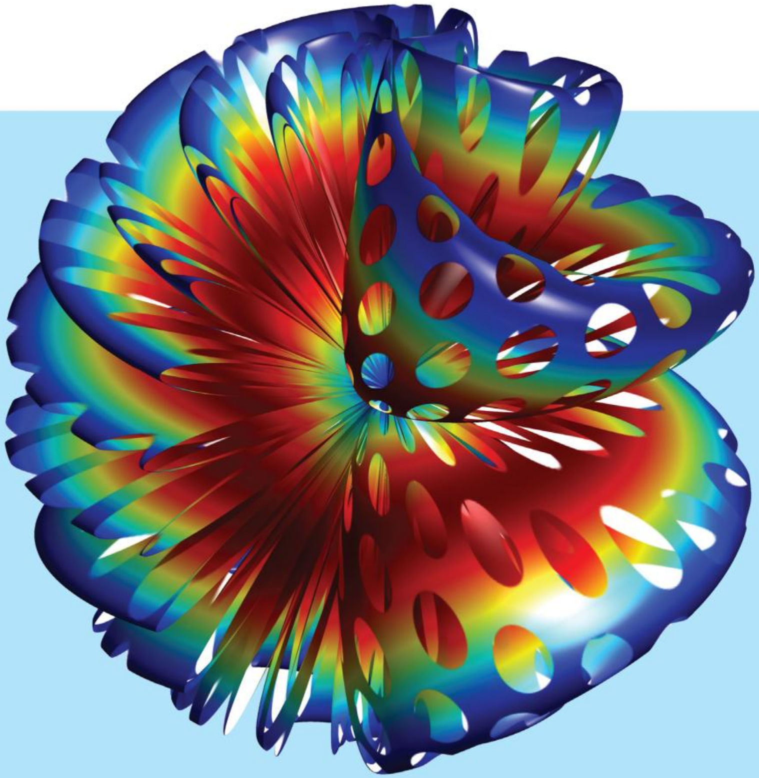
SCIENCE BY SIMULATION

Volume 1: A Mezze of Mathematical Models

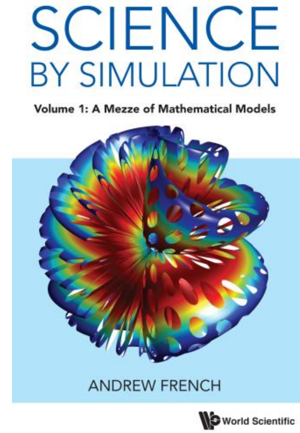


ANDREW FRENCH

World Scientific

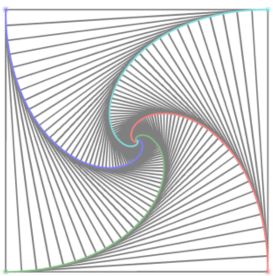
Who?

Dr Andy French. Physics teacher at [Winchester College](), UK.



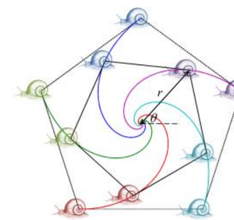
What?

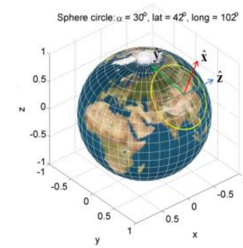
[Book]() / [website]() / educational concept / new [BPhO]() course

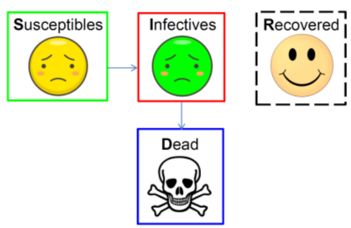


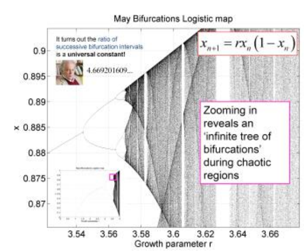
SCIENCE BY SIMULATION

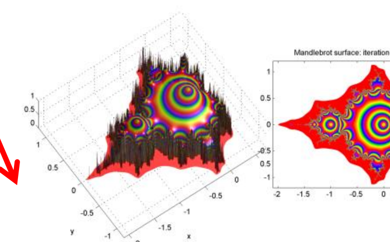
Dr Andrew French andy.french@physics.org www.eclecticon.info/scibysim.htm

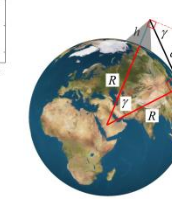






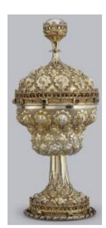


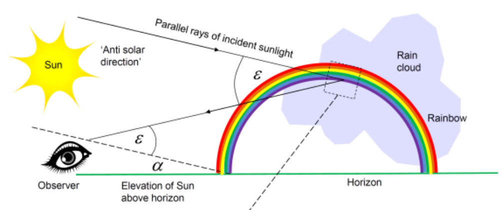




When?

ASE International Day 12/01/22





How?

A selection of example models and contexts

Why?

Dr Andrew French andy.french@physics.org www.eclecticon.info/scibysim.htm

- Many (perhaps *most*) future jobs will be performed by robots / artificial intelligence and not humans. **I'd like my students to have a good chance to become the programmers. The alternative doesn't sound nearly so interesting.**

- But we have a widening skills gap. For both students *and* teachers. Mathematical content of A-Level Physics has been steadily removed (in the UK) in the past few decades. IT, and access to IT, is mostly pervasive, but ***creative* IT**, **model building**, **experimental experience**, **data analysis**, **datalogging** and **numerical methods** are often outside the scope of an increasingly exam-focussed, paired down syllabus.

And yet these skills are *highly* desirable to modern industry

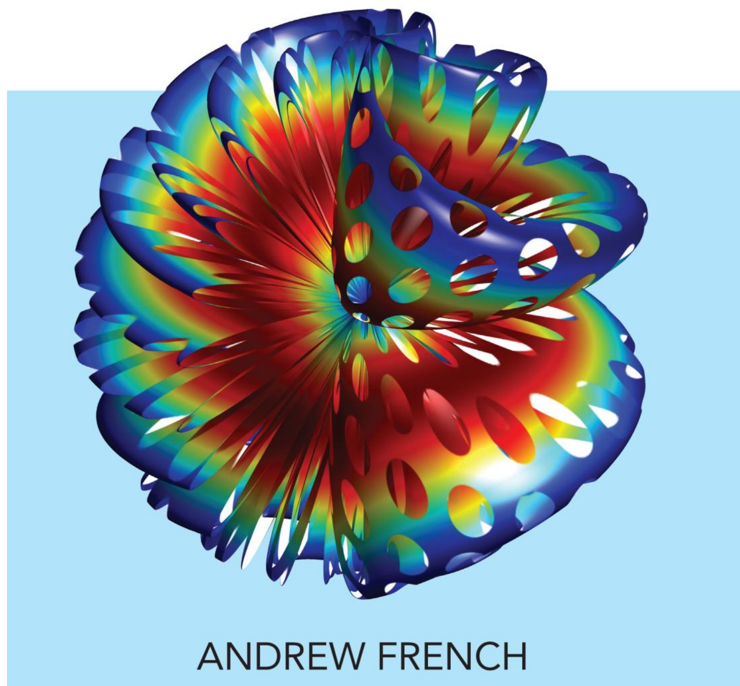
Learn to build mathematical models

SCIENCE BY SIMULATION

Volume 1: A Mezze of Mathematical Models

The power of context

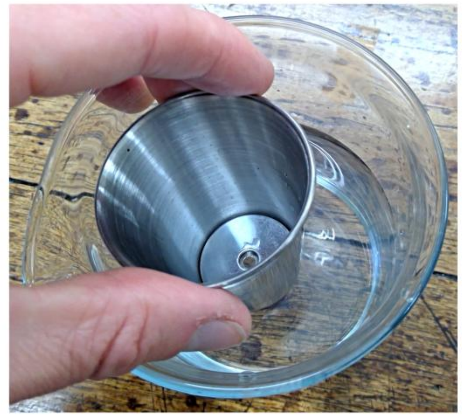
Science by storytelling!

