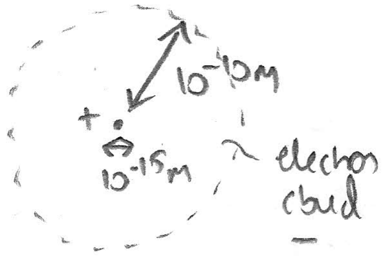
PRE-U REVISION NOTES. QUANTUM & ATOMIC PHYSICS AF. 3/19

QUANTUM MECHANICS I: BLACK BODY RADIATION & PLANCK LAW PHOTOELECTRIC EFFECT, ELECTRON DIFFRACTION, BOHR ATOM & H SPECTRA

- Democritus (460BC – 370BC) proposes matter consists of 'uncuttable' atomos components ATOMS
- This does not become scientific orthodoxy till the 20^{th} century AD!
- RUTHERFORD experiment showed atoms comprise of a tiny +ve charged nucleus ($\sim 10^{-15}$ m) surrounded by a 'cloud' of −ve charged electrons. Most of the mass is in the nucleus.



So if nucleus is a meter rule the electrons extend from Winchester to London! (100 km = 10^5 m)

Note # of atoms in a cube 'marble sized' of width 3.6 cm

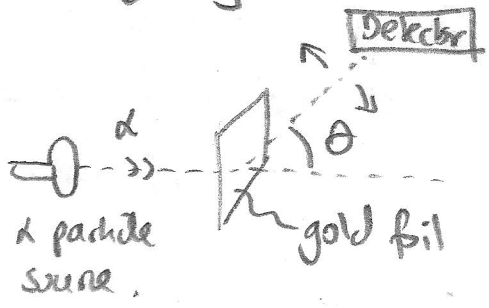
$$\simeq \left(\frac{3.6\times 10^{-2}}{10^{-10}}\right)^3 \simeq \boxed{5\times 10^{25}}$$

This is similar to the # marbles that would form an Earth!

$$\left(\frac{1.28\times 10^{7}\,\text{m}}{3.6\times 10^{-2}}\right)^3 \simeq \boxed{4.4\times 10^{25}}$$

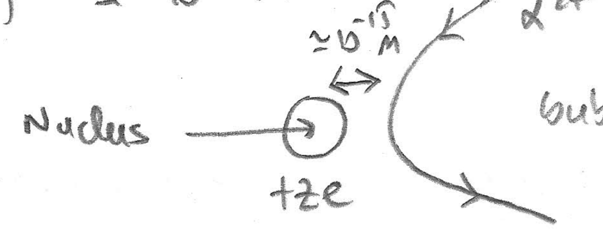
[Earth diameter is 1.28×10^7 m]

.... So ATOMS ARE <u>VERY</u> SMALL!



$\boxed{\text{Rutherford exp.}}$ determined the statistics* of α particles (He nuclei from radioactive decay) scattered (by θ) by a sheet of very thin gold foil. The scattering law is consistent with the nucleus behaving $\boxed{\text{like a point particle}}$ of charge Ze ($Z = 79$ for gold), with closest approach (without richochet) $\simeq 10^{-15}$ m

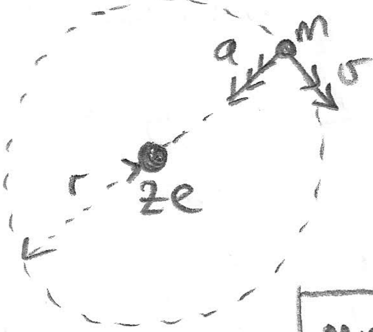
[Detector moves in a circular arc]



Coulomb repulsion of α particle

* See Nuclear notes.

So electrons orbit a nucleus? Unfortunately this is a problem for classical physics, since to travel in a circle one must accelerate, and accelerating charges RADIATE



Consider an electron orbiting a nucleus of charge Ze. The electron has mass M and charge e.

