1/ Caesium-133 produces radiation of frequency

$f = 9{,}192{,}631{,}770$ Hz $= 9.1926 \times 10^9$ Hz

(i) $\lambda = \frac{c}{f} = \frac{2.998 \times 10^8}{9.1926 \times 10^9} = 3.261 \times 10^{-2}$

ie, $\simeq \boxed{3.26 \text{ cm}}$ This type of radiation is $\boxed{\text{Microwave}}$

(1 MHz $\rightarrow$ 10 GHz)

300m $\rightarrow$ 0.1 cm

(ii) $eV = hf$ { V is equivalent accelerating voltage acting on one electron }

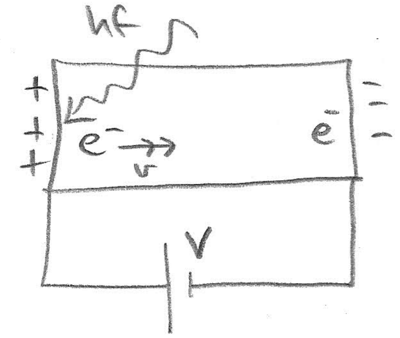
So energy in electron-volts is $\frac{hf}{e} = \frac{6.63 \times 10^{-34} \times 9.1926 \times 10^9}{1.60 \times 10^{-19}}$

$= \boxed{3.81 \times 10^{-5}}$ eV

Now the ionization energy of Hydrogen is 13.6 eV, and typical photoelectric effect work functions for metals are ~ 5 eV.

So this transition is $\boxed{\text{electronic,}}$ between two possibly large quantum number orbitals. Nuclear transitions are ~ MeV

2/ (i) Photoelectric effect



Energy of electron emitted due to absorption of a photon of frequency f is

$E = hf - W$ ← work function of metal

$\therefore$ if "stopping potential" is V

$\boxed{eV = hf - W}$

(i) when $\boxed{hf < W}$, no electrons are emitted.

if $f_{min} = \frac{c}{300 \times 10^{-9}} = 9.993 \times 10^{14}$ Hz, $\boxed{hf_{min} = W}$

$\Rightarrow W = \frac{2.998 \times 10^8 \times 6.63 \times 10^{-34}}{300 \times 10^{-9} \times 1.60 \times 10^{-19}} = \boxed{4.14 \text{ eV}}$

(iii) $P = 100W$ (laser power) 10% efficiency

$\lambda = 200nm$ (laser wavelength)

Energy per laser photon is $hf = \frac{hc}{\lambda}$

Energy of photoelectron is $\frac{hc}{\lambda} - W$

(W from above is 4.141 eV)

If 10% efficient, 10 W results in photoelectrons

$\therefore$ # electrons per second $= \dfrac{10}{\frac{hc}{\lambda} - W}$

$\therefore$ Current is $\boxed{\dfrac{10e}{\frac{hc}{\lambda} - W} = I}$

$$I = \frac{10 \times 1.6 \times 10^{-19}}{\frac{6.63 \times 10^{-34} \times 2.998 \times 10^{8}}{200 \times 10^{-9}} - 4.141 \times 1.6 \times 10^{-19}}$$

$\boxed{I = 4.83 \text{ A}}$

kinetic energy of H_2O

3/ $\lambda = \dfrac{h}{p}$ $\dfrac{p^2}{2M} = 3 \times \frac{1}{2} k_B T$ $\leftarrow$ Average KE of H_2O

$p = \sqrt{3k_B T M}$

$\therefore \lambda = \dfrac{h}{\sqrt{3k_B T M}}$ Now $M \simeq \underbrace{2 \times M_p}_{2 \times {}^1H} + \underbrace{8 \times M_p + 8 \times M_n}_{{}^{16}_{8}O}$

$M \simeq 2 \times 1.6726 \times 10^{-27} + 8 \times 1.6726 \times 10^{-27} + 8 \times 1.6749 \times 10^{-27}$

