Stefan's law states the power per unit area Φ radiated by a body of uniform absolute temperature T is $\Phi = \sigma T^4$ where the Stefan-Bolzmann constant $\sigma = 5.67 \times 10^{-8} \mathrm{Wm^{-2}K^{-1}}$. *Wien's law* states the wavelength $\lambda_{\max}$ corresponding to the peak of the Planck 'Black Body' radiation spectrum is related to the absolute temperature *T* of the radiation source by the equation $\lambda_{\max} T = 2.90 \times 10^{-3} \mathrm{m} \times \mathrm{K}$.

In order to explain the shape of radiation spectrum, electromagnetic radiation must be *quantized* in an energy sense. *Photons* have energy $E = hf$ where f is the frequency (in Hz) and $h = 6.63 \times 10^{-34}$ Js is *Planck's constant.*

If a metal surface is illuminated by light, the maximum energy of *photoelectrons* emitted is given by $E = hf - \phi$ where ϕ is the *work function* of the metal. Typical values are a few (e.g. between 2.0 and 5.0) eV. $1\mathrm{eV} = 1.60 \times 10^{-19}$ J.

The de-Broglie relation relates the momentum p of a particle to its associated wavelength λ : $p = h/\lambda$. This means electrons, protons etc can exhibit wave-like properties such as *diffraction*.

In Special Relativity $p = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} mv$, and energy $E = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} mc^2$ for particles with mass m. The *Energy-Momentum invariant* $E^2 - p^2c^2 = m^2c^4$, so this means for massless particles (like photons) $p = \frac{hf}{c} = \frac{h}{\lambda}$ i.e. de-Broglie's relation is general.

In the *Bohr theory of the Hydrogen atom*, electrons can be thought of as *standing waves*, with their de-Broglie wavelength constrained by a 'circular orbit' of radius r. i.e. $n\lambda = 2\pi r$ where n is a positive integer. This also means that *angular momentum* of the electron is quantized. Note if the electron were to 'actually' orbit, the acceleration would cause it to *radiate*, and therefore spiraling into the proton nucleus within about 1 microsecond. In the Bohr model of a Hydrogen atom, the electron is only allowed to have energies given by the formula $E_n = -\frac{13.6\mathrm{eV}}{n^2}$ where *n* is a positive integer. The wavelength of photons which correspond to transitions between energy level n and energy level m is given by the *Balmer Formula*: $\lambda_{nm} = \frac{8\varepsilon_0 h^3 c}{me^4} \frac{1}{Z^2} \left(\frac{1}{m^2} - \frac{1}{n^2}\right)^{-1} \approx 91.13\mathrm{nm} \frac{1}{Z^2} \left(\frac{1}{m^2} - \frac{1}{n^2}\right)^{-1}$. Z is the number of protons in *Hydrogenic* nucleus.[1]

Question1

(i) To an order of magnitude (i.e. power of ten), what is the radius of an atom (in metres)? How does this compare to the radius of the nucleus of an atom?

(ii) A large blue marble has a diameter of about 3.6cm. Contrast the number of atoms that constitute the marble with the number of marbles that could fill an Earth-sized container. (Earth diameter is about 1.28×10^7 m).

Question 2

Describe briefly *the Rutherford experiment* (with a suitable diagram) and what we know about atomic structure from the conclusions drawn by Geiger and Marsden.

Question 3

(i) An old 100W light bulb produces light by a glowing Tungsten filament. The filament can be modelled as a thin strip of length 20cm and width 0.14mm. Calculate the temperature of the filament in degrees Celsius. What have you assumed in this calculation? Do you think this a reasonable assumption?

(ii) The Sun has a radius of 695,500km and has a surface temperature of about 5778K. Calculate its luminosity (in watts).

[1] i.e. an atom with Z protons and a single electron.

(iii) Calculate the width of a square solar panel (in m) that delivers a maximum of 100W of solar energy on the surface of Mars. Mars is 227.9 million km from the Sun. Assume the solar panel is 20% efficient.

(iv) Human core body temperature is about 37°C and surface skin temperature about 34°C. Determine λ_{max} in nm for the EM radiation given off by human skin. What part of the EM spectrum is this?

(v) Sketch the radiation spectrum for the Sun, vs a cooler red star, and a hotter blue star. Calculate λ_{max} for the Sun (in nm).

(vi) In several billion years, the Sun will probably expand and become a Red Giant. If at the start of this process, the Sun still radiates with the same luminosity as now, calculate the expansion ratio of the solar radius, if $\lambda_{max} = 680\text{nm}$.

Question 4

(i) Silver has a work function of 4.3eV. Calculate the maximum wavelength of light (in nm) that will enable photoelectrons to be produced.

(ii) UV light of wavelength 200nm is shone onto a silver cathode. Calculate the energy of the photoelectrons produced (in J).

(iii) The UV light power received by the silver is 1.00 micro-watts. Assuming all the light is absorbed by the silver, calculate number of photons absorbed per second.

(iv) If one photo-electron is produced per UV photon, calculate the photocurrent (in A). The charge on the electron is $e = 1.60 \times 10^{-19}\text{C}$

(v) Calculate the stopping voltage required to reduce the photocurrent to zero.

Question 5 An electron of mass $m = 9.109 \times 10^{-31}\text{kg}$ and charge *e* is accelerated in a vacuum tube which has a potential difference of V volts.

(i) Find an expression for the final velocity *v* of the electron in terms of m, e, V

(ii) Derive an inequality for *V* such that the electron speed is less than 1% of the speed of light. $c = 2.998 \times 10^{8}\text{ms}^{-1}$. What does this mean we can ignore?

(iii) In the *Classical limit*, show that the de-Broglie wavelength of the electron is given by: $\lambda = \dfrac{h}{\sqrt{2meV}}$

(iv) An electron beam is directed into a carbon disc target, which is thought to have layers of atoms about $1.23 \times 10^{-10}\text{m}$ apart. Sketch a diagram of electron intensity on a screen, vs angle from the original beam direction. Explain briefly *why* it has this shape, and what this tells you (perhaps mysteriously!) about the physical characteristics of electrons. [To do this properly, consider *Bragg scattering* from alternate atomic layers in the carbon disc].

(v) The first peak of the intensity pattern is at an angle of 10.0^{o}. Calculate the accelerating voltage to achieve this.

(vi) Calculate the angles of all the other peaks, and hence make a more accurate sketch than in part (iv)

Question 6 The *Lyman* series of (emission) spectral lines is a transition from E_n to E_1 where $n \geq 2$. Calculate the wavelength (in nm) of the spectral line when $n = 3$. Repeat the calculation for a *Balmer* line (E_4 to E_2), and a *Pfund* line (E_7 to E_5). Which parts of the EM spectrum are these?