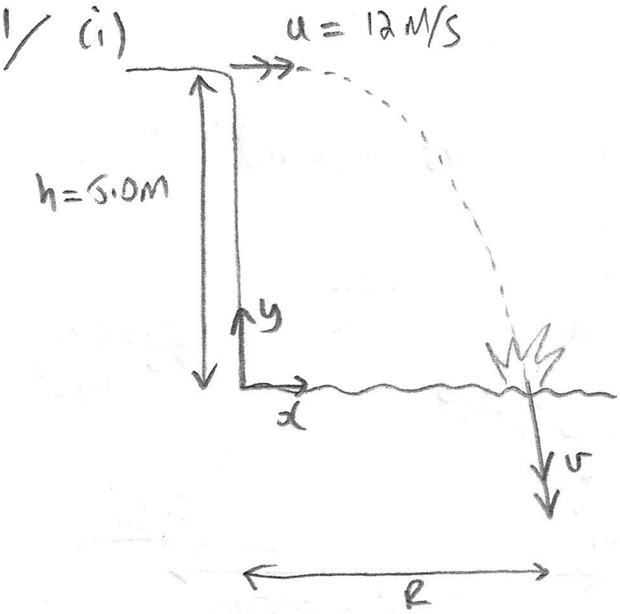
Projectile Motion

1/ (i)



$g = 9.81\,m/s^2$

Projectile motion with no air resistance

$$\boxed{\begin{aligned} &x = ut \\ &y = -\tfrac{1}{2}gt^2 + h \\ &v_x = u \qquad v_y = -gt \end{aligned}}$$

a) $y = 0$ when $h = \frac{1}{2}gt^2$

$\therefore$ Time of flight $t = \sqrt{\frac{2h}{g}}$

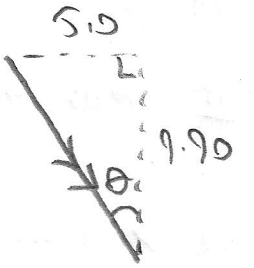
$t = \sqrt{\frac{2 \times 5.0}{9.81}} = \boxed{1.01\,s}$

b) $x = R$ when $t = 1.01s$

so $R = 5.0 \times \sqrt{\frac{2 \times 5.0}{9.81}} = \boxed{5.05\,m}$

d) $v_x = 5.0$ m/s $\qquad v_y = -9.81 \times \sqrt{\frac{2 \times 5.0}{9.81}} = \boxed{-9.90\ m/s}$

(At impact with sea after 1.01s)

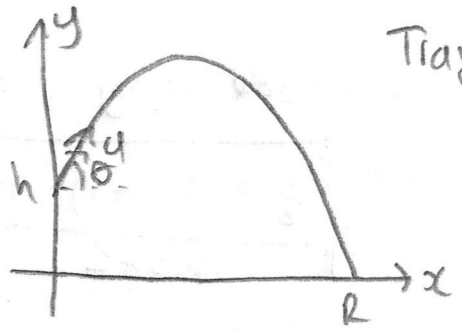


$\theta = \tan^{-1}\left(\frac{5.0}{9.90}\right) = \boxed{26.8^\circ}$ (from vertical)

c) The impact speed is

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{5.0^2 + 9.90^2} = \boxed{11.1\,m/s}$

(ii) a)



Trajectory equation: $\boxed{y = h + x\tan\theta - \frac{g}{2u^2}(1+\tan^2\theta)x^2}$

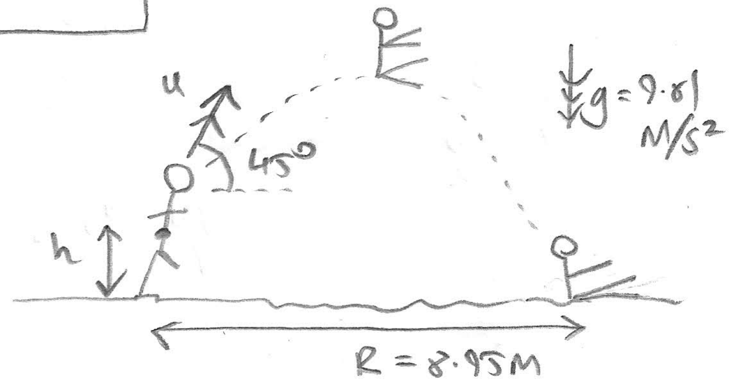
when $y = 0$, $x = R$

$\therefore \frac{g}{2u^2}(1+\tan^2\theta)R^2 = h + R\tan\theta$

$$\therefore \quad u^2 = \frac{g}{2} \frac{(1+\tan^2\theta) R^2}{h + R\tan\theta}$$

$$\therefore \quad \boxed{u = \sqrt{\frac{1}{2} \frac{g(1+\tan^2\theta)R^2}{h + R\tan\theta}}}$$

as required.



(b) World Long Jump record:

$h = 0.94M$
(Mike Powell's COM)

Using the formula for u above:

$$u = \sqrt{\frac{\frac{1}{2} 9.81 (1+1) 8.95^2}{0.94 + 8.95 \times 1}}$$

$\tan 45^\circ = 1$

$$= \boxed{8.91 \, M/s}$$

At this speed he would run 100M in

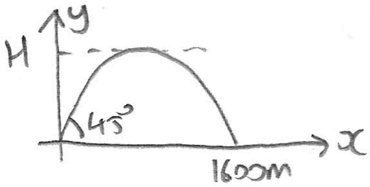
$t = \frac{100M}{8.91M/s} = \boxed{11.2s}$ ie not that fast.

[Usain Bolt's top speed $\approx 12M/s$] →

So could the long jump record be > 9.0M?
[Air resistance likely to be significant here, as is difficulty of turning → speed into $\nearrow$ velocity]

(ii)

"Basilica"
(1453)



a) Muzzle velocity is:

$$u = \sqrt{\frac{\frac{1}{2} 9.81 (1+1) \times 1600^2}{1600 \times 1}}$$

$$\boxed{u = 125 \, M/s}$$