

KINETIC THEORY & THE BOLTZMANN FACTOR

Q1/ Mean free path of a molecule

$$l = \frac{1}{\pi\sqrt{2}d^2n}$$

molecules / unit volume

(i) d is molecular diameter = $0.3 \times 10^{-9} \text{ m}$.

If an ideal gas: $pV = \underbrace{\frac{nV}{N_A}}_{\text{\# mols}} RT$ and $k_B = \frac{R}{N_A}$

$$\therefore n = \frac{p}{k_B T}$$

$$l = \frac{k_B T}{\pi\sqrt{2}d^2 p}$$

$$\begin{aligned} \text{So } l &= \frac{1.381 \times 10^{-23} \times (20+273)}{\pi\sqrt{2} (0.3 \times 10^{-9})^2 \times 101325} \\ &= \boxed{9.99 \times 10^{-8} \text{ m}} \end{aligned}$$

$$kn = \frac{l}{d} = \boxed{333} \quad \text{so a statistical approach is valid.}$$

i.e. the mean free path is \gg than molecular dimensions.
So random collisions between molecules, with plenty of empty space in between.

(ii) $\frac{1}{2} m \overline{v^2} = \frac{3}{2} k_B T$ for an O_2 molecule.

Mean kinetic energy

$$\therefore v_{\text{rms}} = \sqrt{v^2} = \sqrt{\frac{3k_B T}{m}}$$

$$= \sqrt{\frac{3 \times 1.381 \times 10^{-23} \times (20+273)}{15.999 \times 1.661 \times 10^{-27}}}$$

$$= \boxed{687 \text{ m/s}} \quad \text{Pretty fast!}$$

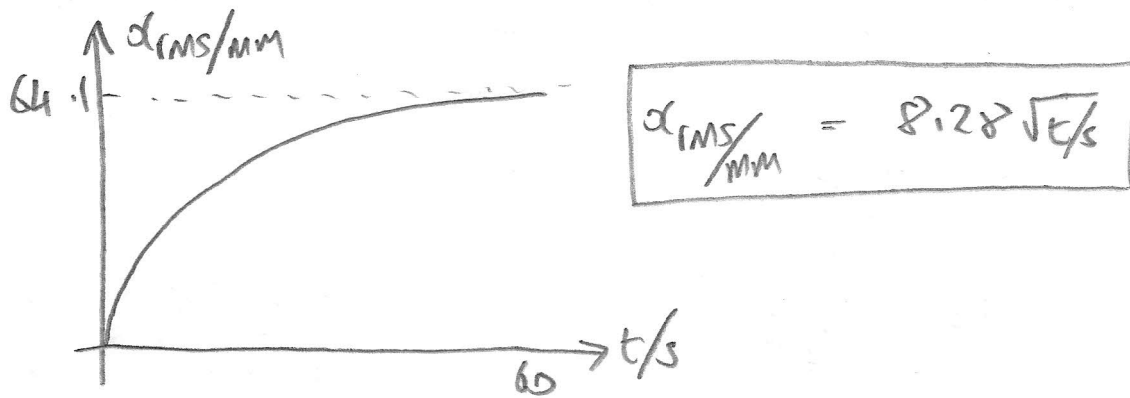
(iii) RMS distance $\sqrt{\langle x^2 \rangle} = \sqrt{l \langle v \rangle t}$

$$\sqrt{l \langle v \rangle} = \sqrt{9.99 \times 10^{-8} \times 687} = 8.28 \times 10^{-3}$$

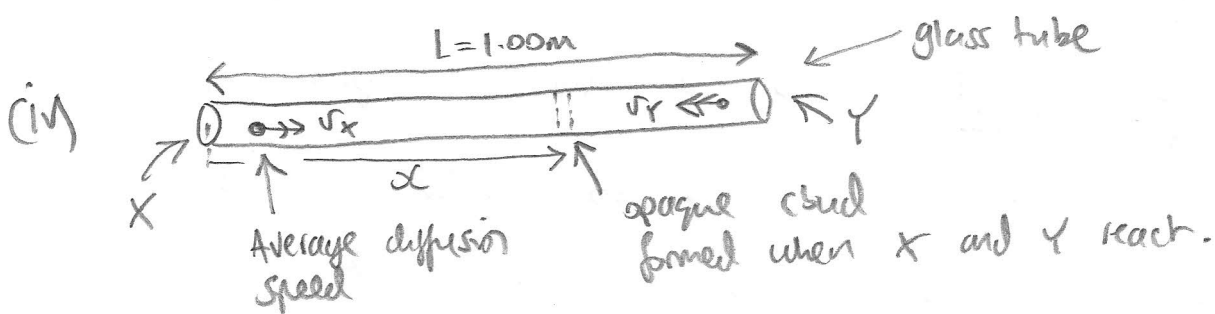
$$\text{So } \boxed{x_{\text{rms}} = 8.28 \times 10^{-3} \sqrt{(t/s)}} \quad (\text{m})$$

In 60s, the O_2 molecule will travel an RMS distance:

$$\begin{aligned} \alpha_{rms} &= 8.28 \times 10^{-3} \times \sqrt{60} \quad (\text{cm}) \\ &= \boxed{64.1 \text{ mm}} \end{aligned}$$



i.e. about 6.41 cm. So not much really.



Constant temperature \therefore all molecules have the same average KE

$$\therefore \frac{1}{2} M_X v_X^2 = \frac{1}{2} M_Y v_Y^2 \quad \Rightarrow \quad \boxed{\frac{v_X}{v_Y} = \sqrt{\frac{M_Y}{M_X}}}$$

Let cloud form after t seconds at α metres from the entry part of gas X.

$$\left. \begin{aligned} \alpha &= v_X t \\ \alpha &= L - v_Y t \end{aligned} \right\} \therefore \begin{aligned} v_X t &= L - v_Y t \\ (v_X + v_Y) t &= L \end{aligned} \quad \Bigg| \quad \therefore \boxed{t = \frac{L}{v_X + v_Y}}$$

\therefore opaque cloud forms at: $\alpha = \frac{v_X L}{v_X + v_Y}$

$$\alpha = \frac{v_X/v_Y}{v_X/v_Y + 1} L \quad \Rightarrow \quad \boxed{\alpha = \frac{\sqrt{M_Y/M_X}}{\sqrt{M_Y/M_X} + 1} L}$$

