Mathematics I, II and III (9465, 9470, and 9475)

General Introduction

There are two syllabuses, one for Mathematics I and Mathematics II, the other for Mathematics III. The syllabus for Mathematics I and Mathematics II is based on a single subject Mathematics Advanced GCE. Questions on Mathematics II are intended to be more challenging than questions on Mathematics I. The syllabus for Mathematics III is wider.

In designing the syllabuses, the specifications for all the UK Advanced GCE examinations, the Scottish Advanced Higher and the International Baccalauriat were consulted.

Each of Mathematics I, II and III will be a 3-hour paper divided into three sections as follows:

Section A (Pure Mathematics)	eight questions
Section B (Mechanics)	three questions
Section C (Probability and Statistics)	two questions

All questions will carry the same weight. Candidates will be assessed on the six questions best answered; no restriction will be placed on the number of questions that may be attempted from any section. Normally, a candidate who answers at least four questions well will be awarded a grade 1.

The marking scheme for each question will be designed to reward candidates who make good progress towards a complete solution.

Syllabuses

The syllabuses given below are for the guidance of both examiners and candidates. The following points should be noted.

1. Questions may test candidates' ability to apply mathematical knowledge in novel and unfamiliar ways.
2. Solutions will frequently require insight, ingenuity, persistence and the ability to work through substantial sequences of algebraic manipulation.
3. Some questions may be largely independent of particular syllabus topics, being designed to test general mathematical potential.
4. Questions will often require knowledge of several different syllabus topics.
5. Examiners will aim to set questions on a wide range of topics, but it is not guaranteed that every topic will be examined every year.
6. Questions may be set that require knowledge of elementary topics, such as basic geometry, mensuration (areas of triangles, volume of sphere, etc), sine and cosine formulae for triangles, rational and irrational numbers.

Formula booklets and calculators

Candidates may use an Advanced GCE booklet of standard formulae, definitions and statistical tables. The inter-Board agreed list of mathematical notation will be followed. Calculators are not permitted (or required).

MATHEMATICS I (9465) and MATHEMATICS II (9470)

Section A: Pure Mathematics

This section comprises the Advanced GCE common core (typically, modules C1 — C4 of an Advanced GCE specification) broadly interpreted, together with a few additional items ∗enclosed in asterisks∗.

Specification	Notes
General	
Mathematical vocabulary and notation	including: equivalent to; necessary and sufficient; if and only if; $\Rightarrow$; $\Leftrightarrow$; $\equiv$.
Methods of proof	including proof by contradiction and disproof by counterexample; ∗including, in simple cases, proof by induction∗.
Algebra	
Indices and surds	including rationalising denominators.
Quadratics	including proving positivity by completing a square.
The expansion for $(a+b)^n$	including knowledge of the general term; notation: $\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$.
Algebraic operations on polynomials and rational functions.	including factorisation, the factor theorem, the remainder theorem; including understanding that, for example, if $$x^3 + bx^2 + cx + d \equiv (x-\alpha)(x-\beta)(x-\gamma),$$ then $d = -\alpha\beta\gamma$.
Partial fractions	including denominators with a repeated or quadratic factor.
Sequences and series	including use of, for example, $a_{n+1} = \mathrm{f}(a_n)$ or $a_{n+1} = \mathrm{f}(a_n, a_{n-1})$; including understanding of the terms convergent, divergent and periodic in simple cases; including use of $\sum_{k=1}^{n} k$ to obtain related sums.
The binomial series for $(1+x)^k$, where k is a rational number.	including understanding of the condition $\lvert x\rvert < 1$.
Arithmetic and geometric series	including sums to infinity and conditions for convergence, where appropriate.
∗Inequalities∗	including solution of, eg, $\frac{1}{a-x} > \frac{x}{x-b}$; including simple inequalities involving the modulus function; including the solution of simultaneous inequalities by graphical means.