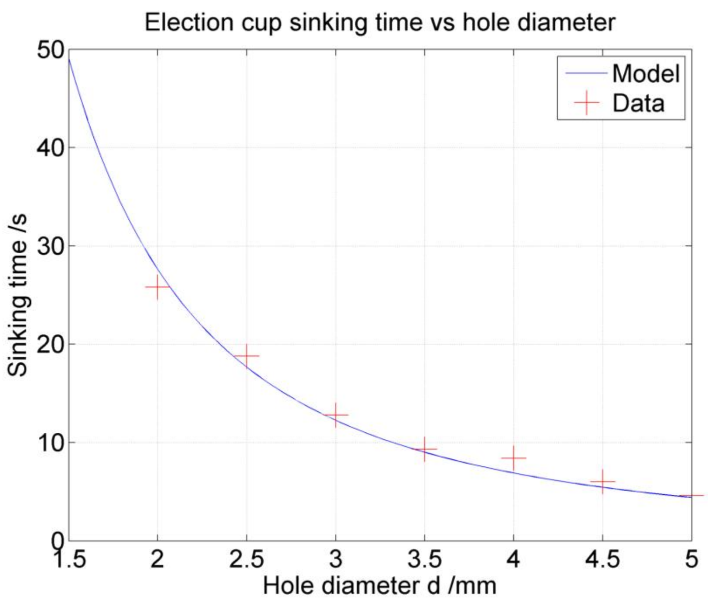


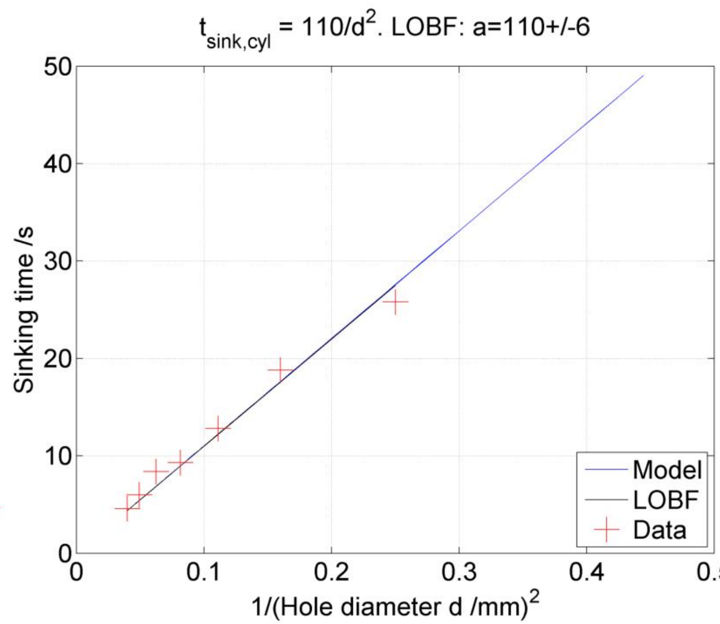
Learn to code dynamic computer simulations

World Scientific

ELECTION CUPS







ELECTION PHYSICS PRACTICAL 2020
ELECTION CUPS
A.French & A. Chesters 21/9/2019

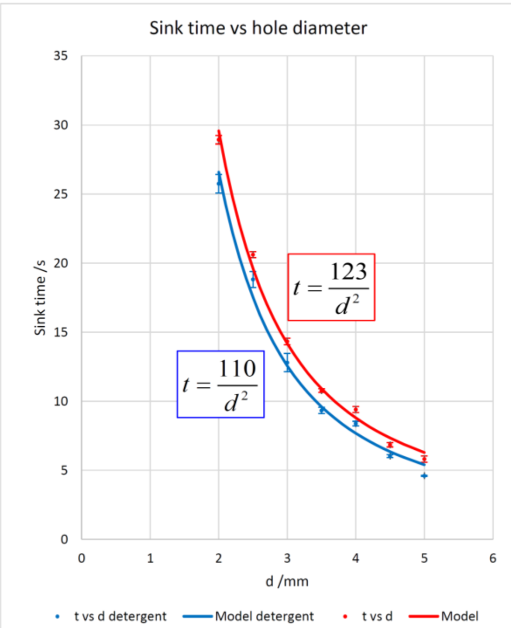
400ml of water in a 600ml beaker
Depth of water 1.2cm above rim of submerged cup

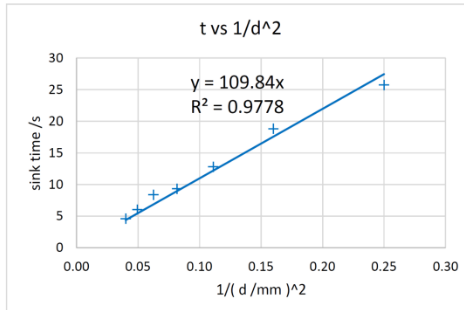
WITH DETERGENT

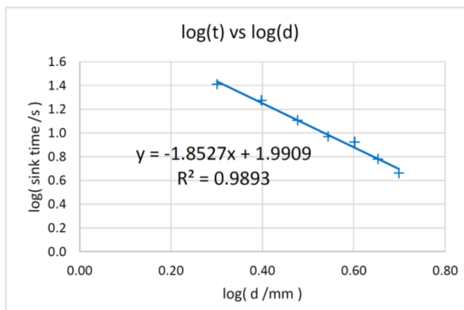
(a)

Hole diameter /mm (d)	Cup mass + 2x washers /g	Sink time /s: 1	2	3	Mean /s (t)	Mean of squares	Uncertainty /s (dt)	ln(d)	ln(t)	1/d^2
2	41.8	25.01	26.32	25.92	25.75	663.36	0.67	0.301	1.411	0.250
2.5	41.5	19.47	18.63	18.36	18.82	354.42	0.58	0.398	1.275	0.160
3	42.4	13.56	12.45	12.38	12.80	164.05	0.66	0.477	1.107	0.111
3.5	42.5	9.44	9.48	9.07	9.33	87.08	0.23	0.544	0.970	0.082
4	41	8.54	8.4	8.21	8.38	70.30	0.17	0.602	0.923	0.063
4.5	43	6.02	5.94	6.14	6.03	36.41	0.10	0.653	0.781	0.049
5	42.2	4.57	4.65	4.58	4.60	21.16	0.04	0.699	0.663	0.040
unknown	42.4	9.68	9.58	10.22	9.83	96.64	0.34			

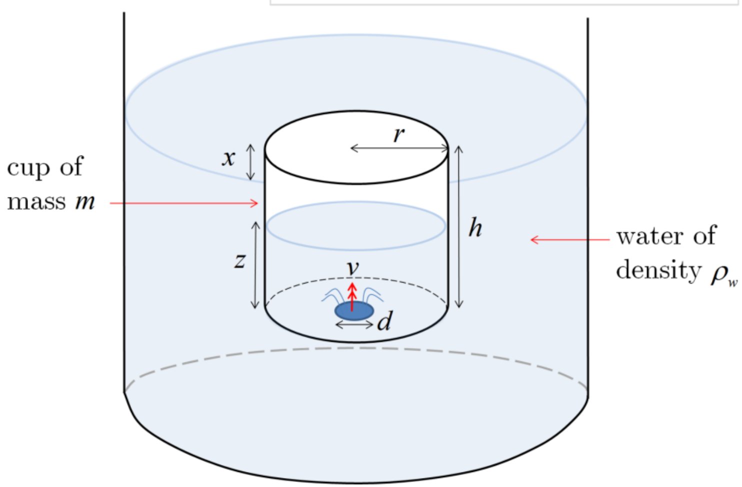
MODEL	
d /mm	t /s
2	26.58
2.1	24.24
2.2	22.21
2.3	20.43
2.4	18.88
2.5	17.51
2.6	16.29
2.7	15.21
2.8	14.24
2.9	13.36
3	12.58
3.1	11.87
3.2	11.22
3.3	10.63
3.4	10.10
3.5	9.61
3.6	9.16
3.7	8.74
3.8	8.36
3.9	8.00
4	7.68
4.1	7.37
4.2	7.09
4.3	6.83
4.4	6.58
4.5	6.35
4.6	6.14
4.7	5.94
4.8	5.75
4.9	5.58
5	5.41

