RADIOACTIVITY

1/ Activity A of Rn gas obeys decay law:

(i)
$$A(t) = A_0 / 2^{t/t_{1/2}}$$

so
$$2^{t/t_{1/2}} = A_0/A$$

$$\frac{t}{t_{1/2}} \ln 2 = \ln(A_0/A)$$

$$\therefore \boxed{t_{1/2} = \frac{t \ln 2}{\ln(A_0/A)}}$$

$$\therefore t_{1/2} = 22 \text{ days} \times \frac{\ln 2}{\ln\left(\frac{1}{0.02}\right)}$$

$$\boxed{t_{1/2} = 3.9 \text{ days}}$$

[Rn-222 has a half life of 3.82 days]

(ii) Carbon-14 activity $A(t) = A_0 / 2^{t/t_{1/2}}$

$$\Rightarrow \boxed{t = \frac{\ln(A_0/A)}{\ln 2} t_{1/2}}$$

{ is same as analysis in (i)}

Fresh biomatter : 238 Bq / kg = A

Reindeer horn hammer : 1.05 Bq/kg = A_0

$t_{1/2}$ for C-14 is 5370 years

$$\therefore \text{ hammer is } \simeq \frac{\ln\left(\frac{238}{1.05}\right)}{\ln 2} \times 5370 \text{ years}$$

$$= \boxed{42{,}000 \text{ years old}}$$

iii) $\boxed{I = I_0 e^{-\mu x}} = \boxed{I_0 / 2^{x/x_{1/2}}}$

so $e^{\mu x} = 2^{x/x_{1/2}}$

$\therefore \mu x = \frac{x}{x_{1/2}} \ln 2$

$\therefore \boxed{\mu = \frac{\ln 2}{x_{1/2}}}$

$I_0 = 200\,Bq$. $I = 10\,Bq$. $x = 2.0\,mm$

$I_0/I = e^{\mu x}$ $\therefore$ $\mu = \frac{1}{x} \ln\left(\frac{I_0}{I}\right)$

$= \frac{1}{2.0\,mm} \ln\left(\frac{200}{10}\right)$

$= \boxed{1.50\,mm^{-1}}$

$x_{1/2} = \frac{\ln 2}{\mu} = \frac{2.0\,mm \times \ln 2}{\ln 20} = \boxed{0.46\,mm}$

(iv) $t_{1/2}$ for Ra-226 is 1602 years. $A = 9{,}900\,Bq = -\frac{dN}{dt}$

$\frac{dN}{dt} = -\lambda N$

$\lambda = \frac{\ln 2}{t_{1/2}}$ so $A = \frac{\ln 2\, N}{t_{1/2}}$

$\Rightarrow N = t_{1/2} A / \ln 2$

Now mass of one Ra-226 atom $\approx$ 218 u

So mass of Ra-226 $\simeq \boxed{218\,u\, t_{1/2} A / \ln 2}$

$= 226 \times 1.661 \times 10^{-27} \times 1602 \times 9{,}900 / \ln 2 \times 365 \times 24 \times 3600$

$= \boxed{2.71 \times 10^{-10}\,kg}$

Repeat for U - 235:

$$M = 235u\, t_{\frac{1}{2}} A / \ln 2$$

$$M = 235 \times 1.661 \times 10^{-27} \times 2.22 \times 10^{16} \times 9{,}100 / \ln 2$$

$$= \boxed{1.24 \times 10^{-4} \text{ kg}} \qquad \text{(i.e. about } 0.124 \text{ mg)}$$

So for 1 kg of radioactive material: $M = (Z+N) u t_{\frac{1}{2}} A / \ln 2$

$$\therefore \boxed{A = \frac{M \ln 2}{(Z+N) u t_{\frac{1}{2}}}}$$

So for Radium - 226: $A/\text{kg} = \frac{\ln 2}{226} \frac{1}{1.661 \times 10^{-27}} \frac{1}{5.052 \times 10^{10}}$

$$= \boxed{3.65 \times 10^{13} \text{ Bq/kg}}$$

whereas for U-235 this is:

$$A/\text{kg} = \frac{\ln 2}{235} \frac{1}{1.661 \times 10^{-27}} \frac{1}{2.22 \times 10^{16}}$$

$$= \boxed{8.00 \times 10^{7} \text{ Bq/kg}}$$

(iv) ${}^{231}_{91}Pa \rightarrow {}^{227}_{89}Ac + {}^{4}_{2}\alpha \qquad t_{\frac{1}{2}} = 32{,}800$ years.

