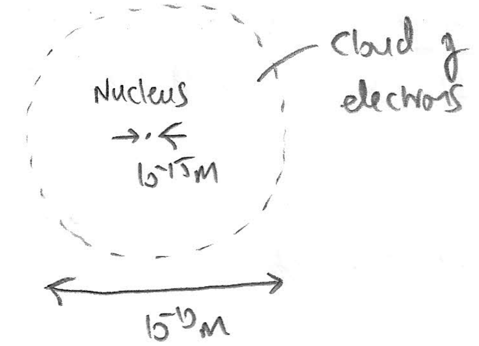
Quantum mechanics problems

1/

(i)



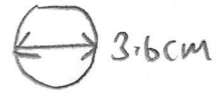
So if a 1m ruler was an atomic nucleus, the electron cloud would extend 100km (!) (ie London from Winchester)

ATOMS ARE MOSTLY EMPTY SPACE

(ii) # atoms in marble is $N \approx \dfrac{\frac{4}{3}\pi\left(\frac{3.6\times10^{-2}}{2}\right)^3}{\frac{4}{3}\pi\left(\frac{1}{2}\times10^{-10}\right)^3}$

$= 3.6^3 \times 10^{-6+30}$

$= \boxed{4.7\times10^{25}}$

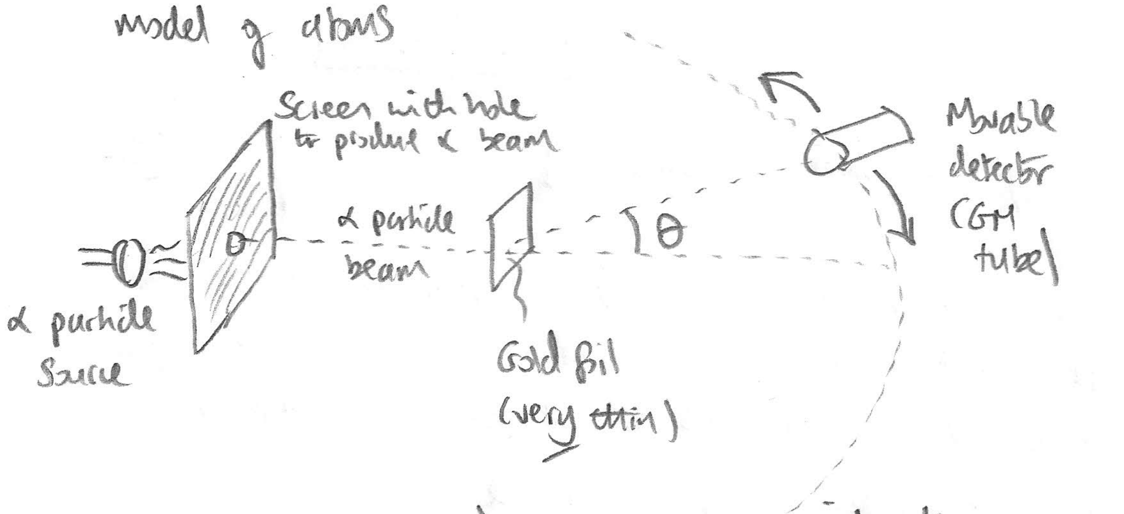


marbles in an Earth sized container (ignore space between marbles)

$$\simeq \frac{4}{3}\pi\left(1.28\times10^{7}/2\right)^3 \Big/ \frac{4}{3}\pi\left(\frac{3.6\times10^{-2}}{2}\right)^3 = \boxed{4.5\times10^{25}}$$

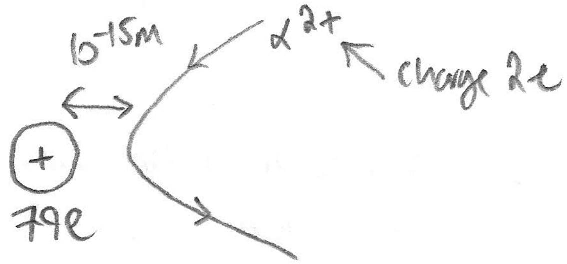
So "number of atoms in a large marble $\simeq$ # marbles that would make an Earth". ATOMS ARE VERY SMALL!