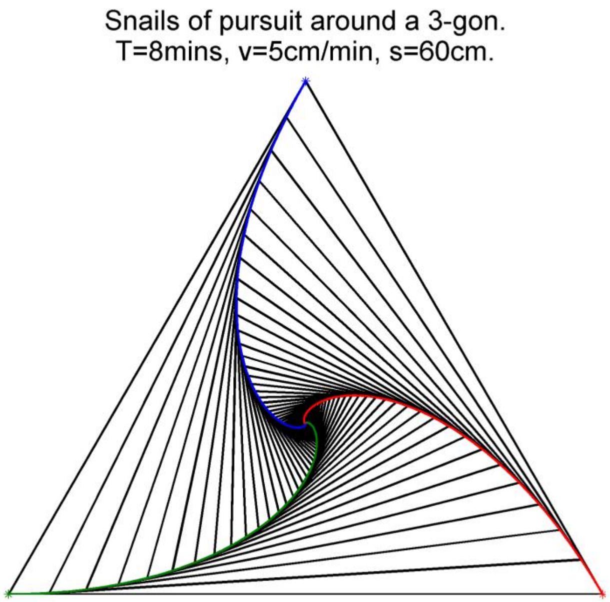




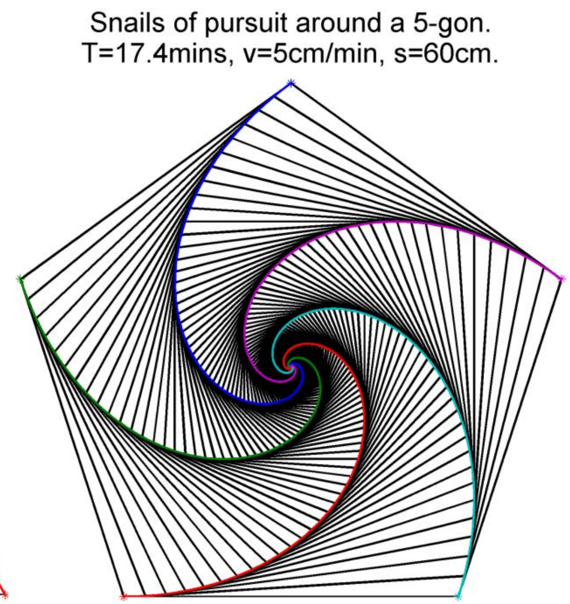


(b)

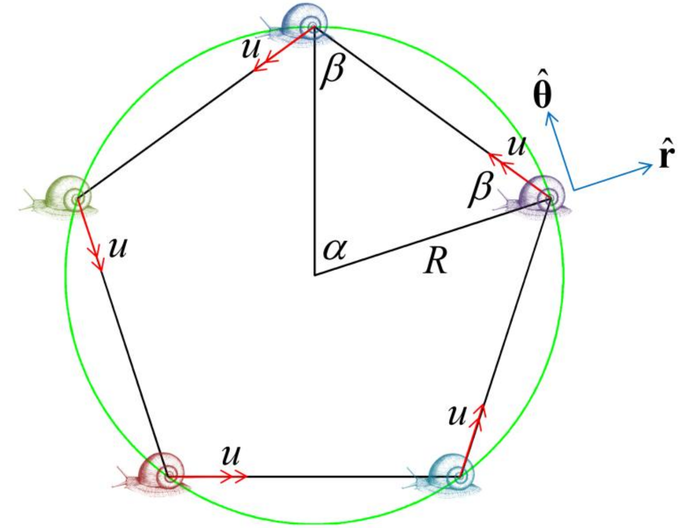


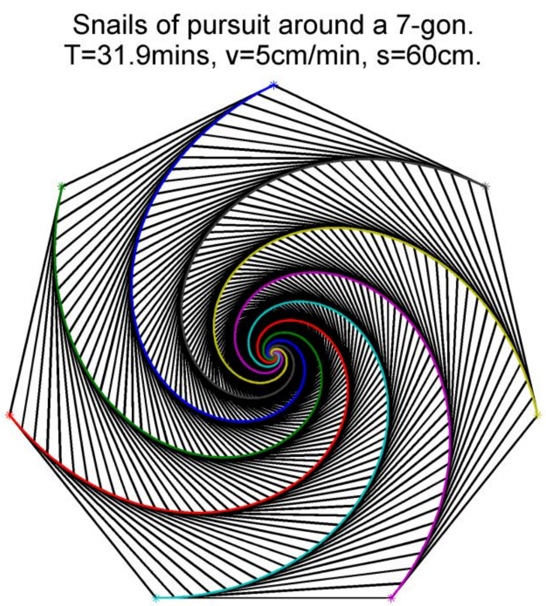

Snails of pursuit around a 3-gon.
T=8mins, v=5cm/min, s=60cm.

(a)

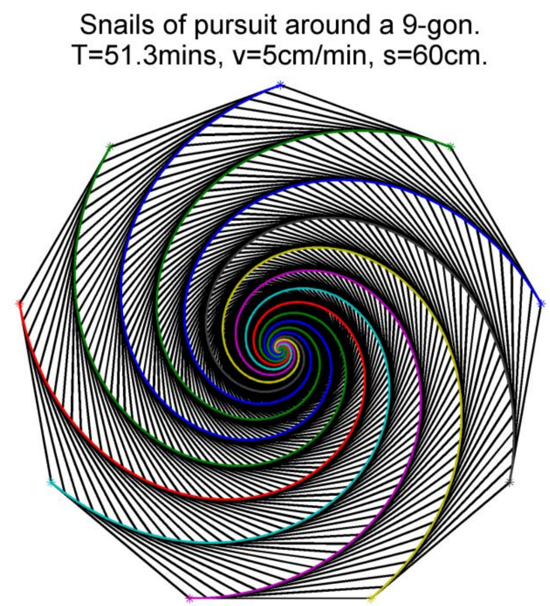

Snails of pursuit around a 5-gon.
T=17.4mins, v=5cm/min, s=60cm.

(b)

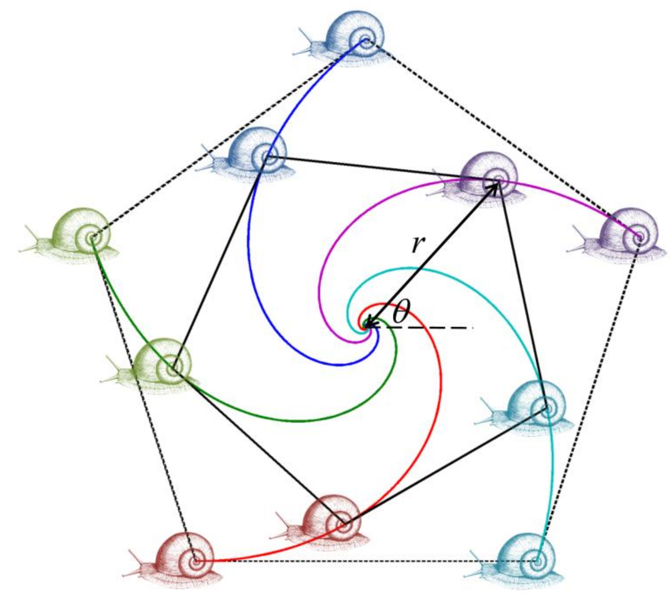

u
β
θ̂
r̂
α
R


Snails of pursuit around a 7-gon.
T=31.9mins, v=5cm/min, s=60cm.

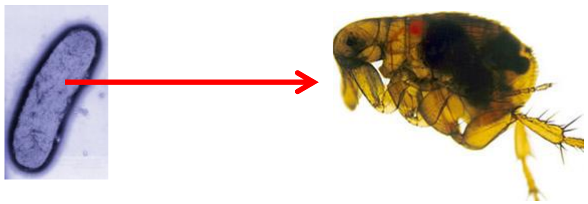
(c)


Snails of pursuit around a 9-gon.
T=51.3mins, v=5cm/min, s=60cm.