$m = 3.0125 \times 10^{-26}$ kg

$$\therefore \quad \lambda = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 1.38 \times 10^{-23} \times 300 \times 3.0125 \times 10^{-26}}}$$

$$\boxed{\lambda = 3.43 \times 10^{-11} \text{ m}} \qquad (0.034 \text{ nm})$$

4/ (i) $eV = \frac{1}{2} m_e v^2$ let $v = c/2$

$\therefore \quad eV = \frac{1}{2} m_e \, c^2/4$

$8eV = m_e c^2$

$$\boxed{V = \frac{m_e c^2}{8e}}$$

$$\therefore \quad V = \frac{9.109 \times 10^{-31} \times (2.998 \times 10^{8})^2}{8 \times 1.60 \times 10^{-19}}$$

$V = 6.40 \times 10^{4}$ Volts

ie $\boxed{64 \text{ kV}}$

(ii) $E = (\gamma - 1) m_e c^2 = eV$

$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$ let $\frac{v}{c} = \frac{1}{2}$

$$\therefore \quad \gamma = \left(1 - \frac{1}{4}\right)^{-\frac{1}{2}} = \left(\frac{3}{4}\right)^{-\frac{1}{2}} = \left(\frac{4}{3}\right)^{\frac{1}{2}} = \boxed{\frac{2}{\sqrt{3}}}$$

$$\therefore \quad V = \left(\frac{2}{\sqrt{3}} - 1\right) m_e c^2/e$$

$$V = \left(\frac{2}{\sqrt{3}} - 1\right) \times \frac{9.109 \times 10^{-31} \times (2.998 \times 10^{8})^2}{1.60 \times 10^{-19}}$$

$$= \boxed{79 \text{ kV}} \qquad (7.92 \times 10^{4} \text{ Volts})$$

(iii) $\lambda = \frac{h}{p}$ de-Broglie

$\frac{p^2}{2M} = eV$ classical $\therefore$ $p = \sqrt{2meV}$

$\therefore$ $\boxed{\lambda = \frac{h}{\sqrt{2meV}}}$ Quantum / classical result.

$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.109 \times 10^{-31} \times 1.6 \times 10^{-19} \times 6.40 \times 10^{4}}}$ [Classical version]

classical accelerating voltage.

$= \boxed{4.85 \times 10^{-12}}$ m

$p^2c^2 + M_e^2c^4 = (M_ec^2 + eV)^2$ Relativistic result

Energy Momentum invariant.

$\therefore$ $p = \sqrt{\frac{(M_ec^2 + eV)^2 - M_e^2c^4}{c^2}}$

So $\lambda = \frac{h}{p} \Rightarrow \lambda = \frac{hc}{\sqrt{(M_ec^2 + eV)^2 - M_e^2c^4}}$

$$\boxed{\lambda = \frac{hc}{\sqrt{2m_ec^2eV + e^2V^2}}}$$

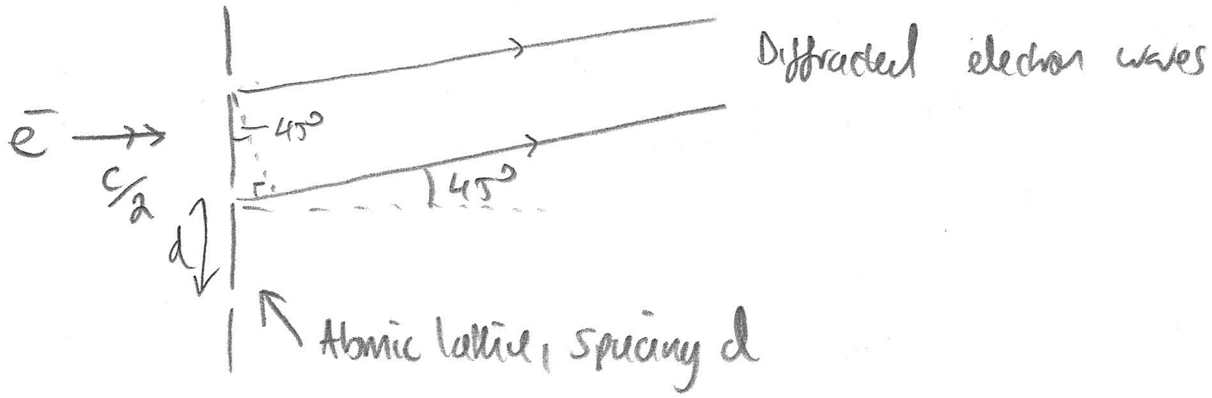
$\lambda = \frac{6.63 \times 10^{-34} \times 2.998 \times 10^{8}}{\sqrt{2 \times 9.109 \times 10^{-31} \times (2.998 \times 10^{8})^2 \times 1.6 \times 10^{-19} \times 7.92 \times 10^{4} + (1.6 \times 10^{-19} \times 7.92 \times 10^{4})^2}}$

$= \boxed{4.20 \times 10^{-12} \text{ m}}$

use relativistic voltage.

