PRE-U REVISION NOTES

THERMAL PHYSICS :   IDEAL GASES, FIRST & SECOND LAWS OF THERMODYNAMICS.
     (II)   HEAT ENGINES. KINETIC THEORY. BOLTZMANN
           STATISTICS. PLANCK RADIATION DISTRIBUTION

MACROSCOPIC ← → MICROSCOPIC ---- And limit to 'ideal gas' model

(REAL ENGINES ON HUMAN SCALE)   (Molecular, atomic scale)

Treat gases, liquids as a continuum ie ignore molecular properties

Comprehend MACRO properties such as heat, pressure, temperature, radiation diffusion, entropy from averages g molecular interactions

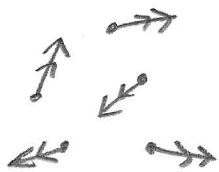Temperature is a measure g the mean kinetic energy g molecules in random motion. The internal energy g an ideal gas is

$$U \propto T$$

Energy /S ↗   ↖ Absolute temperature in kelvin

Ideal gas

* molecules are point-like objects that are in random motion and collide elastically
* Mean-free paths between molecular collisions >> molecular size
* molecules move so fast that can ignore intermolecular forces, so U = kinetic energy and no potential energy due to electric and magnetic fields

A HEAT ENGINE is a system which converts some g the flow g energy from HOT to COLD reservoirs into useful work. A reservoir is a body large enough to not change temperature when heat is added or subtracted.

Is not all!

HOT RESERVOIR T_H

↓ Q_in

HEAT ENGINE → W

↓ Q_out

COLD RESERVOIR T_C
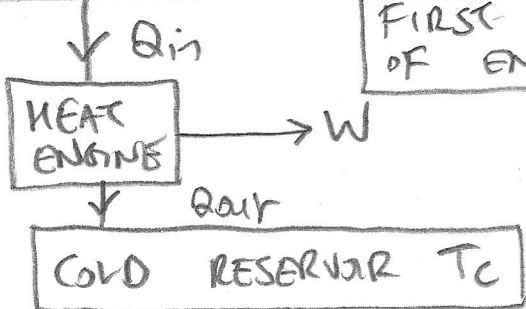
FIRST LAW OF THERMODYNAMICS - CONSERVATION OF ENERGY

$$Q_{in} = Q_{out} + W$$

↑ Heat input to engine   ↑ Heat output g engine   ↖ work done by engine

(1)

SECOND LAW OF THERMODYNAMICS — TOTAL ENTROPY OF THE UNIVERSE CANNOT DECREASE FOR ANY CHANGE

Entropy change S $\qquad$ $dS = \dfrac{dQ}{T}$

So for the heat engine, since T_H and T_C don't change when heat added/subtracted

$$\Delta S_H = -\,Q_{in}/T_H \qquad\qquad \Delta S_C = Q_{out}/T_C$$

$\therefore$ $\boxed{\Delta S_{total} = \Delta S_H + \Delta S_C}$ and 2nd law $\Rightarrow$ $\boxed{\Delta S_{TOTAL} \geqslant 0}$

$\therefore$ $\quad -\dfrac{Q_{in}}{T_H} + \dfrac{Q_{out}}{T_C} \geqslant 0$

Now from 1st law
$$\boxed{Q_{in} = Q_{out} + W}$$
$\therefore$ $Q_{out} = Q_{in} - W$

$\therefore$ $\quad \dfrac{Q_{in} - W}{T_C} - \dfrac{Q_{in}}{T_H} \geqslant 0$

$Q_{in}\left(\dfrac{1}{T_C} - \dfrac{1}{T_H}\right) \geqslant \dfrac{W}{T_C}$

$$\boxed{\dfrac{W}{Q_{in}} \leq 1 - \dfrac{T_C}{T_H}}$$

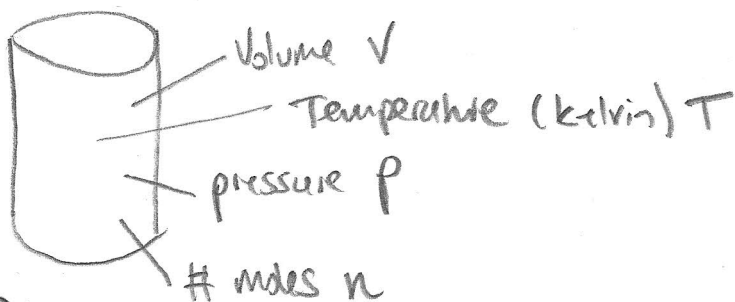Define heat engine $\boxed{\text{efficiency}}$ $\eta = \dfrac{\text{Work done}}{\text{heat input}}$

ie $\boxed{\eta = W/Q_{in}}$ Hence $\boxed{\eta \leq 1 - T_C/T_H}$

So <u>maximum</u> efficiency of a heat engine is $1 - T_C/T_H$

IDEAL GASES — EQUATION OF STATE, WORK, INTERNAL ENERGY

- Volume V
- Temperature (kelvin) T
- pressure P
- # moles n

$\boxed{pV = nRT}$ Ideal gas equation

Area A

↓ F
↕ dx
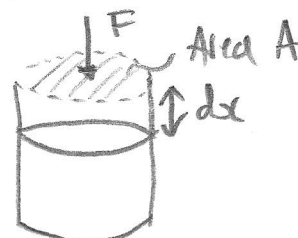
Work done compressing a gas is
$\boxed{dW = Fdx}$

Now pressure acting on the gas is p and F = pA.
change in volume dV = -A dx :

So $dW = pA \times -\dfrac{dV}{A}$ or $\boxed{dW = -pdV}$

So for gases: 1st law is $\boxed{dU = dQ - pdV}$

change in internal energy

Heat added

Work done on gas.

Internal energy of ideal gas

$$\boxed{U = \tfrac{1}{2}\alpha nRT}$$

α

moles

molar gas constant $R = 8.314\,J\,mol^{-1}k^{-1}$

"degrees of freedom" of molecular motion (eg $\alpha = 3$ if only x, y, z translation. $T\uparrow$ $\Rightarrow$ other modes eg vibration rotation possible).

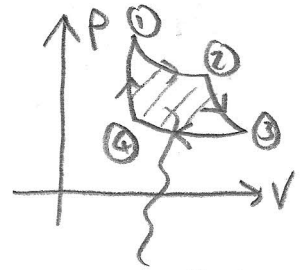So Since pV = nRT

$\Rightarrow \boxed{U = \tfrac{1}{2}\alpha pV}$

involving ideal gases

$\boxed{\text{HEAT ENGINES}}$ { i.e petrol, diesel, steam.....}
can be analysed as a cyclical process in P,V space

Work done by gas is $\displaystyle\int_{cycle}(-dW) \Rightarrow \boxed{\oint pdV = W}$

Heat inputs Q_{in} and outputs Q_{out} at various stages of cycle

$W = \oint pdV$

$\boxed{\text{Special cases: } \#1 \text{ Constant volume (ISOCHORIC) process}}$

dV = 0 $\therefore$ 1st law dU = dQ

$dU = \tfrac{1}{2}\alpha nRdT$
$dQ = C_V dT$

C_V is constant volume heat capacity

$\therefore \boxed{C_V = \tfrac{1}{2}\alpha nR}$

if M is the molar mass / kg

the constant volume specific heat capacity
$\leftarrow$ lower case!
$\boxed{C_V = \tfrac{1}{2}\alpha \dfrac{R}{M}}$

So $\boxed{\Delta Q = C_V M \Delta T}$
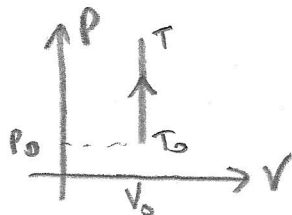
Ideal gas eqn
$pV_0 = nRT_0$
$pV_0 = nRT$

$\therefore \boxed{\dfrac{P}{P_0} = \dfrac{T}{T_0}}$

③

#2: Constant pressure (ISOBARIC) process

Cp is constant pressure heat capacity

$$dU = \tfrac{1}{2} \alpha n R dT$$

$$\boxed{dQ = Cp\, dT}$$

From $pV = nRT$ and $dW = -p\, dV$

Since $p =$ constant $\quad p\, dV = nR\, dT \quad \Rightarrow \quad \boxed{dW = -nR\, dT}$

So from 1st law: $dU = dQ + dW \quad \Rightarrow \quad \tfrac{1}{2}\alpha n R dT = Cp\, dT - nR\, dT$

$\therefore\; Cp = \tfrac{1}{2}\alpha n R + nR \qquad$ or $\qquad \boxed{Cp = C_V + nR}$ Mayer relationship

$$\boxed{\Delta Q = m\, cp\, \Delta T}$$

$$\boxed{\Delta W = -p\, \Delta V}$$

$$\boxed{cp = \dfrac{Cp}{nm}} \;\leftarrow\; \text{specific heat capacity}$$
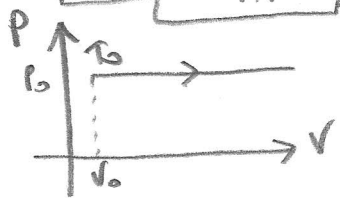
$P_0 V_0 = nRT_0$

$P_0 V = nRT$

$$\boxed{\dfrac{V}{V_0} = \dfrac{T}{T_0}}$$

#3: Constant temperature (ISOTHERMAL) process

$dU = 0$ Since $dT = 0$

1st law $dQ = p\, dV$

$\therefore\; dQ = \underbrace{\dfrac{nRT}{V}}_{P} dV \quad \Rightarrow \quad \Delta Q = nRT \displaystyle\int_{V_0}^{V} \dfrac{1}{V} dV$

$$\Rightarrow \boxed{\Delta Q = nRT \ln \dfrac{V}{V_0}} \;\leftarrow\; \text{Heat supplied to gas.}$$

