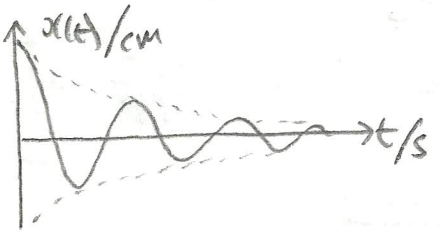
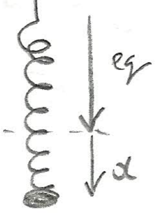
(vii) old car suspension

Corrections to Q1 (vii)





$$\boxed{\begin{aligned} x(t) &= Ae^{-\gamma t}\cos\left(t\sqrt{\omega^2-\gamma^2}-\phi\right) \\ \dot{x} &= -\gamma A e^{-\gamma t}\cos\left(t\sqrt{\omega^2-\gamma^2}-\phi\right) \\ &\quad - \sqrt{\omega^2-\gamma^2}\,Ae^{-\gamma t}\sin\left(t\sqrt{\omega^2-\gamma^2}-\phi\right)\end{aligned}}$$

let $\gamma = \frac{1}{3}\omega$

$x(0) = 10$ (cm)

$\dot{x}(0) = 0$

so $x_0 = A\cos(-\phi)$ $\qquad 0 = -\gamma A\cos(-\phi) - \sqrt{\omega^2-\gamma^2}\,A\sin(-\phi)$

$\therefore$ $\boxed{A = \dfrac{x_0}{\cos\phi}}$ $\qquad \gamma \cancel{A}\cos\phi = \sqrt{\omega^2-\gamma^2}\,\cancel{A}\sin\phi$

so $\boxed{\tan^{-1}\left(\dfrac{\gamma}{\sqrt{\omega^2-\gamma^2}}\right) = \phi}$

only if $\dot{x}(0)=0$

Note $A^2 = \dfrac{x_0^2}{\cos^2\phi} = x_0^2(1+\tan^2\phi)$

$$= x_0^2\left(1+\frac{\gamma^2}{\omega^2-\gamma^2}\right)$$

$\therefore$ $$\boxed{A = x_0\left(1+\frac{\gamma^2}{\omega^2-\gamma^2}\right)^{\frac{1}{2}}}$$

so $A = 10\left(1+\dfrac{\frac{1}{9}\omega^2}{\omega^2-\frac{1}{9}\omega^2}\right) = 10\left(1+\dfrac{1/9}{8/9}\right)$

$= 10\left(1+\frac{1}{8}\right)$

$= \boxed{11.25}$

$\phi = \tan^{-1}\left(\dfrac{1/3}{\sqrt{1-1/9}}\right) = \boxed{19.5^\circ}$

$\left[\ \dfrac{1/3}{\sqrt{1-1/9}} = \dfrac{1/3}{\sqrt{8/9}} = \dfrac{1}{\sqrt{8}} = \dfrac{1}{2\sqrt{2}}\ \right]$

so $\boxed{x(t) = 11.25\cos\left(t\omega\frac{\sqrt{8}}{3} - \tan^{-1}\left(\frac{1}{2\sqrt{2}}\right)\right)\times e^{-\frac{1}{3}\omega t}}$

Now assume $2\pi f = \sqrt{\omega^2 - \gamma^2} = \frac{\sqrt{8}}{3}\omega$ (0.943ω)

$\therefore$ if $f = 5$ Hz, $\omega = \frac{2\pi \times 5 \times 3}{\sqrt{8}} = \frac{30\pi}{\sqrt{8}} = \boxed{\frac{15\pi}{\sqrt{2}}}$

$\boxed{\omega = 33.3 \text{ rad s}^{-1}}$

So $x(0.1) \approx -0.329$ m

$\dot{x}(0.3) \approx 0$

A nicer plot if $\gamma = \frac{\omega}{10}$, and evaluate at $t = 0.15$.