PRE-U REVISION NOTES MECHANICS AF/3118

COSMOLOGY: DISTANCE MEASUREMENT, RADIAL VELOCITY MEASUREMENT USING DOPPLER, COSMOLOGICAL MODELS, HUBBLE LAW

THE COSMOS "is all that is or ever was or ever will be" (Carl Sagan 1934–1996 Cosmos pp20)

↳ The universe on the largest of scales. ie

Planets → Stars → Galaxies → Clusters → Universe

Key questions for Science:

1. How big is the universe? How many stars?
2. How far away are the stars?
3. How fast are the stars moving?
4. How old is the universe? Did it have a 'beginning'? Will it have an 'end'?
5. What are the physical properties of the cosmos? ie average density, temperature, matter/radiation composition.

And importantly! How can we measure these things? → (2)–(5)

ASTRONOMICAL DISTANCES (and times and speeds)

"planetary" units →

$\boxed{1AU = 1.496 \times 10^{11} m}$ Average Earth–Sun separation.

$\boxed{1Yr} = 365 \times 24 \times 3600 s \approx \pi \times 10^7 s$ orbital period of Earth–Sun system

$v_\oplus = \dfrac{2\pi \times 1AU}{1Yr} = \boxed{29.8 \text{ km/s}}$ Earth orbital speed about the Sun

Distance to nearest star beyond solar system (Proxima centauri) $\approx$ $\boxed{4 \text{ light years}}$

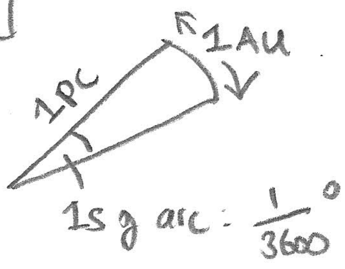
$1 Ly = 2.998 \times 10^8 m/s \times 365 \times 24 \times 3600 s = \boxed{9.461 \times 10^{15} m}$

Distance of Sun to centre of galaxy $\approx$ $\boxed{25{,}000 \text{ Ly}}$

Distance to nearest galaxy $\approx$ $\boxed{2 \times 10^6 \text{ Ly}}$

" " distant quasars $\approx$ $\boxed{10 \times 10^9 \text{ ly}}$

Most measurements from telescopes involve angles, so a $\boxed{\text{Parsec}}$ is useful:
pc

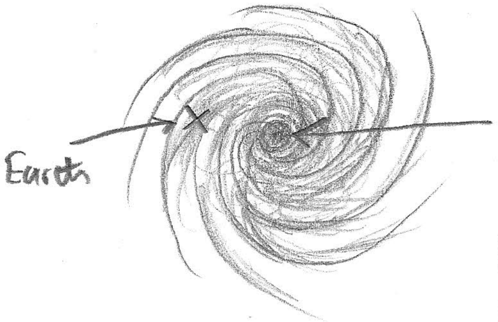


$\therefore 1 pc = \dfrac{1.496 \times 10^{11} m}{\pi/180 \times 3600} = \boxed{3.086 \times 10^{16} m} = \boxed{3.26 \text{ ly}}$

Note a 'Mega-parsec' (MPc) is a common large scale distance

0.75 Mpc ≃ distance to nearest galaxy. $\boxed{1\,\text{Mpc} = 10^6\,\text{pc} = 3.086\times10^{22}\,\text{m}}$

HOW MANY STARS ARE THERE?



Most galaxies are thought to contain a **Supermassive Black Hole** at the centre. Sagittarius A* is ours.