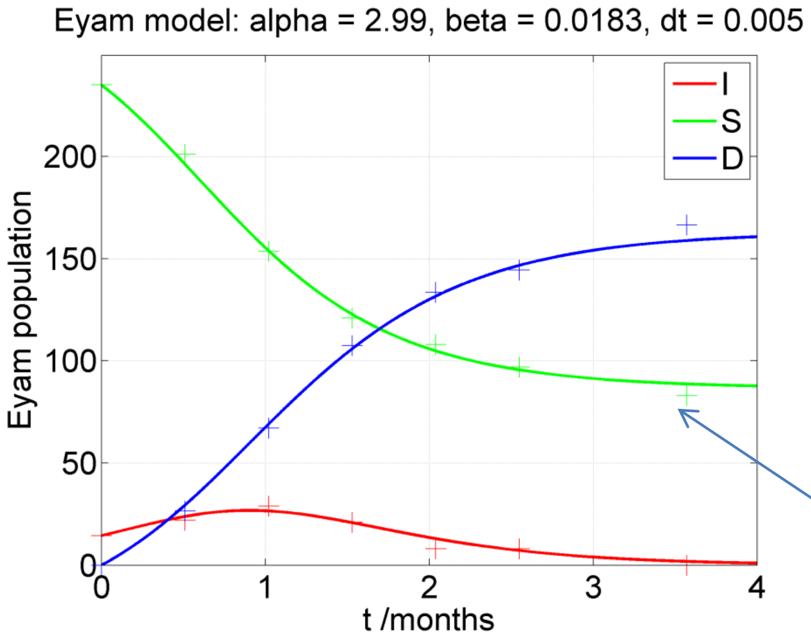
(d)

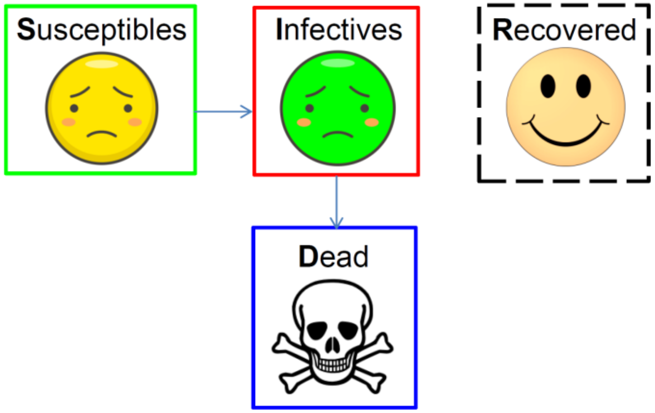

r
θ

1665. A bale of damp cloth is delivered to the Derbyshire village of **Eyam**... George Viccars, the tailor's assistant, dries the cloth and releases fleas infected with *Yersinia Pestis* bacteria – **Plague**



Rector **William Mompesson** *quarantines* Eyam and records **Infected**, **Susceptible** and **Dead** populations *as time progresses*





Can we develop a mathematical model to predict I,S,D vs time? What does this tell us about *Epidemiology* in general?

e.g Flu, Ebola

Calculus methods, differential equations numerical methods, line of best fit, iteration, loops ...

$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI - \alpha I$$

$$\frac{dD}{dt} = \alpha I$$

Leonhard Euler
1707-1783

Euler numerical *iterative* solution scheme

$$\alpha = 2.894, \quad \beta = \frac{\alpha}{163.3}$$

$$t_0 = 0,\ S_0 = 235,\ I_0 = 14.5,\ D_0 = 0$$

$$t_{n+1} = t_n + \Delta t$$

$$S_{n+1} = S_n - \beta S_n I_n \Delta t$$

$$I_{n+1} = I_n + \left(\beta S_n I_n - \alpha I_n\right)\Delta t$$

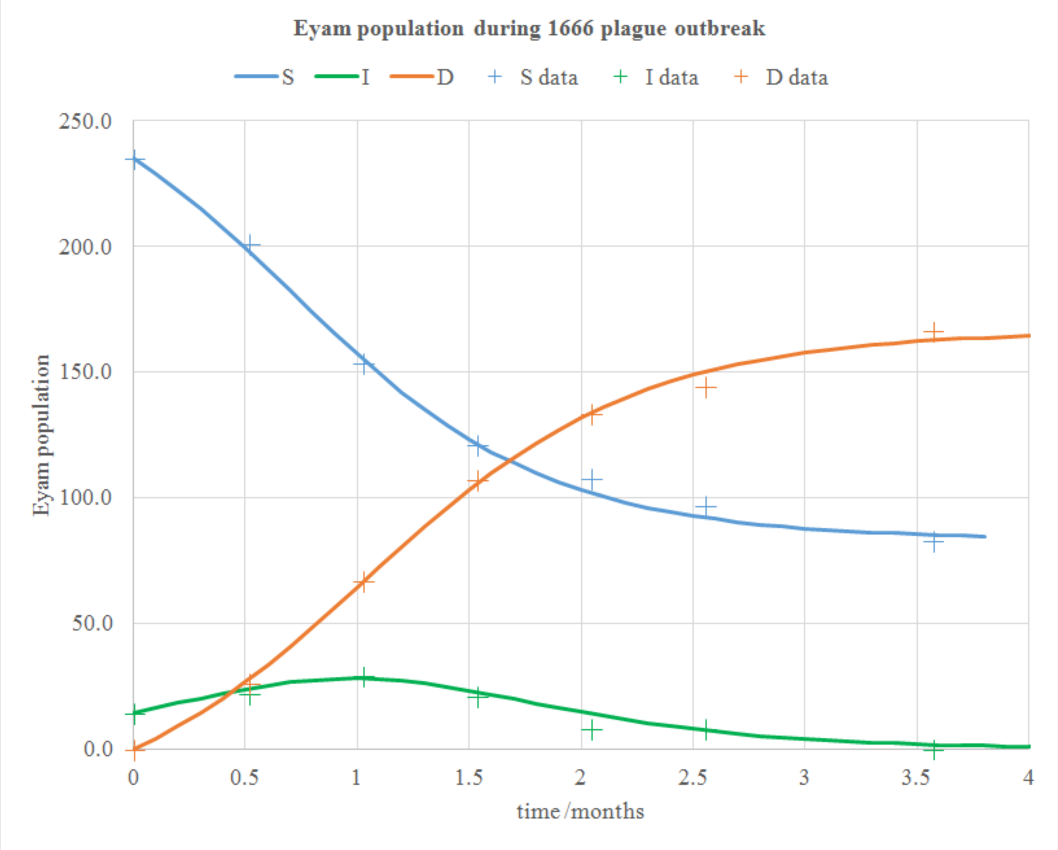
$$D_{n+1} = D_n + \alpha I_n \Delta t$$

We performed the Eyam analysis in **Python**, then in **MATLAB**.
You can also construct an Euler model via a spreadsheet (**Excel**).

Black Death Epidemiological model using the Eyam data
Andy French & John Cullerne. 24th February 2018.

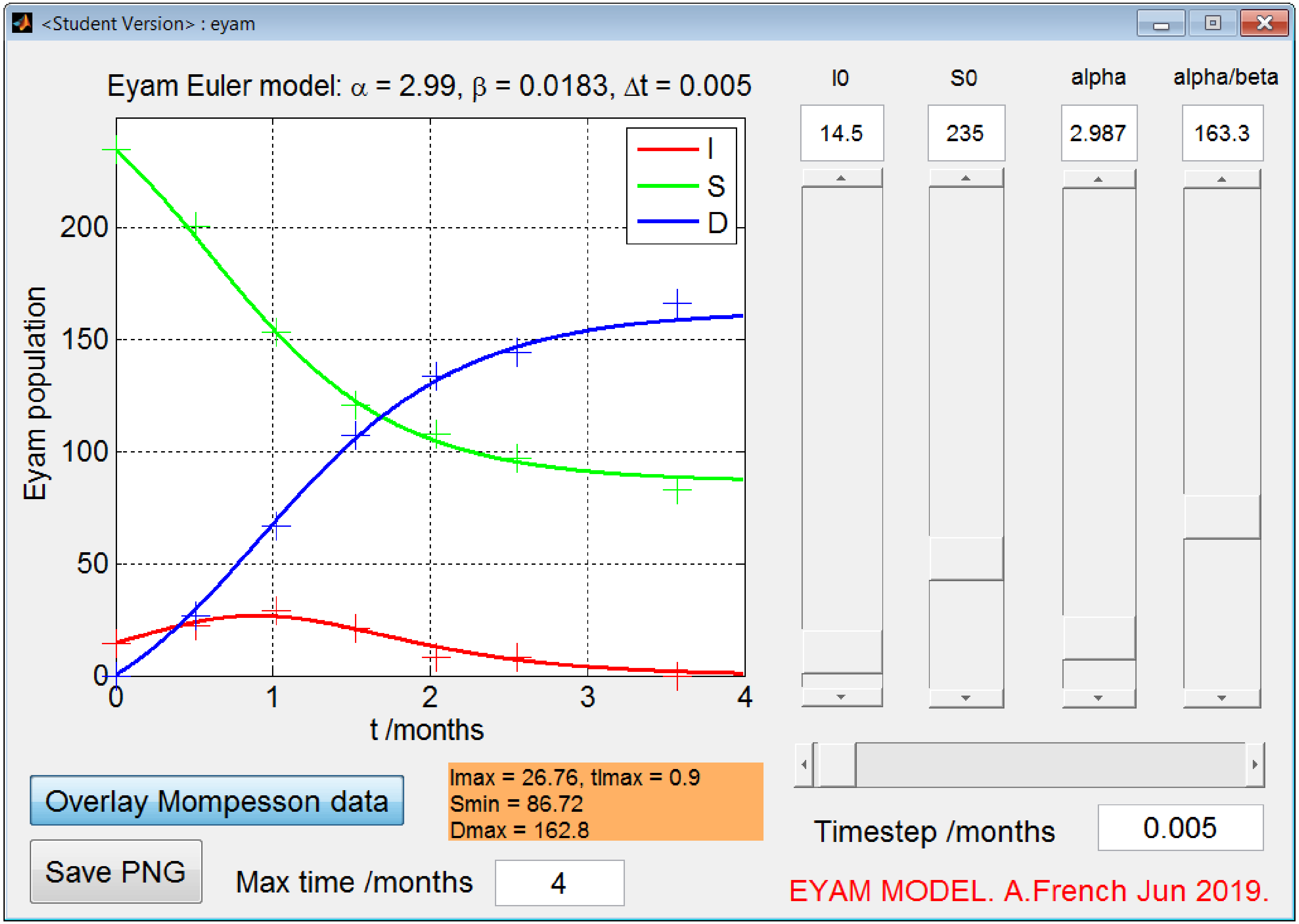
Initial population N0	249.5
Initial number of succeptables S0	**235**
Initial number of infectives I0	**14.5**
Transmission rate constant beta	0.017759
Death rate constant alpha	**2.9**
timestep dt /months	**0.1**