Functions

Domain, range, composition, inverse	including use of functional notation such as $y = \mathrm{f}(ax+b)$, $x = \mathrm{f}^{-1}(y)$ and $z = \mathrm{f}\big(\mathrm{g}(x)\big)$.
Increasing and decreasing functions	both the common usages of the term ‘increasing’ (i.e. $x > y \Rightarrow \mathrm{f}(x) > \mathrm{f}(y)$ or $x > y \Rightarrow \mathrm{f}(x) \geqslant \mathrm{f}(y)$) will be acceptable.
Exponentials and logarithms	including $x = a^y \Leftrightarrow y = \log_a x$, $x = \mathrm{e}^y \Leftrightarrow y = \ln x$; ∗including the exponential series $$\mathrm{e}^x = 1 + x + \cdots + x^n/n! + \cdots \ . \qquad *$$
The effect of simple transformations	such as $y = a\mathrm{f}(bx+c)+d$.
The modulus function.	
Location of roots of $\mathrm{f}(x) = 0$ by considering changes of sign of $\mathrm{f}(x)$.	
Approximate solution of equations using simple iterative methods.	

Curve sketching

General curve sketching	including use of symmetry, tranformations, behaviour as $x \to \pm\infty$, points or regions where the function is undefined, turning points, asymptotes parallel to the axes.

Trigonometry

Radian measure, arc length of a circle, area of a segment.	
Trigonometric functions	including knowledge of standard values, such as $\tan(\pi/4)$, $\sin 30^\circ$; including identities such as $\sec^2\phi - \tan^2\phi = 1$; including application to geometric problems in two and three dimensions.
Double angle formulae	including their use in calculating, eg, $\tan(\pi/8)$.
Formulae for $\sin(A \pm B)$ and $\cos(A \pm B)$	including their use in solving equations such as $$a\cos\theta + b\sin\theta = c.$$
Inverse trigonometric functions	definitions including domains and ranges; notation: $\arctan\theta$, etc

Coordinate geometry

Straight lines in two-dimensions	including the equation of a line through two given points, or through a given point and parallel to a given line or through a given point and perpendicular to a given line; including finding a point which divides a segment in a given ratio.
Circles	using the general form $(x-a)^2 + (y-b)^2 = R^2$; including points of intersection of circles and lines.
Cartesian and parametric equations of curves and conversion between the two forms.	

Calculus

Interpretation of a derivative as a limit and as a rate of change	including knowledge of both notations $\mathrm{f}'(x)$ and $\frac{\mathrm{d}y}{\mathrm{d}x}$.
Differentiation of standard functions	including algebraic expressions, trigonometric (but not inverse trigonometric) functions, exponential and log functions.
Differentiation of composite functions, products and quotients and functions defined implicitly.	
Higher derivatives	including knowledge of both notations $\mathrm{f}''(x)$ and $\frac{d^2y}{\mathrm{d}x^2}$; including knowledge of the notation $\frac{d^ny}{\mathrm{d}x^n}$.
Applications of differentiation to gradients, tangents and normals, stationary points, increasing and decreasing functions	including finding maxima and minima which are not stationary points; including classification of stationary points using the second derivative.
Integration as reverse of differentiation	
Integral as area under a curve	including area between two graphs; including approximation of integral by the rectangle and trapezium rules.
Volume within a surface of revolution	
Knowledge and use of standard integrals	including the forms $\int \mathrm{f}'(\mathrm{g}(x))\mathrm{g}'(x)\mathrm{d}x$ and $\int \mathrm{f}'(x)/\mathrm{f}(x)\mathrm{d}x$; including transformation of an integrand into standard (or some given) form; including use of partial fractions; not including knowledge of integrals involving inverse trigonometric functions.
Definite integrals	$*$including calculation, without justification, of simple improper integrals such as $\int_0^\infty \mathrm{e}^{-x}\mathrm{d}x$ and $\int_0^1 x^{-\frac{1}{2}}\mathrm{d}x$ (if required, information such as the behaviour of $x\mathrm{e}^{-x}$ as $x \to \infty$ or of $x\ln x$ as $x \to 0$ will be given).$*$
Integration by parts and by substitution	including understanding their relationship with differentiation of product and of a composite function; including application to $\int \ln x \mathrm{d}x$.
Formulation and solution of differential equations	formulation of first order equations; solution in the case of a separable equation or by some other method given in the question.

