Stefan's law states the power per unit area Φ radiated by a body of uniform absolute temperature T is $\Phi = \sigma T^4$ where the Stefan-Bolzmann constant $\sigma = \frac{2\pi^5 k_B^4}{15h^3c^2} = 5.67\times10^{-8}\,\mathrm{Wm^{-2}K^{-1}}$. *Wien's law* states the wavelength $\lambda_{\max}$ corresponding to the peak of the Planck 'Black Body' radiation spectrum is related to the absolute temperature *T* of the radiation source by the equation $\lambda_{\max}T = 2.90\times10^{-3}\,\mathrm{m}\times\mathrm{K}$.

In order to explain the shape of radiation spectrum, electromagnetic radiation must be *quantized* in an energy sense. *Photons* have energy $E = hf$ where f is the frequency (in Hz) and $h = 6.63\times10^{-34}\,\mathrm{Js}$ is *Planck's constant.*

If a metal surface is illuminated by light, the maximum energy of *photoelectrons* emitted is given by $E = hf - \phi$ where ϕ is the *work function* of the metal. Typical values are a few (e.g. between 2.0 and 5.0) eV. $1\mathrm{eV} = 1.60\times10^{-19}\,\mathrm{J}$.

The de-Broglie relation relates the momentum p of a particle to its associated wavelength λ : $p = h/\lambda$. This means electrons, protons etc can exhibit wave-like properties such as *diffraction*. In Special Relativity $p = \left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} mv$, and total energy $E = \left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} mc^2$ for particles with mass m. The *Energy-Momentum invariant* $E^2 - p^2c^2 = m^2c^4$, so this means for massless particles (like photons) $p = \frac{hf}{c} = \frac{h}{\lambda}$ i.e. de-Broglie's relation is general.

In the *Bohr theory of the Hydrogen atom*, electrons can be thought of as *standing waves*, with their de-Broglie wavelength constrained by a 'circular orbit' of radius r. i.e. $n\lambda = 2\pi r$ where n is a positive integer. This also means that *angular momentum* of the electron is quantized. Note if the electron were to 'actually' orbit, the acceleration would cause it to *radiate*, and therefore spiraling into the proton nucleus within about 1 microsecond. In the Bohr model of a Hydrogen atom, the electron is only allowed to have energies given by the formula $E_n = -\frac{13.6\mathrm{eV}}{n^2}$ where *n* is a positive integer. The wavelength of photons which correspond to transitions between energy level n and energy level m is given by the *Balmer Formula*: $\lambda_{nm} = \frac{8\varepsilon_0 h^3 c}{me^4}\frac{1}{Z^2}\left(\frac{1}{m^2}-\frac{1}{n^2}\right)^{-1} \approx 91.13\mathrm{nm}\frac{1}{Z^2}\left(\frac{1}{m^2}-\frac{1}{n^2}\right)^{-1}$. Z is the number of protons in *Hydrogenic* nucleus.[1]

Physical constants Avogadro's number $N_A = 6.022140857\times10^{23}$ is the number of atoms per mole

Planck's constant	$h = 6.63\times10^{-34}\,\mathrm{Js}$	Speed of light	$c = 2.998\times10^{8}\,\mathrm{ms^{-1}}$
Electron mass	$m_e = 9.109\times10^{-31}\,\mathrm{kg}$	Atomic mass unit	$u = 1.661\times10^{-27}\,\mathrm{kg}$
Proton mass	$m_p = 1.6726\times10^{-27}\,\mathrm{kg}$	Neutron mass	$m_n = 1.6749\times10^{-27}\,\mathrm{kg}$
Electron charge	$e = 1.60\times10^{-19}\,\mathrm{C}$	Permittivity of free space	$\varepsilon_0 = 8.85\times10^{-12}\,\mathrm{m^{-3}kg^{-1}s^4A^2}$
Boltzmann's constant	$k_B = 1.38\times10^{-23}\,\mathrm{JK^{-1}}$	Earth radius	$R_\oplus = 6.378\times10^{6}\,\mathrm{m}$
Earth mass	$M_\oplus = 5.974\times10^{24}\,\mathrm{kg}$	Sun radius	$R_\odot = 6.960\times10^{8}\,\mathrm{m}$
Sun mass	$M_\odot = 1.989\times10^{30}\,\mathrm{kg}$	Earth-Sun mean separation (1AU)	$= 1.496\times10^{11}\,\mathrm{m}$
Gravitational constant	$G = 6.67\times10^{-11}\,\mathrm{Nm^2kg^{-2}}$	Moon mass	$M_m = 7.348\times10^{22}\,\mathrm{kg}$
Moon-Earth separation	$r_{\oplus m} = 384{,}400\mathrm{km}$	Temperature of the surface of the Sun	$T_\odot = 5778\mathrm{K}$

[1] i.e. an atom with Z protons and a single electron.

Question 1 (Adapted from Squires, *Problems in Quantum Mechanics* pp2)

The idea that a specific atomic transition produces a particular frequency of electromagnetic radiation, can be used to *define* the second as 9,192,631,770 periods of radiation produced by Caesium-133.

(i) Calculate the wavelength of this radiation. What part of the electromagnetic spectrum is this?

(ii) Calculate the energy change in electron-volts which corresponds to this emission. Is this an electronic or nuclear change?