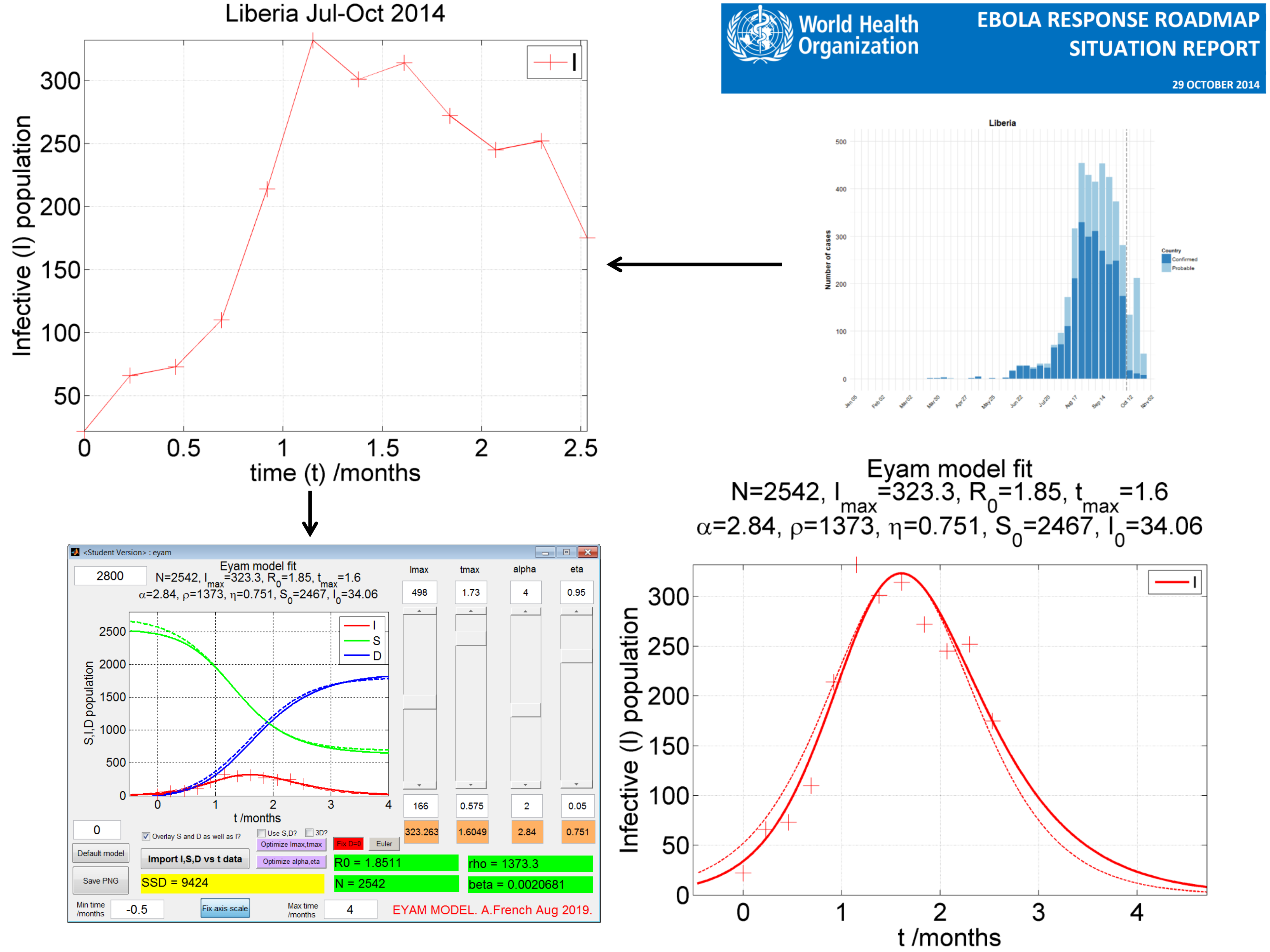
t /months	S	I	D	N	N+D = N0
0	235.0	14.5	0.0	249.5	249.5
0.1	228.9	16.3	4.2	245.3	249.5
0.2	222.3	18.3	8.9	240.6	249.5
0.3	215.1	20.2	14.2	235.3	249.5
0.4	207.4	22.0	20.1	229.4	249.5
0.5	199.3	23.7	26.5	223.0	249.5
0.6	190.9	25.3	33.4	216.1	249.5
0.7	182.3	26.5	40.7	208.8	249.5
0.8	173.7	27.4	48.4	201.1	249.5
0.9	165.3	27.9	56.3	193.2	249.5
1	157.1	28.0	64.4	185.1	249.5
1.1	149.3	27.7	72.5	177.0	249.5
1.2	141.9	27.0	80.6	168.9	249.5
1.3	135.1	26.0	88.4	161.1	249.5
1.4	128.9	24.7	95.9	153.6	249.5
1.5	123.3	23.2	103.1	146.4	249.5
1.6	118.2	21.5	109.8	139.7	249.5