(4)

iv)



$\therefore \quad d\sin 45° = n\lambda \quad$ for constructive interference $(n \in \mathbb{Z})$

let $\quad n = 1 \quad . \quad \sin 45° = \frac{1}{\sqrt{2}}$

$\therefore \quad \boxed{d = \lambda\sqrt{2}}$

So for relativistic situation (most accurate)

$d = \sqrt{2} \times 4.20 \times 10^{-12}$

$\boxed{d = 5.95 \times 10^{-12} \text{ m}}$

This is a bit small for atomic spacing $\approx 10^{-10}$m

if $\quad d = 10^{-10}$ m

$10^{-10} \sin\theta = 4.20 \times 10^{-12}$

$\sin\theta = 4.20 \times 10^{-2}$

$\boxed{\theta \approx 2.4°}$

$\Rightarrow$ Question change to 2° diffraction angle

$\Rightarrow d = \dfrac{4.20 \times 10^{-12}}{\sin 2°}$

$\approx \boxed{1.2 \times 10^{-10} \text{ m}}$

5/ Bohr model:

$$\boxed{E_n \simeq -13.6\,\text{eV} \times Z^2/n^2}$$

To ionize $\boxed{E = 13.6\,\text{eV} \times Z^2}$

Ionizing photons

Now $E = hf = hc/\lambda$

$\therefore \lambda = hc/E$

$$\therefore \boxed{\lambda = \frac{hc}{13.6\,e\,Z^2}}$$

(i) a)

$$\lambda_H = \frac{6.63\times10^{-34} \times 2.998\times10^{8}}{13.6 \times 1.6\times10^{-19}}$$

$$= \boxed{91.3\ \text{nm}}$$

b) $\lambda_{He} = \dfrac{91.3}{2^2} = \boxed{22.8\ \text{nm}}$

c) $\lambda_{Rn} = \dfrac{91.3}{86^2} = \boxed{0.0124\ \text{nm}}$

(ii)

	Element	Ionization energy (kJ mol^{-1})	Ionization energy (eV per atom)	Bohr ionization energy (eV per atom)	λ_{actual} /nm	λ_{Bohr} /nm
z=1	H	1312	13.6	13.6	91.3	91.3
z=2	He	2372.3	24.6	54.4	50.5	22.8
z=86	Rn	1037	10.8	100,586	115.0	0.0124

$$\boxed{\frac{E_{mol}}{N_A} \cdot \frac{1}{e}}$$ is Ionization energy in eV

ie $$\boxed{E/\text{eV} \simeq E/\text{kJmol}^{-1} \times 1.0378\times10^{-2}}$$

$6.022140857 \times 10^{23}$
Avogadro's number

use $E = \frac{hc}{\lambda} \Rightarrow \boxed{\lambda = \frac{hc}{E}}$ to find the wavelength of ionizing radiation

$$\Rightarrow \boxed{\lambda \cong \frac{1242\ \text{nm}}{E/\text{eV}}}$$

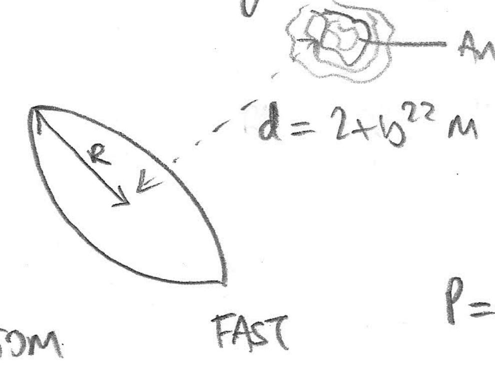
This 'practical conversion formula' idea is very useful. Common in Engineering.

So as $Z \uparrow$, the Bohr model becomes worse and worse.

