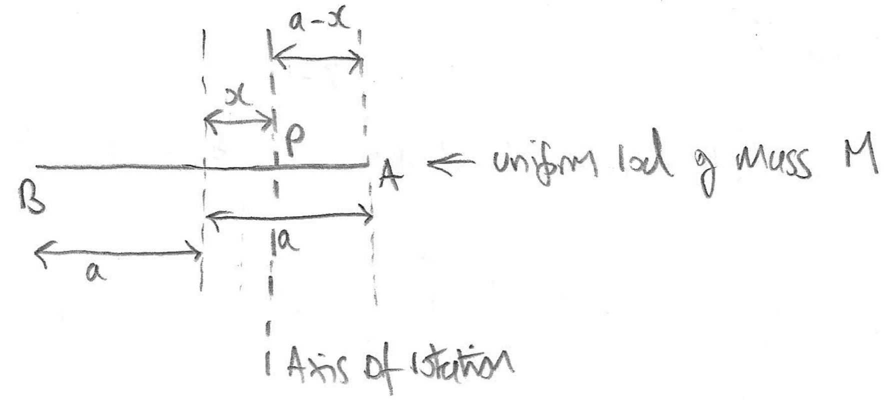
10/



Moment of inertia about P is $\int_0^{a-x} z^2 \rho \, dz + \int_0^{a+x} z^2 \rho \, dz = I$

$\rho = \frac{M}{2a}$ ie mass per unit length

ie "$\sum mr^2$"

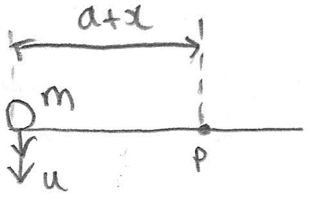
$$I = \frac{1}{3}\frac{M}{2a}\left((a-x)^3 + (a+x)^3\right)$$

$$I = \frac{1}{3}\frac{M}{2a}\left(a^3 - \cancel{3a^2x} + 3ax^2 - \cancel{x^3} + a^3 + \cancel{3a^2x} + 3ax^2 + \cancel{x^3}\right)$$

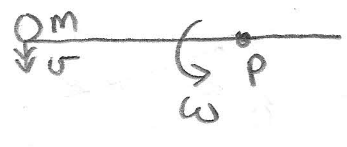
$$I = \frac{1}{3}\frac{M}{2a}\left(2a^3 + 6ax^2\right)$$

$$\boxed{I = \frac{1}{3}M\left(a^2 + 3x^2\right)}$$

BEFORE



AFTER



Conservation of angular momentum

$$mu(a+x) = mv(a+x) + \frac{1}{3}M(a^2+3x^2)\omega$$

Restitution $e = \frac{\text{Speed of separation}}{\text{Speed of approach}} = \frac{(a+x)\omega - v}{u}$

So $v = (a+x)\omega - eu$

$\therefore \quad mu(a+x) = m\left((a+x)\omega - eu\right)(a+x) + \frac{1}{3}M(a^2+3x^2)\omega$

$mu(a+x) + meu(a+x) = \omega\left(m(a+x)^2 + \frac{1}{3}M(a^2+3x^2)\right)$

$$\therefore \quad \omega = \frac{mu(a+x)(1+e)}{m(a+x)^2 + \frac{1}{3}M(a^2+3x^2)}$$

$$\boxed{\omega = \frac{3mu(1+e)(a+x)}{M(a^2+3x^2) + 3m(a+x)^2}}$$ as required

let $m = 2M$ $\quad \therefore \quad \omega = \dfrac{6u(1+e)(a+x)}{a^2+3x^2+6(a^2+2ax+x^2)}$

$$\boxed{\omega = \frac{6u(1+e)(a+x)}{7a^2+9x^2+12ax}}$$

$$\frac{d\omega}{dx} = \frac{(7a^2+9x^2+12ax)(6u(1+e)) - 6u(1+e)(a+x)(18x+12a)}{(7a^2+9x^2+12ax)^2}$$

$\frac{d\omega}{dx} = 0$ when ω is maximized (guess stationary point is a maximum)

ie $7a^2 + 9x^2 + \cancel{12ax} - 18ax - 18x^2 - 12a^2 - \cancel{12ax} = 0$

$\Rightarrow \quad 9x^2 + 18ax + 5a^2 = 0$

$(3x+5a)(3x+a) = 0$

So $\boxed{x = -\frac{a}{3} \text{ or } -\frac{5a}{3}}$

Now $a - x \geq 0$ Since P must be between B and A

$\therefore$ $x \leq a$ ✓

Also $a - x \leq 2a$ $\Rightarrow x \geq -a$

Now $-\frac{5}{3}a < -a$ So only $\boxed{x = -\frac{a}{3}}$ is feasible

$$\therefore \quad \omega_{max} = \frac{6u(1+e)(a - \frac{a}{3})}{7a^2 + 9\left(-\frac{a}{3}\right)^2 + 12a\left(-\frac{a}{3}\right)}$$

$$= \frac{6u(1+e)a \times \frac{2}{3}}{7a^2 + a^2 - 4a^2}$$

$$= \frac{4au(1+e)}{4a^2}$$

$$= \boxed{\frac{u(1+e)}{a}}$$ as required.