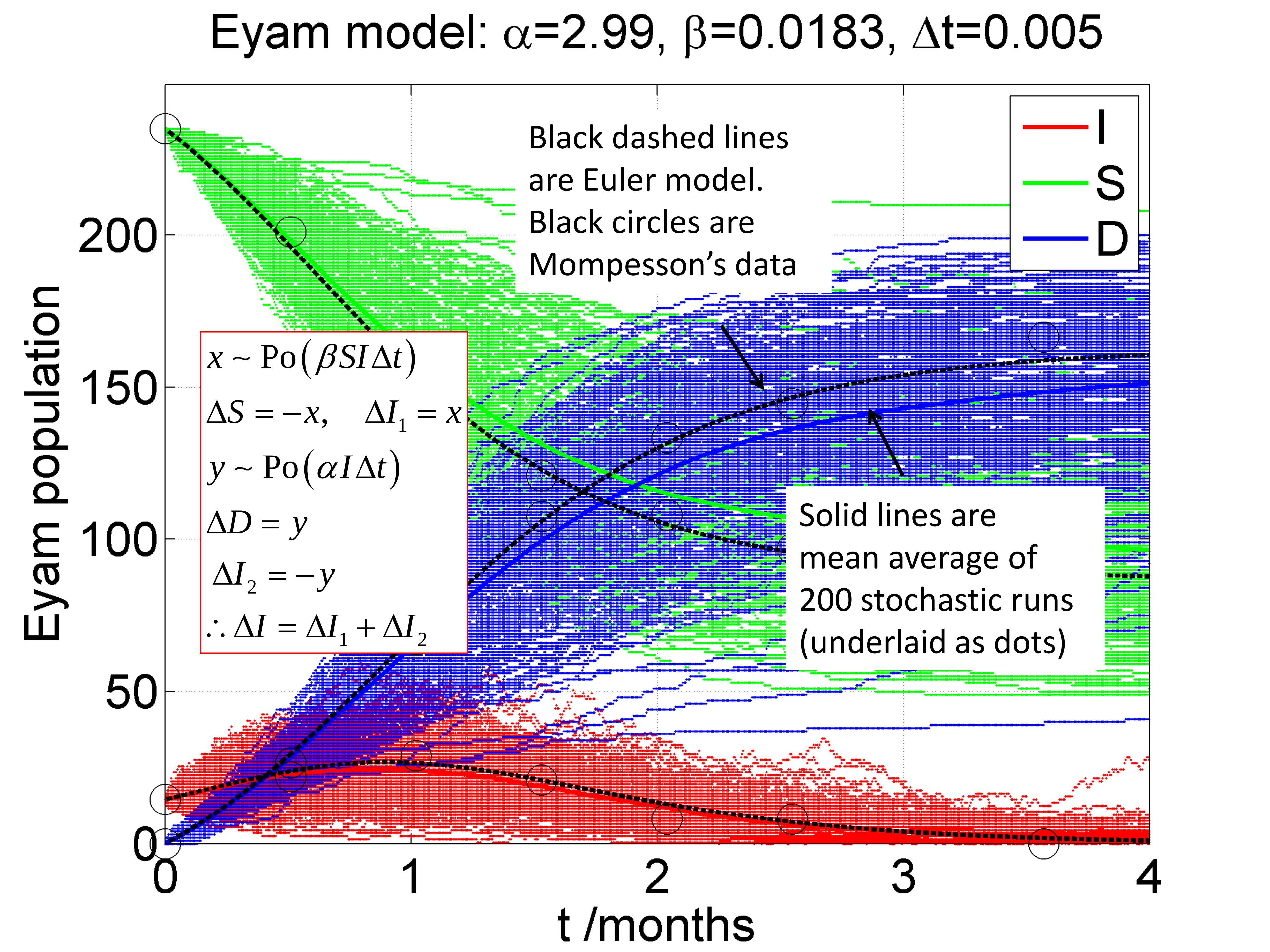
$$\frac{dS}{dt} = -\beta SI \qquad \frac{dI}{dt} = \beta SI - \alpha I \qquad \frac{dD}{dt} = \alpha I$$

Euler Eyam solver implemented in MATLAB with a Graphical User Interface (GUI).
Change the inputs via the sliders or edit boxes, and the curves are computed automatically.

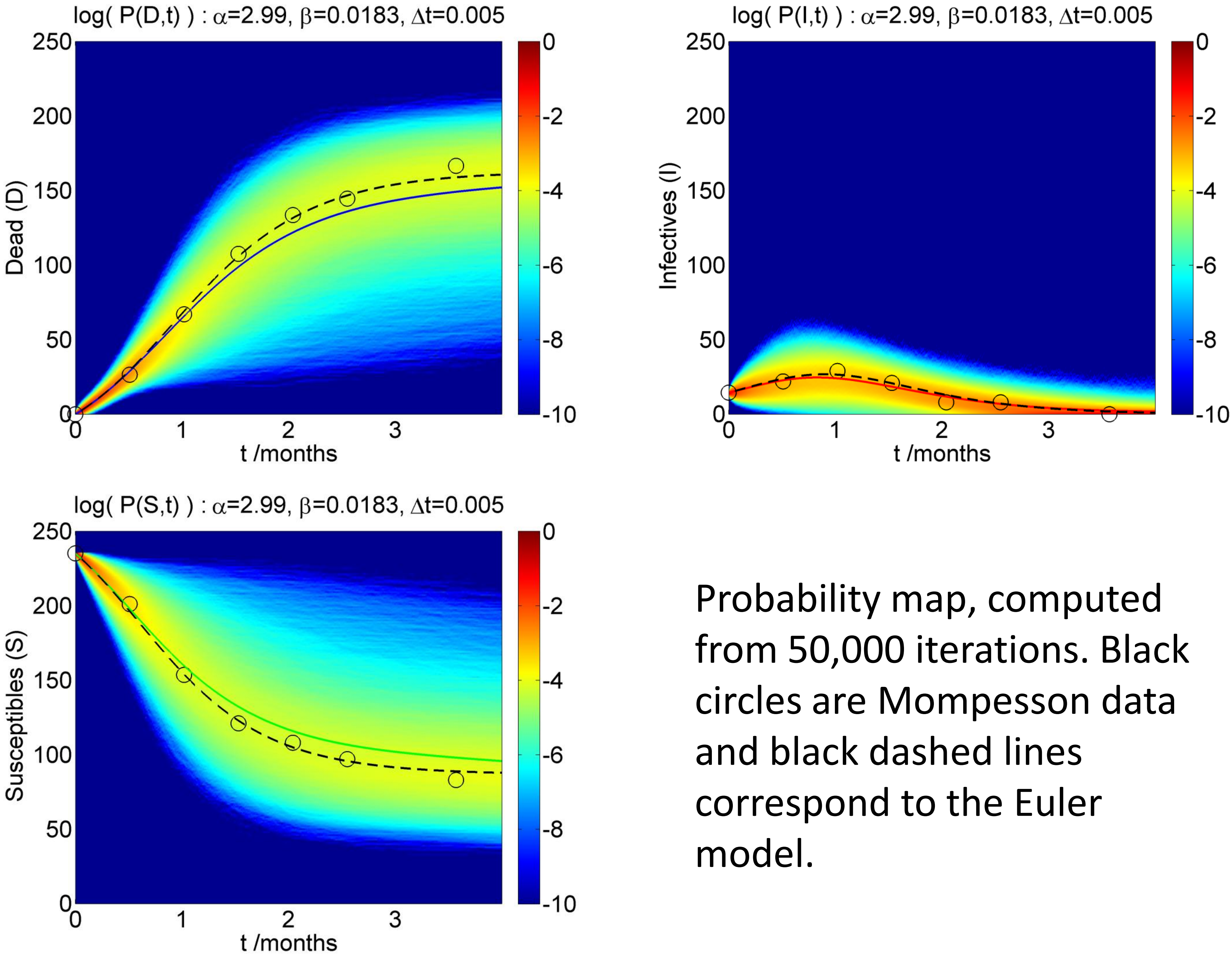




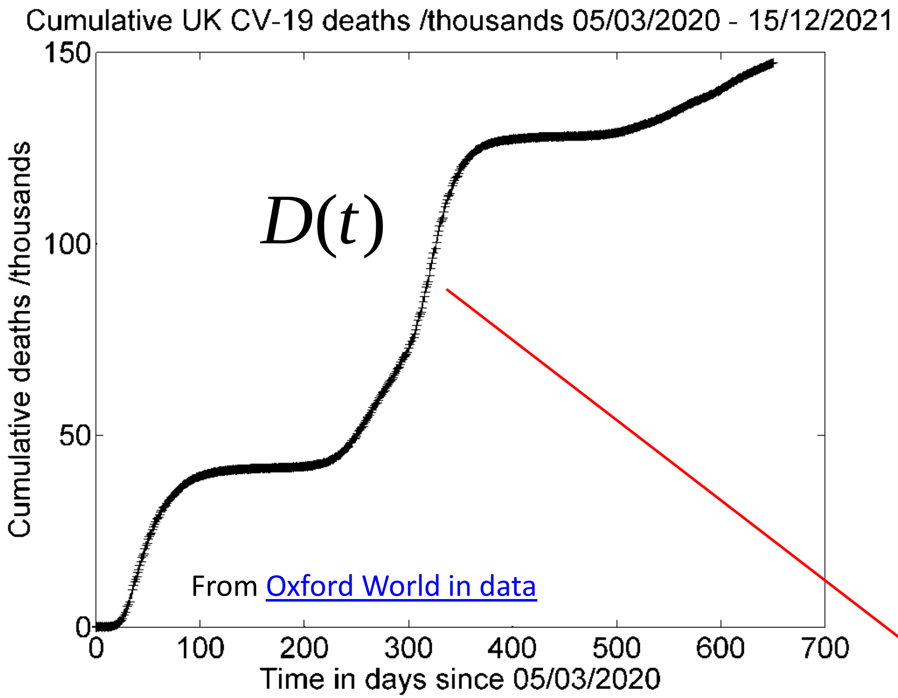
Liberia Jul-Oct 2014
Infective (I) population
time (t) /months
I
World Health Organization
EBOLA RESPONSE ROADMAP
SITUATION REPORT
29 OCTOBER 2014
Liberia
Number of cases
Country
Confirmed
Probable
<Student Version> : eyam
Eyam model fit
N=2542, Imax=323.3, R0=1.85, tmax=1.6
α=2.84, ρ=1373, η=0.751, S0=2467, I0=34.06
Imax
tmax
alpha
eta
S,I,D population
t /months
Overlay S and D as well as I?
Use S,D?
3D?
Optimize Imax,tmax
Fix D=0
Euler
Default model
Import I,S,D vs t data
Optimize alpha,eta
R0 = 1.8511
rho = 1373.3
Save PNG
SSD = 9424
N = 2542
beta = 0.0020681
Min time /months
Fix axis scale
Max time /months
EYAM MODEL. A.French Aug 2019.
Eyam model fit
N=2542, Imax=323.3, R0=1.85, tmax=1.6
α=2.84, ρ=1373, η=0.751, S0=2467, I0=34.06
Infective (I) population
t /months