$\Delta U = \Delta Q + \Delta W$ so work done on gas $\boxed{\Delta W = -nRT \ln\left(\dfrac{V}{V_0}\right)}$

$p = \dfrac{nRT}{V}$

$P_0 = \dfrac{nRT}{V_0}$
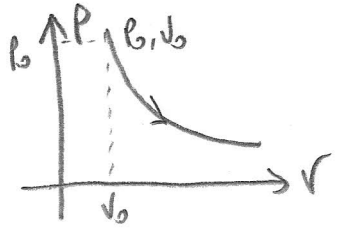
$\therefore\; \boxed{\dfrac{P}{P_0} = \dfrac{V_0}{V}}$

ie $\boxed{PV = \text{constant}}$

(Boyle's law)

(4)

#4: NO HEAT EXCHANGED (ADIABATIC OR ISENTROPIC) process

$dQ = 0$ ∴ first law: $dU = -p\,dV$ $\left[\begin{array}{l} U = \frac{1}{2}\alpha n RT \\ pV = nRT \end{array}\right]$

$U = \frac{1}{2}\alpha pV$ ∴ $dU = \frac{1}{2}\alpha(p\,dV + V\,dp)$

So $\frac{1}{2}\alpha(p\,dV + V\,dp) = -p\,dV$

$p\,dV\left(\frac{1}{2}\alpha + 1\right) = -\frac{1}{2}\alpha V\,dp$

$\left(1 + \frac{2}{\alpha}\right)\frac{dV}{V} = -\frac{dp}{p}$

$\left(1 + \frac{2}{\alpha}\right)\int\frac{dV}{V} = -\int\frac{dp}{p}$

$\left(1 + \frac{2}{\alpha}\right)\ln V = -\ln p + \text{const}$

$\ln V^{1 + 2/\alpha} = \ln p^{-1} + \text{const}$

$V^{1 + 2/\alpha} = \dfrac{\text{Another const}}{p}$

$\left[e^{\ln p^{-1} + \text{const}} = \frac{1}{p}e^{\text{const}}\right]$

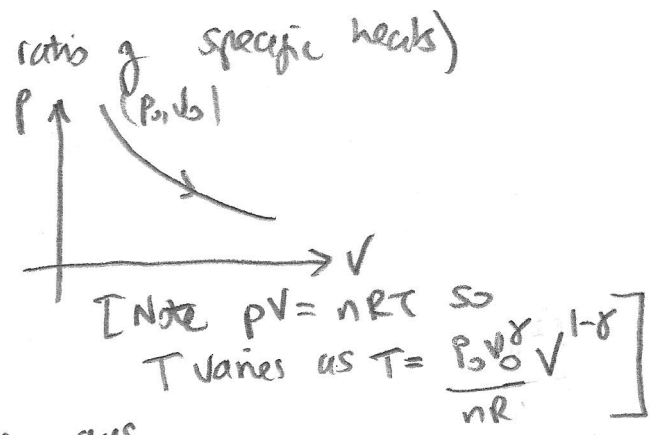So $pV^{1 + 2/\alpha} = \text{constant}$.

Now from Isochoric & Isobaric processes $C_p = \frac{1}{2}\alpha nR + nR$

$C_v = \frac{1}{2}\alpha nR$

So $\boxed{\gamma = \dfrac{C_p}{C_v} = 1 + \dfrac{2}{\alpha}}$

(Not this is also $\gamma = \dfrac{C_p}{C_v}$ ie. ratio of specific heats)

∴ $\boxed{pV^\gamma = \text{constant}}$

$\boxed{pV^\gamma = p_0 V_0^\gamma}$

[Note $pV = nRT$ so T varies as $T = \dfrac{p_0 V_0^\gamma}{nR}V^{1-\gamma}$]

Now $dW = -p\,dV$ work done on gas

∴ $\Delta W = -p_0 V_0^\gamma \displaystyle\int_{V_0}^{V}\frac{dV}{V^\gamma}$ since $p = \dfrac{p_0 V_0^\gamma}{V^\gamma}$

$\Delta W = -p_0 V_0^\gamma\left[\dfrac{V^{1-\gamma}}{1-\gamma}\right]_{V_0}^{V}$

[$\gamma > 1$ from Mayr so it makes sense to write it this way ↓]

$\Delta W = \dfrac{p_0 V_0^\gamma}{\gamma-1}\left[V^{1-\gamma} - V_0^{1-\gamma}\right] = \boxed{\dfrac{p_0 V_0}{\gamma-1}\left[\left(\dfrac{V_0}{V}\right)^{\gamma-1} - 1\right]}$

Idealized heat engines: CARNOT CYCLE
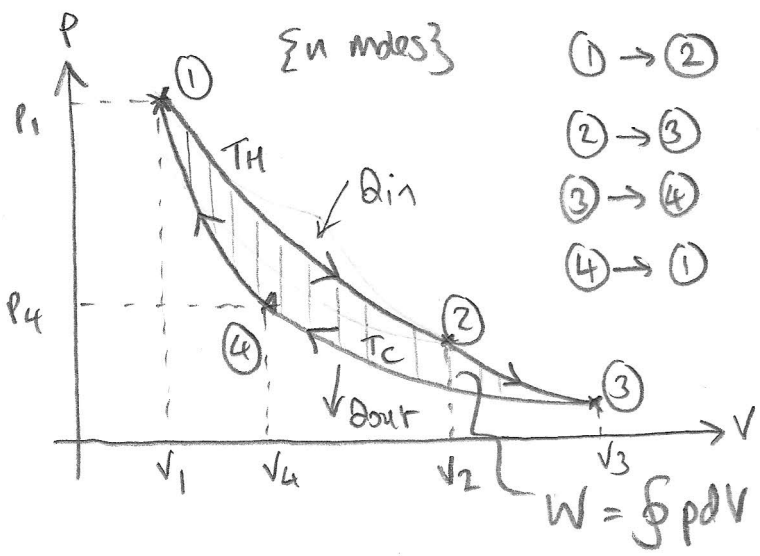
{n moles}

① → ②   Iso thermal expansion
② → ③   Adiabatic expansion
③ → ④   Isothermal compression
④ → ①   Adiabatic compression

{Define W as work done by gas (not on gas)}

$$W = \oint p\, dV$$

From isothermal processes:

$$Q_{in} = nRT_H \ln \frac{V_2}{V_1}$$

$$Q_{out} = nRT_C \ln \frac{V_3}{V_4}$$

and since the other stages are adiabatic (ie no more heat exchanged) and $\Delta U = 0$ for whole cycle ($U = \frac{1}{2}\alpha pV$ and p,V return to same values (p_1, V_1))

$$\Rightarrow \quad W = Q_{in} - Q_{out}$$

$$\Rightarrow \quad W = nRT_H \ln \frac{V_2}{V_1} - nRT_C \ln \frac{V_3}{V_4}$$

Now for adiabatic stages:

$$P_2 V_2^{\gamma} = P_3 V_3^{\gamma}$$

$$P_1 V_1^{\gamma} = P_4 V_4^{\gamma}$$

So $\quad \dfrac{P_2}{P_1}\left(\dfrac{V_2}{V_1}\right)^{\gamma} = \dfrac{P_3}{P_4}\left(\dfrac{V_3}{V_4}\right)^{\gamma} \quad \Rightarrow \quad \dfrac{P_2 V_2}{P_1 V_1}\left(\dfrac{V_2}{V_1}\right)^{\gamma-1} = \dfrac{P_3 V_3}{P_4 V_4}\left(\dfrac{V_3}{V_4}\right)^{\gamma-1}$

Now $\quad P_2 V_2 = nRT_H \qquad P_3 V_3 = nRT_C \qquad$ from ideal gas
$\qquad\quad P_1 V_1 = nRT_H \qquad P_4 V_4 = nRT_C \qquad$ equation

So $\quad \left(\dfrac{V_2}{V_1}\right)^{\gamma-1} = \left(\dfrac{V_3}{V_4}\right)^{\gamma-1} \quad \Rightarrow \quad \boxed{\dfrac{V_2}{V_1} = \dfrac{V_3}{V_4}}$

$$\therefore \quad \boxed{W = nR(T_H - T_C)\ln\left(\frac{V_2}{V_1}\right)}$$

So efficiency of Carnot cycle is: $\eta = \dfrac{W}{Q_{in}}$

$\Rightarrow \eta = \dfrac{nR(T_H - T_C)\ln\left(\frac{V_2}{V_1}\right)}{nRT_H \ln \frac{V_2}{V_1}} = \boxed{1 - \dfrac{T_C}{T_H}}$

So since for $\underline{any}$ heat engine $\eta \leq 1 - \frac{T_C}{T_H}$

$\Rightarrow$ Carnot engine is $\uparrow$ the most efficient possible.

$\boxed{\text{one example}}$

$\boxed{\text{NOTE ON ENTROPY}}$

Work done $\underline{by}$ gas $\swarrow$

$dU = dQ - \boxed{dW}$

$\dfrac{dQ}{T} = dS$

Now for a heat engine $\oint dU = 0$

So if we plot a heat cycle in T, S space, the area enclosed is the work done per cycle.

$\therefore \quad dU = TdS - dW$

So $\boxed{\oint TdS = \oint dW}$

$\oint dW = \oint TdS$

For the $\boxed{\text{Carnot cycle}}$, no entropy changes in adiabatic phases, and since T = constant in isothermal stages $\Rightarrow$ Carnot cycle is a $\boxed{\text{rectangle}}$ in T, S space!