For Radon, it is out (λ or E) by about 10^4

why? well the model of one electron exposed to Ze charge is incorrect for larger atoms. The repulsion from other electrons must be significant, especially as Z (and $\therefore$ the number of electrons) increases.

6/



If Andromeda radiates isotropically at $P = 8\times10^{27}$ W at 1420 MHz

Power per unit area a distance d away is $P/4\pi d^2$

$\therefore$ Power received by FAST is $\pi R^2 \times \frac{P}{4\pi d^2} = \boxed{\frac{1}{4}P\left(\frac{R}{d}\right)^2}$

Each photon has energy hf

$\therefore$ # photons received by FAST per second

$$= \frac{\text{Energy received per second}}{\text{photon energy}}$$

$$= \frac{\frac{1}{4} P \left(\frac{R}{d}\right)^2}{hf}$$

$$= \boxed{\frac{PR^2}{4d^2hf}} = \frac{8\times10^{27} \times 250^2}{4\times(2\times10^{22})^2\times6.63\times10^{-34}\times1420\times10^6}$$

$$= \boxed{3.3\times10^{11}} \text{ photons / second}$$

Now a 24mm × 36mm DSLR receives energy

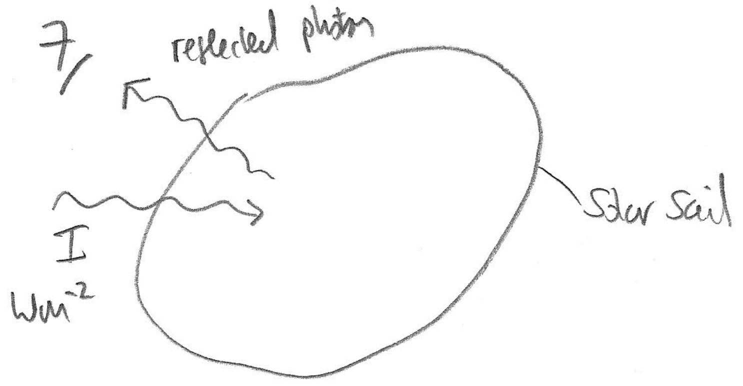
$$\underbrace{24\times10^{-3}\times36\times10^{-3}}_{\substack{\text{Area of}\\ \text{CCD}}} \times \underbrace{100}_{\substack{\text{Power}\\ (\text{Js}^{-1}/\text{m}^2)\\ \text{of sun}\\ \text{per m}^2}} \times \underbrace{\frac{1}{500}}_{\substack{\text{Shutter}\\ \text{opening}\\ \text{time}}} = 1.728\times10^{-4}\ \text{J}.$$

$$\therefore \frac{1.728\times10^{-4}}{\underbrace{6.63\times10^{-34}}_{h}\times\underbrace{\frac{2.998\times10^8}{500\times10^{-9}}}_{f=\frac{c}{\lambda}}} = \boxed{4.35\times10^{14}\text{ photons}}$$

So to take an equivalent "exposure" with FAST, it will need to stare at Andromeda for 1317s or $\simeq$ $\boxed{22 \text{ minutes}}$

hf is energy of (500 nm) solar photon

7/



(i) $\lambda = \frac{h}{p}$ $\therefore$ $p = \frac{h}{\lambda}$ p is Momentum of photon of wavelength λ

Now reflected photon has it's direction reversed, $\therefore$ the sail must provide an impulse of $2p$ to it. $\therefore$ each photon exerts and $\boxed{\text{impulse of } 2p}$ on the sail. ie impulse $\boxed{\Delta p = \frac{2h}{\lambda}}$

(ii) The energy of each photon is $hf = \frac{hc}{\lambda}$

$\therefore$ # photons striking the sail per second is

$$\frac{I \times A}{hc/\lambda}$$ ← total energy

$\therefore$ Total impulse per second is (which is the force upon the sail)

$$F = \underbrace{\frac{IA\lambda}{hc}}_{\text{\#photons per second}} \times \underbrace{\frac{2h}{\lambda}}_{\Delta p \text{ impulse on sail per photon.}}$$

$\boxed{F = 2IA/c}$ (ie h, λ cancel. If Pressure $P = F/A$ $\therefore$ $\boxed{P = \frac{2I}{c}}$ is the formula for radiation pressure.

