An atomic element ${}_{Z}^{A}X$ has an nucleus comprising Z protons and $A-Z$ neutrons. *Atomic number* Z defines the element (and a unique letter X) , whereas differing *mass number* A defines the *isotope* of an element. The mass of an isotope is *approximately* the mass number multiplied by $u = 1.661 \times 10^{-27}\,\text{kg}$. (*u* is the mass of $1/12$ of the mass of Carbon-12). The difference is due to the *binding energy* between the nucleons, and it is the changes in the binding energy (effectively a loss or gain of atomic mass) which is the basis of nuclear fission or fusion.

Radioactive decay is the spontaneous fragmentation of an atomic nucleus. This is a *random* process, with the probability of decay at any given time being constant for each *isotope*. However, the probability varies enormously between isotopes.

1Bq (Becquerel) = 1 decay per second. 1Ci (Curie) = $3.7 \times 10^{10}\,\text{Bq}$. $1\text{MeV} = 1.602 \times 10^{-19}\,\text{J}$.

The rate of change ('*activity*') of N radioactive particles is: $dN/dt = -\lambda N$ which has solution: $N(t) = N_0 e^{-\lambda t}$.

When $N = \frac{1}{2}N_0, t = t_{1/2}$ which is the *half-life* of a radioactive isotope. $\lambda = \frac{\ln 2}{t_{1/2}}$ i.e. $N(t) = \frac{N_0}{2^{t/t_{1/2}}}$

Alpha decay: ${}_{Z}^{Z+N}\text{X} \rightarrow {}_{Z-2}^{Z+N-4}\text{Y} + \alpha$ e.g. ${}_{90}^{229}\text{Th} \rightarrow {}_{88}^{225}\text{Ra} + \alpha$

Beta+ decay: ${}_{Z}^{Z+N}\text{X} \rightarrow {}_{Z-1}^{Z+N}\text{Y} + e^+ + \nu_e$ e.g. ${}_{11}^{22}\text{Na} \rightarrow {}_{10}^{22}\text{Ne} + e^+ + \nu_e$

Beta - decay: ${}_{Z}^{Z+N}\text{X} \rightarrow {}_{Z+1}^{Z+N}\text{Y} + e^- + \overline{\nu}_e$ e.g. ${}_{6}^{14}\text{C} \rightarrow {}_{7}^{14}\text{N} + e^- + \overline{\nu}_e$

Electron capture: ${}_{Z}^{Z+N}\text{X} + e^- \rightarrow {}_{Z-1}^{Z+N}\text{Y} + \nu_e$ e.g. ${}_{11}^{22}\text{Na} + e^- \rightarrow {}_{10}^{22}\text{Ne} + \nu_e$

ν_e is an *electron neutrino* and $\overline{\nu}_e$ an *electron-antineutrino*. These are very weakly interacting and almost massless particles, that nonetheless convey energy and momentum. Unlike alpha particles, beta particles have a spectrum of energies between 0.01MeV to 10MeV due to the random apportionment between beta and neutrino.

Transitions from a 'excited' nucleus state can also result in the emission of **gamma rays** i.e. energy release but with no change in number of neutrons or protons.

I'm not aware of any rule which predicts half-life from A, Z , but for Alpha decay, a correlation (*The Geiger-Nuttall law*) exists between half life $t_{1/2}$, atomic number and the energy of alpha particles emitted: $\log t_{\frac{1}{2}} = a + \frac{bZ}{\sqrt{E_\alpha}}$. Alpha particle kinetic energy E_α is typically about 5MeV.

The activity of alpha, beta, gamma radiation will attenuate with distance due to *ionization* of molecules. Gamma is weakly ionizing and therefore highly penetrating. The opposite is true for alpha. The intensity I of radiation varies with penetration depth x via $I = I_0 e^{-\mu x}$. In a similar way to half-life, we can define a 'half thickness' $x_{1/2}$ when $I = \frac{1}{2}I_0$,

i.e. $x_{1/2} = \ln 2/\mu$.