Since: $Q_{in} = nRT_H \ln \frac{V_2}{V_1}$

$Q_{out} = nRT_C \ln \frac{V_3}{V_4} = nRT_C \ln \frac{V_3}{V_4}$

$\Delta S = \frac{\Delta Q}{T}$

$\boxed{\Delta S = nR \ln \frac{V_2}{V_1}}$

$\boxed{\text{Note}} \quad \boxed{\dfrac{Q_{in}}{T_H} = \dfrac{Q_{out}}{T_C}}$ since this cycle is rectangular

and since $Q_{in} = nRT_H \ln \frac{V_2}{V_1} \Rightarrow \dfrac{Q_{out}}{T_C} = nR \ln \frac{V_2}{V_1}$

So you can get $\eta = 1 - \frac{T_C}{T_H}$ without having to work out $\frac{V_3}{V_4} = \frac{V_2}{V_1}$ as above.
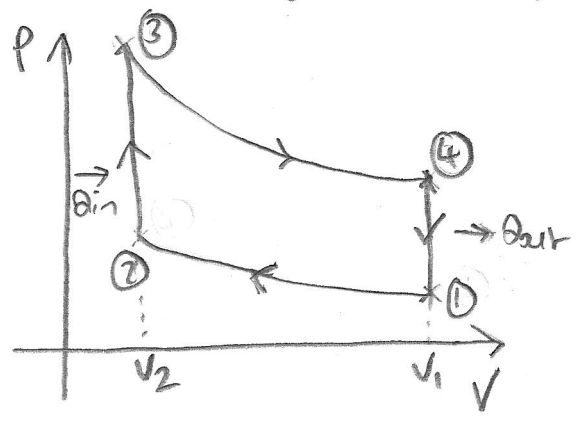
⑦

other classic heat cycles   (see Eclecticon notes on Heat Engines for details)

Otto cycle   (spark ignition petrol engines)

①→②   Adiabatic compression

②→③   Isochoric heating (spark explosion of gas in piston!)

③→④   Adiabatic expansion

④→①   isochoric cooling (heat released when piston at max. expansion)

[Note at ① there is air exchange (exhaust) at constant pressure with atmosphere.]

One can show:

$$\eta = 1 - \frac{1}{r^{\gamma - 1}}$$

where

$$\gamma = \frac{c_p}{c_v}$$

$$r = \frac{V_1}{V_2}$$

Compression ratio

Diesel cycle   (Air compression sufficient to ignite fuel vapors)
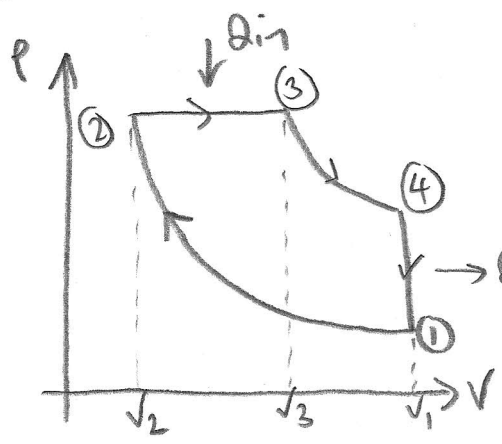
①→② Adiabatic compression

②→③ Isobaric heating

③→④ Adiabatic expansion

④→① Isochoric cooling

one can show:

$$\eta = 1 - \frac{1}{r^{\gamma - 1}}\left(\frac{s^{\gamma} - 1}{\gamma (s - 1)}\right)$$

$$r = \frac{V_1}{V_2} \qquad s = \frac{V_3}{V_2}$$

So possibility for greater efficacy than a petrol engine — which is why most container ships use diesel, in huge engines.

→ [Example Excel sheets for Carnot, otto & diesel cycles]

⑧

Carnot Cycle model
Dr A. French. September 2017

Nicolas
Léonard Sadi
Carnot
(1796-1832)

Input parameters

Hot reservoir temperature /Celsius	150
Cold reservoir temperature /Celsius	20
Mass of gas /g	1.00
Volume of gas at lowest volume and hightest pressure /litres	0.40
Volume of gas after isothermal expansion /litres	1.00
Degrees of freedom of molecular motion	3
Molar mass of gas /gmol^-1	28.966

Outputs

Heat input during isothermal expansion /kJ	0.111
Heat output during isothermal compression /kJ	0.077
Total work done by gas on surroundings /kJ	0.034
Entropy change during isothermal stages /JK^-1	0.263
Efficiency (work done / heat input)	0.307

Theoretical efficiency	0.307

$$\eta = 1 - \frac{T_C}{T_H}$$

Note all temperatures incorporated into calculations
will be converted to Kelvin first - i.e. add 273 to Celsius number.

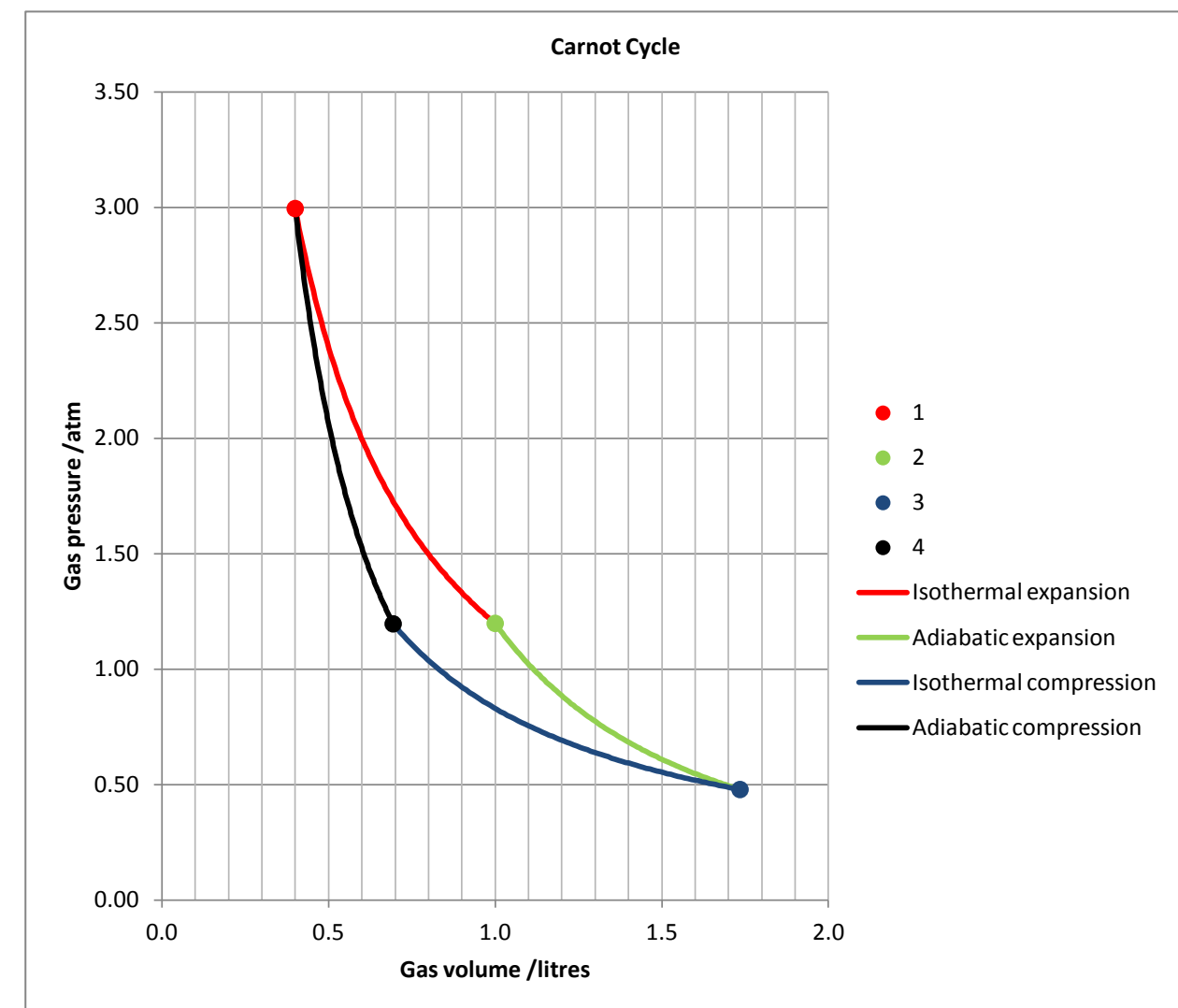**Pressure, volume coordinates of heat cycle**

p1	3.00
V1	0.4
p2	1.20
V2	1.0
p3	0.48
V3	1.73
p4	1.20
V4	0.7

Note all pressures are quoted in atmospheres. 1atm = 101,325 Pa. Volumes in litres. T in K.
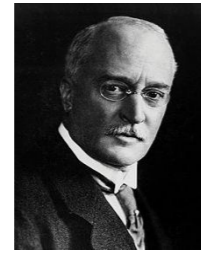
Reservoir temperatures in K

