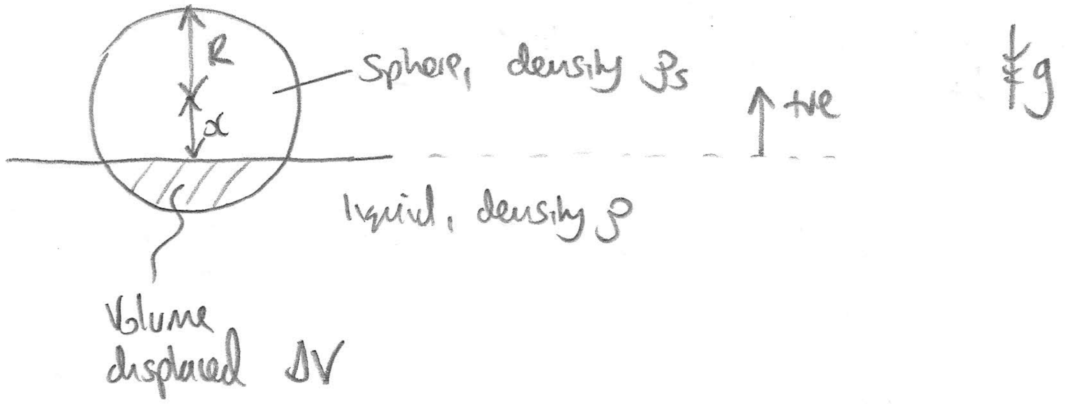
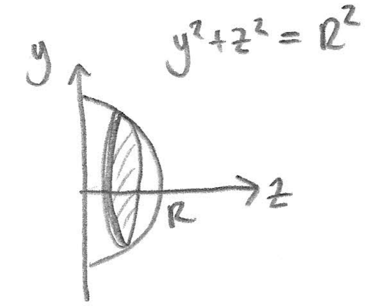
9/



$$\Delta V = \tfrac{2}{3}\pi R^3 - \int_0^x \pi y^2\,dz = \tfrac{2}{3}\pi R^3 - \int_0^x \pi(R^2 - z^2)\,dz$$



$$= \tfrac{2}{3}\pi R^3 - \pi\left[zR^2 - \tfrac{1}{3}z^3\right]_0^x$$

$$= \boxed{\frac{\pi}{3}\left\{2R^3 - 3xR^2 + x^3\right\}}$$

Newton II :

$$\underbrace{\tfrac{4}{3}\pi R^3\rho_s\,\ddot{x}}_{\text{mass} \times \text{acceleration}} = \underbrace{-\tfrac{4}{3}\pi R^3\rho_s g}_{\text{weight}} + \underbrace{\rho\Delta V g}_{\text{upthrust}}$$

$$\therefore \boxed{4R^3\rho_s(g+\ddot{x}) = (2R^3 - 3xR^2 + x^3)\rho g}$$

Sphere is in equilibrium when $x = \frac{1}{2}R$ ie $\ddot{x} = 0$

$$\therefore 4R^3\rho_s g = \left(2R^3 - \tfrac{3}{2}R^3 + \tfrac{R^3}{8}\right)\rho g$$

$$\frac{\rho_s}{\rho} = \frac{2 - 3/2 + 1/8}{4} \Rightarrow \boxed{\rho_s = \tfrac{5}{32}\rho}$$

Consider small oscillations about equilibrium position

let $x = \frac{1}{2}R + w$

$$\therefore 4R^3\rho_s(g+\ddot{w}) = \left(2R^3 - 3\left(\tfrac{1}{2}R + w\right)R^2 + \left(w + \tfrac{R}{2}\right)^3\right)\rho g$$

$$4R^3 + \frac{5}{32}(g+\ddot{w}) = g\left(2R^3 - \frac{3R^3}{2} - 3wR^2 + w^3 + \frac{3w^2R}{2} + \frac{3wR^2}{4} + \frac{R^3}{8}\right)$$

$$\frac{5}{8}R^3(g+\ddot{w}) = g\left(\frac{5}{8}R^3 - \frac{9}{4}wR^2 + \frac{3R}{2}w^2 + w^3\right)$$

$$\frac{5}{8}R^3\ddot{w} = g\left(-\frac{9}{4}wR^2 + \frac{3R}{2}w^2 + w^3\right)$$

$$\boxed{\ddot{w} = \frac{8}{5}\frac{g}{R^3}\left(-\frac{9}{4}wR^2 + \frac{3R}{2}w^2 + w^3\right)}$$

For small oscillations, ignore w^2 and w^3 terms

Since assume $w^3 \ll w^2 \ll w$

$$\therefore \ddot{w} \approx -\frac{8g}{5R^3}\left(\frac{9}{4}wR^2\right)$$

$$\ddot{w} \approx -\frac{18g}{5R}w$$

SHM with period T such that

$$\frac{18g}{5R} = \left(\frac{2\pi}{T}\right)^2$$

$$\therefore T = 2\pi\sqrt{\frac{5R}{18g}}$$

$$T = \frac{2}{3}\pi\sqrt{\frac{5R}{2g}}$$

$$\boxed{T = \frac{\pi}{3}\sqrt{\frac{10R}{g}}}$$