Our **galaxy** ("The Milky Way")

Contains ≃ 200-400 billion stars

Mass ≈ $0.8 - 1.5 \times 10^{12}$ solar masses

Diameter 100,000 – 120,000 ly

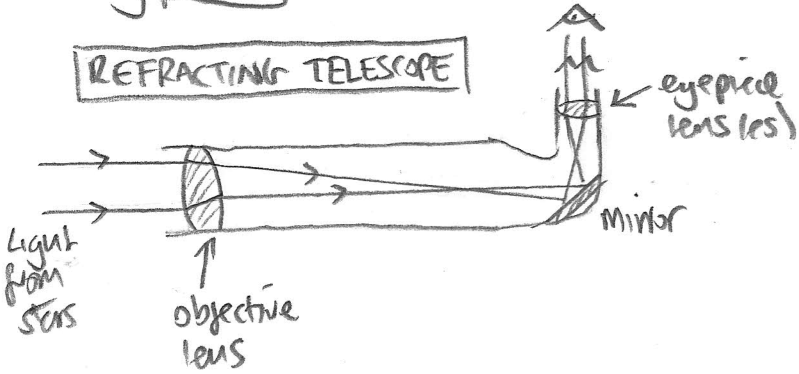
Rotation period ≃ 225 million years "Galactic year"

* THERE ARE THOUGHT TO BE BILLIONS of GALAXIES IN THE COSMOS! *

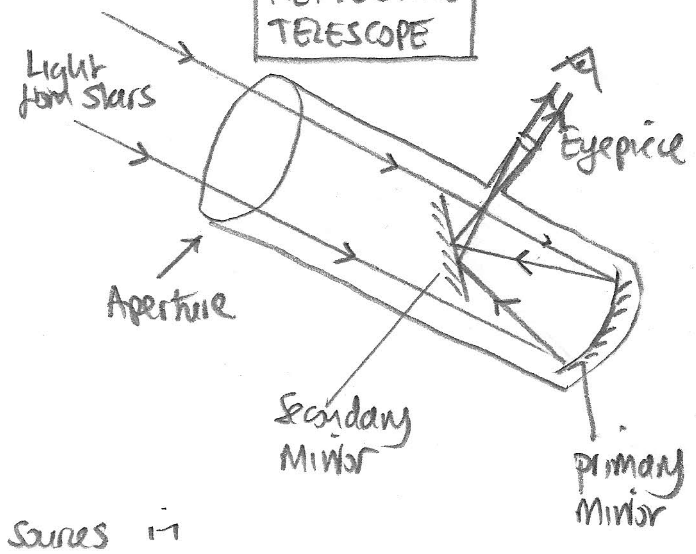
HOW DO WE SEE INDIVIDUAL STARS & GALAXIES?

Since Galileo and Newton, **telescopes** allow light from faint stars to be focussed into an eyepiece. Early telescopes were refracting i.e. using **lenses** whereas most telescopes are based upon mirrors, ie **reflection**

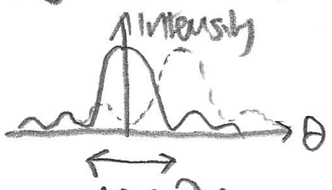
REFRACTING TELESCOPE



REFLECTING TELESCOPE

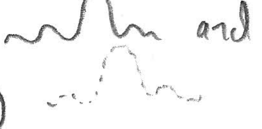


Any optical system is **diffraction limited** by the size of the aperture, d



$\Delta\theta \approx \lambda/d$

Diffraction patterns of Two sources



To resolve two point sources in an angular sense we must obey the **Rayleigh Criterion**

$$\boxed{\Delta\theta \simeq \lambda/d}$$

Angular resolution in radians

For the "Canaries Great Telescope" $d \simeq 10.4\,\text{m}$. So for visible light $\lambda \simeq 500\,\text{nm} \simeq 5\times10^{2}\times10^{-9} = \boxed{5\times10^{-7}\,\text{m}}$