As Rees suggests (Physics by Example pp142) This could be derived from classical wave theory.

Newton II: $Ma = F$

$\therefore$ $\boxed{a = \frac{2IA}{Mc}}$

a: acceleration, M: spacecraft mass.

(iii) If light has intensity of $1\,kW\,m^{-2} = I$
(500 nm doesn't matter, only total power) ↑ ie $10^3\,Wm^{-2}$

$m = 10 \times 10^3\,kg = 10^4\,kg$

$A = 100 \times (10^3\,m)^2 = 10^8\,m^2$

∴ using classical physics, $v = at$ ← time; ↑ velocity; acceleration

∴ $t = v/a$

∴ $\boxed{t = v \times \frac{mc}{2IA}}$

So if $v = c/100$

∴ $t = \frac{mc^2}{200IA} = \frac{10^4 \times (2.998 \times 10^8)^2}{200 \times 10^3 \times 10^8}$

$= \boxed{4.49 \times 10^7 \text{ seconds}}$

ie $\approx \boxed{1 \text{ year, } 5 \text{ months}}$

8/ $\boxed{I = \sigma T^4}$ Stefan Boltzmann law

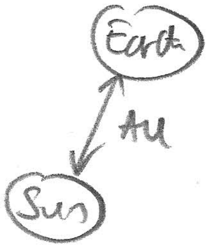
(T is absolute temperature / K

$\sigma = \dfrac{2\pi^5 k_B^4}{15 h^3 c^2} \approx 5.67 \times 10^{-8}\ W\,m^{-2}\,k^{-1}$

I is power / m^2

(i) International Space Station receives 1361 W/m^2 from Sun. If Sun radiates isotropically, and $L_\odot$ is the total power emitted (its "Luminosity")

$L_\odot = 1361 \times 4\pi \times AU^2$



$L_\odot = 1361 \times 4\pi \times (1.496 \times 10^{11})^2$

$\boxed{L_\odot = 3.83 \times 10^{26}\ W}$

If the Sun shines for 10 billion years (10^{10} years) this means energy radiated is

$E = 3.83 \times 10^{26} \times 10^{10} \times 365 \times 24 \times 3600$

$\boxed{E = 1.21 \times 10^{44}\ J}$ $\therefore$ if $E = \Delta M c^2 \Rightarrow \Delta M = \boxed{1.34 \times 10^{27}\ kg}$

$M_\odot = 1.989 \times 10^{30}\ kg$ $\therefore$ % loss in

Mass is $\dfrac{1.34 \times 10^{27}}{1.989 \times 10^{30}} \times 100 \simeq \boxed{0.07\%}$

(ii) Sirius : T = 9940 K.

a) $\dfrac{I_{Sirius}}{I_{Sol}} = \left(\dfrac{9940}{5778}\right)^4 = 8.759$ $\therefore$ expect 1361×8.759 $= \boxed{11.9\ kW/m^2}$

↑ Solar surface temperature

(b) $$L_{Sirius} = \left(\frac{9940}{5778}\right)^4 \times L_{\odot} \times \left(\frac{R_{Sirius}}{R_{Sol}}\right)^2$$

$$= \left(\frac{9940}{5778}\right)^4 \times 3.83e^{26} \times \left(\frac{1.711}{1}\right)^2$$

$$= \boxed{9.82e^{27}\ Js^{-1}}$$

(c) If Sirius loses 0.07% of its mass over lifetime t

$$\underbrace{9.82e^{27}\, t}_{\text{Total energy radiated}} = \overbrace{\frac{0.07}{100} \times 2.02 \times 1.989e^{30} \times c^2}^{\text{mass of Sirius lost} \times c^2}$$

$$\therefore\ t = 2.57e^{16}\ s$$

$$\simeq \boxed{0.82 \text{ billion years}}$$

(iii) Power received by a 10 m² solar panel at 1 AU = $1.496e^{11}$ m from Sirius is

$$P = 10 \times \frac{\overbrace{9.82e^{27}}^{\text{Sirius luminosity}}}{4\pi \times (1.496e^{11})^2} = \boxed{349\ kW}$$

