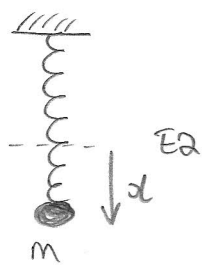


SIMPLE HARMONIC MOTION (SHM) AND SMALL OSCILLATIONS

1/ (i)



$$\omega = \sqrt{\frac{k}{m}}$$

Since from Newton II

$$m\ddot{x} = -kx$$

$$\therefore \ddot{x} = -\frac{k}{m}x$$

[Compare to SHM equation $\ddot{x} = -\omega^2 x$]

$$\omega = \frac{2\pi}{T} \quad \therefore \quad \frac{T}{2\pi} = \sqrt{\frac{m}{k}} \quad \Rightarrow \quad \boxed{T = 2\pi \sqrt{\frac{m}{k}}}$$

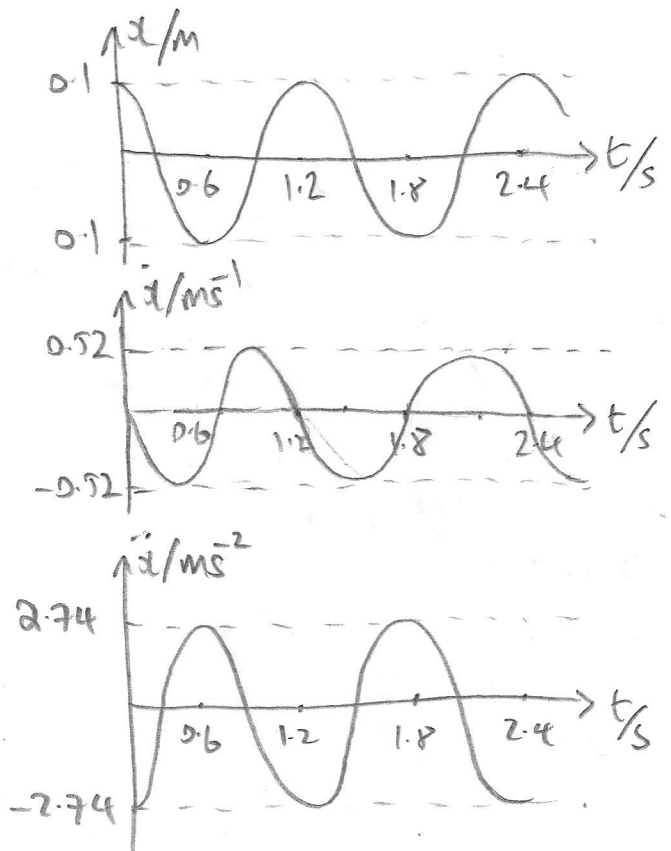
$$\text{So } \left(\frac{T}{2\pi}\right)^2 = \frac{m}{k} \quad \therefore \quad \boxed{k = m \left(\frac{2\pi}{T}\right)^2} \quad \left\{ \begin{array}{l} \text{Quick way:} \\ k = m\omega^2 \end{array} \right\}$$

$$\text{So } k = 0.2 \text{ kg} \times \left(\frac{2\pi}{1.2\text{s}}\right)^2 = \boxed{5.48 \text{ kg s}^{-2}}$$

$$(ii) \quad \boxed{x = x_0 \cos\left(\frac{2\pi t}{T}\right)}$$

$$x_0 = 0.1 \text{ m}, \quad T = 1.2 \text{ s}$$

$$\begin{aligned} \dot{x} &= -\frac{2\pi}{T} x_0 \sin\left(\frac{2\pi t}{T}\right) \\ \ddot{x} &= -\left(\frac{2\pi}{T}\right)^2 x_0 \cos\left(\frac{2\pi t}{T}\right) \end{aligned}$$