Coulomb's law + Newton II:

$$\boxed{\frac{Mv^2}{r} = Ma = \frac{Ze^2}{4\pi\varepsilon_0 r^2}} \qquad \therefore\ a = \frac{Ze^2}{4\pi\varepsilon_0 M r^2}$$

acceleration radially inwards

Now power radiated by an accelerating charge (e) is $\boxed{\dot{E} = \frac{-e^2 a^2}{6\pi\varepsilon_0 c^3}}$

From Newton II: $KE = \boxed{\frac{1}{2}Mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r}}$

So $\boxed{\text{lifetime}}$ of electron is $\tau \simeq \frac{\frac{1}{2}Mv^2}{|\dot{E}|} \simeq \boxed{\frac{12\pi^2\varepsilon_0^2 c^3 M^2 r^3}{Ze^4}}$

[Approximation since $r \downarrow$ as electron loses KE and spirals into nucleus]

using $r \sim 10^{-10}\,m \Rightarrow \boxed{\tau \simeq 5 \times 10^{-11}\,s}$

So atoms should not exist!

To 'solve' this problem requires a DIFFERENT concept of what an electron is, in the vicinity of an atomic nucleus in particular. The major leap is to consider electrons to have $\boxed{\text{wave-like characteristics}}$

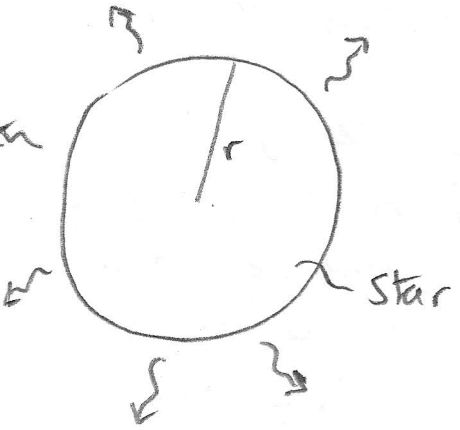
$c = 2.998 \times 10^{8}\ m/s$

$M = 9.109 \times 10^{-31}\ kg$ $\quad e = 1.602 \times 10^{-19}\ C$ $\quad \varepsilon_0 = 8.85 \times 10^{-12}\ Fm^{-1}$

Before we can 'solve' this problem – we need to understand three perplexing experiments that defied classical physics in the 19th century:

① BLACK BODY RADIATION
② PHOTO ELECTRIC EFFECT
③ ELECTRON DIFFRACTION

BLACK BODY RADIATION



A body of absolute temperature T (kelvin) will radiate with power I watts/m² of surface area according to Stefan's law

$$\boxed{I = \varepsilon \sigma T^4}$$

I: power flux (Wm^{-2})

ε: Emmisivity

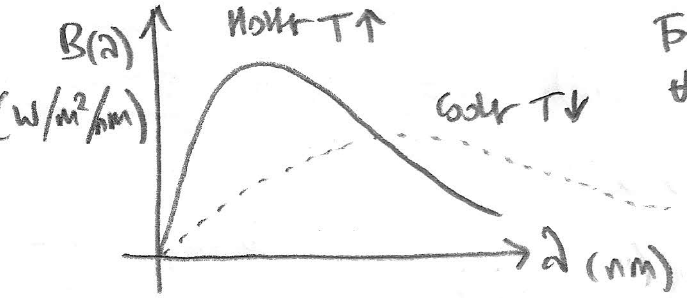
σ: $\boxed{5.67 \times 10^{-8}\ Wm^{-2}K^{-1}}$ Stefan Boltzmann constant.

$\varepsilon = 1$ if the object produces all it's radiation (like the sun) or re-radiates ≈ 100% of absorbed radiation (like a black painted sphere)

So Luminosity L of a star is (J/s)

$$\boxed{L = 4\pi r^2 \sigma T^4}$$

Now if you record the power flux as a function of radiation wavelength λ you get the following curve:

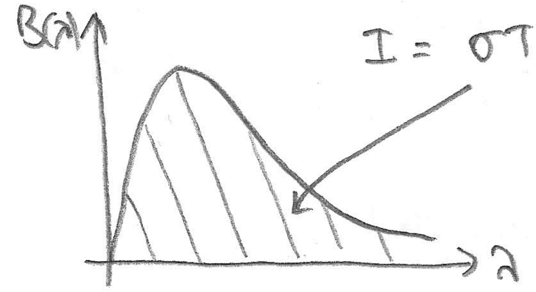


For temperatures ≃ few 1000 K the peak is in the visible range (400 – 700 nm). This is why really hot metals glow.