T_H	423
T_C	293

Number of moles of gas in engine	0.035
Ratio of specific heats gamma	1.667

Carnot Cycle

	1 to 2		2 to 3		3 to 4		4 to 1	
	Isothermal expansion		Adiabatic expansion		Isothermal compression		Adiabatic compression	
V diff fraction	p	V	p	V	p	V	p	V
0	2.996	0.400	1.198	1.000	0.478	1.735	1.196	0.694
0.01	2.951	0.406	1.184	1.007	0.481	1.724	1.205	0.691
0.02	2.908	0.412	1.169	1.015	0.484	1.714	1.213	0.688
0.03	2.867	0.418	1.155	1.022	0.487	1.703	1.222	0.685
0.04	2.826	0.424	1.142	1.029	0.490	1.693	1.231	0.682
0.05	2.787	0.430	1.128	1.037	0.493	1.683	1.240	0.679
0.06	2.748	0.436	1.115	1.044	0.496	1.672	1.249	0.676
0.07	2.711	0.442	1.102	1.051	0.499	1.662	1.258	0.673
0.08	2.675	0.448	1.089	1.059	0.503	1.651	1.267	0.670
0.09	2.639	0.454	1.077	1.066	0.506	1.641	1.276	0.667
0.1	2.605	0.460	1.065	1.073	0.509	1.631	1.286	0.664
0.11	2.571	0.466	1.053	1.081	0.512	1.620	1.295	0.662
0.12	2.539	0.472	1.041	1.088	0.516	1.610	1.305	0.659
0.13	2.507	0.478	1.029	1.096	0.519	1.599	1.315	0.656
0.14	2.476	0.484	1.018	1.103	0.522	1.589	1.324	0.653
0.15	2.445	0.490	1.007	1.110	0.526	1.579	1.334	0.650
0.16	2.416	0.496	0.996	1.118	0.529	1.568	1.345	0.647
0.17	2.387	0.502	0.985	1.125	0.533	1.558	1.355	0.644
0.18	2.359	0.508	0.974	1.132	0.536	1.547	1.365	0.641
0.19	2.331	0.514	0.964	1.140	0.540	1.537	1.376	0.638
0.2	2.304	0.520	0.953	1.147	0.544	1.526	1.386	0.635
0.21	2.278	0.526	0.943	1.154	0.547	1.516	1.397	0.632
0.22	2.252	0.532	0.933	1.162	0.551	1.506	1.408	0.629
0.23	2.227	0.538	0.924	1.169	0.555	1.495	1.419	0.626
0.24	2.203	0.544	0.914	1.176	0.559	1.485	1.430	0.623
0.25	2.179	0.550	0.905	1.184	0.563	1.474	1.441	0.620
0.26	2.155	0.556	0.895	1.191	0.567	1.464	1.453	0.617
0.27	2.132	0.562	0.886	1.198	0.571	1.454	1.465	0.615
0.28	2.110	0.568	0.877	1.206	0.575	1.443	1.476	0.612
0.29	2.088	0.574	0.868	1.213	0.579	1.433	1.488	0.609
0.3	2.066	0.580	0.860	1.220	0.584	1.422	1.500	0.606
0.31	2.045	0.586	0.851	1.228	0.588	1.412	1.512	0.603
0.32	2.024	0.592	0.843	1.235	0.592	1.402	1.525	0.600
0.33	2.004	0.598	0.834	1.242	0.597	1.391	1.537	0.597
0.34	1.984	0.604	0.826	1.250	0.601	1.381	1.550	0.594
0.35	1.964	0.610	0.818	1.257	0.606	1.370	1.563	0.591
0.36	1.945	0.616	0.810	1.264	0.610	1.360	1.576	0.588
0.37	1.926	0.622	0.803	1.272	0.615	1.350	1.589	0.585
0.38	1.908	0.628	0.795	1.279	0.620	1.339	1.603	0.582
0.39	1.890	0.634	0.787	1.287	0.625	1.329	1.616	0.579
0.4	1.872	0.640	0.780	1.294	0.630	1.318	1.630	0.576
0.41	1.855	0.646	0.773	1.301	0.635	1.308	1.644	0.573
0.42	1.838	0.652	0.765	1.309	0.640	1.298	1.658	0.570
0.43	1.821	0.658	0.758	1.316	0.645	1.287	1.672	0.567
0.44	1.805	0.664	0.751	1.323	0.650	1.277	1.687	0.565
0.45	1.788	0.670	0.744	1.331	0.655	1.266	1.702	0.562
0.46	1.773	0.676	0.738	1.338	0.661	1.256	1.717	0.559
0.47	1.757	0.682	0.731	1.345	0.666	1.245	1.732	0.556
0.48	1.742	0.688	0.724	1.353	0.672	1.235	1.747	0.553
0.49	1.727	0.694	0.718	1.360	0.678	1.225	1.763	0.550
0.5	1.712	0.700	0.711	1.367	0.684	1.214	1.778	0.547
0.51	1.697	0.706	0.705	1.375	0.689	1.204	1.794	0.544
0.52	1.683	0.712	0.699	1.382	0.695	1.193	1.811	0.541

Legend:
- 1
- 2
- 3
- 4
- Isothermal expansion
- Adiabatic expansion
- Isothermal compression
- Adiabatic compression

Diesel Cycle model
Dr A. French. September 2017

Rudolf Diesel
(1858-1913)

Input parameters

Temperature T1 of air draw into piston /Celsius	25
Low pressure state p1 /atm	1.00
Volume V1 of uncompressed gas /litres	1820
Volume V2 of compressed gas /litres	79
Volume V3 of compressed gas after isobaric heating /litres	170
Degrees of freedom of molecular motion	3
Molar mass of gas /gmol^-1	28.966

Outputs

Heat input during isobaric heating /kJ	4267
Heat output during isochoric cooling /kJ	710
Total work done by gas on surroundings /kJ	3557
Efficiency (work done / heat input)	0.834

Theoretical efficiency	0.834

$$\eta = 1 - \frac{1}{r^{\gamma-1}}\left(\frac{s^{\gamma}-1}{\gamma(s-1)}\right)$$

$$r = \frac{V_1}{V_2} \quad s = \frac{V_3}{V_2} \quad \gamma = \frac{c_P}{c_V}$$

Note all temperatures incorporated into calculations
will be converted to Kelvin first
- i.e. add 273 to Celsius number.

Pressure, volume, temperature coordinates of heat cycle

p1	1.0
V1	1820
T1	298
p2	186.0
V2	79.1
T2	2410
p3	186.0
V3	169.7
T3	5168
p4	3.6
V4	1820.0
T4	1063

Single cylinder power output

Engine RPM	84
Power output /kW	4,980

Number of cylinders	14
Total power output /kW	69,717

Note all pressures are quoted in atmospheres. 1atm = 101,325 Pa. Volumes in litres, T in K

Number of moles of gas in engine	74
Ratio of specific heats gamma	1.667
Constant volume specific heat capacity /Jkg^-1K^-1	431
Constant pressure specific heat capacity /Jkg^-1K^-1	718

Note real petrol engines have an efficiency of more like 20%, whereas diesels can be
up to 40%. In other words, significant losses!

Diesel Cycle

Gas pressure /atm vs Gas volume /litres

Legend:
- 1 (red)
- 2 (green)
- 3 (blue)
- 4 (black)
- Adiabatic compression
- Isobaric heating
- Adiabatic expansion
- Isochoric cooling
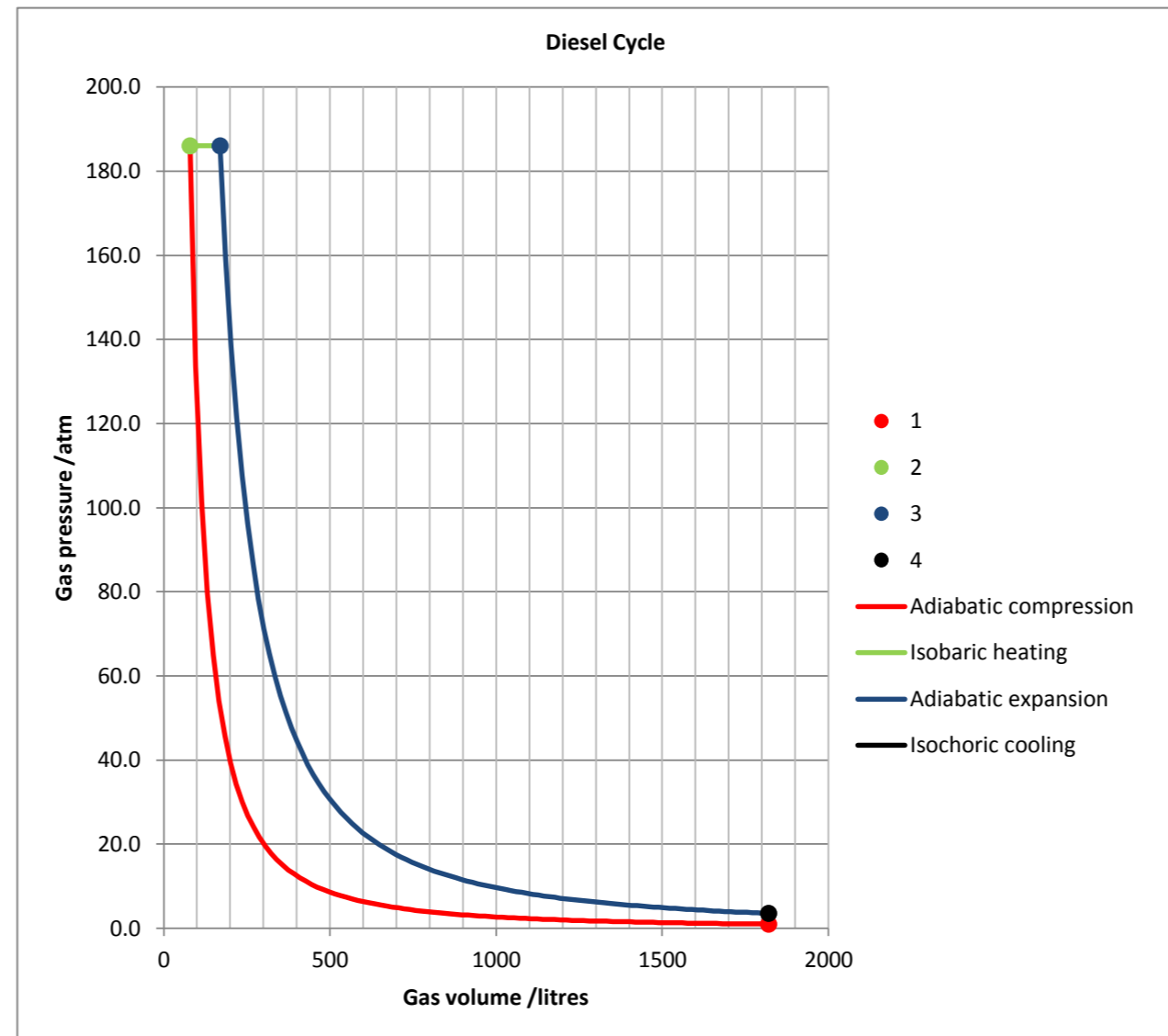
World's largest container ship in 2014
MV CSCL Globe