Question 2

(i) It is observed that no electrons are emitted from a metal until light of wavelength 300nm or less is shone upon it. Use this information to calculate the work function of the metal in electron-volts.

(ii) A 100W 200nm laser is shone upon the metal. Assuming the photo-electron current constitutes 10% of the incident power, calculate the size of the current which results from emitted electrons.

Question 3

Estimate the de-Broglie wavelength of a water molecule at 300K. You may assume the kinetic energy of the molecule is equal to $E = 3\times\frac{1}{2}k_B T$ where T is the absolute temperature. Assume the atomic structure is $2\times {}^{1}_{1}\mathrm{H} + {}^{16}_{8}\mathrm{O}$ and assume the total mass of water is the sum of its proton and neutron parts. Ignore the mass of electrons, and the mass-reducing effect of the *binding energy*. i.e. the total mass of water will actually be *less* than the sum of its proton and neutron parts. *This fact, and the idea that the mass deficit is accounted for by energy changes, is the fundamental idea behind Nuclear power and Radioactivity.*

Question 4

(i) Assuming Classical Physics, calculate the voltage required to accelerate an electron to a velocity of half the speed of light. Repeat the calculation using the relativistic formula for Kinetic Energy:

$$E = (\gamma - 1)mc^2, \quad \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

(ii) Calculate the de-Broglie wavelength of the electron in both Classical and Relativistic cases.

(iii) A beam of electrons travelling at half of the speed of light strikes an atomic lattice and is diffracted by an angle of two degrees. Assuming this is the first ($n = 1$) diffraction ring, calculate the spacing which gives rise to the diffraction pattern.

Question 5

Use the Bohr model to predict the maximum wavelength of electromagnetic radiation that will ionize:

(a) Hydrogen (atomic number 1); (b) Helium (atomic number 2); (c) Radon (atomic number 86)

(ii) The actual ionization energies are the following:[2]

Hydrogen: 1312.0 $\mathrm{kJmol^{-1}}$; Helium: 2372.3 $\mathrm{kJmol^{-1}}$; Radon: 1037 $\mathrm{kJmol^{-1}}$

(a) Comment on the differences between the measured values and those of the Bohr model. What is assumed about the Bohr model that is significantly different from the meaning of the ionization energies above?

(b) Work out the wavelength of photons associated with the actual ionization energies.

[2] https://en.wikipedia.org/wiki/Molar_ionization_energies_of_the_elements

Question 6 (Adapted from Rees, *Physics by Example* pp141)

The Chinese Five hundred metre Aperture Spherical Telescope (FAST) has a diameter of 500m. It is pointed at the Andromeda Nebula, which is at a distance of $2\times10^{22}\,\mathrm{m}$. The telescope is tuned to receive signals at 1420MHz. Calculations based upon Nebula size and temperature (inferred from the spectrum of light received) predict the nebula radiates at $8\times10^{27}\,\mathrm{W}$ at 1420MHz.

Use the information above to predict the number of photons which will be received per second by FAST. Then compare this to the number of photons received when a 24mm x 36mm Digital SLR is illuminated with sunlight at 100 Wm^{-2} for 1/500 of a second. (This is a typical 'shutter speed' for general photography). How long must FAST stare at Andromeda to obtain an equivalent exposure to a DSLR in sunlight? *Use 500nm as the typical wavelength of Sunlight.*

Question 7

A solar sail of area A square metres reflects incident light (with assumed 100% efficiency) which is of intensity I watts per square metre.

(i) If light is of average wavelength λ, use de-Broglie's relation to write an equation for the impulse provided by the light on the sail per second.

(ii) A spaceship, which uses a solar sail as its method of propulsion has mass m, (which includes the solar sail). Use part (i) to determine a equation for its acceleration due to *radiation pressure.* Assume the velocity of the spacecraft is such that relativistic effects can be ignored.

(iii) If light at 500nm has intensity 1kWm^{-2}, what is the predicted acceleration of a 10 tonne spacecraft with solar sails of area 100km^2 ? How long will it take to reach a 100th of the speed of light?

(Note at this point we would have to modify Newton's Second Law to take into account relativistic effects - i.e. the spacecraft's velocity will not continue to increase at the same rate. But the rate of time passing for the astronauts will also slow ....)

Question 8

(i) A maximum of 1,361Wm^{-2} of solar radiation is received by the International Space Station. Use this information to estimate the total power emitted by the Sun every second. A *Main Sequence* star like the Sun will typically spend about 10 billion years in this phase. Assuming the Sun has been at the same temperature, use $E = mc^2$ to calculate the % change in its mass over this time period.

(ii) Sirius, the brightest star in the night sky, is estimated to have a surface temperature of 9,940K. It has a radius of 1.711 solar radii and a mass of 2.02 solar masses.

(a) If the Earth were orbiting Sirius at the same distance as the Sun, calculate the maximum radiation power that would be received per square metre on Earth.

(b) Calculate the Luminosity of Sirius i.e. energy radiated per second

(c) If Sirius loses about the same proportion of its mass as the Sun in its lifetime, how long will it last in this phase?

(iii) The specific heat capacity of water is about 4,181 $\mathrm{Jkg}^{-1}\mathrm{K}^{-1}$. *DogStar* heating elements claim to have "equivalent power to a 10m^2 solar panel at one astronomical unit from Sirius!"

How long will they take to boil one litre of water initially at $20^{\circ}\mathrm{C}$?