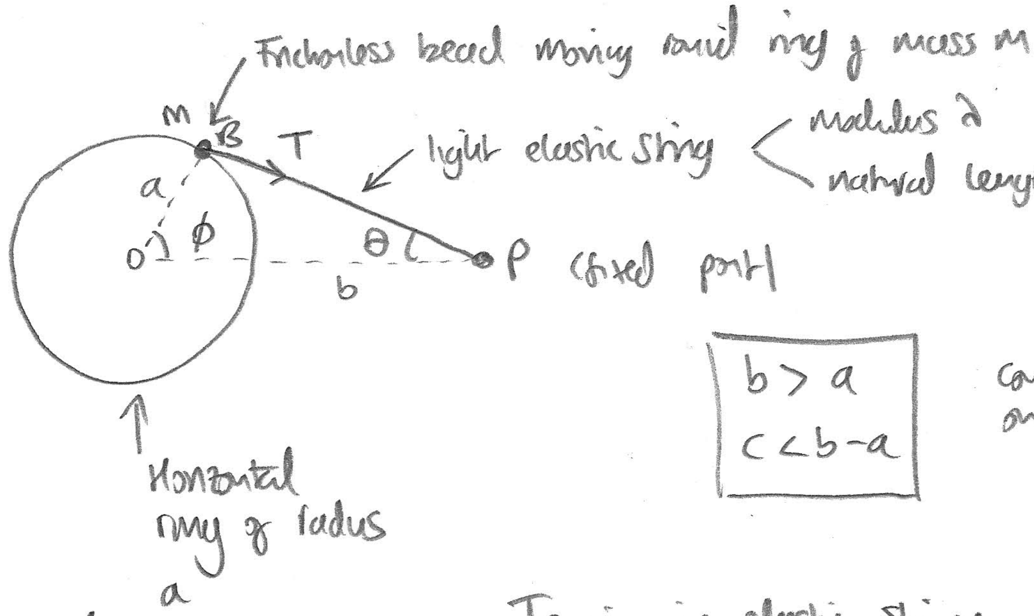
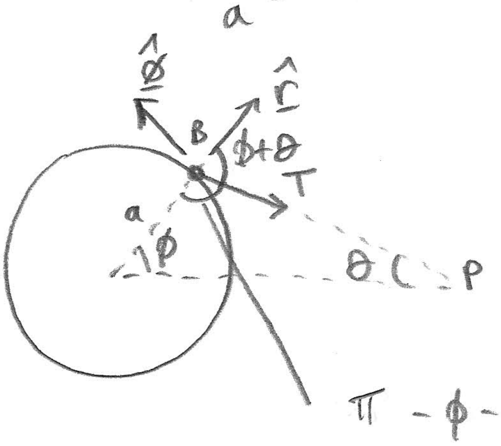
10/



$$\boxed{\begin{array}{l} b > a \\ c < b-a \end{array}}$$ constraints on a, b, c



Tension in elastic string

$$T = \lambda \frac{PB - c}{c}$$

Sine rule $\quad \dfrac{PB}{\sin\phi} = \dfrac{a}{\sin\theta}$

$$\therefore \quad \boxed{T = \frac{\lambda}{c}\left(a\frac{\sin\phi}{\sin\theta} - c\right)}$$

Newton II for bead

$/\!/ \ \hat{r} \quad : \quad ma\dot{\phi}^2 = -T\cos(\phi+\theta)$

$/\!/ \ \hat{\phi} \quad : \quad ma\ddot{\phi} = T\sin(\phi+\theta)$

$$\therefore \quad ma\ddot{\phi} = -\frac{\lambda}{c}\left(a\frac{\sin\phi}{\sin\theta} - c\right)\sin(\phi+\theta)$$

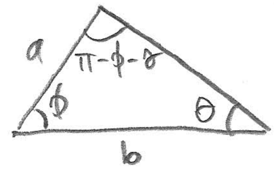
(A) $\quad \boxed{ma\ddot{\phi} = -\lambda\left(\dfrac{a\sin\phi}{c\sin\theta} - 1\right)\sin(\phi+\theta)}$ as required

Now when ϕ and θ are small $\quad \sin\phi \simeq \phi \quad \sin\theta \simeq \theta$
$\qquad \cos\phi \simeq 1 \quad \cos\theta \simeq 1$

$\therefore \ \sin(\phi+\theta) = \sin\phi\cos\theta + \cos\phi\sin\theta$

$\qquad \simeq \phi + \theta \quad$ when ϕ, θ small (obvious from Really)

Now from Sine rule



$$\frac{b}{\sin(\pi-\phi-\theta)} = \frac{a}{\sin\theta}$$

$$\sin(\pi-(\phi+\theta)) = \sin\pi\cos(\phi+\theta) - \cos\pi\sin(\phi+\theta)$$

$$\cos\pi = -1$$
$$\sin\pi = 0$$

$$\therefore \sin(\pi-(\phi+\theta)) = \sin(\phi+\theta)$$

$$\therefore \frac{b}{\sin(\phi+\theta)} = \frac{a}{\sin\theta}$$

So if θ, ϕ small

$$\frac{b}{\phi+\theta} = \frac{a}{\theta}$$

or $\boxed{a(\theta+\phi) = b\theta}$

as required

$\therefore$ Substituting into $(*)$

$$ma\ddot{\phi} = -\lambda\left(\frac{a\phi}{c\theta} - 1\right)(\theta+\phi)$$

Now $a(\theta+\phi) = b\theta \Rightarrow \theta(b-a) = a\phi$

$$\boxed{\theta = \frac{a\phi}{b-a}}$$

$$\therefore ma\ddot{\phi} = -\lambda\left(\frac{a\phi}{ca\phi}(b-a) - 1\right)\left(\frac{a\phi}{b-a} + \phi\right)$$

$$ma\ddot{\phi} = -\lambda\left(\frac{b-a}{c} - 1\right)\left(\frac{a}{b-a} + 1\right)\phi$$

$$ma\ddot{\phi} = \frac{-\lambda(b-a-c)(a+b-a)}{(b-a)c}\phi$$

$$\boxed{ma\ddot{\phi} = -\frac{\lambda(b-a-c)b}{(b-a)c}\,\phi}$$

Compare to SHM equation ie let solutions be of form

$$\phi = \phi_0 \cos(2\pi t/T)$$

$$\ddot{\phi} = -\left(\frac{2\pi}{T}\right)^2 \phi$$

where T is the oscillation period

$$\therefore \left(\frac{2\pi}{T}\right)^2 = \frac{\lambda(b-a-c)b}{ma(b-a)c}$$

$$\therefore \boxed{T = 2\pi\sqrt{\frac{ma(b-a)c}{\lambda(b-a-c)b}}}$$

$(\approx)$

ie only valid when ϕ, θ are both small ($\ll 1$ radian)