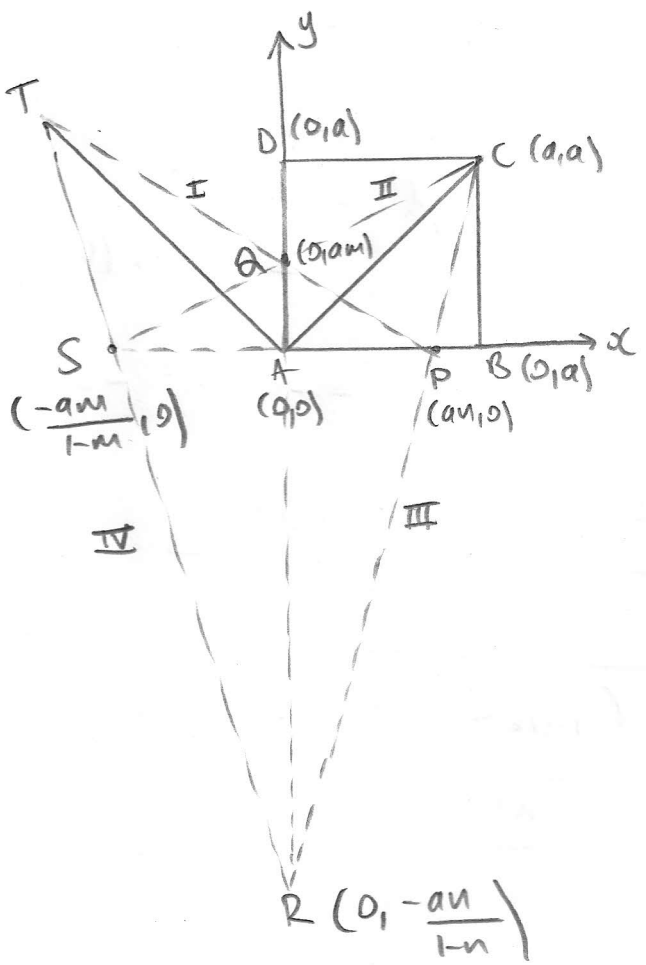
5/ First question is to show $T\hat{A}C$ is 90°

line TQP has equation (I) $\boxed{y_I = -\frac{m}{n}x + am}$

II: $y = \frac{a(1-m)}{a}x + am$

$\boxed{y_{II} = (1-m)x + am}$



[0 < m < n < 1]

III: $y = \frac{a}{a-an}x + c$

$y = \frac{x}{1-n} + c$

using (a,a)

$a = \frac{a}{1-n} + c$

$\therefore \quad c = a\left(1 - \frac{1}{1-n}\right)$

$c = \frac{a(1-n-1)}{1-n}$

$\boxed{c = -\frac{an}{1-n}}$

[So R is $(0, -\frac{an}{1-n})$]

$\therefore$ III: $\boxed{y_{III} = \frac{x}{1-n} - \frac{an}{1-n}}$

Point S is when y_{II} is zero

ie $0 = (1-m)x + am \Rightarrow x = \frac{-am}{1-m}$

$\therefore$ S is $\left(\frac{-am}{1-m}, 0\right)$

So IV is $y = \frac{-\frac{an}{1-n}}{\frac{am}{1-m}}x - \frac{an}{1-n}$

$\boxed{y_{IV} = -\left(\frac{1-m}{1-n}\right)\frac{n}{m}x - \frac{an}{1-n}}$

T is the intersection of IV and I.

i.e. $$-\left(\frac{1-m}{1-n}\right)\frac{n}{m}x - \frac{an}{1-n} = -\frac{n}{m}x + am$$

$$-(1-m)nx - an^2m = -m^2x(1-n) + am^2(1-n)$$

$$-(1-m)n^2x - an^2m = -m^2x(1-n) + am^2n(1-n)$$

$$\therefore x\left(m^2(1-n) - n^2(1-m)\right) = am^2n(1-n) + an^2m$$

$$\therefore \boxed{x = \frac{am^2n(1-n) + an^2m}{m^2(1-n) - n^2(1-m)}}$$

using y_I: $$y = -\frac{m}{n}\left\{\frac{am^2n(1-n) + an^2m}{m^2(1-n) - n^2(1-m)}\right\} + am$$

$$= \frac{-\frac{m}{n}\left(am^2n(1-n) + an^2m\right) + am\left(m^2(1-n) - n^2(1-m)\right)}{m^2(1-n) - n^2(1-m)}$$

$$= \frac{-m^3a(1-n) - m^2an + m^3a(1-n) - man^2(1-m)}{m^2(1-n) - n^2(1-m)}$$

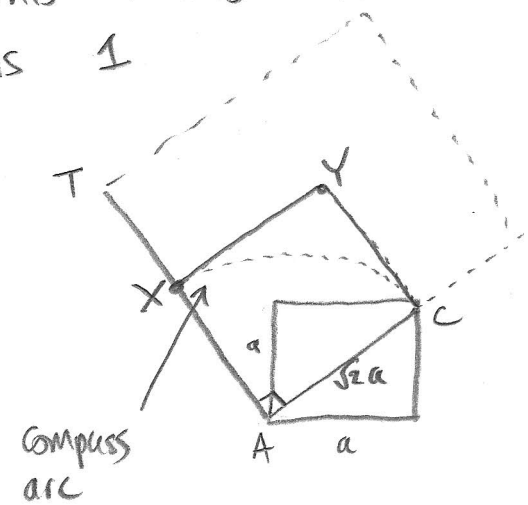
$$= \frac{-m^2an - man^2 + m^2an^2}{m^2(1-n) - n^2(1-m)}$$

$$= \frac{am^2n(-1+n) - an^2m}{m^2(1-n) - n^2(1-m)}$$

$$= \boxed{-x}$$

So gradient of $\overrightarrow{AT}$ is -1

This means $T\hat{A}C$ is 90°, since the gradient of $\overrightarrow{AC}$ is 1



A square of area $2a^2$ has sides $\sqrt{2}a$

To construct such a square (e.g. ACYX)

- Find line AT as per the construction above
- Draw an arc using a compass with radius AC where this crosses AT you have another vertex X.
- You could repeat the whole process with C rather than A to find Y and then join the dots.

However this all seems rather a waste of time!

- Draw a line through AC ✓ "Walk compass AC distances along AC to achieve this"
- Find perpendicular bisectors through A and C
- Draw arcs of length AC from A and then C to find X, Y.