$\frac{dN}{dt} = -\lambda N$ where N is the # of Pa 231 atoms

So $\boxed{N(t) = N_0 e^{-\lambda t}}$ # lead atoms is $N_0 - N$

So proportion Pb is $\frac{N_0 - N}{N_0} = 1 - \frac{N}{N_0} = 1 - e^{-\lambda t}$

$e^{-\lambda t} = 2^{-t/t_{\frac{1}{2}}}$ So $0.9 = 1 - 2^{-t/32{,}800 \text{ years}}$

$\Rightarrow 2^{t/32{,}800 \text{ yr}} = \frac{1}{0.1} \Rightarrow t = 32{,}800 \text{ yrs} \times \frac{\ln 10}{\ln 2}$

$\Rightarrow$ $\boxed{t = 109{,}000 \text{ years}}$

2/ (i) $^{123}_{53}I + {}^{0}_{-1}e^{-} \longrightarrow {}^{123}_{52}Te + \nu_e$ (+ γ ← 159 keV)

(ii) * γ rays emitted can easily be detected outside the body - it can $\therefore$ be used as a biological tracker.

* γ rays are weakly ionizing, so anticipate lower risk of cell damage (certainly compared to α, β of similar energies)

* Half life of 13.22 hours is long enough to run the medical diagnostic, but not too long such that a radioactive source remains in the body.
$^{123}_{52}Te$ is stable and $\therefore$ not radioactive.

(iii) $\boxed{A = A_0 / 2^{t/t_{1/2}}}$ so $A(24) = 2.5 \times 10^7 / 2^{24/13.22}$

$= \boxed{7.1 \times 10^6 \text{ Bq}}$

The activity in the thyroid gland will be less since I-123 will have dispersed in the body.

(iv) $A = -\frac{dN}{dt} = +\lambda N$ $\left\{ \frac{dN}{dt} = -\lambda N \right\}$

$\lambda = \frac{\ln 2}{t_{1/2}}$ so $N_0 = A_0 / \lambda$ $\Rightarrow$ $\boxed{N_0 = \frac{A_0 t_{1/2}}{\ln 2}}$

$N_0 = \frac{2.5 \times 10^7 \times 13.22 \times 3600}{\ln 2}$ $= \boxed{1.72 \times 10^{12} \text{ atoms}}$

(v) Total energy $= \boxed{N_0 \times 159 \text{ keV}}$ (ie if all N_0 decay)

$= 1.72 \times 10^{12} \times 159 \times 10^3 \times 1.602 \times 10^{-19}$ J

$= \boxed{0.044 \text{ J}}$

(vi) $\underset{P}{\text{Power}} / Js^{-1} = 159\text{ keV} \times \underbrace{A}_{\text{activity}}$

So when $t=0$: $P = 159 \times 10^3 \times 1.602 \times 10^{-19} \times 2.5 \times 10^7$ J/s

$= \boxed{6.37 \times 10^{-7} \text{ J/s}}$

compare to $\dfrac{N_0 \times 159\text{ keV}}{48 \times 3600} = \boxed{2.53 \times 10^{-7} \text{ J/s}}$

(Assume $\simeq$ all atoms have decayed after 48 hours.

The more accurate calculation is:

$$\frac{N_0 - N_0/2^{48/13.22}}{48 \times 3600} \times 159\text{ keV} = \underbrace{0.92}_{1 - \frac{1}{2^{48/13.22}}} \times 2.53 \times 10^{-7} \text{ J/s}$$

$= \boxed{2.33 \times 10^{-7} \text{ J/s}}$).

So initial power is $\simeq$ 2.7 $\times$ average power over 48 hours.

3/ (i) $^{241}_{95}Am \rightarrow {}^{237}_{93}Np + {}^{4}_{2}\alpha$ $\quad E_\alpha = 5.486$ MeV.

$t_{\frac{1}{2}} = 432.2$ years.

(ii) mass of Am-241 atom is:

$\simeq 241 \times 1.661 \times 10^{-27}$ kg $\quad$ (ie 241u).