Question 1

(i) Naturally occurring radon (Rn) gas can build up in basements and other underground enclosures. A sample of air is taken from a recently unsealed mine in Cornwall, and it takes 22 days for the radon to decay to 2% of original levels, into a cocktail of isotopes of polonium, lead and bismuth. Determine the half-life of the radon gas (in days).

(ii) Carbon-14 has a half life of about 5370 years. When living tissue dies, it ceases to exchange carbon with the biosphere, hence the amount of Carbon-14 decays with time since death. A comparative measurement of Carbon-14 decay rate can therefore be used to date a sample. 1kg of fresh biomatter produces about 238Bq from C-14 decay. A hammer made from reindeer horn was discovered in a cave in Lithuania in 2016 with activity C-14 of 1.05Bq /kg. How old was the hammer?

(iii) A Strontium-90 (Sr-90) beta source has an activity of 200Bq. This is measured by a Geiger-Muller (GM) tube next to the source. When 2.00 mm of aluminum is placed between the source and the GM tube, the activity drops to 10Bq. Calculate the half-thickness, and attenuation constant μ, of aluminum for attenuation of Sr-90 beta particles.

(iv) Radium-226 has a half life of about 1,602 years. A Ra-226 source has an activity of 9,900Bq. What is the mass of Ra-226 in the source? What would the equivalent mass be of Uranium-235 (half-life of $2.22 \times 10^{16}\,\mathrm{s}$) ?

(iv) Protactinium-231 (Pa-231) decays via an alpha emission to Actinium-227 with a half-life of 32,800 years. Ac-227 then decays (via a chain of relatively short-lived isotopes) to lead (Pb-207), which is stable. A pure Pa-231 statuette of Otto Hahn and Lise Meitner (the discoverers of Pa-231 in 1917) is encased in a vacuum flask and blasted into space. This rather bizarre (and expensive!) message-in-a-bottle is eventually discovered by a future alien civilization, and it is now 90% lead. How long has the statuette been floating in space?

Question 2

Iodine-123 has a half-life of 13.22 hours and decays via electron capture to tellurium-123 (Te), which is very stable. The reaction releases gamma radiation of energy 159keV. The atomic number for Iodine is 53.

(i) Write a decay equation for this reaction

(ii) What properties of Iodine-123 make it useful for medical applications?

(iii) An initial activity of $2.5 \times 10^{7}\,\mathrm{Bq}$ is recommended for thyroid imaging. Calculate the total activity after 24 hours. Why will the activity in the thyroid gland be less than this?

(iv) Calculate the number of I-123 atoms in a dose of initial activity $2.5 \times 10^{7}\,\mathrm{Bq}$.

(v) Determine (in J) the total energy absorbed due to gamma emissions

(vi) Determine the power (J/s) of gamma emissions immediately after an I-123 dose has been administered. Compare this to the average power over a 48 hour period.

Question 3

Americium-241 (Am-241) is an alpha emitter, which decays with a half-life of 432.2 years to Neptunium. The alpha particle emitted has an energy of 5.486MeV. About 0.33 μg of Americium Oxide (AmO_2) form the active element of most modern smoke detectors.

(i) Write a decay equation for the decay of Am-241 to Neptunium (Np). The atomic number of Americium is 95.

(ii) Calculate the activity of Am-241 (in Bq, and Ci) of a freshly made smoke detector. Assume the mass number of Oxygen is 16.

(iii) The ionization energy of air (i.e. a mixture of Nitrogen, Oxygen etc) is about 34eV. How many ionizations will a fresh Am-241 source cause per second? Hence calculate the current in a (smokeless) detector.