2/ The Rutherford experiment (1908–1913), performed by Hans Geiger and Ernest Marsden provided convincing evidence for the modern nuclear model of atoms



The experiment determined the statistics of scattered α particles (He nuclei from radioactive decay) with angle θ, by a very thin gold foil.

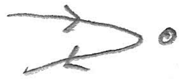
The scattering law (with θ) is consistent with the nucleus behaving like a point particle of charge Ze ($Z=79$ for gold), where $e = 1.6\times10^{-19}$ C, with closest approach $\times10^{-15}$ m



ie Coulomb repulsion between the +ve nucleus and the α particle

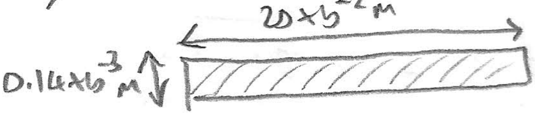
Most particles passed through the gold undeflected 

occasionally deflected 

and very occasionally a 'richochet' 

→ This tells us about the relative size of the nucleus (which contains all the +ve charge and most of the mass of an atom) compared to the atom. ie 10^{-15}m : 10^{-10}m.

3/ power P emitted by light bulb is $\boxed{P = A\sigma T^4}$



=  filament in 

(i) $A = 0.14\times10^{-3} \times 20\times10^{-2}\ m^2 = \boxed{2.8\times10^{-5} m}$

so $\boxed{T = \left(\frac{P}{A\sigma}\right)^{\frac{1}{4}}}$ ∴ if $P = 100$ W

$\Rightarrow T = \left(\frac{100}{2.8\times10^{-5}\times5.67\times10^{-8}}\right)^{\frac{1}{4}}$

$T = 2820$ K

$\boxed{T = 2540\,°C}$

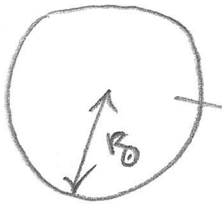
[Melting point of Tungsten ≃ 3422°C]

Assumption is 100W are radiated. Probably a reasonable assumption for total power, but likely most of the light will be outside the narrow visible range 400–700 nm.

so lightbulb won't produce 100W of visible light.

[$\lambda_{max} \approx 2.90\times10^{-3}/2820 = 1030$ nm. ie peak of Black Body spectrum in IR range]
(m)

(ii)



Sun

$T_{\odot} = 5778\,K$

$R_{\odot} = 695{,}500 \times 10^3\ m$

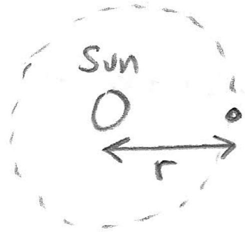
$$\boxed{L_{\odot} = 4\pi R_{\odot}^2 \sigma T^4}$$

Luminosity

$$\therefore\quad L_{\odot} = 4\pi\left(695500\times10^3\right)^2 \times 5.67\times10^{-8} \times 5778^4$$

$$\boxed{L_{\odot} = 3.84\times10^{26}\ J/s} \qquad (\text{or } W)$$

(iii)



Power received by solar panel of area a^2 from Sun, on Mars, is

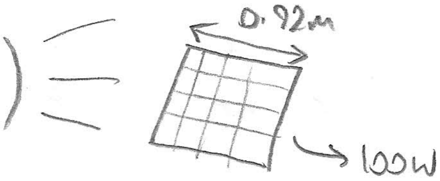
$$\frac{L_{\odot}}{4\pi r^2}\times a^2.$$

let output power P be fraction β of this.