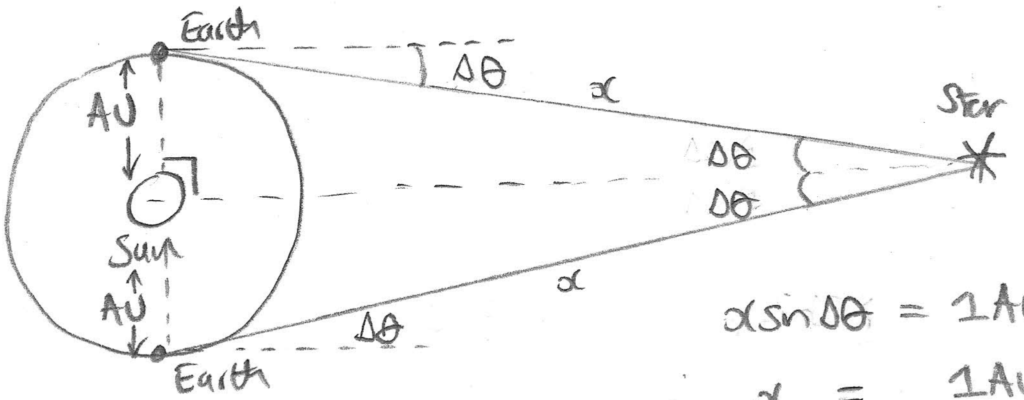
$\therefore \Delta\theta \simeq 4.8\times10^{-8}$ radians $\simeq \boxed{9.9\times10^{-3}\text{ arc seconds}}$

1 rad = $180\times3600/\pi$ arc seconds

MEASURING THE DISTANCES OF THE STARS USING PARALLAX

The Earth orbits the Sun every year. This means the position of stars will $\boxed{\text{vary in an angular sense}}$ throughout the year. Since the stars are > light years away, the angular deviations due to the motion of the stars themselves (say relative to our Sun) are deemed negligible over a year. If the Earth-Sun distance (1 AU) is known, the angular shift can be used to determine distance.

[Note parallax is sometimes $\Delta\theta/2$, sometimes $\Delta\theta$ — watch out! Define in a diagram...]



$$x \sin \Delta\theta = 1\text{AU}$$

$$\therefore \quad x = \frac{1\text{AU}}{\sin \Delta\theta}$$

ie observe star ✱ six months apart and record total angular deviation $2\Delta\theta$.

if $\Delta\theta \ll 1$ radian $\quad \sin\Delta\theta \simeq \Delta\theta$

so $\boxed{x \approx \text{AU}/\Delta\theta}$

Note $x = 1$ PARSEC when $\Delta\theta = 1$ arc second

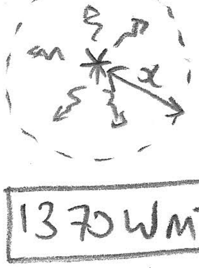
ie $\Delta\theta = \dfrac{\pi}{180 \times 3600}$ radians

$\boxed{1 \text{ Parsec} = 3.086 \times 10^{16}\text{m}}$

LUMINOSITY, CEPHEID VARIABLES AND ANOTHER MEANS OF COMPUTING DISTANCE

The $\boxed{\text{Luminosity}}$ of a star is its total radiated power L. For most stars, this is uniform in all directions. Hence the power received / unit area ("flux") is $\boxed{\Phi = \dfrac{L}{4\pi x^2}}$

ie $4\pi x^2$ is the area of a sphere of radius x surrounding the star.



For our Sun $L_\odot = 3.846 \times 10^{26}$ W, so at 1 AU, $\Phi_\oplus = \boxed{1370 \text{ Wm}^{-2}}$

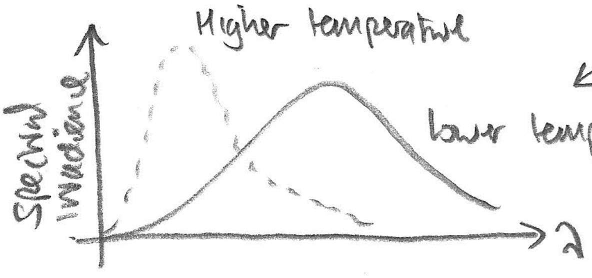
Earth

Now if we measure Φ for a star, and we know L, we can hence determine x from $\boxed{x = \sqrt{\dfrac{L}{4\pi\Phi}}}$

✱ L can be inferred from the radius r and temperature T of a star using Stefan's law $\boxed{L = 4\pi r^2 \sigma T^4}$

Stefan - Boltzmann constant $\boxed{\sigma = 5.67\times10^{-8}\,Wm^{-2}K^{-4}}$ $\left[\sigma = \frac{2\pi^5 k_B^4}{15c^2h^3}\right]$

We can determine the **temperature** of a star from the **shape** of it's **spectral irradiance**, ie the flux within a range of wavelength.