$\therefore$ # atoms initially $= N_0 \simeq \dfrac{0.33 \times 10^{-6} \times 10^{-3}}{241 \times 1.661 \times 10^{-27}}$

$= \boxed{8.24 \times 10^{14}}$

Initial Activity $\boxed{A_0 = \dfrac{\ln 2}{t_{\frac{1}{2}}} N_0} = \dfrac{\ln 2}{432.2 \times 365 \times 24 \times 3600} \times 8.24 \times 10^{14}$ Bq

$(A = -\frac{dN}{dt} = \lambda N)$ $\quad = \boxed{4.2 \times 10^{4} \text{ Bq}}$ $\quad$ (42 kBq)

(iii) one α particle will cause $\simeq \frac{5.486 \times 10^6}{34}$

$= \boxed{1.61 \times 10^5}$ ionizations. Assume these happen

on timescales $\ll 1s$.

The initial activity is 4.2×10^4 Bq {use 41904.56... in calc}

So $\therefore$ expect $4.2 \times 10^4 \times 1.61 \times 10^5$

$= 6.76 \times 10^9$ ionizations / s.

$\therefore$ a current of $6.76 \times 10^9 \times 1.602 \times 10^{-19} \times 2$ A

$= \boxed{2.17 \times 10^{-9} \text{ A}}$

α particle has a 2+ charge

(iv) Assume thermal equilibrium:

So $\frac{1}{2} M \overline{v_s^2} = \frac{1}{2} M_{air} \overline{v_{air}^2} = \frac{3}{2} k_B T$

So $M = M_{air} \times \frac{\overline{v_{air}^2}}{\overline{v_s^2}}$ $\quad M_{air} = 28.97$ g/mol

$\boxed{\text{Now might expect the current to be } \propto \sqrt{v_{air}}}$

So if current in (mostly smoke) is 0.2 of what it was before $\Rightarrow \sqrt{\overline{v_s^2}} \simeq 0.2 \sqrt{\overline{v_{air}^2}}$

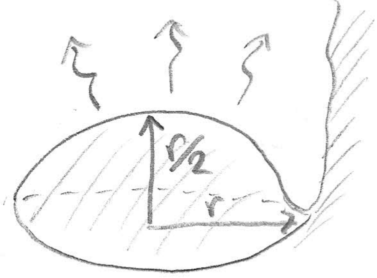
RMS speeds

So $\overline{v_s^2} = 0.04 \, \overline{v_{air}^2}$

$\therefore$ $M \simeq M_{air} \frac{1}{0.04}$

$M \simeq \frac{28.97 \text{ g/mol}}{0.04} = \boxed{724.3 \text{ g/mol}}$

4/



E of radiation particles is 5 MeV.

$\boxed{\text{Elephant's foot}}$

$r \approx 1.0\,m$

(i) A $(10cm)^3$ cube of water has a mass $\approx$ 1kg.

So it should absorb $\frac{80J}{3600s}$

ie $A = \dfrac{\frac{80J}{3600s}}{5\times10^6 \times 1.602\times10^{-19}}$ 22mW

particles of radiation absorbed /s

$= \boxed{2.77\times10^{10}\ Bq}$ *

(ii) Surface area of foot (including base) is

$\pi\left(r^2 + \left(\frac{r}{2}\right)^2\right) + \pi r^2$

$= \pi r^2\left(1 + 1 + \frac{1}{4}\right)$

$= \frac{9\pi r^2}{4}$

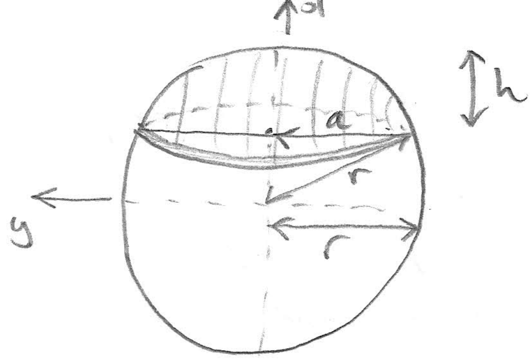
$= \frac{9\pi}{4} \times 1.0\,m^2$

$= \boxed{7.07\,m^2}$

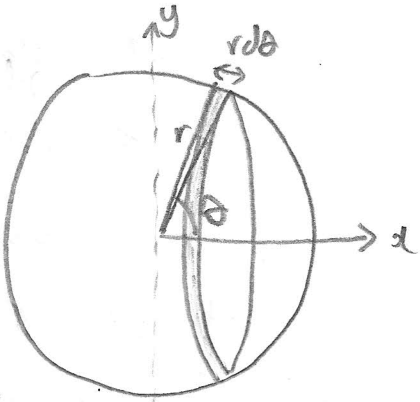
Cube has a 'foot facing' side of area $(10^{-1}m)^2$