$$\therefore\quad P = \frac{\beta a^2 L_{\odot}}{4\pi r^2} \qquad \therefore\quad \boxed{a = \sqrt{\frac{4\pi P r^2}{\beta L_{\odot}}}}$$

$$\text{so}\quad a = \sqrt{\frac{4\pi\times100\times\left(227.9\times10^6\times10^3\right)^2}{0.2\times3.84\times10^{26}}} \qquad (m)$$

$$\boxed{a = 0.92\ m}$$



(iv)

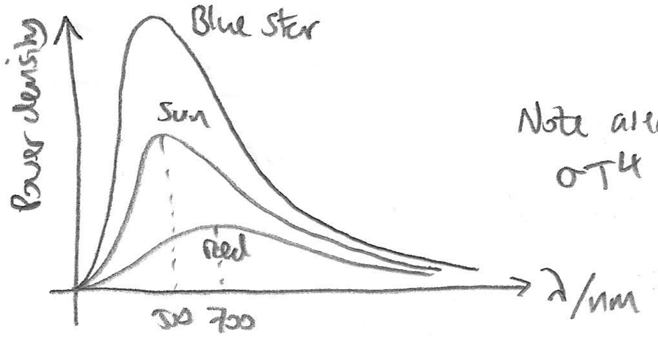
$$\lambda_{max} = \frac{2.90\times10^{-3}}{34+273}\cdot\frac{1}{10^{-9}} \qquad (nm)$$

$$= \boxed{9450\ nm}$$

ie IR radiation is given off by human skin (when human is alive!)

[This explains why IR cameras are used to determine the 'heat signatures' of animals, buildings etc].

(v)



Note area under graph is σT^4, so much greater area for higher temperatures.

[Planck radiation spectra: $\boxed{B(\lambda) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1}}$

$\sigma = \frac{2\pi^5 k_B^4}{15c^2h^3} \approx 5.67 \times 10^{-8}\ Wm^{-2}K^{-1}$]

(vi) Now: $\lambda_{max} = \frac{2.9 \times 10^{-3}}{5778}$ m (for Sun)

$= \boxed{502\ nm}$ (ie yellow)

(Weis: $\lambda_{max} T = 2.9 \times 10^{-3}$ mK)

At red giant phase: $\lambda_{max} = 680$ nm

$\Rightarrow T_{RG} = 2.9 \times 10^{-3} / 680 \times 10^{-9} = \boxed{4265\ K}$ → Calc. memory.

Now $L_\odot = 4\pi R_\odot^2 \sigma T^4$ If this is a constant and R_{RG} is Red Giant radius:

$\Rightarrow R_{RG}^2 T_{RG}^4 = R_\odot^2 T_\odot^4$

So expansion ratio: $R_{RG}/R_\odot = \left(\frac{T_\odot}{T_{RG}}\right)^2$

$= \left(\frac{5778}{4265}\right)^2 = \boxed{1.84}$

4/ (i) W for Silver (Ag) is 4.3eV

To generate photoelectrons, photon energy $\frac{hc}{\lambda} > W$

$\therefore \lambda < \frac{hc}{e} \frac{1}{(W/eV)}$

$\lambda < \frac{6.63 \times 10^{-34} \times 2.998 \times 10^{8}}{1.60 \times 10^{-19} \times 10^{-9}} \frac{1}{(W/eV)}$ (nm)

$\boxed{\lambda < \frac{1242 \text{ nm}}{W/eV}}$ So $\lambda < \frac{1242 \text{nm}}{4.3}$

$\boxed{\lambda < 289 \text{ nm}}$ (ie UV)

(ii) $\boxed{E_k = \frac{hc}{\lambda} - W}$

$\frac{E_k}{eV} = \frac{6.63 \times 10^{-34} \times 2.998 \times 10^{8}}{200 \times 10^{-9} \times 1.6 \times 10^{-19}} - 4.3 = \boxed{1.91 \text{ eV}}$