The curve is the **planck law**

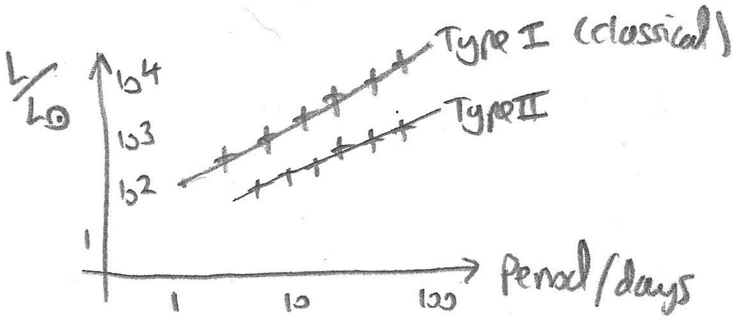
$$\boxed{B(\lambda,T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda k_B T}-1}}$$

$k_B = 1.381\times10^{-23}\,m^2\,kg\,s^{-2}\,K^{-1}$

$h = 6.626\times10^{-34}\,m^2\,kg\,s^{-1}$

$c = 2.998\times10^{8}\,ms^{-1}$

Even more accurate measurements of luminosity can be found from **cepheid variable** stars, who have a known luminosity vs period variation*. The period is associated with a sinusoidal variation of luminosity, which can occur on timescales of days → months. Cepheid variables are known as **"Standard candles"**



* Due to a pulsation mechanism.

[Note log scales!]
↑
ie power law correlation.

Note the change in a stars luminosity can also indicate the transit of an **extra solar planet** - ie a partial eclipse!

The depth of the change in L can indicate the speed and radius of the planet.
(and width in time)

DOPPLER SHIFT AND ITS USES

For radial motion at $v \ll c$, the signals from a moving source will appear to be frequency shifted in proportion to the ratio of the source to the wave speed. For starlight

Wavelength shift → $\boxed{\frac{\Delta\lambda}{\lambda} \cong \frac{v}{c}}$ ← Emmitted wavelength

$\Delta\lambda >$ +ve if the star ✶ recedes [REDSHIFT]. BLUESHIFT if $\Delta\lambda < 0$ ie star is approaching.

How do we measure Doppler? The light from a star is mostly generated by quantum transitions in the energy levels of electrons in the $\boxed{\text{Hydrogen atom}}$. These occur at discrete frequencies. At a first approximation, we can use the $\boxed{\text{Bohr model}}$ to work out what these are

Balmer formula

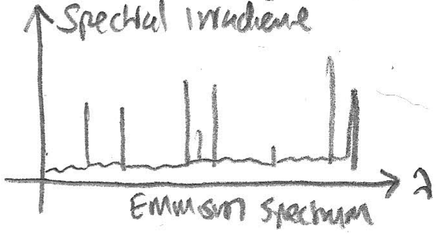
$$\boxed{\lambda_{nm} = \frac{8\varepsilon_0^2 h^3 c}{m_e e^4}\left(\frac{1}{m^2} - \frac{1}{n^2}\right)^{-1}}$$

91.13 nm

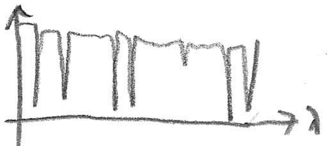
$n > m$

n, m are integers 1, 2, 3 . . . .