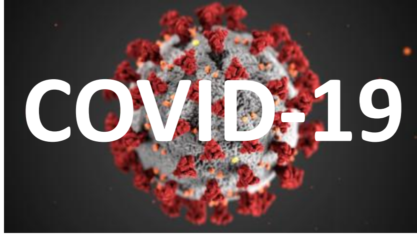
Eyam model: α=2.99, β=0.0183, Δt=0.005
Black dashed lines are Euler model.
Black circles are Mompesson's data
Solid lines are mean average of 200 stochastic runs (underlaid as dots)
$x \sim \mathrm{Po}(\beta S I \Delta t)$
$\Delta S = -x, \quad \Delta I_1 = x$
$y \sim \mathrm{Po}(\alpha I \Delta t)$
$\Delta D = y$
$\Delta I_2 = -y$
$\therefore \Delta I = \Delta I_1 + \Delta I_2$
I
S
D
Eyam population
t /months
0
50
100
150
200
0
1
2
3
4



Probability map, computed from 50,000 iterations. Black circles are Mompesson data and black dashed lines correspond to the Euler model.



One can *estimate* the number of CV-19 **infectives** from the cumulative deaths:



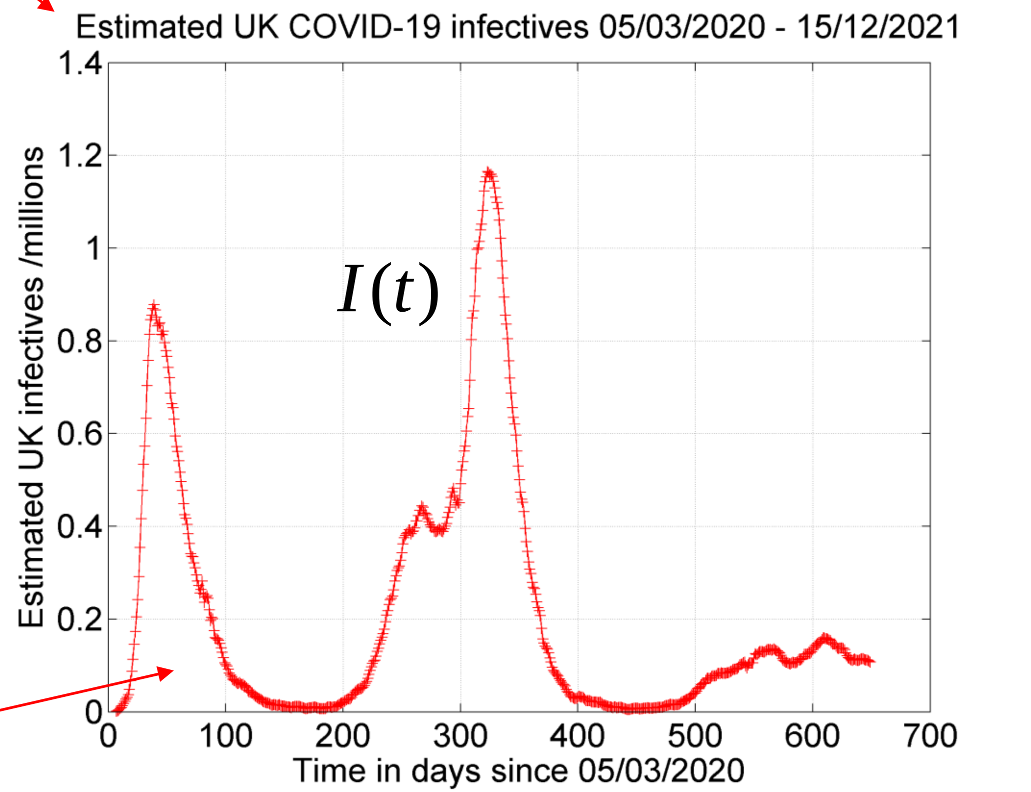
$$I_n = \frac{1}{k\alpha}\frac{dD}{dt} \approx \frac{1}{k\alpha}\frac{D_{n+1} - D_{n-1}}{t_{n+1} - t_{n-1}}$$

Find the gradient and scale by:

$$k\alpha = 0.01 \times \frac{1}{9.32} \text{ days}^{-1}$$

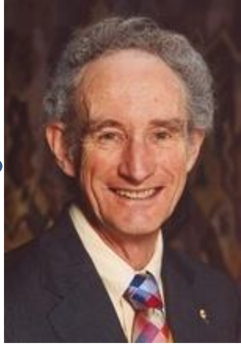
Note *mortality fraction* k and *disease time constant* α may vary considerably within a population and indeed post-vaccination – so treat with caution!

Note: as per the 'daily death rate' graphs in *World in Data*, we also apply a **seven-day moving average** to smooth the numerical derivative.



The logistic map and population modelling

I published this
model in 1976



Robert May
1936-

Assume an ecosystem can support a maximum number of rabbits.
Let x be the fraction of this maximum at year n.

To account for **reproduction**, next year's population is proportional to the previous.

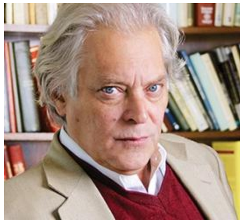
To account for **starvation**, next year's population is *also proportional* to the fraction of the maximum population as yet unfilled.

$$\boxed{x_{n+1} = r x_n (1 - x_n)}$$

Growth parameter

The population next year is predicted using this **iterative equation** called a **logistic map**

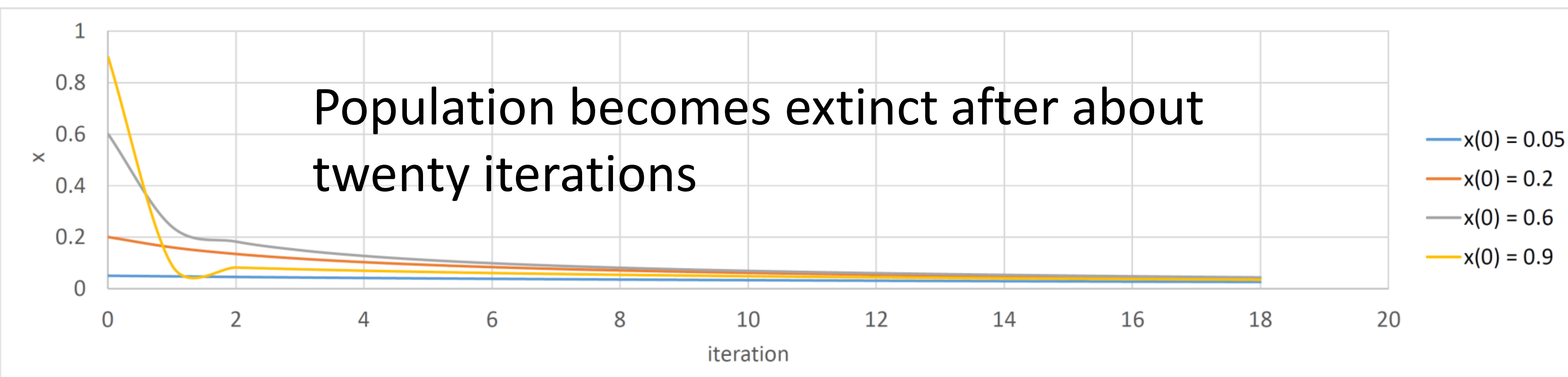
The pattern of x values with n is not always simple .....