MAN B&W 12S90ME-C Mark 9.2 diesel engine. 69,720kW at 84RPM

Assume about 1820 litre
cylinder volume V1

Max compression ratio:
V1/V2 = 23

V3 guessed at:
V1/10.726

	1 to 2		2 to 3		3 to 4		4 to 1	
V or p diff fraction	Adiabatic compression		Isobaric heating		Adiabatic expansion		Isochoric cooling	
	p	V	p	V	p	V	p	V
0	1.000	1820.000	186.014	79.130	186.014	169.681	3.566	1820.000
0.01	1.016	1802.591	186.014	80.036	159.354	186.184	3.540	1820.000
0.02	1.033	1785.183	186.014	80.941	138.322	202.688	3.514	1820.000
0.03	1.050	1767.774	186.014	81.847	121.404	219.191	3.489	1820.000
0.04	1.067	1750.365	186.014	82.752	107.569	235.694	3.463	1820.000
0.05	1.085	1732.957	186.014	83.658	96.095	252.197	3.437	1820.000
0.06	1.104	1715.548	186.014	84.563	86.461	268.700	3.412	1820.000
0.07	1.122	1698.139	186.014	85.469	78.285	285.203	3.386	1820.000
0.08	1.142	1680.730	186.014	86.374	71.279	301.707	3.360	1820.000
0.09	1.162	1663.322	186.014	87.280	65.225	318.210	3.335	1820.000
0.1	1.182	1645.913	186.014	88.186	59.954	334.713	3.309	1820.000
0.11	1.204	1628.504	186.014	89.091	55.332	351.216	3.284	1820.000
0.12	1.225	1611.096	186.014	89.997	51.256	367.719	3.258	1820.000
0.13	1.248	1593.687	186.014	90.902	47.639	384.223	3.232	1820.000
0.14	1.271	1576.278	186.014	91.808	44.414	400.726	3.207	1820.000
0.15	1.295	1558.870	186.014	92.713	41.525	417.229	3.181	1820.000
0.16	1.319	1541.461	186.014	93.619	38.925	433.732	3.155	1820.000
0.17	1.344	1524.052	186.014	94.524	36.577	450.235	3.130	1820.000
0.18	1.370	1506.643	186.014	95.430	34.447	466.739	3.104	1820.000
0.19	1.397	1489.235	186.014	96.335	32.508	483.242	3.078	1820.000
0.2	1.425	1471.826	186.014	97.241	30.739	499.745	3.053	1820.000
0.21	1.453	1454.417	186.014	98.146	29.119	516.248	3.027	1820.000
0.22	1.483	1437.009	186.014	99.052	27.631	532.751	3.001	1820.000
0.23	1.513	1419.600	186.014	99.957	26.261	549.254	2.976	1820.000
0.24								20.000
0.25								20.000
0.26								20.000
0.27								20.000
0.28								20.000
0.29								20.000
0.3								20.000
0.31								20.000
0.32								20.000
0.33								20.000
0.34								20.000
0.35								20.000
0.36								20.000
0.37								20.000
0.38								20.000
0.39								20.000
0.4								20.000
0.41								20.000
0.42								20.000
0.43								20.000
0.44								20.000
0.45								20.000
0.46								20.000
0.47								20.000
0.48								20.000
0.49	2.869	966.974	186.014	123.500	10.034	978.337	2.309	1820.000
0.5	2.957	949.565	186.014	124.406	9.758	994.841	2.283	1820.000
0.51	3.050	932.157	186.014	125.311	9.494	1011.344	2.257	1820.000
0.52	3.147	914.748	186.014	126.217	9.241	1027.847	2.232	1820.000
0.53	3.250	897.339	186.014	127.122	8.999	1044.350	2.206	1820.000
0.54	3.358	879.930	186.014	128.028	8.767	1060.853	2.180	1820.000

Otto Cycle model

Dr A. French. September 2017

Nikolaus Otto
(1832-1891)

Input parameters

Temperature T1 of air draw into piston /Celsius	20
Low pressure state p1 /atm	1.00
High pressure state p3 /atm	100
Volume V1 of uncompressed gas /litres	1
Volume V2 of compressed /litres	0.1
Degrees of freedom of molecular motion	3
Molar mass of gas /gmol^-1	28.966

Outputs

Heat input during isochoric heating /kJ	0.814
Heat output during isochoric cooling /kJ	0.175
Total work done by gas on surroundings /kJ	0.639
Efficiency (work done / heat input)	0.785
Theoretical efficiency	0.785

$$\eta = 1 - \frac{1}{r^{\gamma-1}}$$
$$r = \frac{V_1}{V_2} \qquad \gamma = \frac{c_P}{c_V}$$

Note all temperatures incorporated into calculations
will be converted to Kelvin first - i.e. add 273 to Celsius number.

Pressure, volume, temperature coordinates of heat cycle

	p1	1.00
	V1	1.000
	T1	293
	p2	46.42
	V2	0.100
	T2	1360
	p3	100.00
	V3	0.100
	T3	2930
	p4	2.15
	V4	1.000
	T4	631

Power per cylinder

Engine RPM	6500
Power output /kW	69.21979
Number of cylinders	1
Total power output /kW	69.22

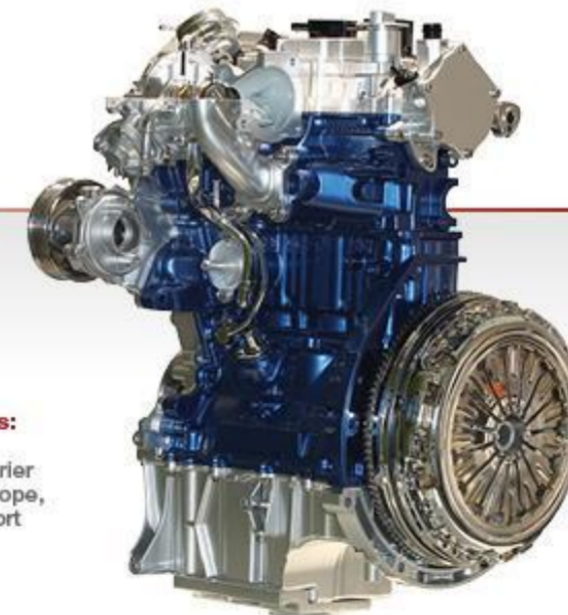Note real petrol engines have an efficiency of more like 20%, whereas diesels can be
up to 40%. In other words, significant losses!

Note all pressures are quoted in atmospheres. 1atm = 101,325 Pa. Volumes in litres. T in K.

Number of moles of gas in engine	0.042
Ratio of specific heats gamma	1.667
Constant volume specific heat capacity /Jkg^-1K^-1	431
Constant pressure specific heat capacity /Jkg^-1K^-1	718
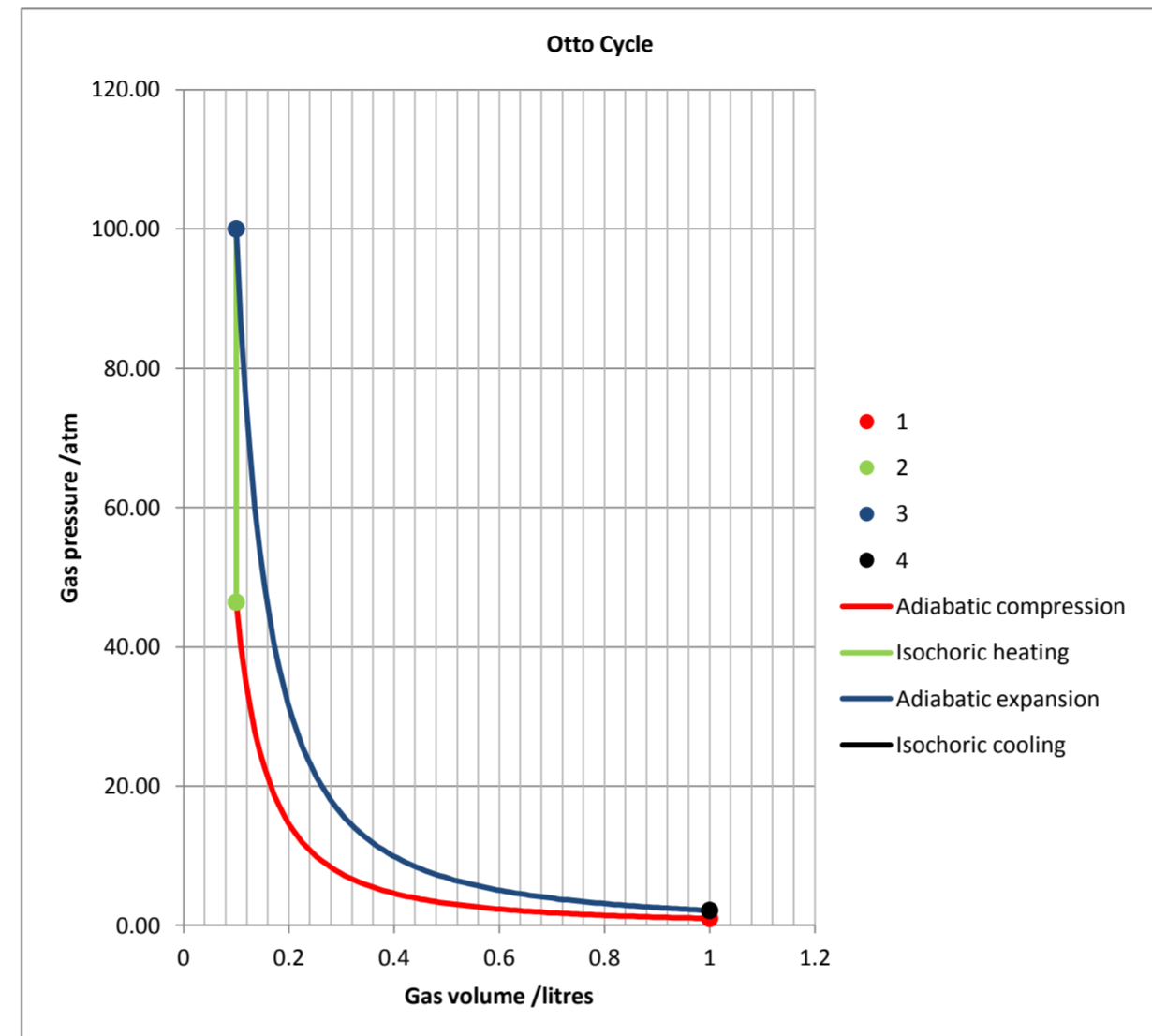
FORD MOTOR CO.
1.0L ECOBOOST
DOHC DI I-3

