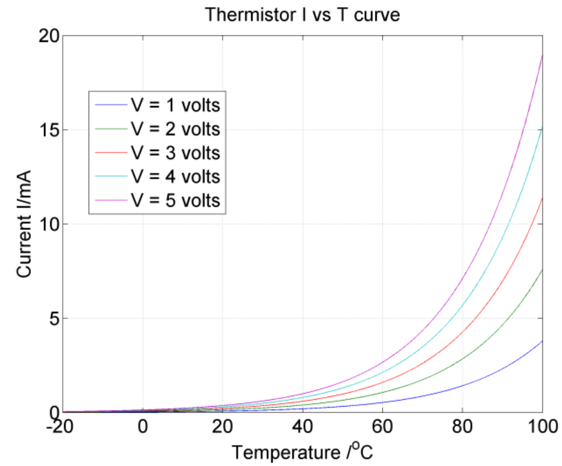
A simple electrical circuit can be used as a **sensor** if the resistance of one of the components in the circuit varies *significantly* with a change in the parameter being measured. For example, the resistance of a *thermistor* will change with temperature, and the consequential change in current drawn through a circuit can be used to measure the temperature, assuming a *calibration process* has occurred to relate the current measured to temperature.



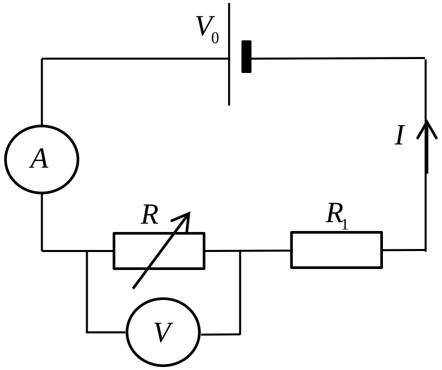




So apply a fixed voltage to the circuit, measure current using a sensitive ammeter and then use the graph to determine the temperature measurement.

Note a typical thermistor is has a *highly non-linear* resistance vs temperature relationship.

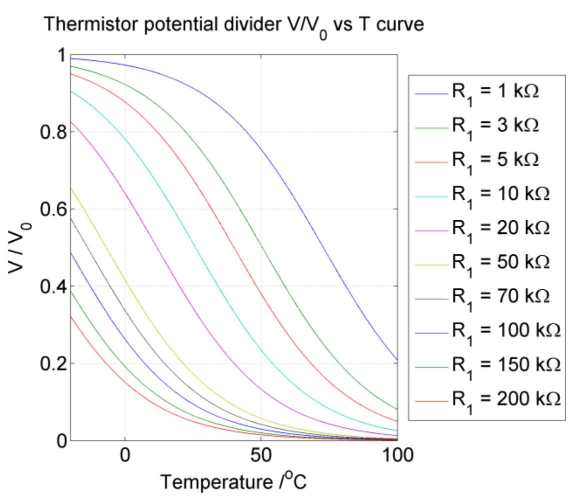
An alternative electrical measurement scheme is to use *voltage* rather than current. This may be preferable if the resistance of the sensor is very low, which means there may be a risk of drawing dangerously large currents through the sensor circuit. To vary voltage as resistance of the sensor changes, we use a **potential divider**.



$V_0 = I(R + R_1)$ Ohm's Law

$V = IR$

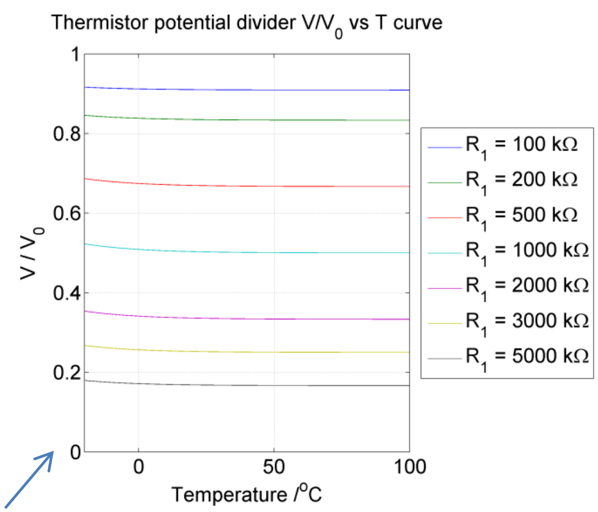
$$\therefore \boxed{\frac{V}{V_0} = \frac{R}{R + R_1}}$$



The sensor can be said to ***sensitive*** if the electrical measurement (voltage in this case) variation is *large* given a variation in the parameter being measured. i.e. in our case the (*T*,*V*) graph varies significantly (i.e. a large % change) over the temperature range -20°C to 100°C.