$$\frac{2\pi}{T} x_0 = \frac{2\pi}{1.2} \times 0.1 = \boxed{0.52 \text{ m/s}}$$

$$\left(\frac{2\pi}{T}\right)^2 x_0 = \boxed{2.74 \text{ m/s}^2}$$

$$\begin{aligned} \dot{x}(2.0) &= -\frac{2\pi}{1.2} \times 0.1 \sin\left(\frac{2\pi \times 2.0}{1.2}\right) \\ &= \boxed{-0.45 \text{ m/s}} \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad KE &= \frac{1}{2} m \dot{x}^2 \\
 &= \frac{1}{2} m \left(-\frac{2\pi}{T} x_0 \sin\left(\frac{2\pi t}{T}\right) \right)^2 \\
 &= \frac{1}{2} m \times \frac{4\pi^2}{T^2} x_0^2 \sin^2\left(\frac{2\pi t}{T}\right)
 \end{aligned}$$

$$= \boxed{\frac{2\pi^2 M x_0^2}{T^2} \sin^2\left(\frac{2\pi t}{T}\right)}$$

So twice maxima per period T .

$$EPE = \frac{1}{2} k x^2 = \frac{1}{2} k x_0^2 \cos^2\left(\frac{2\pi t}{T}\right)$$

Now from (i) : $k = m \left(\frac{2\pi}{T}\right)^2$

So $EPE = \frac{1}{2} m \times \frac{4\pi^2}{T^2} x_0^2 \cos^2\left(\frac{2\pi t}{T}\right)$

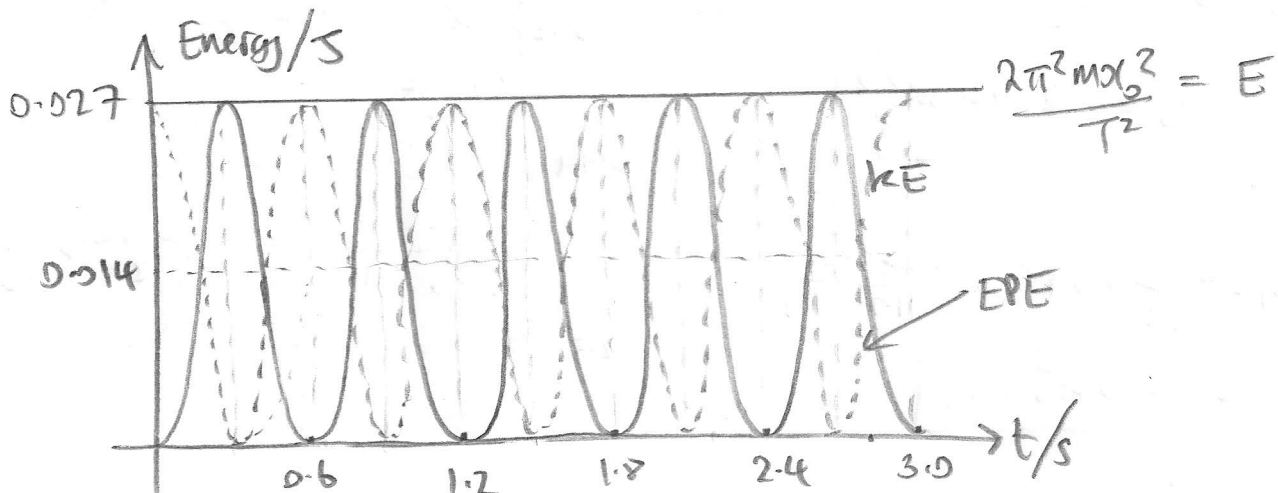
$$= \boxed{\frac{2\pi^2 m x_0^2}{T^2} \cos^2\left(\frac{2\pi t}{T}\right)}$$

Also.