Displacement: 999 cc
Block / head material: cast iron / aluminum
Horsepower (SAE net): 123 @ 6,000 rpm
Torque: 125 lb.-ft. (169 Nm) @ 2,500 rpm
Specific output: 123 hp/L
Bore x stroke: 71.9 x 82 mm
Compression ratio: 10.0:1

EPA city / highway: 31 / 43 mpg
Assembly site: Cologne, Germany; Craiova, Romania
Application tested: '14 Ford Fiesta SE
Additional applications: Ford Focus, C-Max, Mondeo, Transit Courier (Europe); B-Max (Europe, Asia/Pacific); EcoSport (Asia/Pacific)

Otto Cycle

Legend:
- 1 (red)
- 2 (green)
- 3 (blue)
- 4 (black)
- Adiabatic compression
- Isochoric heating
- Adiabatic expansion
- Isochoric cooling

V or p diff fraction	1 to 2 Adiabatic compression p	V	2 to 3 Isochoric heating p	V	3 to 4 Adiabatic expansion p	V	4 to 1 Isochoric cooling p	V
0	1.000	1.000	46.416	0.100	100.000	0.100	2.154	1.000
0.01	1.015	0.991	46.952	0.100	86.621	0.109	2.143	1.000
0.02	1.031	0.982	47.488	0.100	75.892	0.118	2.131	1.000
0.03	1.047	0.973	48.023	0.100	67.142	0.127	2.120	1.000
0.04	1.063	0.964	48.559	0.100	59.901	0.136	2.108	1.000
0.05	1.080	0.955	49.095	0.100	53.834	0.145	2.097	1.000
0.06	1.097	0.946	49.631	0.100	48.693	0.154	2.085	1.000
0.07	1.115	0.937	50.167	0.100	44.295	0.163	2.074	1.000
0.08	1.133	0.928	50.703	0.100	40.500	0.172	2.062	1.000
0.09	1.151	0.919	51.238	0.100	37.199	0.181	2.051	1.000
0.1	1.170	0.910	51.774	0.100	34.309	0.190	2.039	1.000
0.11	1.190	0.901	52.310	0.100	31.762	0.199	2.027	1.000
0.12	1.210	0.892	52.846	0.100	29.505	0.208	2.016	1.000
0.13	1.230	0.883	53.382	0.100	27.494	0.217	2.004	1.000
0.14	1.252	0.874	53.918	0.100	25.693	0.226	1.993	1.000
0.15	1.273	0.865	54.454	0.100	24.074	0.235	1.981	1.000
0.16	1.296	0.856	54.989	0.100	22.613	0.244	1.970	1.000
0.17	1.319	0.847	55.525	0.100	21.288	0.253	1.958	1.000
0.18	1.343	0.838	56.061	0.100	20.083	0.262	1.947	1.000
0.19	1.367	0.829	56.597	0.100	18.984	0.271	1.935	1.000
0.2	1.392	0.820	57.133	0.100	17.978	0.280	1.924	1.000
0.21	1.418	0.811	57.669	0.100	17.054	0.289	1.912	1.000
0.22	1.444	0.802	58.204	0.100	16.205	0.298	1.900	1.000
0.23	1.472	0.793	58.740	0.100	15.421	0.307	1.889	1.000
0.24	1.500	0.784	59.276	0.100	14.696	0.316	1.877	1.000
0.25	1.529	0.775	59.812	0.100	14.024	0.325	1.866	1.000
0.26	1.559	0.766	60.348	0.100	13.400	0.334	1.854	1.000
0.27	1.590	0.757	60.884	0.100	12.819	0.343	1.843	1.000
0.28	1.622	0.748	61.419	0.100	12.277	0.352	1.831	1.000
0.2								00
0.								00
0.3								00
0.3								00
0.3								00
0.3								00
0.3								00
0.3								00
0.3								00
0.3								00
0.3								00
0.								00
0.41	2.154	0.631	68.385	0.100	7.610	0.469	1.681	1.000
0.42	2.206	0.622	68.921	0.100	7.373	0.478	1.670	1.000
0.43	2.261	0.613	69.457	0.100	7.147	0.487	1.658	1.000
0.44	2.317	0.604	69.993	0.100	6.932	0.496	1.646	1.000
0.45	2.376	0.595	70.529	0.100	6.727	0.505	1.635	1.000
0.46	2.437	0.586	71.065	0.100	6.532	0.514	1.623	1.000
0.47	2.501	0.577	71.600	0.100	6.346	0.523	1.612	1.000
0.48	2.567	0.568	72.136	0.100	6.168	0.532	1.600	1.000
0.49	2.636	0.559	72.672	0.100	5.998	0.541	1.589	1.000
0.5	2.709	0.550	73.208	0.100	5.835	0.550	1.577	1.000
0.51	2.784	0.541	73.744	0.100	5.680	0.559	1.566	1.000
0.52	2.863	0.532	74.280	0.100	5.530	0.568	1.554	1.000
0.53	2.946	0.523	74.815	0.100	5.387	0.577	1.543	1.000

Kinetic theory A statistical theory of the motion of molecules. A macroscopic (i.e. human sized) quantity of gas has physical properties that can be explained by considering averages of molecular properties. LARGE numbers of molecules here!

Molecular motion is essentially a random process e.g. **Brownian Motion** of pollen grains. (unseen air molecules)

* 1 m³ (solid) has ~ $\left(\frac{1}{10^{-10}}\right)^3 = \boxed{10^{30} \text{ atoms}}$

[$1\mathring{A} = 10^{-10}$ m ≈ atomic size]

* $\boxed{1 \text{ mole} = 6.02 \times 10^{23}}$ molecules

↑ Avogadro's number N_A

MEAN FREE PATH ℓ "average distance between molecular collisions"

$$\ell = \frac{\text{distance travelled in time } t}{\# \text{ molecules it will collide with}} = \frac{\sqrt{\langle v^2 \rangle}\, t}{n \times \text{"interaction volume"}}$$

↑ # molecules/unit volume

$\sqrt{\langle v^2 \rangle}$ is the root-mean-squared (RMS) speed

Interaction volume

○ ↕ d

Molecular diameter
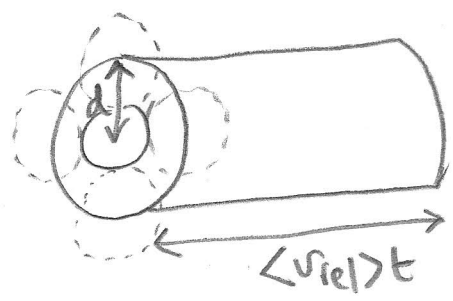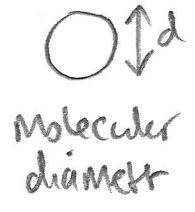
A kind of 'collision tube' between molecules

$$\langle v_{rel} \rangle = \sqrt{\langle |\underline{v}_i - \underline{v}_j|^2 \rangle}$$

$$= \sqrt{\langle v_i^2 + v_j^2 - 2\underline{v}_i \cdot \underline{v}_j \rangle}$$

$\langle v_i^2 \rangle = \langle v^2 \rangle$, $\langle \underline{v}_i \cdot \underline{v}_j \rangle = 0$ ∴ $\boxed{\langle v_{rel} \rangle \sqrt{2}\sqrt{\langle v^2 \rangle}}$

So interaction volume $= \pi d^2 \sqrt{2} \sqrt{\langle v^2 \rangle}$

∴ $\boxed{\ell = \dfrac{1}{\pi \sqrt{2}\, d^2 n}}$

Define Knudsen's number $\boxed{kn = \dfrac{\ell}{d}}$

$k \gg 1$ Statistical mechanics
$k \ll 1$ Continuum mechanics

So $\boxed{kn = \dfrac{1}{\pi \sqrt{2}\, n d^3}}$

For ideal gas and $\boxed{\dfrac{R}{N_A} = k_B}$

$pV = \dfrac{nV}{N_A} RT$ ← # moles

Boltzmann's constant.