Problem is, to explain the shape of this curve involved treating radiation as composed of DISCRETE 'QUANTA' of energy $\boxed{E = hc/\lambda \text{ or } hf}$ (Planck's law)

$h = 6.63 \times 10^{-34}\ m^2\, kg\, s^{-1}$ Planck's constant.

[RED: 620–750 nm YELLOW: 570–590 nm BLUE: 450–495 nm]



$I = \sigma T^4 = \int_0^\infty B(\lambda)\,d\lambda$

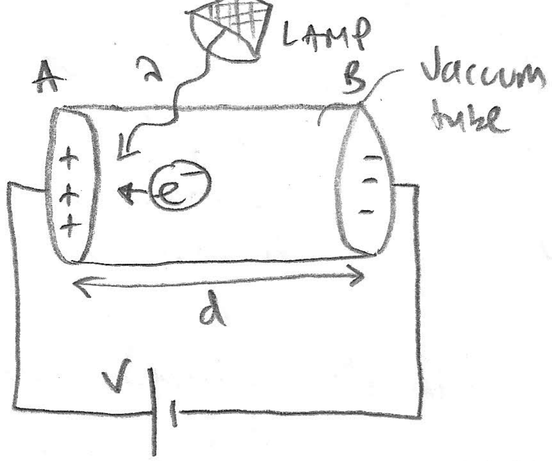
$\boxed{\text{PLANCK RADIATION SPECTRA}}$

$$\boxed{B(\lambda) = \frac{2hc^2}{\lambda^5}\,\frac{1}{e^{hc/\lambda k_B T}-1}}$$

$$\boxed{\sigma = \frac{2\pi^5 k_B^4}{15 c^2 h^3} \simeq 5.67\times10^{-8}\,W m^{-2} K^{-1}}$$

$k_B = 1.38\times10^{-23}\,m^2\,kg\,s^{-2}\,K^{-1}$
Boltzmann's constant.

we can make use of 'photon' energy $\boxed{E = hf}$ to explain the $\boxed{\text{photoelectric effect}}$



* Shine light of intensity I and wavelength λ on a metal plate in a vaccum tube. * Electrons are ejected from the metal <u>ONLY</u> when λ < some critical wavelength, regardless of intensity I.

* once e^- are emitted, # of electrons produced scales with I
* We can apply a $\boxed{\text{reverse bias}}$ V to "just" reduce the photoelectron current from A to B to zero.

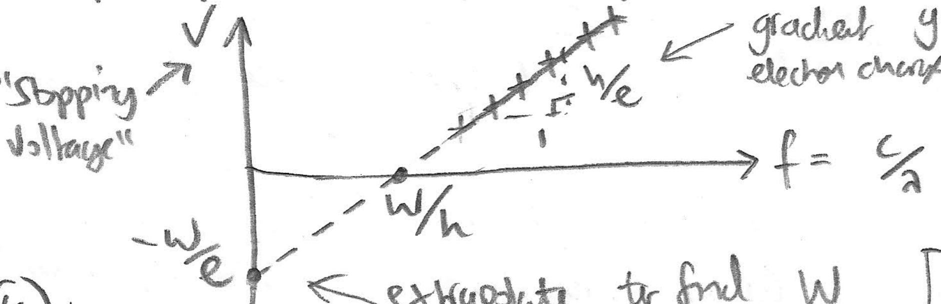
$\boxed{\text{Explanation:}}$ electron leaves A with KE $\boxed{E = hf - W}$

photon energy (hf), Work to extract e^- from metal (W)

So when $\boxed{eV = E}$, photocurrent → 0

Can use this (Millikan 1868–1953) to find h if repeat exp. for different λ.

