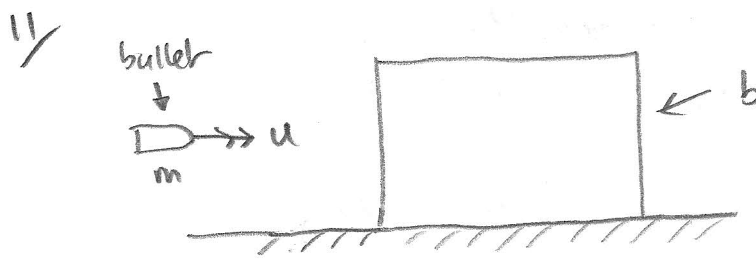
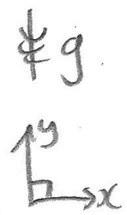
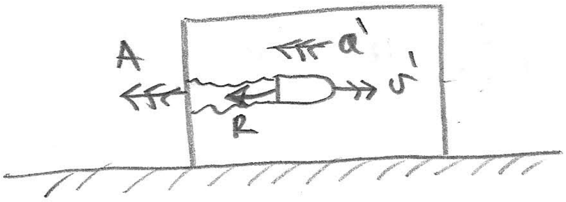
11/





(i)



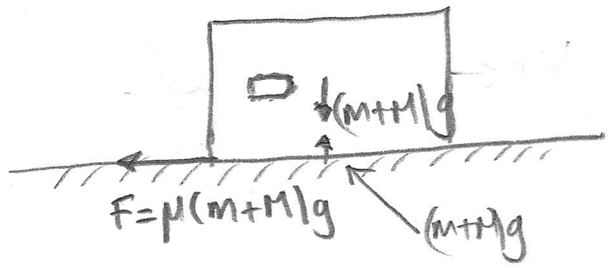
Forces on bullet

a' is acceleration relative to fixed coordinates

[$R > (M+m)\mu g$]

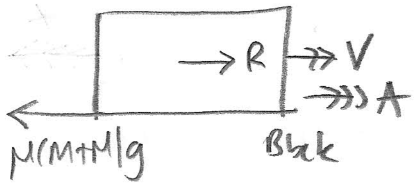
Forces on bullet + block ensemble

(assume no mass is 'ejected' by bullet)



$\Rightarrow$ So friction force on the block is also $F = \mu(m+M)g$

Newton II $\parallel x$



bullet : $ma' = R$

block : $MA = -\mu(m+M)g + R$

let relative deceleration of bullet compared to block be

$a = a' + A$

$\therefore$ $$\boxed{a = \frac{R}{m} + \frac{R - \mu g(m+M)}{M}}$$ as required

(ii) The relative block and bullet acceleration is constant, and since the bullet does not emerge from the block (we are told this), the bullet comes to rest* after t seconds when $u = at$

ie $t = \left(\frac{R}{m} + \frac{R - \mu g(m+M)}{M}\right)^{-1} u$

* relative to the block

Now clearly the block accelerates when it is struck by the bullet.

$V = At$, since the block starts from rest.

Let v be the block velocity at time t
ie the common velocity of the block and bullet

$$v = \frac{R - \mu(m+M)g}{M}\, u \underbrace{\frac{mM}{RM + Rm - m\mu g(m+M)}}_{\frac{1}{a}}$$

$$\left[A = \frac{R - \mu(m+M)g}{M} \right]$$

$$\therefore \boxed{va = \frac{Ru - (m+M)\mu g u}{M}}$$ as required

(iii) The distance moved by the block is x

Now $V^2 = 2Ax$ (since initial velocity $= 0$)

$\therefore v^2 = 2Ax$, where x in this case is the distance moved by the block while the bullet is moving.

$$\therefore x = \frac{v^2}{2A}$$

$$\therefore x = \frac{v^2}{2} \underbrace{\frac{M}{R - \mu(m+M)g}}_{\frac{1}{A}}$$

$$x = \frac{v^2}{2}\, \frac{Mu}{Ru - (m+M)\mu g u} = \frac{v^2}{2} u \frac{1}{va}$$

$$\therefore \boxed{x = \frac{uv}{2a}}$$

(iv) Now since the block and bullet travel at the same speed (v), deceleration is a''

Newton II: $$(M+m)a'' = \underbrace{\mu(M+m)g}_{\text{friction force}}$$

$$\therefore \quad a'' = \mu g$$

$\therefore$ let X be total distance travelled by the block after bullet and block are moving at the same speed

$$0 = v^2 - 2a''X$$

$$\therefore \quad X = \frac{v^2}{2a''} = \frac{v^2}{2\mu g}$$

$\therefore$ Total distance travelled by block is $x + X = x_{tot}$

ie $$\frac{uv}{2a} + \frac{v^2}{2\mu g}$$

$$x_{tot} = \frac{muv}{2\mu g}\left(\frac{\mu g}{ma} + \frac{v}{Mu}\right)$$

[want to eliminate a to get final answer]

$$x_{tot} = \frac{muv}{2\mu g}\left(\frac{\mu g u + av}{mau}\right) = \frac{muv}{2\mu g}\frac{1}{k}$$

From above $$ma = R + \left(R - (M+m)\mu g\right)\frac{m}{M}$$

$$av = \frac{Ru - (M+m)\mu g u}{M}$$

$$\therefore \quad k = \frac{Ru + (R - (M+m)\mu g)\frac{m}{M}u}{\mu g u + \frac{Ru - (M+m)\mu g u}{M}}$$

$$h = \frac{MRu + mRu - mu(m+M)\mu g}{M\mu g u + Ru - (M+m)\mu g u}$$

$$h = \frac{MR + mR - (m+M)\, mu\mu g}{Ru - m\mu g u}$$

$$\boxed{h = m+M}$$

$$\therefore \boxed{x_{TOT} = \frac{muv}{2(M+m)\mu g}} \quad \text{as required}$$

Now if $R < (M+m)\mu g$, lets see what happens to the block

From Newton II: $MA = R - \mu(m+M)g$

$\therefore$ $R = MA + \mu(M+M)g$

$\therefore$ Substituting into the inequality above

$$MA + \mu(m+M)g < (M+m)\mu g$$

$$\Rightarrow \boxed{A < 0}$$

Now this means the block would go backwards when hit. Since this is physically impossible we may conclude that the block cannot move if $R < (M+m)\mu g$

The bullet decelleration is R/m so it will penetrate x where $0 = u^2 - \frac{2R}{m}x \Rightarrow \boxed{x = \frac{mu^2}{2R}}$