8/ $S_n = \sum_{r=0}^{n-1} e^{2i(\alpha + r\pi/n)}$ α is fixed and $n \geq 2$

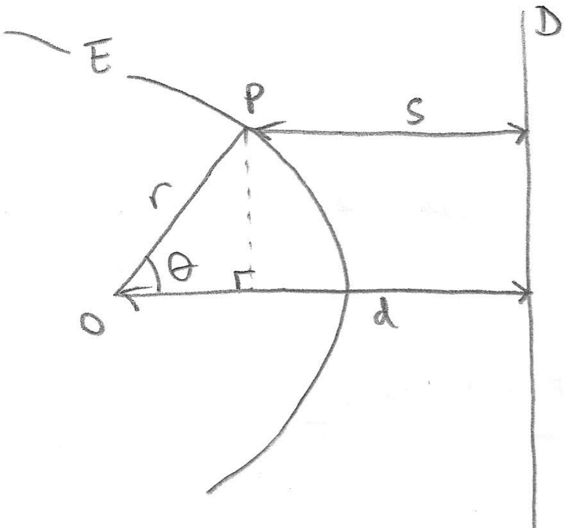
$$= e^{2i\alpha} + e^{2i\alpha} e^{\frac{2i\pi}{n}} + e^{2i\alpha}\left(e^{\frac{2i\pi}{n}}\right)^2 + e^{2i\alpha}\left(e^{\frac{2i\pi}{n}}\right)^3 + \dots$$

ie a geometric series with first term $e^{2i\alpha}$ and common ratio $e^{2\pi i/n}$

$$\therefore S_n = \frac{e^{2i\alpha}\left(1 - e^{2\pi i}\right)}{1 - e^{2\pi i/n}}$$

Now since $n \geq 2$, $e^{2\pi i/n} \neq 1$ so $1 - e^{2\pi i/n} \neq 0$

$e^{2\pi i} = 1$ so $\boxed{S_n = 0}$



$r\cos\theta + s = d$ $\therefore \boxed{s = d - r\cos\theta}$

Curve E is such that $r = ks$

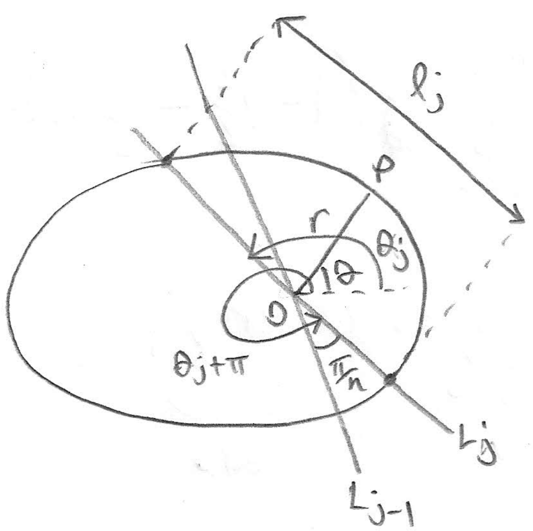
where $0 < k < 1$

So $r = kd - kr\cos\theta$

$r(1 + k\cos\theta) = kd$

$$\therefore \boxed{r = \frac{kd}{1 + k\cos\theta}}$$

(ie an ellipse - hence the name)



$\theta_j = \alpha + (j-1)\frac{\pi}{n}$

$$l_j = \frac{kd}{1 + k\cos\theta_j} + \frac{kd}{1 + k\cos(\theta_j + \pi)}$$

Now $\cos(\theta_j + \pi) = \cos\theta_j \cos\pi - \sin\theta_j \sin\pi$

$= -\cos\theta_j$

$$\therefore l_j = kd\left(\frac{1 - k\cos\theta_j + 1 + k\cos\theta_j}{1 - k^2\cos^2\theta_j}\right)$$

$$\boxed{l_j = \frac{2hd}{1-h^2\cos^2\theta_j}}$$

$$\therefore \sum_{j=1}^{n} \frac{1}{l_j} = \sum_{j=1}^{n} \frac{1-h^2\cos^2\theta_j}{2kd}$$

$$= \sum_{j=1}^{n} \frac{1-h^2\cos^2\left(\alpha+(j-1)\pi/n\right)}{2hd}$$

The first bit is clearly useful (!)

$$\sum_{r=0}^{n-1} \cos\left(2\alpha+2r\frac{\pi}{n}\right) + i\sin\left(2\alpha+2r\frac{\pi}{n}\right) = 0$$

So $$\sum_{r=0}^{n-1} \cos\left(2\alpha+2r\pi/n\right) = 0$$

Now can we make the α above have the same meaning?

$$\cos^2\left(\alpha+(j-1)\pi/n\right) = \frac{1+\cos\left(2\alpha+2(j-1)\pi/n\right)}{2}$$

let $r = j-1$

So $$\sum_{j=1}^{n} \frac{1-h^2\cos^2\left(\alpha+(j-1)\frac{\pi}{n}\right)}{2hd} = \sum_{r=0}^{n-1} \frac{1}{2hd}\left(1-\frac{h^2}{2}-\frac{h^2}{2}\cos\left(2\alpha+2r\frac{\pi}{n}\right)\right)$$

$$= \sum_{r=0}^{n-1} \frac{2-h^2}{4hd} - \frac{h}{4d}\underbrace{\sum_{r=0}^{n-1}\cos\left(2\alpha+2r\pi/n\right)}_{\text{zero}}$$

$$\therefore \sum_{j=1}^{n} \frac{1}{l_j} = \boxed{\frac{2-h^2}{4hd}\,n}$$ as required.