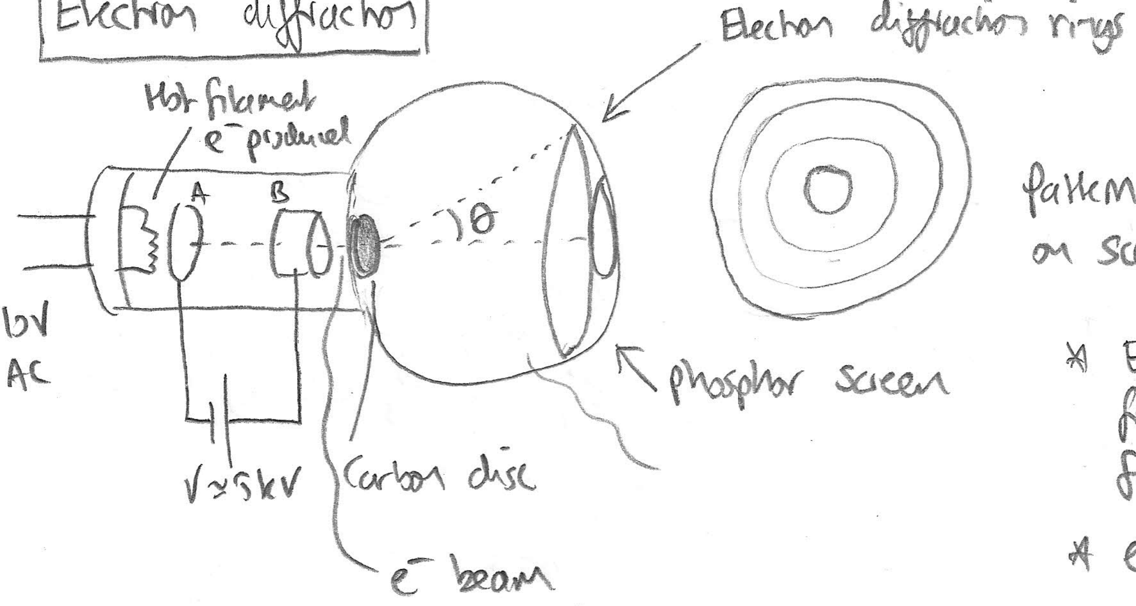
gradient yields h if know electron charge e



$$\boxed{V = \frac{hf}{e} - \frac{W}{e}}$$

← extrapolate to find W [Copper: $W \simeq 4.7\,eV$]

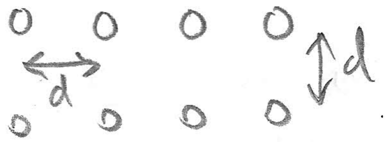
Electron diffraction



pattern of rings observed on screen.

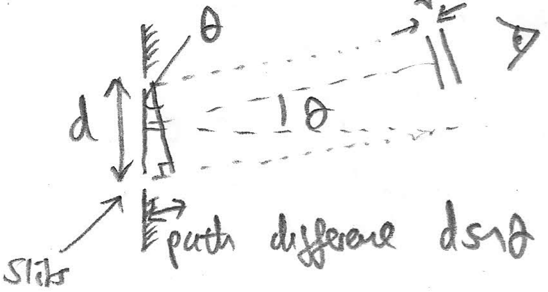
* Electrons produced from AC applied to filament
* e^- accelerated A → B

* e^- beam produced, which collides with a carbon disc with atomic spacing d



plane waves at θ
↓ $\boxed{r \gg d^2/\lambda}$

In $\boxed{\text{far field}}$ of an aperture with slits of spacing d



constructive interference when

$$\frac{2\pi}{\lambda} \times d\sin\theta = 2\pi m \leftarrow \text{integer}$$

ie $\boxed{\sin\theta = \frac{m\lambda}{d}}$

So electrons appear to act like <u>waves</u> of wavelength λ

$\boxed{\text{de-Broglie's idea}}$

Energy - momentum invariant (*Special Relativity)

$$\boxed{E^2 - p^2c^2 = m^2c^4}$$

So for photons, $m = 0$ ∴ momentum $\boxed{p = E/c}$

So from Planck: $E = hf = hc/\lambda$ ∴ $p = \frac{hc}{\lambda c}$

∴ * $\boxed{p = \frac{h}{\lambda}}$ *   So let's assume this works IN GENERAL for massive particles too!