Vectors

Vectors in two and three dimensions	including use of column vector and $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ notation.
Magnitude of a vector	including the idea of a unit vector.
Vector addition and multiplication by scalars	including geometrical interpretations.
Position vectors	including application to geometrical problems.
The distance between two points.	
Vector equations of lines	including the finding the intersection of two lines; understanding the notion of skew lines (knowledge of shortest distance between skew lines is not required).
The scalar product	including its use for calculating the angle between two vectors.

Section B: Mechanics

This section is roughly equivalent to two GCE modules M1 and M2; candidates who have only studied one module of mechanics are likely to lack both the knowledge and the experience of mechanics required to attempt the questions. Questions may involve any of the material in the Pure Mathematics syllabus.

Force as a vector	including resultant of several forces acting at a point and the triangle or polygon of forces; including equilibrium of a particle; forces include weight, reaction, tension and friction.
Centre of mass	including obtaining the centre of mass of a system of particles, of a simple uniform rigid body (possible composite) and, in simple cases, of non-uniform body by integration.
Equilibrium of a rigid body or several rigid bodies in contact	including use of moment of a force; for example, a ladder leaning against a wall or on a movable cylinder; including investigation of whether equilibrium is broken by sliding, toppling or rolling; including use of Newton's third law; excluding questions involving frameworks.
Kinematics of a particle in a plane	including the case when velocity or acceleration depends on time (but excluding use of acceleration $= v\frac{\mathrm{d}v}{\mathrm{d}x}$) ; questions may involve the distance between two moving particles, but detailed knowledge of relative velocity is not required.
Energy (kinetic and potential), work and power	including application of the principle of conservation of energy.
Collisions of particles	including conservation of momentum, conservation of energy (when appropriate); coefficient of restitution, Newton's experimental law; including simple cases of oblique impact (on a plane, for example); including knowledge of the terms *perfectly elastic* ($e = 1$) and *inelastic* ($e = 0$); questions involving successive impacts may be set.
Newton's first and second laws of motion	including motion of a particle in two and three dimensions and motion of connected particles, such as trains, or particles connected by means of pulleys.
Motion of a projectile under gravity	including manipulation of the equation $$y = x\tan\alpha - \frac{gx^2}{2V^2\cos^2\alpha},$$ viewed, possibly, as a quadratic in $\tan\alpha$; not including projectiles on inclined planes.

Section C: Probability and Statistics

The emphasis, in comparison with Advanced GCE and other comparable examinations, is towards probability and formal proofs, and away from data analysis and use of standard statistical tests. Questions may involve use of any of the material in the Pure Mathematics syllabus.

Probability

Permutations, combinations and arrangements	including sampling with and without replacement.
Exclusive and complementary events	including understanding of $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$, though not necessarily in this form.
Conditional probability	informal applications, such as tree diagrams.

Distributions

Discrete and continuous probability distribution functions and cumulative distribution functions.	including calculation of mean, variance, median, mode and expectations by explicit summation or integration for a given (possibly unfamiliar) distribution (eg exponential or geometric or something similarly straightforward); notation: $\mathrm{f}(x) = \mathrm{F}'(x)$.
Binomial distribution	including explicit calculation of mean.
Poisson distribution	including explicit calculation of mean; including use as approximation to binomial distribution where appropriate.
Normal distribution	including conversion to the standard normal distribution by translation and scaling; including use as approximation to the binomial or Poisson distributions where appropriate; notation: $X \sim \mathrm{N}(\mu, \sigma^2)$.

Hypothesis testing

Basic concepts in the case of a simple null hypothesis and simple or compound alternative	including knowledge of the terminology *null hypothesis* and *alternative hypothesis*, *one* and *two tailed tests.*

MATHEMATICS III (9475)

In each section knowledge of the corresponding parts of the syllabus for Mathematics I and II is assumed, and may form the basis of some questions.

Section A: Pure Mathematics

Algebra

Summation of series	including use of method of differences, recognition of Maclaurin's series (including binomial series) and partial fractions; including investigation of convergence, by consideration of the sum to n terms.
Polynomial equations	including complex solutions (and the fact that they occur in conjugate pairs if the cooefficients are real); including the relationship between symmetric functions of the roots and the coefficients; including formation of a new equation whose roots are related to the roots of the old equation.
Induction	including application complicated situations such as divisibility problems.