(iv) The molar mass of dry air is about 28.97g/mol. In a smoke detector, the charged particles caused by alpha ionization tend to stick to the heavier, larger smoke molecules. The current in a smoke detector drops to 20% of its (smokeless) value. Use this to estimate the molar mass of the smoke particles. Assume the smoke is at the same temperature as the dry air in the smokeless scenario.

Question 4 *The Elephant's Foot* is a large mass of *corium* glass lava that formed underneath the remains of the Chernobyl reactor in 1986. About 8 months after its formation, radiation of 80 *grays per hour* were measured near its surface. 1 gray is 1J of absorbed radiation per kg of matter. For humans, whole-body exposure to 5 grays or more is likely to be fatal.

(i) If a typical alpha or beta particle has energies of 5MeV, calculate the number of particles per second that a $10\text{cm}\times10\text{cm}\times10\text{cm}$ cube of water would absorb if placed close to the Elephant's Foot when the radiation was 80 grays /hour. How would the flux of particles change if the water was moved three times as far away from the foot?

(ii) From the famous (1996) photograph of the Elephant's Foot by Artur Korneyev, we may estimate the dimensions to be hemispherical with radius 1.0m and height 0.5m. If one assumes the flux of radiation (i.e. particles per m^2) from the foot to be the same as the particles absorbed by the water cube in (a), estimate the total activity of the foot. Assume the surface area of the foot in 1996 is the same as in 1986. (In actual fact is would have flowed somewhat during this time). The curved surface area of a spherical cap of radius a and height h is $A=\pi\left(a^2+h^2\right)$. [EXTRA FOR THE KEEN - prove this relationship using calculus].

(iii) Assume 4.2mm of lead is needed to reduce gamma radiation from the Elephant's Foot (in 1986) by a factor of two. If the gamma activity measured by an unshielded detector is $2.77\times10^8\,\text{Bq}$, calculate the thickness of lead required to reduce the gamma count rate to a reasonable background level of about 5Bq.

Question 5 In the film (and book) *The Martian*, astronaut Mark Watney uses a *radioisotope thermal generator* (RTG) to stay warm. The RTG contains about 4.8kg of plutonium-238. Pu-238 has a half-life of 87.7 years and decays to Uranium-234 via alpha emission. The kinetic energy of alpha particles emitted are 5.593MeV.

(i) Calculate how many Pu-238 atoms are in 4.8kg of Pu-238. Hence determine the initial activity and power (J/s) in the form of electrical energy (and ultimately heat) produced by the ionization of molecules in the RTG casing. Assume 100% conversion of alpha particle kinetic energy to thermal energy produced by the RTG.

(ii) Perform appropriate calculations and hence sketch (or plot) a graph of heat power /kg of a Pu-238 RTG vs time /years.

Question 6

(i) Construct a spreadsheet and verify the *Geiger-Nuttall law* for alpha decay using the following data[1] :

Isotope	Atomic number Z	Half life $t_{1/2}$ /s	Alpha energy E_α / MeV
Polonium-212	84	2.99×10^{-7}	8.955
Polonium-214	84	1.60×10^{-4}	7.680
Radon-218	86	3.50×10^{-2}	7.263
Astatine-217	85	3.20×10^{-2}	7.000
Bismuth-213	83	2.74×10^{3}	5.870
Radium-224	88	3.14×10^{5}	5.789
Thorium-238	90	6.03×10^{7}	5.520
Protactinium-231	91	1.03×10^{12}	5.150
Neptunium-237	93	6.75×10^{13}	4.959
Thorium-232	90	4.43×10^{17}	4.081

(ii) Verify from (a) that $\log_{10}\left(t_{\frac{1}{2}}/s\right)\approx\dfrac{1.51\times Z}{\sqrt{\left(E_\alpha/\text{MeV}\right)}}-49.3$, and use this to predict the half life of Francium-221, (Z = 87) given $E_\alpha=6.3\text{MeV}$.

[1] http://web.vu.lt/ff/a.poskus/files/2013/06/NP_No11.pdf