For electron: $\lambda = \frac{h}{p}$ Classical physics $eV = \underbrace{\frac{p^2}{2m}}_{KE}$

$[p = mv \quad E_k = \frac{1}{2}mv^2 = p^2/2m]$

∴ $p = \sqrt{2meV}$ so $\boxed{\lambda = \dfrac{h}{\sqrt{2meV}}}$

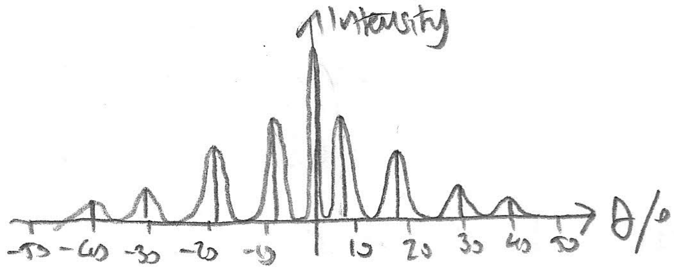
For $V = 5\,kV$, $\boxed{\lambda = 1.7\times10^{-11}\,m}$ if $d \approx 10^{-10}\,m$

$\Rightarrow \dfrac{\lambda}{d} \approx 0.17$ ∴ $\theta = \sin^{-1}\left(\dfrac{m\lambda}{d}\right)$

$= 0°, 9.8°, 19.9°, 30.7°, 42.8°$

$58.2°$

$m=3$, $m=4$, $m=5$



∴ you should be able to see ≈ 5 rings + spot at 0°.

RECAP: * particles (like electrons) can also be waves where $\boxed{\lambda = h/p}$ * light is quantized in energy $\boxed{E = hf}$

We can use this to adapt the classical model of the atom

THE BOHR ATOM

Imagine an electron as a STANDING WAVE around the "orbit" of a nucleus at radius r.

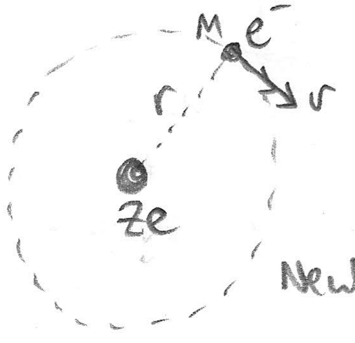
This is how we can get around $\dot{E} \approx 0$ if no radiation!

So $\boxed{2\pi r = n\lambda}$

Now from de-Broglie $\lambda = h/p = \dfrac{h}{mv}$

∴ $2\pi r = \dfrac{nh}{mv}$ ∴ $\boxed{mvr = n\hbar}$

angular momentum



ANGULAR MOMENTUM IS QUANTIZED

Now let's add this to a classical model

Newton II: $\boxed{\dfrac{mv^2}{r} = \dfrac{Ze^2}{4\pi\varepsilon_0 r^2}}$ Energy: $\boxed{\tfrac{1}{2}mv^2 - \dfrac{Ze^2}{4\pi\varepsilon_0 r}} = E$

[$\hbar = h/2\pi$] ← Note $p = h/\lambda$, $k = \dfrac{2\pi}{\lambda}$ ∴ $\boxed{p = \hbar k}$]

$\therefore$ Total energy is, since $\frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r}$

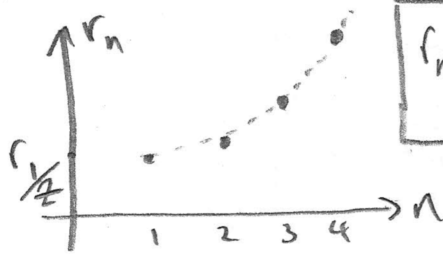
$$\boxed{E = \frac{-Ze^2}{8\pi\varepsilon_0 r}}$$

