

QUANTUM MECHANICS 3 (Measurement, wavefunction, uncertainty principle)

i) $\lambda = \frac{h}{p}$ where $p = m_e v$ (classical physics)

(ii) $v = \frac{1}{137} c$ $\therefore \lambda = \frac{h}{m_e c} \times 137$

$$\lambda = \frac{6.63 \times 10^{-34} \times 137}{9.109 \times 10^{-31} \times 2.998 \times 10^8} \quad (\text{m})$$

$$= \boxed{3.3 \times 10^{-10} \text{ m}} \quad \text{i.e. atom sized!}$$

(unsurprisingly, since in Bohr model, $2\pi r = n\lambda$ where r is the 'atomic radius').

If using Special Relativity: $p = (1 - \frac{v^2}{c^2})^{-\frac{1}{2}} m_e v$

$$p = (1 - \frac{1}{137^2})^{-\frac{1}{2}} m_e \frac{c}{137}$$

$$\Rightarrow \boxed{\lambda = 3.3 \times 10^{-10} \text{ m}} \quad (\text{so to 2 sf}$$

S.R effects make no difference)

(iii) If $\lambda_e = \lambda_n \Rightarrow \frac{h}{p_e} = \frac{h}{p_n} \Rightarrow m_e v_e = m_n v_n$

$$\Rightarrow \boxed{\frac{v_e}{v_n} = \frac{m_n}{m_e}}$$

$$\therefore \frac{v_e}{v_n} = \frac{1.6749 \times 10^{-27}}{9.109 \times 10^{-31}}$$

$$\frac{v_e}{v_n} \approx \boxed{1837}$$

Now $E = \frac{p^2}{2m}$ $p = \frac{h}{\lambda}$

$$\text{So } \boxed{E = \frac{h^2}{2\lambda^2 m}}$$

$$\text{So } \boxed{\frac{E_e}{E_n} = \frac{m_n}{m_e} = 1837}$$

\uparrow if $\lambda_e = \lambda_n$

So if $\lambda_e = \lambda_n$, ratio of speeds
and KE is the ratio of the masses.

$$(iii) \quad \Delta x \Delta p \approx \frac{h}{2} \quad \Delta E \Delta t \approx \frac{h}{2} \quad \Delta p \approx mc \quad \Delta E \approx mc^2$$

$$\therefore \boxed{\Delta x \approx \frac{h}{2mc}} \quad \text{and} \quad \boxed{\Delta t \approx \frac{h}{2mc^2}}$$

$$\begin{aligned} Z \text{ boson: } m &= 91.2 \text{ GeV}/c^2 = \frac{91.2 \times 10^9 \times 1.602 \times 10^{-19}}{(2.998 \times 10^8)^2} \text{ kg} \\ &= \boxed{1.626 \times 10^{-25} \text{ kg}} \end{aligned}$$

$$\therefore \Delta x = \frac{6.626 \times 10^{-34}}{2\pi \times 2 \times 1.626 \times 10^{-25} \times 2.998 \times 10^8} = \boxed{1.08 \times 10^{-18} \text{ m}}$$

$$\Delta t = \Delta x/c = \boxed{3.61 \times 10^{-27} \text{ s}}$$

∴ Sub nuclear length scales ($\ll 10^{-15} \text{ m}$) and very short times. { if $c \approx \frac{\Delta x}{\Delta t}$ then like $c = \lambda/T$ for a very high energy γ ray }

$(iv) \quad \psi(x,t) = A \sin(kx - \omega t)$ $\frac{\partial \psi}{\partial x} = k A \cos(kx - \omega t)$ $\frac{\partial^2 \psi}{\partial x^2} = -k^2 A \sin(kx - \omega t)$ $\therefore \frac{\partial^2 \psi}{\partial x^2} = -k^2 \psi$	$\frac{\partial \psi}{\partial t} = -\omega A \cos(kx - \omega t)$ $\frac{\partial^2 \psi}{\partial t^2} = -\omega A (-\sin(kx - \omega t))(-\omega)$ $= -\omega^2 A \sin(kx - \omega t)$ $\therefore \frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi$
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Now $k = \frac{2\pi}{\lambda}$ and $\omega = 2\pi f$ and $c = f\lambda$

so $\omega = \frac{2\pi c}{\lambda} \Rightarrow \boxed{\omega = ck}$ so $\omega^2 = c^2 k^2$

$$\therefore \frac{\partial^2 \psi}{\partial x^2} = -k^2 \psi \quad \frac{\partial^2 \psi}{\partial t^2} = -c^2 k^2 \psi$$

