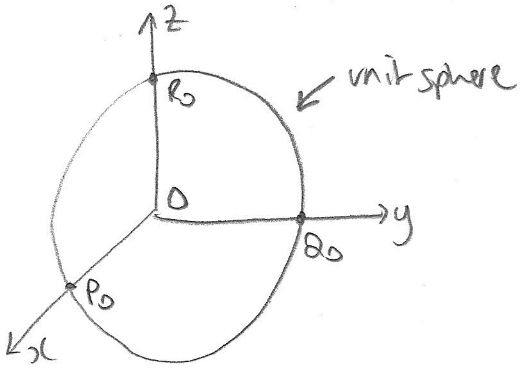
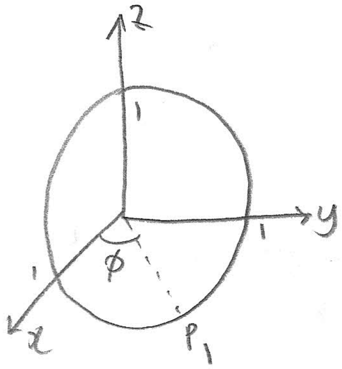
6/

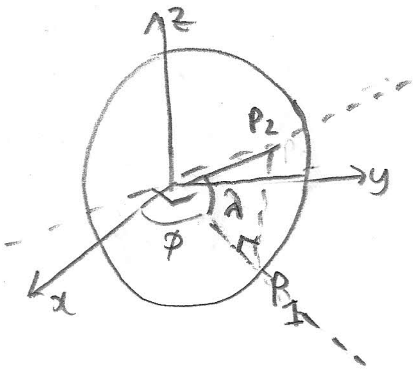
(i)





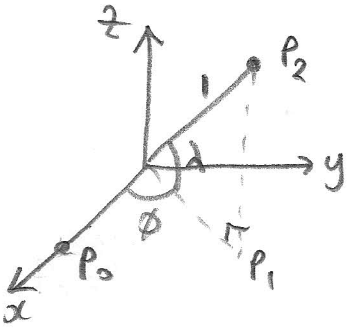
$$\boxed{\overrightarrow{OP_1} = \begin{pmatrix} \cos\phi \\ \sin\phi \\ 0 \end{pmatrix}}$$

(ii)



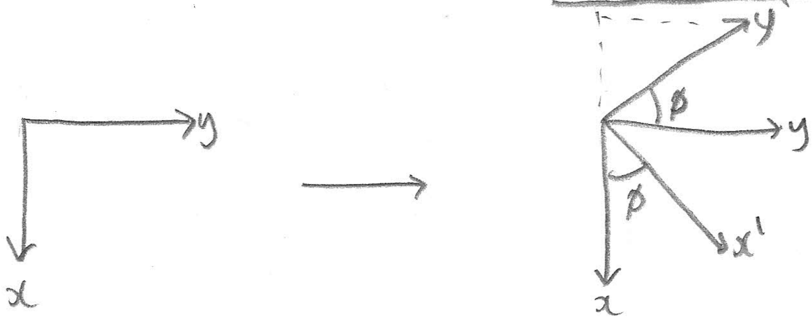
$$\boxed{\overrightarrow{OP_2} = \begin{pmatrix} \cos\lambda\cos\phi \\ \cos\lambda\sin\phi \\ \sin\lambda \end{pmatrix}}$$

(ie rotate about line through O $\perp$ to $\overrightarrow{OP_1}$)



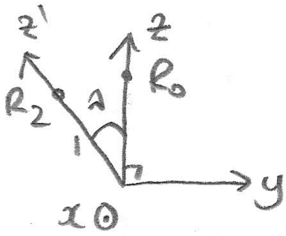
$$\overrightarrow{OQ_0} \longrightarrow \overrightarrow{OQ_1} \longrightarrow \boxed{\overrightarrow{OQ_2}}$$