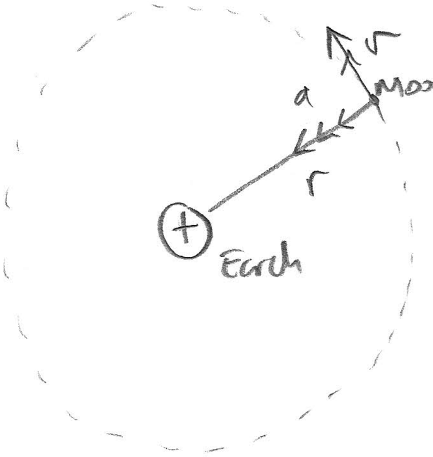
power/m²

1kg → one litre of water at 20°C requires

4181 × 80 = 334.5 kJ to boil (ie reach 100°C) ∴ DogStar heating elements will boil a litre in $\frac{334.5\ kJ}{349\ kJs^{-1}} = \boxed{0.96 \text{ seconds}}$

(ie about a second)

9/ (i) "Bohr model of the Earth - Moon system"



Newton II

$$M_M \frac{v^2}{r} = \frac{G M_\oplus M_M}{r^2}$$

$$\therefore \boxed{v^2 = \frac{GM_\oplus}{r}} \qquad (*)$$

De-Broglie : / wave resonance condition

$$\boxed{2\pi r = n\lambda} \qquad \boxed{\lambda = \frac{h}{M_M v}}$$

$$\therefore \boxed{r = \frac{nh}{2M_M v \pi}}$$

$$\therefore v^2 = GM_\oplus \times \frac{2 m_M v \pi}{nh}$$

$$\boxed{v = \frac{2 G M_\oplus M_M \pi}{nh}} \qquad (a)$$

using $(*)$:

$$r = \frac{GM_\oplus}{v^2}$$

$$r = GM_\oplus \times \frac{n^2 h^2}{4\pi^2 G^2 M_\oplus^2 M_M^2}$$

$$\boxed{r = \frac{n^2 h^2}{4\pi^2 G M_\oplus M_M^2}} \qquad (b)$$

Total energy is $E = \frac{1}{2} M_M v^2 - \frac{G M_\oplus M_M}{r}$

$$E = \frac{1}{2} M_m \frac{G M_\oplus}{r} - \frac{G M_\oplus M_m}{r}$$

$$\boxed{E = -\frac{G M_\oplus M_m}{2r}}$$

$$\therefore \quad E = -\frac{G M_\oplus M_m}{2} \times \frac{4\pi^2 G M_\oplus M_m^2}{n^2 h^2}$$

$$\boxed{E = \frac{-2\pi^2 G^2 M_\oplus^2 M_m^3}{n^2 h^2}} \qquad \text{(c)}$$

iii) using $r = \frac{n^2 h^2}{4\pi^2 G M_\oplus M_m^2} \Rightarrow \boxed{h = \frac{2\pi M_m}{n} \sqrt{G M_\oplus r}}$

and in reality $r = 384{,}400$ km

$G = 6.67 \times 10^{-11}$ Nm^{2}kg^{-2}
$M_\oplus = 5.974 \times 10^{24}$ kg
$M_m = 7.348 \times 10^{22}$ kg

take $n = 1$

$$\therefore \quad h = 2\pi \times 7.348 \times 10^{22} \times \sqrt{6.67 \times 10^{-11} \times 5.974 \times 10^{24} \times 384400 \times 10^3}$$

$$\boxed{h = 1.181 \times 10^{35} \text{ Js}}$$

(iii) Energy of a photon is $E = hc/\lambda$

So # photons from a 100W, 550 nm light bulb every second is

$$\frac{100}{\frac{1.81 \times 10^{35} \times 2.998 \times 10^{8}}{550 \times 10^{-9}}} = 1.01 \times 10^{-48} \quad (!)$$

So we would have to wait 9.9×10^{47} seconds for a single photon to emerge.

Another way of looking at the situation is that photons are about $\frac{1.81 \times 10^{35}}{6.63 \times 10^{-34}} \approx 2.73 \times 10^{68}$ times more energetic in this universe. So a "light bulb" which emits the same # of photons as one on earth would be 2×10^{68} more powerful!

However, 'energy conservation' should be our starting point. If the energy in this universe is equivalent to ours, then photons are likely to be rare. The universe will be very dark.

HF March
2016