Trigonometry

Formula for $\sin A \sin B$ (etc) and $\sin A + \sin B$ (etc)	

Coordinate geometry

Ellipse, parabola, hyperbola and rectangular hyperbola	including standard Cartesian forms and parametric forms; including geometrical understanding, but not detailed knowledge of, the terms *focus*, *directrix*, *eccentricity*.
Polar coordinates	including relation with Cartesian coordinates; including ability to sketch simple graphs; including integral formula for area.

Complex numbers

Algebra of complex numbers	including Cartesian and polar forms.
Argand diagram	including geometrical representation of sums and products; including identification of loci and regions (eg $\lvert z - a\rvert = k\lvert z - b\rvert$).
Euler's identity: $\mathrm{e}^{\mathrm{i}\theta} \equiv \cos\theta + \mathrm{i}\sin\theta$	including the relationship between trigonometrical and exponential functions; including application to summation of trigonometric series.
de Moivre's theorem	including application to finding nth roots.

Hyperbolic functions

Definitions and properties of the six hyperbolic functions	including integrals and derivatives of $\cosh$, $\sinh$, $\tanh$ and $\coth$.
Formulae corresponding to trigonometric formulae	such as $\cosh^2 x - \sinh^2 x = 1$ and $\sinh(x+y) = \cdots$.
Inverse functions	including ability to derive logarithmic forms and graphs; including derivatives.

Calculus

Integrals resulting in inverse trig. or hyperbolic functions	including integrands such as $(x^2 - ax)^{-\frac{1}{2}}$.
Maclaurin's series	including knowledge of expansions for $\sin x$, $\cos x$, $\cosh x$, $\sinh x$ and $\ln(1+x)$.
Arc length of a curve expressed in Cartesian coordinates	including curves described parametrically.
First order linear differential equations	including the general form $$\frac{\mathrm{d}y}{\mathrm{d}x} + \mathrm{P}(x)y = \mathrm{Q}(x).$$
Second order linear differential equations with constant coefficients	including the inhomogeneous case $$a\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + b\frac{\mathrm{d}y}{\mathrm{d}x} + cy = \mathrm{f}(x),$$ where $\mathrm{f}(x)$ is an exponential function, a sin/cos function or a polynomial (but excluding resonant cases); including equations that can be transformed into the above form; including initial or boundary conditions.

Section B: Mechanics

Motion of a particle subject to a variable force	including use of $v\frac{\mathrm{d}v}{\mathrm{d}x}$ for acceleration.
Elastic strings and springs	including Hooke's law and the potential energy; notation: $T = kx = \lambda x/l$, where k is stiffness, λ is modulus of elasticity.
Motion in a circle	including motion with non-uniform speed.
Simple harmonic motion	including obtaining from a given model an equation of the form $\ddot{x} + \omega^2 x = a$, where a is a constant, and quoting the solution.
Relative motion	
Moment of inertia	including calculations involving integration (such as a non-uniform rod, or a lamina); including parallel and perpendicular axis theorems.
Motion of a rigid body about a fixed axis	including understanding of angular momentum and rotational kinetic energy and their conservation; including the understanding of couple and the work done by a couple; including calculation of forces on the axis.

Section C: Probability and Statistics

Algebra of expectations	for example, $\mathrm{E}(aX+bY+c)=a\mathrm{E}(X)+b\mathrm{E}(Y)+c$, $\mathrm{Var}\,(X)=\mathrm{E}(X^2)-\mathrm{E}(X)^2$, $\mathrm{Var}\,(aX+b)=a^2\,\mathrm{Var}\,(X)$.
Further theory of distribution functions	including, for example, use of the cumulative distribution function to calculate the distribution of a related random variable (eg X^2).
Generating functions	including the use of generating functions to obtain the distribution of the sum of independent random variables.
Discrete bivariate distributions	including understanding of the idea of independence; including marginal and conditional distributions; including use of $\mathrm{Var}\,(aX+bY)=a^2\,\mathrm{Var}\,(X)+b^2\,\mathrm{Var}\,(Y)+2ab\mathrm{Cov}(X,Y)$.
Sampling	including sample mean and variance; including use of the central limit theorem.