Total energy $E = KE + EPE = \frac{2\pi^2 m x_0^2}{T^2} \left[\underbrace{\cos^2\left(\frac{2\pi t}{T}\right) + \sin^2\left(\frac{2\pi t}{T}\right)}_{=1} \right]$

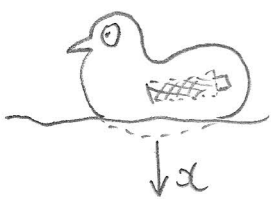
$$\Rightarrow \boxed{E = \frac{2\pi^2 m x_0^2}{T^2}}$$

ie $\boxed{E = \frac{1}{2} m \omega^2 x_0^2}$



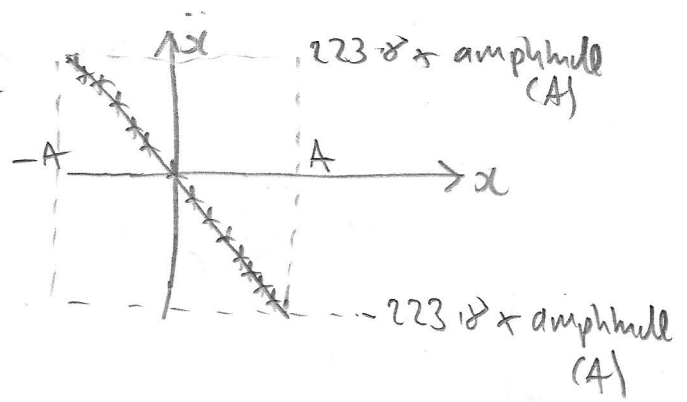
② Total energy = $\frac{2\pi^2 \times 0.2 \times 0.1^2}{1.2^2} = \boxed{0.0275}$

(iv)



$$\ddot{x} = - \left(\frac{2\pi}{T} \right)^2 x$$

$$\ddot{x} = -223.8 x$$

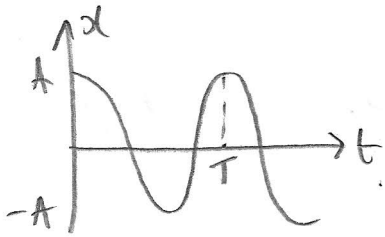


$$\therefore \left(\frac{2\pi}{T} \right)^2 = 223.8$$

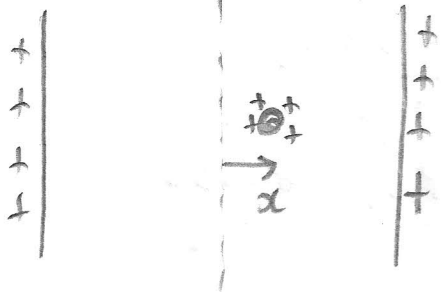
$$\therefore \frac{2\pi}{T} = \sqrt{223.8}$$

$$\therefore T = \frac{2\pi}{\sqrt{223.8}}$$

$$T = 0.42 \text{ s}$$



(v)



$$\begin{aligned} x &= A \cos(\omega t - \phi) \\ \dot{x} &= -A\omega \sin(\omega t - \phi) \\ \ddot{x} &= -\omega^2 x \end{aligned}$$

when $t = 0 \text{ s}$, $x = 2.0$, $\dot{x} = -3.0$
and $\ddot{x} = -4$. (cm, cm/s, cm/s²).

$$\therefore 2 = A \cos(-\phi) \quad (1)$$

$$-3 = -A\omega \sin(-\phi) \quad (2)$$

$$-4 = -\omega^2 \times 2 \quad (3)$$

$$\therefore \omega^2 = 2 \quad \therefore \omega = \sqrt{2} \approx 1.41 \text{ rad/s}$$

$$\frac{(2)}{(1)} : -\sqrt{2} \tan(-\phi) = \frac{-3}{2} \Rightarrow \tan(-\phi) = \frac{3}{2\sqrt{2}}$$

$$\text{So } \phi = -\tan^{-1}\left(\frac{3}{2\sqrt{2}}\right) = -0.815 \text{ radians } (-46.7^\circ)$$

$$\text{In } (1) : A = \frac{2}{\cos(-\phi)} = 2.92 \text{ cm}$$

(3)