$\therefore$ Total activity of foot

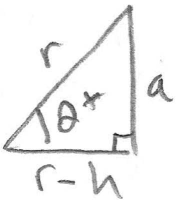
$\approx \dfrac{9\pi/4}{10^{-2}} \times 2.77\times10^{10}\ Bq$



Surface area of a spherical cap



$$A = \int_0^{\theta^*} 2\pi r \sin\theta \times r\,d\theta$$



$r\sin\theta^* = a$
$r\cos\theta^* = r-h$

$$A = 2\pi r^2 \Big[-\cos\theta\Big]_0^{\theta^*}$$

$$A = 2\pi r^2\left[-\cos\theta^* + 1\right]$$

$$A = 2\pi r^2\left(1 - \frac{r-h}{r}\right)$$

$$A = 2\pi\left(r^2 - r(r-h)\right)$$

$$\boxed{A = 2\pi r h}$$

Now $r^2 = a^2 + (r-h)^2$

$\cancel{r^2} = a^2 + \cancel{r^2} - 2rh + h^2$

So $2rh = a^2 + h^2$

$\therefore \boxed{A = \pi(a^2+h^2)}$

ie total activity $\approx$ $\boxed{1.96 \times 10^{13} \text{ Bq}}$

(Possibly slightly less since cube of water would absorb radiation from the sides, especially if γ).

(ii) Cont. Expect an inverse-square law of particle flux from foot. So if $3\times$ away, $\frac{1}{9} \times$ flux. If α, β, get some attenuation too – but if γ, can probably ignore this since μ for γ in air is fairly negligible. ie half-thickness for γ in air is going to be <u>many metres</u>

particularly α.

ie expect $\frac{1}{r^2}$ loss to dominate for a γ source.

(iii) Assume γ detector has a cross section of $\approx 1\text{ cm}^2$ so would receive $\approx 1/100$ of radiation absorbed by the water cube. ie $A_0 = 2.77 \times 10^8$ Bq.

If $x_{\frac{1}{2}} = 4.2$mm of lead is needed to reduce A to $A/2$ (background), what is x s.t. $A = 5$ Bq?

$$A = A_0 \Big/ 2^{x/x_{\frac{1}{2}}} \qquad \therefore \quad 2^{x/x_{\frac{1}{2}}} = A_0/A$$

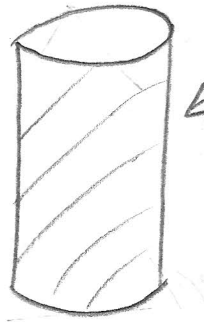
$$\frac{x}{x_{\frac{1}{2}}} \ln 2 = \ln(A_0/A)$$

$$\boxed{x = x_{\frac{1}{2}} \ln(A_0/A) / \ln 2}$$

$$\therefore \quad x = \frac{4.2\text{mm} \times \ln\left(2.77 \times 10^8 / 5\right)}{\ln 2} = \boxed{108 \text{ mm}}$$

(8) (ie 10.8cm of Pb!) So Elephant's foot too dangerous to inspect in 1986.

5)



radioisotope thermal generator (RTG)

$M = 4.8\,kg$ of Pu-238

$t_{\frac{1}{2}} = 87.7$ years

$E_\alpha = 5.593\,MeV$

$${}^{238}_{94}Pu \rightarrow {}^{234}_{92}U + {}^{4}_{2}\alpha$$

(i) # atoms in 4.8kg of Pu-238 are:

$$N_0 = \frac{4.8\,kg}{238 \times 1.661 \times 10^{-27}\,kg} \simeq \boxed{1.21 \times 10^{25}}$$

Initial activity is $A_0 = \frac{\ln 2}{t_{\frac{1}{2}}} N_0$ (ie $\frac{dN}{dt} = -\lambda N$)

So $A_0 = \frac{\ln 2}{t_{\frac{1}{2}}} \frac{M}{238u}$

$$A_0 = \frac{\ln 2 \times 4.8}{87.7 \times 365 \times 24 \times 3600 \times 238 \times 1.661 \times 10^{-27}}$$

$$\boxed{A_0 = 3.04 \times 10^{15}\ Bq}$$

So thermal power (assuming 100% energy conversion from E_α to heat)

is $A_0 E_\alpha$ $= \underbrace{3.04 \times 10^{15}}_{\text{calc memory}} \times 5.593 \times 10^{6} \times 1.602 \times 10^{-19}$ (J/s)