$k_B = 1.38 \times 10^{-23}$ J k^{-1}

⑨

So $p = n k_B T$ $\therefore$ $n = \dfrac{p}{k_B T}$

$\therefore$ $\boxed{\lambda_n = \dfrac{k_B T}{\pi \sqrt{2} d^3 p}}$

$d = 0.3 \times 5^{-9} m$ $T = 293 k$
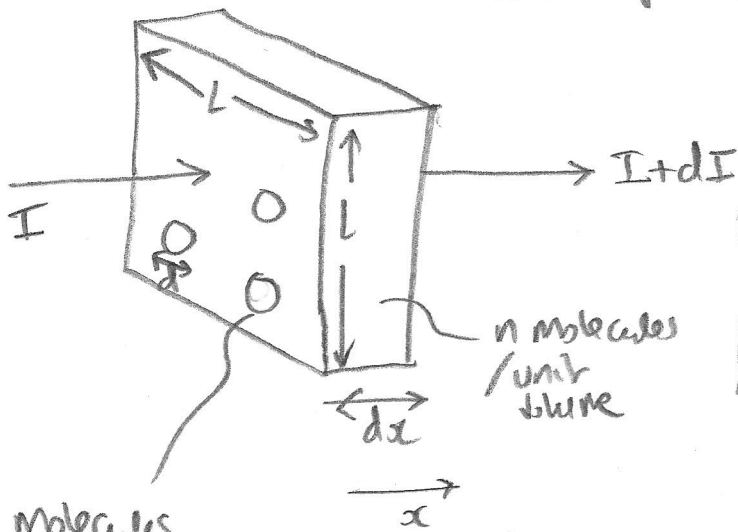
$p = 10^5 Pa$ (atmospheric pressure)

$\Rightarrow$ $\boxed{\lambda_n \approx 333}$ and $\boxed{\ell \approx 1.0 \times 5^{-7} m}$

So Statistical mechanics is justified for air at S.T.P (standard temp & pressure).

$\boxed{\text{Beer lambert law \& mean free path (Similar idea to attenuation of } \gamma \text{ rays)}}$

Another exponential decay!

consider a particle beam of I molecules / unit area passing though a slab of matter of dimensions $L \times L \times dx$.

$\rightarrow I + dI$

I

n molecules / unit volume

dx

$\vec{x}$

$\boxed{dI = -I \times \text{probability of absorption over range } dx}$

P_{absorb}

Molecules with 'colliding cross section $\boxed{\sigma = \sqrt{2} d^2 \pi}$

{ok $\sqrt{2} d^2 \pi$ is a little contrived....!}

$\therefore dI = -I n \sigma dx$

$\Rightarrow \displaystyle\int_{I_0}^{I} \dfrac{dI}{I} = -n\sigma \int_0^x dx$

$P_{absorb} = \text{molecules in } L^2 dx \times \dfrac{\text{molecular area}}{L^2}$

$= n L^2 dx \times \dfrac{\sigma}{L^2}$

$\therefore \ln\left(\dfrac{I}{I_0}\right) = -n\sigma x$

$\boxed{I = I_0 e^{-n\sigma x}}$
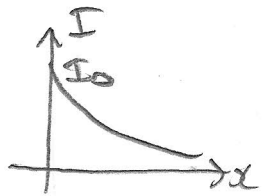
$\therefore$ "Survival probability" (or probability of a molecular collision)

is $\quad p(x) dx = k e^{-n\sigma x} dx$

↑ between x and $x + dx$

$\Rightarrow k = n\sigma$

Now $\displaystyle\int_0^\infty p(x) dx = 1$

So $\boxed{p(x) = n\sigma e^{-n\sigma x}}$

So mean free path $\ell = \displaystyle\int_0^\infty n\sigma x\, e^{-n\sigma x} dx$

ie $\ell = \displaystyle\int_0^\infty x\, p(x) dx$

$$\Rightarrow l = \left[-xn\sigma \frac{1}{n\sigma} e^{-n\sigma x} \right]_0^\infty - \int_0^\infty \left(-\frac{1}{n\sigma} \right) e^{-n\sigma x} n\sigma \, dx$$

$$\boxed{l = \frac{1}{n\sigma}} \qquad \text{so} \qquad \boxed{l = \frac{1}{n\sqrt{2}\pi d^2}} \quad \text{as before}$$

In summary:
$$\boxed{\begin{array}{c} I = I_0 \, e^{-x/l} \\[2mm] P(x) = \dfrac{e^{-x/l}}{l} \end{array}}$$

$$l = \frac{1}{n\sqrt{2}\pi d^2}$$
mean free path.

$\boxed{\text{A random walk}} \xrightarrow{x}$ consider a 1D motion
$\boxed{x = l \sum_{i=1}^{N} d_i}$
Displacement

where step size is mean free path l

and $a_i = \begin{cases} +1 \\ -1 \end{cases}$, randomly selected. N steps.

Now $\sqrt{\langle x^2 \rangle} = l \sqrt{\left\langle \left(\sum_{i=1}^{N} d_i \right)^2 \right\rangle} = l \sqrt{\left\langle \underbrace{\sum_{i=1}^{N} d_i^2}_{N \times 1} + \underbrace{\sum_{i=1,j\neq i}^{N}\sum_{j=1}^{N} d_i d_j}_{\substack{\text{zero since} \\ \text{random choice} \\ \text{of } +, -1}} \right\rangle}$

$$\boxed{\sqrt{\langle x^2 \rangle} = l \sqrt{N}}$$

Now if average speed is $\sqrt{\langle v^2 \rangle}$

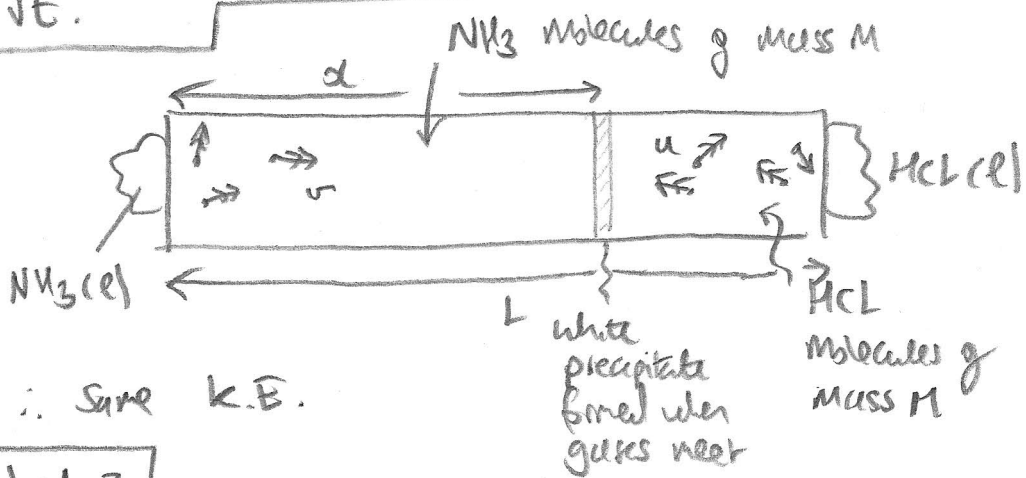$\#$ steps $N = \dfrac{\sqrt{\langle v^2 \rangle}\, t}{l}$ $\qquad \therefore \boxed{\sqrt{\langle x^2 \rangle} = \left(l \sqrt{\langle v^2 \rangle}\, t \right)^{\frac{1}{2}}}$

$\boxed{\text{We expect diffusion of molecules to have RMS diffusion distance } \propto \sqrt{t}.}$

$\boxed{\text{Graham's law}}$

NH$_3$ molecules of mass M

HCL (g)

NH$_3$(g)

HCL molecules of mass M'

L white precipitate formed when gases meet

If two gases are in thermal equilibrium
$\rightarrow$ same temperature $\therefore$ same K.E.

So $\boxed{\frac{1}{2} m v^2 = \frac{1}{2} M u^2}$

(1) v, u average (RMS) speeds

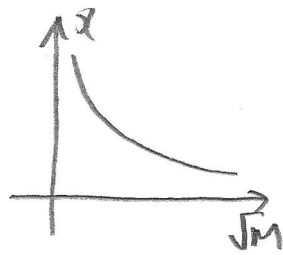Now if precipitate forms after t seconds:

$$x = vt$$
$$L-x = ut$$

$$\therefore \frac{x}{L-x} = \frac{v}{u}$$

Now $\left(\frac{v}{u}\right)^2 = \frac{M}{m}$ $\therefore$ $\boxed{\dfrac{x}{L-x} = \sqrt{\dfrac{M}{m}}}$

So expect diffusion distance $\boxed{x \propto \dfrac{1}{\sqrt{M}}}$

In this example:

$m = 17 \, g/mol$
$M = 36.5 \, g/mol$ $\Rightarrow$ $\dfrac{x}{L-x} = \sqrt{\dfrac{36.5}{17}} \approx 1.47$ $\therefore$ about $3:2$.

$\boxed{\text{Pressure from kinetic theory}}$
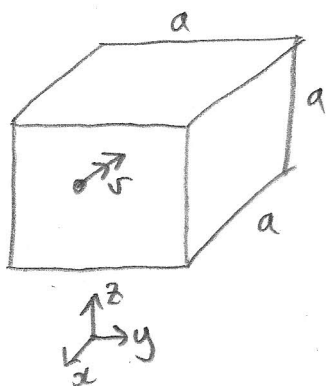
* Assume molecules with velocity $\underline{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$ collide elastically with walls of container of side a.