But what happens if $R = R_0 + \delta R$ where the resistance change $\delta R \ll R_0$?

$$\boxed{\frac{V}{V_0} = \frac{R_0 + \delta R}{R_0 + \delta R + R_1}}$$



In the above example $R_0 = 1000\text{k}\Omega$ i.e. much higher than the variation of the thermistor resistance with temperature (about 100kΩ). Clearly the sensor is *not* vary sensitive in this case as the voltage barely changes over the temperature range.

To correct for this problem we need to make a *differential measurement.* In other words, a voltage response which is **zero** when $R = R_0,\ \delta R = 0$

A **Wheatstone Bridge** circuit can be used to measure *small changes* in the resistance of a variable resistor R_2. This is an effective way of constructing a **sensor system with high sensitivity.** The sensor (e.g. light-depending-resistor, thermistor etc) operates by changing its resistance R_2 via a known curve in response to an environmental variable such as light intensity or temperature. Small changes in R_2 can be transformed into measurable voltages, with zero being the default setting. If the 'default' value of the variable resistor *(R)* is much larger than typical deviations δR, the Wheatstone Bridge gives rise to a voltage proportional to the fractional change $\delta R/R$, rather than the sum $R_2 = R + \delta R$. *This is the justification for the increase in sensitivity.*

Applying *Ohm's law* to the bridge circuit:

$$I_A = \frac{V_0}{R_1 + R_0 + \delta R} \qquad I_C = \frac{V_0}{R_3 + R_4}$$

$$V_A = I_A\left(R_0 + \delta R\right) \qquad V_C = I_C R_4$$

$$V_A = V_0\left(\frac{R_0 + \delta R}{R_1 + R_0 + \delta R}\right) \qquad V_C = V_0\left(\frac{R_4}{R_3 + R_4}\right)$$

$$V_B = V_A - V_C$$

$$\boxed{V_B = V_0\left(\frac{R_0 + \delta R}{R_1 + R_0 + \delta R} - \frac{R_4}{R_3 + R_4}\right)}$$

$$V_B = 0, \quad \delta R = 0$$

To enable a 'zero-offset' when $\delta R = 0$ we must *balance* the bridge.

$$\frac{R_0}{R_1 + R_0} = \frac{R_4}{R_3 + R_4}$$

$$R_0\left(R_3 + R_4\right) = R_4\left(R_1 + R_0\right)$$

$$R_0 R_3 + R R_4 = R_4 R_1 + R_0 R_4$$

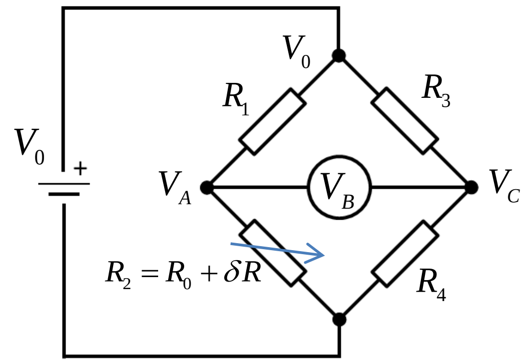
$$\boxed{R_0 = \frac{R_4 R_1}{R_3}}$$

This is called the *Bridge Equation*

Now define *sensitivity* by the gradient of the V_B, δR curve.

$$\frac{\partial V_B}{\partial \delta R} = V_0\left(\frac{\left(R_1 + R_0 + \delta R\right) - \left(R_0 + \delta R\right)}{\left(R_1 + R_0 + \delta R\right)^2}\right) = \frac{V_0 R_1}{\left(R_1 + R_0 + \delta R\right)^2}$$

