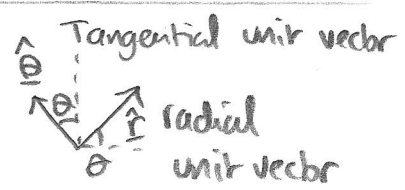
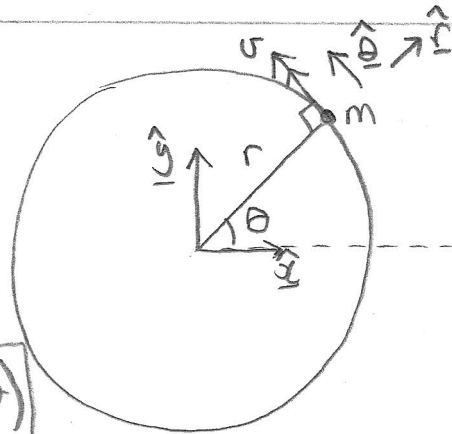


ROTATIONAL MECHANICS : CIRCULAR MOTION, MOMENTS, TORQUE
MOMENTS OF INERTIA, CENTRE OF MASS
ANGULAR MOMENTUM

CIRCULAR MOTION



ie position vector

polar $\underline{r} = r \hat{r}$ $r = \text{constant}$

Cartesian: $\underline{r} = r(\hat{x} \cos\theta + \hat{y} \sin\theta)$

Define $\omega \equiv \dot{\theta}$

Angular velocity (radians/s)

$\dot{\theta} = \frac{d\theta}{dt}$

Now $\hat{r} = \hat{x} \cos\theta + \hat{y} \sin\theta$
 $\hat{\theta} = -\hat{x} \sin\theta + \hat{y} \cos\theta$

$\frac{d\hat{r}}{dt} = (-\hat{x} \sin\theta + \hat{y} \cos\theta) \dot{\theta}$

$\frac{d\hat{r}}{dt} = \dot{\theta} \hat{\theta}$

$\frac{d\hat{\theta}}{dt} = (-\hat{x} \cos\theta - \hat{y} \sin\theta) \dot{\theta}$

$\frac{d\hat{\theta}}{dt} = -\dot{\theta} \hat{r}$

Velocity $\underline{v} = \frac{d}{dt}(r\hat{r}) = r \frac{d\hat{r}}{dt} + \dot{r} \hat{r}$

$\underline{v} = r\dot{\theta} \hat{\theta} + \dot{r} \hat{r}$

if $r = \text{constant}$ $\dot{r} = 0$

$\underline{v} = r\dot{\theta} \hat{\theta}$ or $\underline{v} = r\omega$

Acceleration $\underline{a} = \frac{d\underline{v}}{dt} = \frac{d}{dt}(r\dot{\theta} \hat{\theta} + \dot{r} \hat{r})$

$= r\dot{\theta}(-\dot{\theta} \hat{r}) + \hat{\theta}(r\ddot{\theta} + \dot{\theta} \dot{r}) + \dot{r} \hat{r} + \dot{r} \dot{\theta} \hat{\theta}$

$\underline{a} = \hat{r}(-r\dot{\theta}^2 + \ddot{r}) + \hat{\theta}(2\dot{r}\dot{\theta} + r\ddot{\theta})$

if $\dot{r} = 0$ $\underline{a} = \hat{r}(-r\dot{\theta}^2) + \hat{\theta}(r\ddot{\theta})$

ie radially inward ("centripetal") acceleration $r\omega^2$

tangential acceleration $r\ddot{\theta} = r\dot{\omega}$

Note since $\underline{v} = r\omega$, centripetal acceleration = $\frac{v^2}{r}$