$2\pi r = n\lambda$ & $\lambda = \frac{h}{mv}$

Now since $\boxed{v = \frac{n\hbar}{mr}}$ and $\boxed{mv^2 = \frac{Ze^2}{4\pi\varepsilon_0 r}}$ ← Newton II

$$\therefore \frac{n^2\hbar^2}{m^2r^2} = \frac{Ze^2}{4\pi\varepsilon_0 r m} \Rightarrow \boxed{r_n = \frac{4\pi\varepsilon_0\hbar^2}{me^2}\frac{n^2}{Z}}$$

$$\boxed{r_1 = \frac{4\pi\varepsilon_0\hbar^2}{me^2} \simeq 0.0529\,\text{nm}}$$ is called the $\boxed{\text{Bohr radius}}$



$$\boxed{r_n = \frac{r_1 n^2}{Z}}$$

Hence $v_n = \frac{n\hbar}{m} \times \underbrace{\frac{me^2 Z}{4\pi\varepsilon_0\hbar^2 n^2}}_{\frac{1}{r_n}}$

$$\Rightarrow \boxed{\frac{v_n}{c} = \frac{e^2}{4\pi\varepsilon_0\hbar c} \times \frac{Z}{n}}$$

$\frac{e^2}{4\pi\varepsilon_0\hbar c} \simeq \frac{1}{137}$ FINE STRUCTURE CONSTANT.

So this means if $Z=1, n=1$
$v \simeq \frac{1}{137}c$ ie "not quite relativistic"

[So Classical assumptions of $v \ll c$ and $\therefore$
$E_k = \frac{1}{2}mv^2$, $p = mv$ etc are 'nearly' ok....] ← But might expect subtle relativistic effects!

so $E_n = \frac{-Ze^2}{8\pi\varepsilon_0 r_n}$ (Total energy)

$$\therefore E_n = \frac{-Ze^2}{8\pi\varepsilon_0} \times \frac{Z}{n^2} \times \frac{me^2}{4\pi\varepsilon_0\hbar^2} \quad \therefore \boxed{E_n = -\frac{me^4}{8\varepsilon_0^2 h^2} Z^2 \frac{1}{n^2}}$$

So: $$\boxed{E_n \simeq -\frac{13.6\,\text{eV}}{n^2} \times Z^2}$$

(So Ionization energy for Hydrogen $Z=1$ is $\simeq 13.6\,\text{eV}$)

BALMER FORMULA & HYDROGEN SPECTRA

From spectral analysis of hydrogen – It will only absorb (or emit) radiation at <u>discrete wavelengths</u>

Experimental result →
$$\boxed{\lambda_{nm} = 91.13\,\text{nm} \times \left(\frac{1}{m^2} - \frac{1}{n^2}\right)^{-1}}$$
n, m are integers

[Balmer lines : $n \geq 3, m = 2$].

photon energy absorbed or produced by/from atom

We can interpret this as $\boxed{E = hc/\lambda}$ produced from changes in electron $\boxed{\text{energy levels}}$ $n \to m$.

$$\frac{hc}{\lambda_{nm}} = E_n - E_m = \frac{-me^4}{8\varepsilon_0^2 h^2} Z^2 \left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$
← from above E_n formula.

$$\therefore \quad \boxed{\lambda_{nm} = \frac{8\varepsilon_0^2 h^3 c}{me^4} \frac{1}{Z^2} \left(\frac{1}{m^2} - \frac{1}{n^2}\right)^{-1}}$$

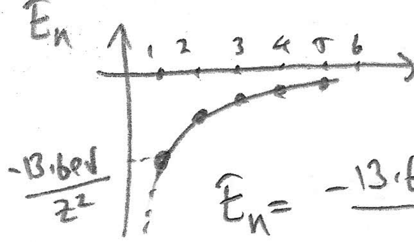
$$\frac{8\varepsilon_0^2 h^3 c}{me^4} \approx 91.13\text{ nm}$$
☺ ✓ It works!

But problems with Bohr model when $Z > 1$ (ie Helium, Lithium etc). Problem is $\boxed{\text{electron repulsion}}$

So Bohr model is ok for 'Hydrogenic atoms' ie single electron, nucleus with charge Ze.

$\boxed{\text{Spectral series / lines:}}$

(Photon produced when $n \to m$. $n > m$ Since E becomes MORE –ve when photon emitted)



$$E_n = \frac{-13.6\text{eV}}{n^2} Z^2$$

Lyman	$n \geq 2, m = 1$	(UV)
Balmer	$n \geq 3, m = 2$	(visible)
Paschen	$n \geq 4, m = 3$	(IR)
Brackett	$n \geq 5, m = 4$	
Pfund	$n \geq 6, m = 5$	