$= 2727$ J/s ie $\simeq \boxed{2.7\,kW}$

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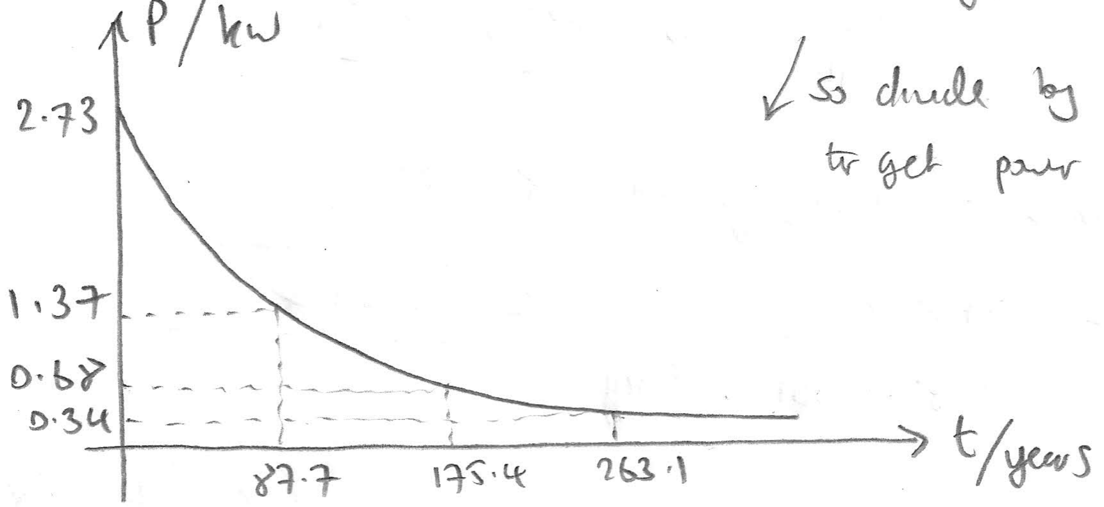
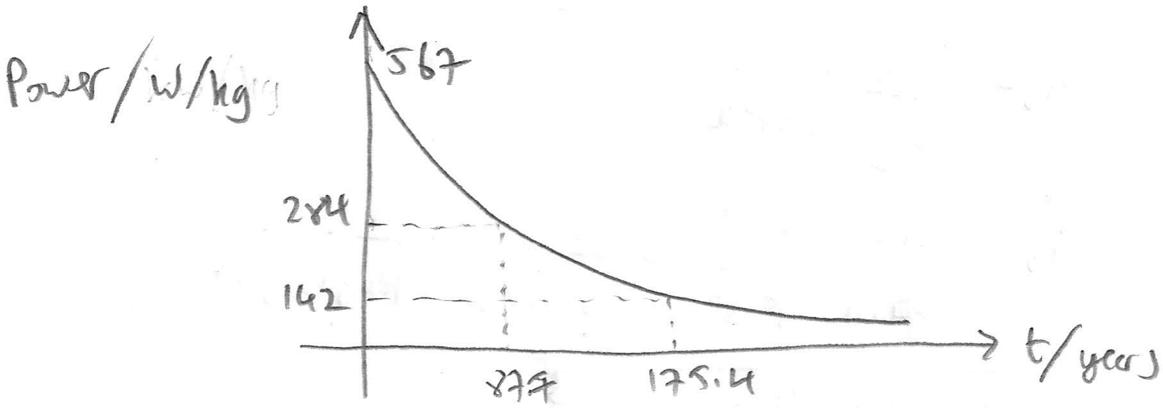
(iii) $P_0 = A_0 E_\alpha \quad \therefore \quad P(t) = A E_\alpha$

$$\therefore \quad \boxed{P(t) = \frac{2.73\ \text{kW}}{2^{t/87.7\ \text{years}}}}$$

This is for 4.8 kg

so divide by 4.8 to get power / kg





6/ See spreadsheet.

Geiger-Nuttall law

If $\boxed{\log_{10}\left(\frac{t_{1/2}}{s}\right) = \frac{151.3}{\sqrt{E_\alpha / \text{MeV}}} - 49.3}$

$^{221}_{87}Fr$ has a ½ life of $t_{\frac{1}{2}} = 10^{\frac{151 \times 87}{\sqrt{6.3}} - 49.3}$

$= 10^{3.04}$ (s)

$= 1094$ s

$\simeq \boxed{18.2 \text{ minutes}}$

(it is actually $\boxed{4.8 \text{ min}}$, which may seem like a large error – but think about the range of $t_{\frac{1}{2}}$!! We are estimating the power of 10).