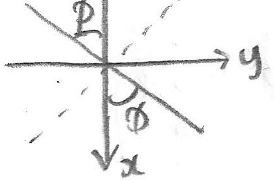
$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{pmatrix} \qquad \boxed{\begin{pmatrix} -\cos\lambda\sin\phi \\ \cos\lambda\cos\phi \\ 0 \end{pmatrix}}$$

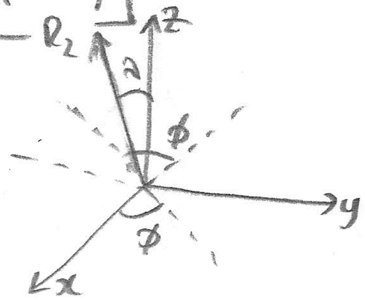


$$\overrightarrow{OR_0} \longrightarrow \overrightarrow{OR_1} \longrightarrow \boxed{\overrightarrow{OR_2}}$$

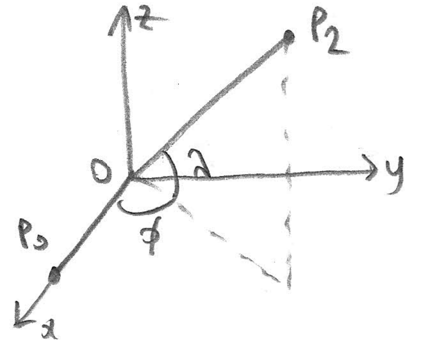
$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \boxed{\begin{pmatrix} -\sin\lambda\cos\phi \\ -\sin\lambda\sin\phi \\ \cos\lambda \end{pmatrix}}$$

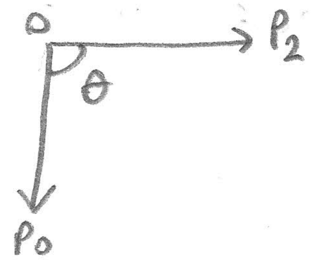






iii) If $\vec{OP_2}$ rotated back to $\vec{OP_0}$





$$\vec{OP_0} \cdot \vec{OP_2} = |\vec{OP_0}|\,|\vec{OP_2}| \cos\theta$$

$|\vec{OP_0}| = 1$

$|\vec{OP_2}| = 1$

$$\vec{OP_0} \cdot \vec{OP_2} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} \cos\lambda\cos\phi \\ \cos\lambda\sin\phi \\ \sin\lambda \end{pmatrix}$$

$$= \cos\lambda\cos\phi$$

$\therefore$ $\boxed{\cos\theta = \cos\lambda\cos\phi}$ as required

Rotation is about $\vec{OP_0} \times \vec{OP_2}$
(of θ)

ie $\hat{x} \times (\cos\lambda\cos\phi\,\hat{x} + \cos\lambda\sin\phi\,\hat{y} + \sin\lambda\,\hat{z})$

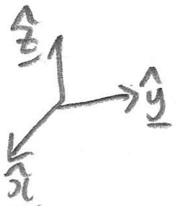
$\hat{x} \times \hat{y} = \hat{z}$

$\hat{y} \times \hat{z} = \hat{x}$

$\hat{z} \times \hat{x} = \hat{y}$

$$= \cos\lambda\sin\phi\,(\hat{x} \times \hat{y}) + \sin\lambda(\hat{x} \times \hat{z})$$

$$= \cos\lambda\sin\phi\,\hat{z} - \sin\lambda\,\hat{y}$$



So rotation vector is $\boxed{\begin{pmatrix} 0 \\ -\sin\lambda \\ \cos\lambda\sin\phi \end{pmatrix}}$

$\hat{x} \times \hat{x} = 0$

$\hat{x} \times \hat{z} = -\hat{z} \times \hat{x}$

From this we can define a $\boxed{\text{rotation matrix}}$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\lambda\cos\phi & -\cos\lambda\sin\phi & -\sin\lambda\cos\phi \\ \cos\lambda\sin\phi & \cos\lambda\cos\phi & -\sin\lambda\sin\phi \\ -\sin\lambda & 0 & \cos\lambda \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$\vec{OP_2}$ $\vec{OP_1}$ $\vec{OP_2}$