Let us choose R_1 such that sensitivity is maximized:



$$\frac{\partial}{\partial R_1}\left(\frac{\partial V_B}{\partial \delta R}\right) = V_0 \frac{\left(R_1 + R_0 + \delta R\right)^2 - 2R_1\left(R_1 + R_0 + \delta R\right)}{\left(R_1 + R_0 + \delta R\right)^4}$$

$$R_1 + R_0 + \delta R > 0$$

$$\therefore \frac{\partial}{\partial R_1}\left(\frac{\partial V_B}{\partial \delta R}\right) = 0 \Rightarrow R_1 + R_0 + \delta R = 2R_1$$

$$\therefore R_1 = R_0 + \delta R$$

Since $\delta R \ll R_0$ and δR is a variable quantity, a practical optimum R_1 is $\boxed{R_1 = R_0}$

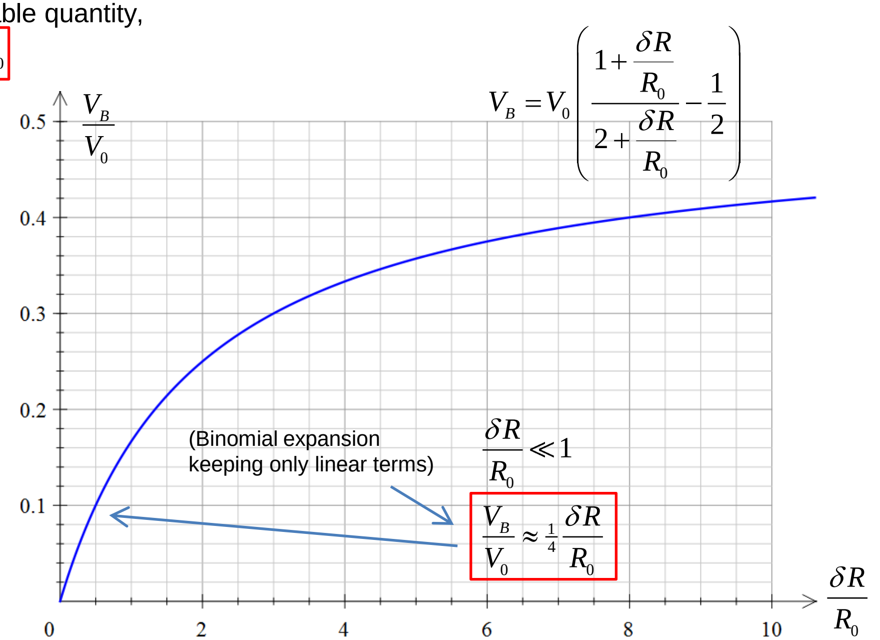
To set $V_B = 0, \quad \delta R = 0$

$$R_0 = \frac{R_4 R_1}{R_3}$$

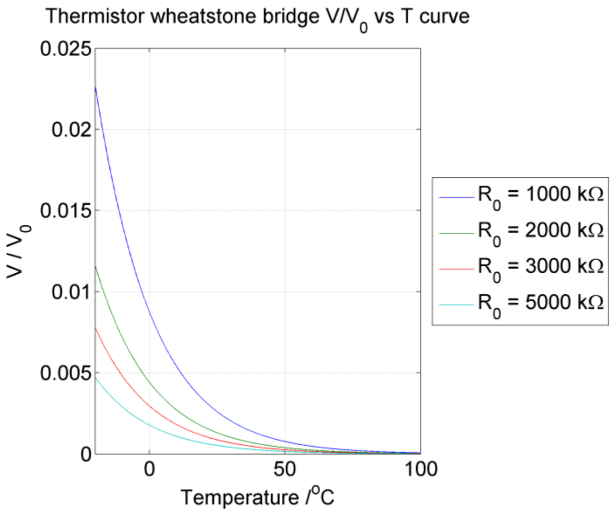
$$R_1 = R_0 \quad \therefore R_4 = R_3$$