* $\therefore$ momentum transferred per collision is $\boxed{\Delta p_{x,y,z} = 2m v_{x,y,z}}$

BEFORE   AFTER

* Time between collisions with the same wall is

$$\boxed{\Delta t_{x,y,z} = \frac{2a}{v_{x,y,z}}}$$

* $\therefore$ force on wall $=$ rate of change of momentum $\times$ # particles in cube N

$$= \Delta p_{x,y,z} / \Delta t_{x,y,z} = \boxed{m v_{x,y,z}^2 N / a}$$

* pressure is force/area

$$P_{x,y,z} = \frac{m v_{x,y,z}^2 N}{a^3}$$

$$\therefore \boxed{P = \tfrac{1}{3} \rho \langle v^2 \rangle}$$

Now $v^2 = v_x^2 + v_y^2 + v_z^2$

So $\langle v^2 \rangle = 3 \langle v_{x,y,z}^2 \rangle$

Also density $\rho = mN/a^3$

THE BOLTZMANN FACTOR $e^{-\varepsilon/k_BT}$ & MOLECULAR SPEEDS

Probability of a particle having energy ε_i (in a large system of particles with which it can exchange energy) is given by:

$$p(\varepsilon_i) \propto e^{-\varepsilon_i/k_BT}$$

T absolute temp/k
$k_B = 1.38 \times 5^{-23} \, JK^{-1}$

{for proof see Echichon notes}

For a gas of molecules of mass m, the probability of molecular speed being between v and $v+dv$ is

$$p(v)\,dv = 4\pi \left(\frac{m}{2\pi k_BT}\right)^{3/2} v^2 e^{-\frac{1}{2}mv^2/k_BT}\,dv$$

ie $p(v)\,dv = $ constant $\times$ ⟨$4\pi v^2 dv$⟩ $\times$ Boltzmann factor e^{-KE/k_BT}

$(4\pi v^2 dv)$ is the volume of a spherical shell in velocity space

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

The number of ways of achieving v^2 for are proportional to this volume, hence so is $p(v)\,dv$.
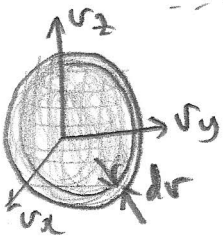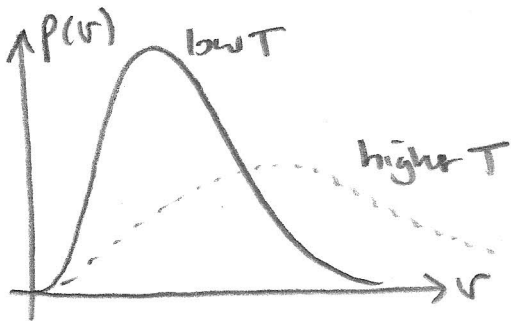
You can find the constant from using some standard integrals of the form

$$\int_0^\infty x^n e^{-ax^2}\,dx = I_n$$

$$\int_0^\infty p(v)\,dv = 1$$

$I_0 = \frac{1}{2}\sqrt{\frac{\pi}{a}}$ $I_1 = \frac{1}{2a}$

$I_2 = \frac{\sqrt{\pi}}{4a^{3/2}}$ $I_n = \frac{n-1}{2a} I_{n-2}$

Now $E[\frac{1}{2}mv^2]$ ie average KE

$$= \frac{1}{2}m \int_0^\infty v^2 p(v)\,dv$$

$$= \boxed{\frac{3}{2}k_BT}$$

(So average KE $\propto T$) **

which indicates the result on page ①

$$U = \frac{\text{\# degrees of freedom}}{(d)} \times \frac{1}{2}k_BT$$

⑬

BOLTZMANN & GIBBS ENTROPY S

* Clausius proposes 2nd law: $ds_{TOT} > 0$ where $\boxed{ds = \dfrac{dQ}{T}}$

* Boltzmann & Gibbs propose a <u>statistical</u> basis for entropy

$$\boxed{S = -k_B \sum_i^N p_i \ln p_i}$$

Gibbs entropy

For a system with N possible states each with probability p_i

$k_B = 1.38 \times 5^{-23}$ JK^{-1}

Simplest case is $\boxed{p_i = \dfrac{1}{W}}$

where W is the number of ways of arranging the system (eg in spatial, or indeed energy terms – or both!)

For N particles with
n_1 of energy level 1
n_2 " " " 2 etc

$$\boxed{W = \dfrac{N!}{\prod_i n_i!}}$$

$\leftarrow$ Think # permutations of VOODOO is $\dfrac{6!}{4!} = 30$

Since $4 \times$ "O" (equivalent to same energy level)

So $S = -k_B \sum_i^N p_i (-\ln W) = k_B \ln W \sum_i^N p_i$ $\sum_i^N p_i = 1$

So $\boxed{S = k_B \ln W}$ Boltzmann entropy

BOLTZMANN FACTOR & CHEMICAL KINETICS

Rate of reaction $\boxed{r = A e^{-\varepsilon/RT}}$

For example 'Sodium Thiosulphate cross' reaction.

$$\boxed{Na_2S_2O_3 (aq) + 2HCl (aq) \rightarrow S(s) + SO_2(g) + H_2O(l) + NaCl (aq)}$$
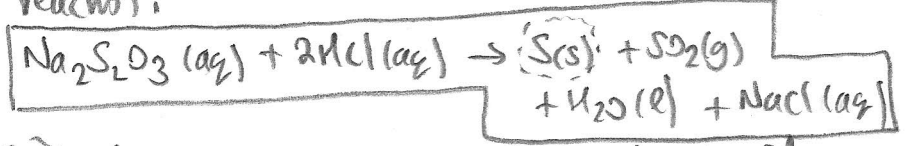
Amorphous sulphur makes the liquid opaque

ε 'Activation energy' J/mol

R Molar gas constant 8.314 $J mol^{-1}k^{-1}$

A Rate constant.

$\boxed{r \simeq \dfrac{1}{t}}$

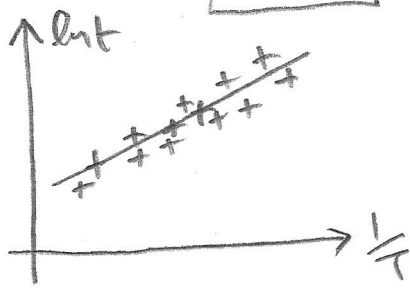Time for cross to vanish.
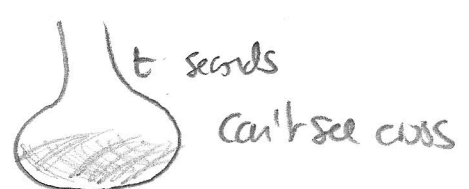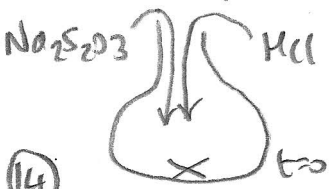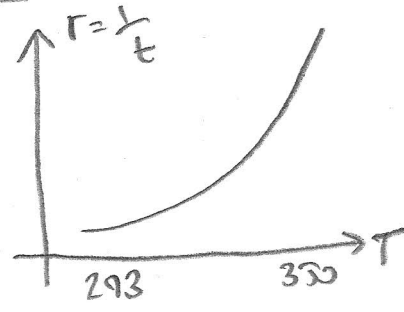
$Na_2S_2O_3$ HCl

X $t=0$

t seconds   Can't see cross

So   if $\boxed{\dfrac{1}{t} = A e^{-\varepsilon/RT}}$ $\ln t = -\ln A + \dfrac{\varepsilon}{RT}$

$\uparrow \ln t$

$\uparrow r = \dfrac{1}{t}$

293   330 $\rightarrow T$

$\{$ 31/1/2019 in Wincall lab $\}$

$\downarrow$

$\boxed{\begin{array}{l} \varepsilon \approx 37.1 \text{ kJ/mol} \\ A \approx 6.14 \times 10^4 \text{ s}^{-1} \end{array}}$

For $50 cm^3$ sodium chlorosulphate, $5cm^3$ g HCl

$\boxed{\text{EVAPORATIVE COOLING AND THE MAXWELL BOLTZMANN SPEED DISTRIBUTION}}$

$\uparrow P(v)$ low T

Higher T

$\rightarrow v/ms^{-1}$

sufficient KE to evaporate
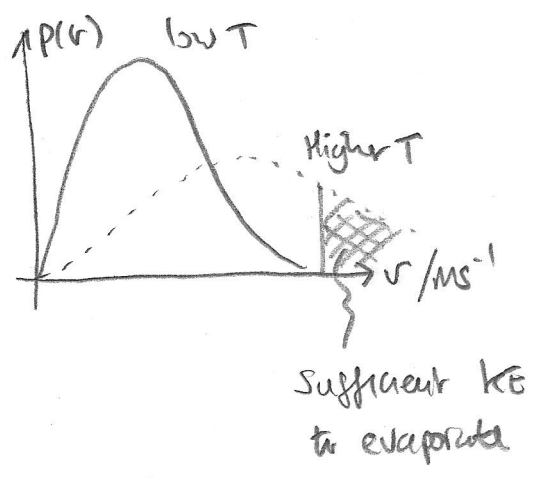
when drying wet clothes, or using air conditioning, idea is to remove the higher energy molecules that have KE sufficient to overcome the latent heat of vaporisation to change state from liquid to gas.

'Removal' is usually via a breeze, open window, and a lack of humidity. (water vapour in the air)

once the most energetic molecules are removed, the average KE $\downarrow$

Since $E[\tfrac{1}{2}mv^2] = \tfrac{3}{2}k_B T$ this will lower the average temperature.

$\boxed{\text{PLANCK RADIATION DISTRIBUTION}}$

$\uparrow B(\lambda)$ Higher T

lower T

$\rightarrow \lambda$

λ_{max}

$\uparrow$ radiation wavelength
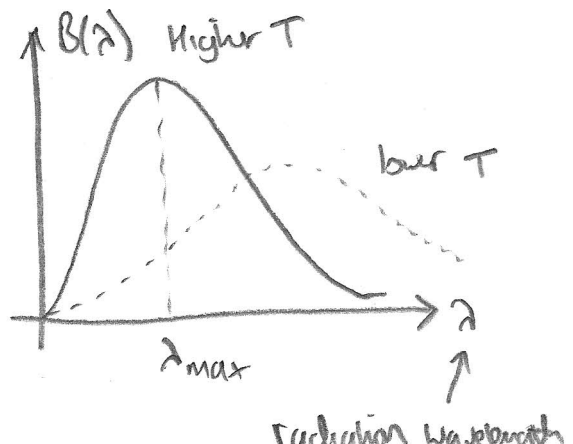
spectral power density of light emitted by a body is:

$$\boxed{B(\lambda) = \frac{2\pi h c^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}}$$

By solving $dB/d\lambda = 0$

$$\boxed{\lambda_{max} \approx \frac{hc}{4.965 \, k_B T}}$$

RED 620 – 750 nm

BLUE 450 – 495 nm

(15)