If we look at the $\boxed{\text{Emission spectrum}}$ of a star we will find peaks corresponding to wavelength λ_{nm}



[The alternative is to look at an $\boxed{\text{absorption}}$ spectrum, where a body of hydrogen gas absorbs light]



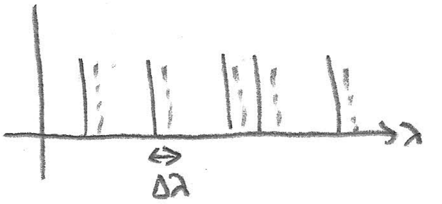
Absorption Spectrum

The idea is to compare these 'spectral bar codes' to the spectra of hydrogen (or other elements such as Oxygen) in the lab on earth. The shift of the spectral lines $\Delta\lambda \Rightarrow v$ via the Doppler formula.

H spectra (star)

- - - -

H spectra (lab)

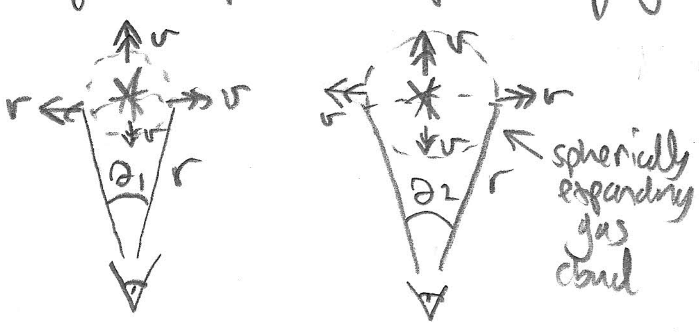


In this case the star is $\boxed{\text{redshifted}}$, ie the star is receding.

Note $\frac{\Delta\lambda}{\lambda} = 3.3 \times 10^{-3}$ (0.33%) for

$\boxed{v = 1000 \text{ km/s}}$ $\left(\frac{1000 \times 10^3 \text{ m/s}}{2.998 \times 10^8 \text{ m/s}} = 3.3 \times 10^{-3}\right)$

$\boxed{\text{Doppler}}$ can also be used to find the $\boxed{\text{distance}}$ of $\boxed{\text{Supernovae}}$ if the spherical expansion of gas clouds can be tracked.