So $\boxed{E_k = 3.06 \times 10^{-19} \text{ J}}$

(iii) # photons / second absorbed by silver is $\frac{1.00 \times 10^{-6} \text{ J/s}}{\frac{hc}{\lambda} \text{ J/photon}}$

$= \frac{1.00 \times 10^{-6}}{\left(\frac{6.63 \times 10^{-34} \times 2.998 \times 10^{8}}{200 \times 10^{-9}}\right)} = \boxed{1.01 \times 10^{12}}$

(iv) $\therefore$ photocurrent $I = 1.01 \times 10^{12} \times 1.6 \times 10^{-19}$ C/s

$\boxed{I = 1.61 \times 10^{-7} \text{ A}}$ (or 0.16 μA)

(v) Since $E_k = 1.91$eV (see above), stopping voltage is $\boxed{1.91 \text{ volts}}$

5/ (i) $\underbrace{\frac{1}{2}mv^2}_{\text{KE gain}} = \underbrace{eV}_{\substack{\text{work done}\\\text{by electric}\\\text{field}}}$ $\therefore$ $\boxed{v = \sqrt{\frac{2eV}{m}}}$

(ii) $v < 0.01c$

so $\sqrt{\frac{2eV}{m}} < 0.01c$

$\frac{2eV}{m} < 10^{-4}c^2$

$V < 10^{-4} \times \frac{mc^2}{2e}$

so for electron: $V < 10^{-4} \times \frac{9.109\times10^{-31} \times (2.998\times10^8)^2}{2 \times 1.6\times10^{-19}}$

$\boxed{V < 25.6 \text{ Volts}}$

This means we can ignore relativistic effects. (i.e. consequences of special relativity).

This voltage is not that high!

i.e. use classical formula

If we use $v = c$ and $v = \sqrt{\frac{2eV}{m}}$ $\Rightarrow V = \frac{mc^2}{2e}$

$= \boxed{256 \text{ kV}}$

so for $V \approx 1\times10^6$ Volts (i.e. in particle accelerators) we will definitely need to use $\boxed{eV = (\gamma - 1)mc^2}$ instead.

If $V = 5\text{kV}$, $\frac{v}{c} = \frac{\sqrt{\frac{2\times1.6\times10^{-19}\times5000}{9.109\times10^{-31}}}}{2.998\times10^8} = \boxed{0.14}$

{So in a school lab, we can 'just about' ignore relativistic effects - but they won't be negligible}

(iii) Now $KE = \boxed{p^2/2m}$ ($p = mv$ is momentum)

in classical limit $v \ll c$ ie $KE = \frac{1}{2}mv^2$

De Broglie: $\boxed{\lambda = h/p}$

So if $eV = p^2/2m$

$\therefore p = \sqrt{2meV}$

$$\therefore \boxed{\lambda = \frac{h}{\sqrt{2meV}}}$$

(iv)

The intensity pattern is a DIFFRACTION PATTERN (or 'interference pattern')

Electrons are behaving like WAVES (of wavelength $\lambda = h/p$) and are diffracted by the spacings between the layers of carbon atoms in the target.

(v) Maxima of intensity pattern when $\boxed{d\sin\theta = \lambda}$

$d = 1.23 \times 10^{-10}$ m for carbon layers

$\lambda = \frac{h}{\sqrt{2meV}}$ $\therefore d^2\sin^2\theta = \frac{h^2}{2meV}$

$$\therefore \boxed{V = \frac{h^2}{2med^2\sin^2\theta}}$$

$$\therefore V = \left(\frac{6.63\times10^{-34}}{1.23\times10^{-10}\times\sin 10^\circ}\right)^2 \times \frac{1}{2\times9.109\times10^{-31}\times1.6\times10^{-19}}$$

