

RADIOACTIVITY

1/ Activity A of Rn gas obeys decay law:

(i) $A(t) = A_0 / 2^{t/t_{1/2}}$

so $2^{t/t_{1/2}} = A_0/A$

$$\frac{t}{t_{1/2}} \ln 2 = \ln(A_0/A)$$

$$\therefore t_{1/2} = \frac{t \ln 2}{\ln(A_0/A)}$$

$$\therefore t_{1/2} = 22 \text{ days} \times \frac{\ln 2}{\ln(1/0.02)}$$

$$t_{1/2} = 3.9 \text{ days}$$

[Rn-222 has a half life of 3.82 days]

(ii) Carbon-14 activity $A(t) = A_0 / 2^{t/t_{1/2}}$

$$\Rightarrow t = \frac{\ln(A_0/A)}{\ln 2} t_{1/2}$$

{ is same as analysis in (i) }

Fresh biomass : 238 Bq/kg = A

Reindeer horn hammer : 1.05 Bq/kg = A_0

$t_{1/2}$ for C-14 is 5370 years

$$\therefore \text{hammer is } \approx \frac{\ln(238/1.05)}{\ln 2} \times 5370 \text{ years}$$

$$= 42,000 \text{ years old}$$

$$\text{iii)} \quad I = I_0 e^{-\mu x} = \frac{I_0}{2^{x/d_{1/2}}}$$

$$\text{so } e^{-\mu x} = 2^{-x/d_{1/2}}$$

$$\therefore \mu x = \frac{x}{d_{1/2}} \ln 2$$

$$\therefore \mu = \frac{\ln 2}{d_{1/2}}$$

$$I_0 = 200 \text{ Bq} \quad I = 10 \text{ Bq} \quad x = 2.0 \text{ mm}$$

$$\frac{I_0}{I} = e^{\mu x} \quad \therefore \mu = \frac{1}{x} \ln \left(\frac{I_0}{I} \right)$$

$$= \frac{1}{2.0 \text{ mm}} \ln \left(\frac{200}{10} \right)$$

$$= \boxed{1.50 \text{ mm}^{-1}}$$

$$d_{1/2} = \frac{\ln 2}{\mu} = \frac{2.0 \text{ mm} \times \ln 2}{\ln 20} = \boxed{0.46 \text{ mm}}$$

$$\text{iv)} \quad t_{1/2} \text{ for Ra-226 is } 1602 \text{ years. } A = 9,900 \text{ Bq} = -\frac{dN}{dt}$$

$$\frac{dN}{dt} = -\lambda N$$

$$\lambda = \frac{\ln 2}{t_{1/2}} \quad \text{so } A = \frac{\ln 2 N}{t_{1/2}}$$

$$\Rightarrow N = t_{1/2} A / \ln 2$$

Now mass of one Ra-226 atom $\approx 218 \text{ u}$

$$\text{so mass of Ra-226} \approx \boxed{218 \text{ u } t_{1/2} A / \ln 2}$$

$$= 226 \times 1.661 \times 10^{-27} \times 1602 \times 9,900 / \ln 2 \times 365 \times 24 \times 3600$$

$$= \boxed{2.71 \times 10^{-6} \text{ kg}}$$

Repeat for U-235:

$$m = 235 u t_{1/2} A / \ln 2$$

$$m = 235 \times 1.661 \times 10^{-27} \times 2.22 \times 10^{16} \times \frac{1.100}{\ln 2}$$

$$= \boxed{1.24 \times 10^{-4} \text{ kg}}$$

(i.e. about 0.124 mg)

So for 1 kg of radioactive material:

$$m = (Z+N) u t_{1/2} A / \ln 2$$

$$\therefore \boxed{A = \frac{m \ln 2}{(Z+N) u t_{1/2}}$$

So for Radium-226:

$$A/\text{kg} = \frac{\ln 2}{226} \frac{1}{1.661 \times 10^{-27}} \frac{1}{5.052 \times 10^{10}}$$

$$= \boxed{3.65 \times 10^{13} \text{ Bq/kg}}$$

whereas for U-235 this is:

$$A/\text{kg} = \frac{\ln 2}{235} \frac{1}{1.661 \times 10^{-27}} \frac{1}{2.22 \times 10^{16}}$$

$$= \boxed{8.00 \times 10^7 \text{ Bq/kg}}$$



$$t_{1/2} = 32,800 \text{ years.}$$

$\frac{dN}{dt} = -\lambda N$ where N is the # of Pa 231 atoms

So $\boxed{N(t) = N_0 e^{-\lambda t}}$ # lead atoms is $N_0 - N$

So proportion Pb is $\frac{N_0 - N}{N_0} = 1 - \frac{N}{N_0} = 1 - e^{-\lambda t}$

$e^{-\lambda t} = 2^{-t/t_{1/2}}$ So $0.9 = 1 - 2^{-t/32,800 \text{ years}}$

$\Rightarrow 2^{t/32,800 \text{ yr}} = \frac{1}{0.1} \Rightarrow t = 32,800 \text{ yrs} \times \frac{\ln 10}{\ln 2}$

③

