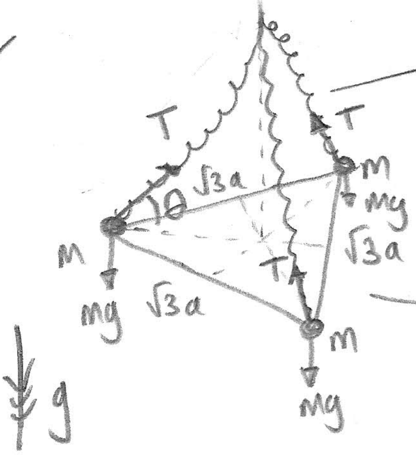
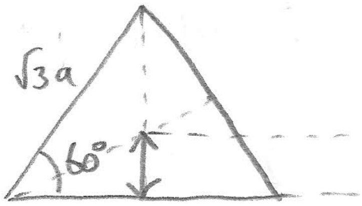
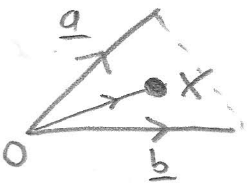
11/



Springs of natural length a and elastic modulus kmg

light rigid rods



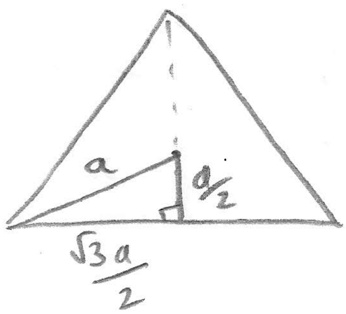


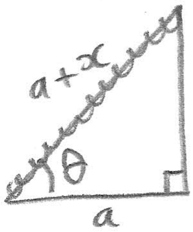
$\vec{OX} = \frac{1}{3}(\underline{a}+\underline{b})$

$\frac{1}{3}\sqrt{3}a \sin 60°$

$= \frac{1}{3}\sqrt{3}a\frac{\sqrt{3}}{2}$

$= \boxed{\frac{a}{2}}$





$(a+x)\cos\theta = a$

$a+x = \frac{a}{\cos\theta}$

$x = \frac{a}{\cos\theta} - a$

x is extension of spring

So tension in spring is (assuming Hookean)

$T = kmg\,x/a$

$T = \frac{kmg\left(\frac{a}{\cos\theta}-a\right)}{a}$

$\boxed{T = \frac{kmg(1-\cos\theta)}{\cos\theta}}$ as required

Now in equilibrium, Newton II resolved vertically

$$\Rightarrow \quad T\sin\theta = mg$$

If $\theta = \frac{\pi}{6}$ at equilibrium

$$\frac{kmg\left(1-\cos\frac{\pi}{6}\right)}{\cos\frac{\pi}{6}}\sin\frac{\pi}{6} = mg$$

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\sin\frac{\pi}{6} = \frac{1}{2}$

$$\therefore \quad \frac{k\left(1-\frac{\sqrt{3}}{2}\right)}{\frac{\sqrt{3}}{2}}\,\frac{1}{2} = 1$$

$$k\left(1-\frac{\sqrt{3}}{2}\right) = \sqrt{3}$$

$$\frac{k(2-\sqrt{3})}{2} = \sqrt{3}$$

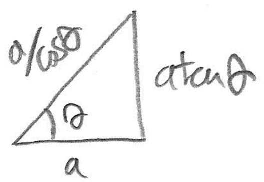
$$k = \frac{2\sqrt{3}}{2-\sqrt{3}}$$

$$k = \frac{2\sqrt{3}(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$$

$$k = \frac{4\sqrt{3}+6}{4-3} \qquad \therefore \quad \boxed{k = 4\sqrt{3}+6}$$

System is released from rest when $\theta = \frac{\pi}{3}$

Compute GPE from hanging part. Total energy is E.



$$\text{So} \quad E = \underbrace{-3mga\tan\tfrac{\pi}{3}}_{\text{"}mgh\text{"}} + \underbrace{\frac{3}{2}\,\frac{kmg}{a}\left(\frac{a}{\cos\frac{\pi}{3}} - a\right)^2}_{\text{"}\frac{1}{2}\frac{\lambda}{\ell}x^2\text{"}}$$

[Note 3 springs]

Now $\tan\frac{\pi}{3} = \sqrt{3}$

$\cos\frac{\pi}{3} = \frac{1}{2}$

$$\therefore \quad E = -3mga\sqrt{3} + \frac{3}{2}\overbrace{(4\sqrt{3}+6)mga}^{k}(2-1)^2$$

$$E = mga(-3\sqrt{3} + 6\sqrt{3}+9)$$

$$\boxed{E = mga(9+3\sqrt{3})}$$

At a general angle assume the masses have velocity (upward!) v

$$\therefore \quad mga(9+3\sqrt{3}) = \underbrace{\frac{3}{2}mv^2}_{\text{3 masses KE}} - \underbrace{3mga\tan\theta}_{\text{GPE}} + \overbrace{3\underbrace{(2\sqrt{3}+3)mga}_{k/2}\left(\frac{1}{\cos\theta}-1\right)^2}^{\text{EPE}}$$

when $\theta = \frac{\pi}{6}$, $v = V$

$$\therefore \quad \frac{3}{2}V^2 = ag\left\{9+3\sqrt{3} + 3\tan\frac{\pi}{6} - 3(2\sqrt{3}+3)\left(\frac{1}{\cos\frac{\pi}{6}}-1\right)^2\right\}$$

$$V^2 = \frac{2ag}{3}\left\{9+3\sqrt{3} + \frac{3}{\sqrt{3}} - 3(2\sqrt{3}+3)\left(\frac{2}{\sqrt{3}}-1\right)^2\right\}$$

$$V^2 = \frac{2ag}{3}\left\{9+3\sqrt{3} + \sqrt{3} - 3(2\sqrt{3}+3)(2-\sqrt{3})^2/3\right\}$$

$$\begin{aligned}(2\sqrt{3}+3)(2-\sqrt{3})^2 &= (2\sqrt{3}+3)(4-4\sqrt{3}+3)\\ &= (2\sqrt{3}+3)(7-4\sqrt{3})\\ &= 14\sqrt{3}+21-24-12\sqrt{3}\\ &= -3+2\sqrt{3}\end{aligned}$$

$$\therefore \quad V^2 = \frac{2ag}{3}\left\{9+4\sqrt{3}+3-2\sqrt{3}\right\} = \boxed{\frac{4ag}{3}(6+\sqrt{3})}$$

as required.