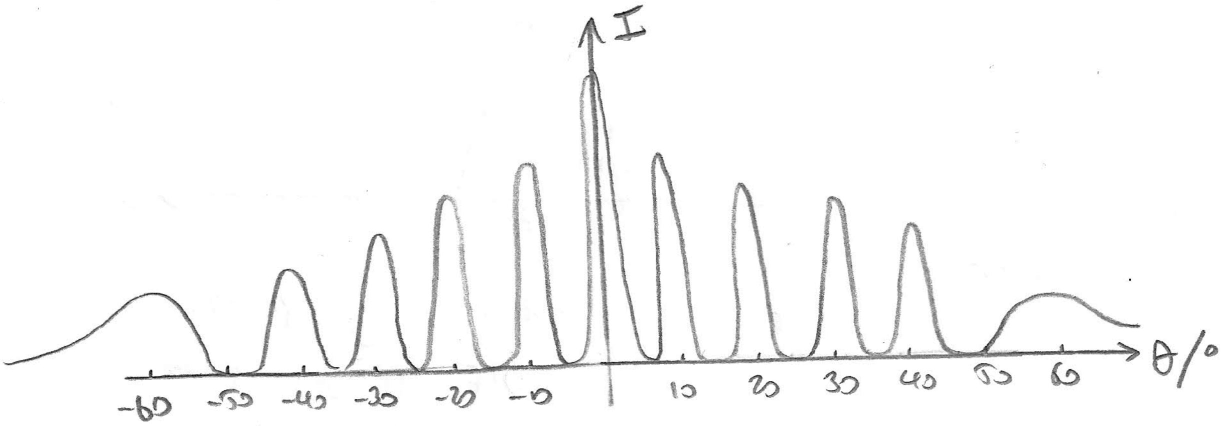
$\boxed{V = 3300 \text{ volts}}$ (ie 3.3 kV)

(vi) other maxima are $\boxed{\theta_n = \sin^{-1}\left(\frac{n\lambda}{d}\right)}$

$$\frac{\lambda}{d} = \frac{h}{d\sqrt{2meV}} = 0.174$$

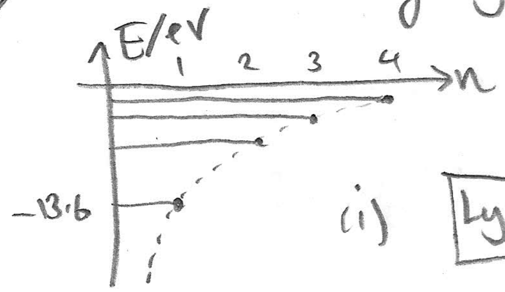
So $\boxed{\theta = 0°, \pm 10°, \pm 20.3°, \pm 31.4°, \pm 44.0°, \pm 60.3°}$

(Note $1/0.174 = 5.76$, so max n is 5)



Note there will be some 'envelope' due to the finite widths of the 'slits' which produce the diffraction pattern.

6/ Bohr model of Hydrogen: $\boxed{E_n = -\frac{13.6\,eV}{n^2}}$ (Energy levels of electron).



(i) $\boxed{\text{Lyman}}$ $\frac{hc}{\lambda} = 13.6\,eV\left(\frac{1}{1} - \frac{1}{3^2}\right)$

$\left[\frac{hc}{13.6e} = 91.3\,nm\right]$

$\therefore \lambda = \frac{91.3\,nm}{1 - 1/3^2} = \boxed{103\,nm}$ ie $\boxed{UV}$

(ii) $\boxed{\text{Balmer}}$ $\lambda = \frac{91.3\,nm}{\frac{1}{2^2} - \frac{1}{4^2}} = \boxed{487\,nm}$ ie $\boxed{\text{Visible}}$

(iii) $\boxed{\text{Pfund}}$ $\lambda = \frac{91.3\,nm}{\frac{1}{5^2} - \frac{1}{7^2}} = \boxed{4660\,nm}$ ie $\boxed{IR}$