$t = 0$ $t = \Delta t$

Now $2v\Delta t = r(\theta_2 - \theta_1) = 2r\Delta\theta$

So $\boxed{r = v\Delta t / \Delta\theta}$

Find v from $\frac{\Delta\lambda}{\lambda} = v/c$

Eg. Δt could be a few months

$\Delta\theta$ is (half) the angular change in spherical gas cloud

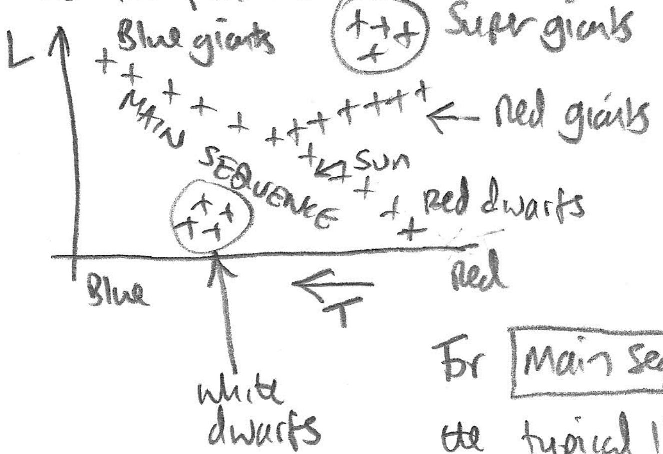
$[2\Delta\theta = \theta_2 - \theta_1]$

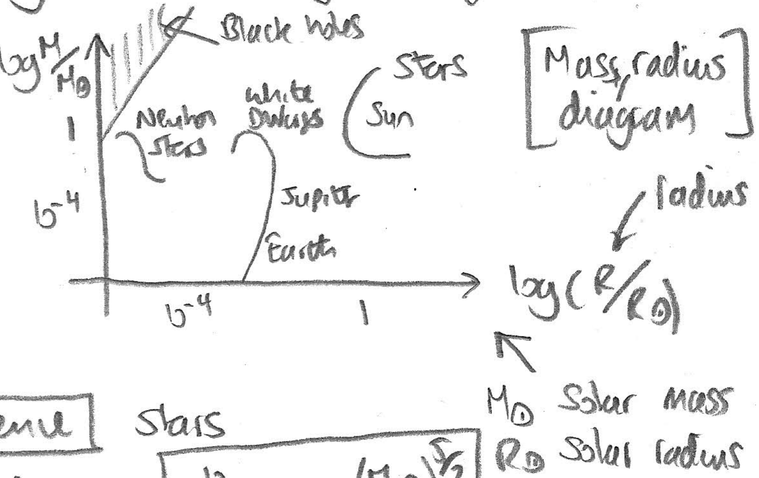
STARS AND THEIR EVOLUTION

Stars are typically spherical accretions of mostly hydrogen, bound via gravity and supported from collapse by the radiation pressure due to EM rays emitted from fusion reactions (eg hydrogen isotopes → helium).

Depending on the mass of stars, they will 'burn' at different rates and expand / contract in radius. Once the fusion fuel is used up, stars will shed matter and form dense objects such as NEUTRON STARS, and in EXTREME CASES, BLACK HOLES.

Stars form a pattern (Hertzsprung - Russell diagram) of luminosity vs temperature





For Main Sequence stars the typical lifetime is $\simeq \boxed{10^{10} \text{ years} \times \left(\frac{M_\odot}{M}\right)^{5/2}}$

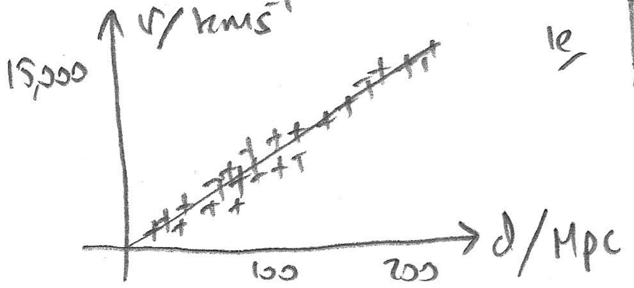
ie the Sun should last about 10^{10} years, less massive stars will last longer (eg red dwarfs, which are cooler and less luminous) and more massive stars (eg blue giants, which are hotter and more luminous) will last less long.

RED DWARFS	L↓	t↑	M↓
BLUE GIANTS	L↑	t↓	M↑

If a star is massive enough, gravity will cause it to collapse such that a SUPERNOVA is formed. This is an extreme explosion which can outshine a galaxy! The energy is such that heavier elements can be created via nuclear fusion. This process NUCLEOSYNTHESIS is how all our elements of life (eg carbon, Nitrogen etc) were formed. We are stardust, supernova residue....

HUBBLE LAW & THE BIG BANG

1889–1953
Edwin Hubble plotted the recessional velocities for many galaxies – most were redshifted – against distance. (Calculated by parallax or luminosity methods). The result was a linear trend...



ie $\boxed{v = H_0 d}$ $\boxed{H_0 \approx 71.9 \text{ km/s/Mpc}}$

"Hubble constant"

This is the modern value.

Since v/d has units of $1/\text{time}$ $\Rightarrow$ $\boxed{1/H_0}$ is a measure of the **AGE OF THE UNIVERSE**, and implies that the **UNIVERSE IS EXPANDING**. George Lemaître called this 'moment of creation' the **BIG BANG**

$$\frac{1}{H_0} = \left(\frac{71.9 \times 10^3 \text{ M/s}}{3.086 \times 10^{22} \text{ m}} \right)^{-1} \approx \boxed{13.6 \text{ billion years}}$$

Modern analysis, using various COSMOLOGICAL MODELS, and observations of the COSMIC MICROWAVE BACKGROUND (CMB) radiation (red shifted remnant of the Big Bang, which pervades all of space) yield the