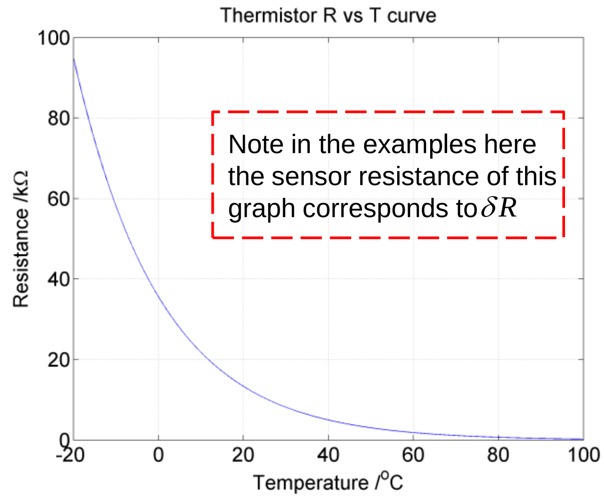
$$V_B = V_0\left(\frac{R_0 + \delta R}{2R_0 + \delta R} - \frac{R_4}{R_4 + R_4}\right)$$

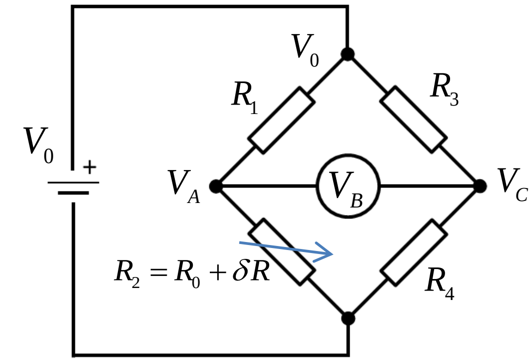
$$\boxed{V_B = V_0\left(\frac{1 + \frac{\delta R}{R_0}}{2 + \frac{\delta R}{R_0}} - \frac{1}{2}\right)}$$



The ***Wheatstone Bridge*** can significantly improve the sensitivity of our thermistor sensor *in series with a large fixed resistor* (which could be the internal resistance of the sensor system). This is because the voltage now changes a small amount from zero, rather than a small deviation from a fixed quantity.







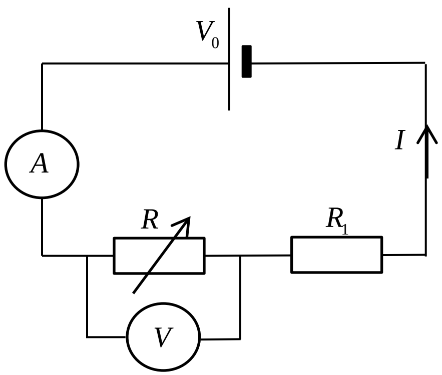
The Wheatstone Bridge circuit

$$V_B = V_0\left(\frac{R_0 + \delta R}{R_1 + R_0 + \delta R} - \frac{R_4}{R_3 + R_4}\right)$$

$$R_1 = R_3 = R_4 = R_0$$

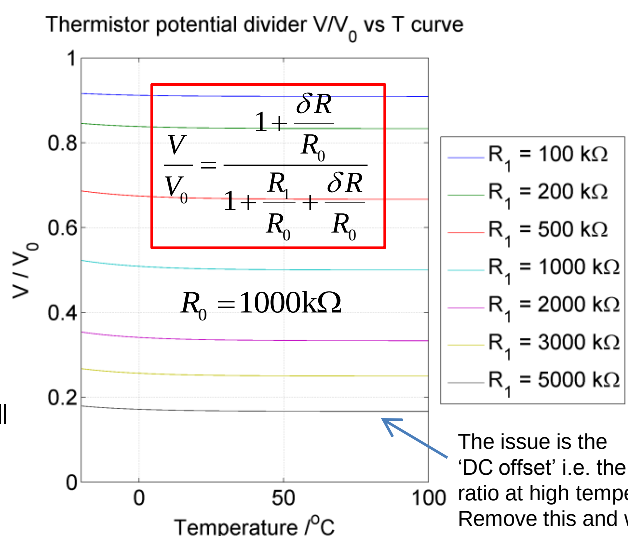
$$\boxed{\therefore V_B = V_0\left(\frac{1 + \frac{\delta R}{R_0}}{2 + \frac{\delta R}{R_0}} - \frac{1}{2}\right)}$$

The downside is the actual voltage ratio may be too small result in a measurable Wheatstone Bridge voltage. The solution here is to *increase* the applied voltage to the system.



If the sensor resistor variation is small compared to the mean, the potential divider circuit is not very sensitive.

$R = R_0 + \delta R \qquad \delta R \ll R_0$



The issue is the 'DC offset' i.e. the voltage ratio at high temperatures. Remove this and we have the Bridge circuit result.