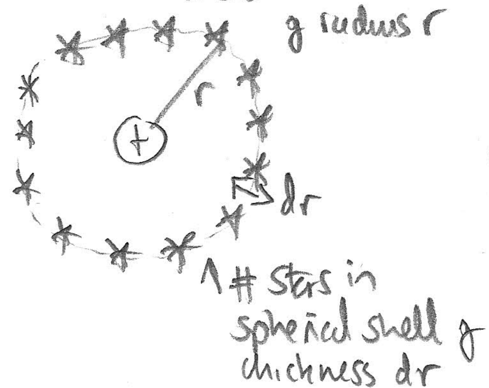
age of the universe $\approx$ 13.8 billion years

The BIG BANG THEORY resolves **Olbers' Paradox** (Heinrich Olbers 1758–1840)

If stars had mean luminosity L, and n stars / unit volume, power per unit area received on Earth should be

$$\boxed{P = \int_{r=0}^{\infty} \underbrace{n \times 4\pi r^2 dr}_{\#\text{ stars in shell of radius } r} \times \frac{L}{4\pi r^2} = nL \int_0^{\infty} dr = \infty}\ !$$

power / m^2



The sky is ~~not~~ not infinitely bright, so the universe must be expanding – light from stars further away take takes longer than d/c to reach us since d increases with time.

↑ distance

COSMOLOGICAL MODELS

There are several models of the expansion of the universe, and simple ones were proposed by $\boxed{\text{Alexander Friedmann}}$ (1888–1925).

See Eclechicon note for 'full' derivation ('Quasi-classically')

"curvature" $-1<k<1$?

$$H^2 = \frac{8}{3}\pi G\rho - \frac{kc^2}{R^2} + \frac{1}{3}\Lambda c^2 \quad (1)$$

$$\dot{H} + H^2 = -\frac{4}{3}\pi G\left(\rho + \frac{3P}{c^2}\right) + \frac{1}{3}\Lambda c^2 \quad (2)$$

Λ: cosmological constant

$$\dot{\rho} = -3\left(\rho + \frac{P}{c^2}\right)H \quad (3)$$

rate of change of density

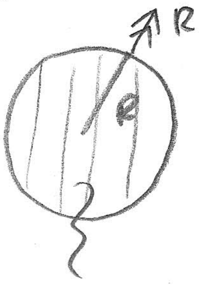
$$\rho = \frac{\rho_0}{(R/R_0)^{3+\varepsilon}}$$

$\varepsilon = 0$ matter dominated
$\varepsilon = 1$ radiation dominated
P is radiation pressure

$H = \dot{R}/R$

(At current epoch (i.e. now))

$H_0 = \dot{R}/R$



Expanding sphere of universe, of density ρ

If radiation dominated $\boxed{R(t) \propto \sqrt{t}}$ with 'characteristic age' $\boxed{t_0 = \frac{1}{2}\frac{1}{H_0}}$

If matter dominated $\boxed{R(t) \propto t^{2/3}}$

with 'characteristic age' $\boxed{t_0 = \frac{2}{3}\frac{1}{H_0}}$

Since the age of the universe $> \frac{1}{H_0}$, this suggest a phase of INFLATION (exponential expansion) in the early universe.

CURRENTLY THERE IS A PROBLEM WITH MASS IN THE UNIVERSE

Using (1) and letting $H = H_0$, $k=0$, $\Lambda = 0$ $\Rightarrow$ $\boxed{\rho = \frac{3H_0^2}{8\pi G}}$

which using $\boxed{H_0 = 71.9 \text{ km/s/Mpc}}$, $\boxed{G = 6.67\times10^{-11}\ \text{m}^3\text{kg}^{-1}\text{s}^{-2}}$

$\Rightarrow$ $\boxed{\rho \approx 5.8 \text{ protons/m}^3}$ $[m_p \approx 1.67\times10^{-27}\text{ kg}]$

Now we observe $\sim$ $\boxed{0.4 \text{ protons/m}^3}$ ..... Current analysis predicts the universe is 4.9% "normal matter" (+ radiation pressure P/c^2 contribution ?) and 25.9% $\boxed{\text{DARK MATTER}}$ and 69.1% $\boxed{\text{DARK ENERGY}}$. At the time of writing the nature of these 'Dark' terms is still a mystery!