$$r = 1 \qquad x_{n+1} = r x_n \left(1 - x_n\right)$$

iteration number n

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
x(n)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0.05	0.0475	0.045244	0.043197	0.041331	0.039623	0.038053	0.036605	0.035265	0.034021	0.032864	0.031784	0.030773	0.029826	0.028937	0.028099	0.02731	0.026564	0.025858
	0.1	0.09	0.0819	0.075192	0.069538	0.064703	0.060516	0.056854	0.053622	0.050746	0.048171	0.045851	0.043749	0.041835	0.040084	0.038478	0.036997	0.035628	0.034359
	0.15	0.1275	0.111244	0.098869	0.089094	0.081156	0.07457	0.069009	0.064247	0.060119	0.056505	0.053312	0.05047	0.047923	0.045626	0.043544	0.041648	0.039914	0.038321
	0.2	0.16	0.1344	0.116337	0.102802	0.092234	0.083727	0.076717	0.070831	0.065814	0.061483	0.057703	0.054373	0.051417	0.048773	0.046394	0.044242	0.042284	0.040496
	0.25	0.1875	0.152344	0.129135	0.112459	0.099812	0.08985	0.081777	0.075089	0.069451	0.064627	0.060451	0.056796	0.053571	0.050701	0.04813	0.045814	0.043715	0.041804
	0.3	0.21	0.1659	0.138377	0.119229	0.105013	0.093986	0.085152	0.077901	0.071833	0.066673	0.062228	0.058355	0.05495	0.05193	0.049234	0.04681	0.044619	0.042628
	0.35	0.2275	0.175744	0.144858	0.123874	0.108529	0.096751	0.08739	0.079753	0.073392	0.068006	0.063381	0.059364	0.05584	0.052722	0.049942	0.047448	0.045197	0.043154
	0.4	0.24	0.1824	0.14913	0.12689	0.110789	0.098515	0.08881	0.080923	0.074374	0.068843	0.064103	0.059994	0.056395	0.053214	0.050383	0.047844	0.045555	0.04348
	0.45	0.2475	0.186244	0.151557	0.128587	0.112053	0.099497	0.089597	0.08157	0.074916	0.069304	0.064501	0.06034	0.056699	0.053485	0.050624	0.048061	0.045751	0.043658
	0.5	0.25	0.1875	0.152344	0.129135	0.112459	0.099812	0.08985	0.081777	0.075089	0.069451	0.064627	0.060451	0.056796	0.053571	0.050701	0.04813	0.045814	0.043715
	0.55	0.2475	0.186244	0.151557	0.128587	0.112053	0.099497	0.089597	0.08157	0.074916	0.069304	0.064501	0.06034	0.056699	0.053485	0.050624	0.048061	0.045751	0.043658
	0.6	0.24	0.1824	0.14913	0.12689	0.110789	0.098515	0.08881	0.080923	0.074374	0.068843	0.064103	0.059994	0.056395	0.053214	0.050383	0.047844	0.045555	0.04348
	0.65	0.2275	0.175744	0.144858	0.123874	0.108529	0.096751	0.08739	0.079753	0.073392	0.068006	0.063381	0.059364	0.05584	0.052722	0.049942	0.047448	0.045197	0.043154
	0.7	0.21	0.1659	0.138377	0.119229	0.105013	0.093986	0.085152	0.077901	0.071833	0.066673	0.062228	0.058355	0.05495	0.05193	0.049234	0.04681	0.044619	0.042628
	0.75	0.1875	0.152344	0.129135	0.112459	0.099812	0.08985	0.081777	0.075089	0.069451	0.064627	0.060451	0.056796	0.053571	0.050701	0.04813	0.045814	0.043715	0.041804
	0.8	0.16	0.1344	0.116337	0.102802	0.092234	0.083727	0.076717	0.070831	0.065814	0.061483	0.057703	0.054373	0.051417	0.048773	0.046394	0.044242	0.042284	0.040496
	0.85	0.1275	0.111244	0.098869	0.089094	0.081156	0.07457	0.069009	0.064247	0.060119	0.056505	0.053312	0.05047	0.047923	0.045626	0.043544	0.041648	0.039914	0.038321
	0.9	0.09	0.0819	0.075192	0.069538	0.064703	0.060516	0.056854	0.053622	0.050746	0.048171	0.045851	0.043749	0.041835	0.040084	0.038478	0.036997	0.035628	0.034359
	0.95	0.0475	0.045244	0.043197	0.041331	0.039623	0.038053	0.036605	0.035265	0.034021	0.032864	0.031784	0.030773	0.029826	0.028937	0.028099	0.02731	0.026564	0.025858
	1	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16

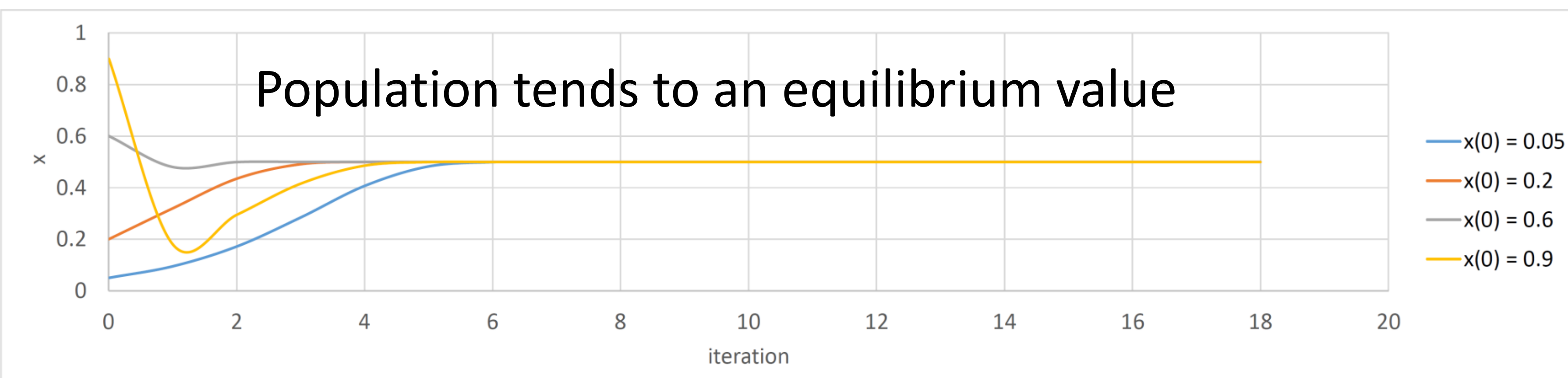


$$r = 2 \qquad x_{n+1} = r x_n \left(1 - x_n\right)$$

iteration number n

x(n)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.05	0.095	0.17195	0.284766	0.407349	0.482832	0.49941	0.499999	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.1	0.18	0.2952	0.416114	0.485926	0.499604	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.15	0.255	0.37995	0.471176	0.498338	0.499994	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.2	0.32	0.4352	0.491602	0.499859	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.25	0.375	0.46875	0.498047	0.499992	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.3	0.42	0.4872	0.499672	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.35	0.455	0.49595	0.499967	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.4	0.48	0.4992	0.499999	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.45	0.495	0.49995	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.55	0.495	0.49995	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.6	0.48	0.4992	0.499999	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.65	0.455	0.49595	0.499967	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.7	0.42	0.4872	0.499672	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.75	0.375	0.46875	0.498047	0.499992	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.8	0.32	0.4352	0.491602	0.499859	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.85	0.255	0.37995	0.471176	0.498338	0.499994	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.9	0.18	0.2952	0.416114	0.485926	0.499604	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.95	0.095	0.17195	0.284766	0.407349	0.482832	0.49941	0.499999	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
1	-4.4E-16	-8.9E-16	-1.8E-15	-3.6E-15	-7.1E-15	-1.4E-14	-2.8E-14	-5.7E-14	-1.1E-13	-2.3E-13	-4.5E-13	-9.1E-13	-1.8E-12	-3.6E-12	-7.3E-12	-1.5E-11	-2.9E-11	-5.8E-11

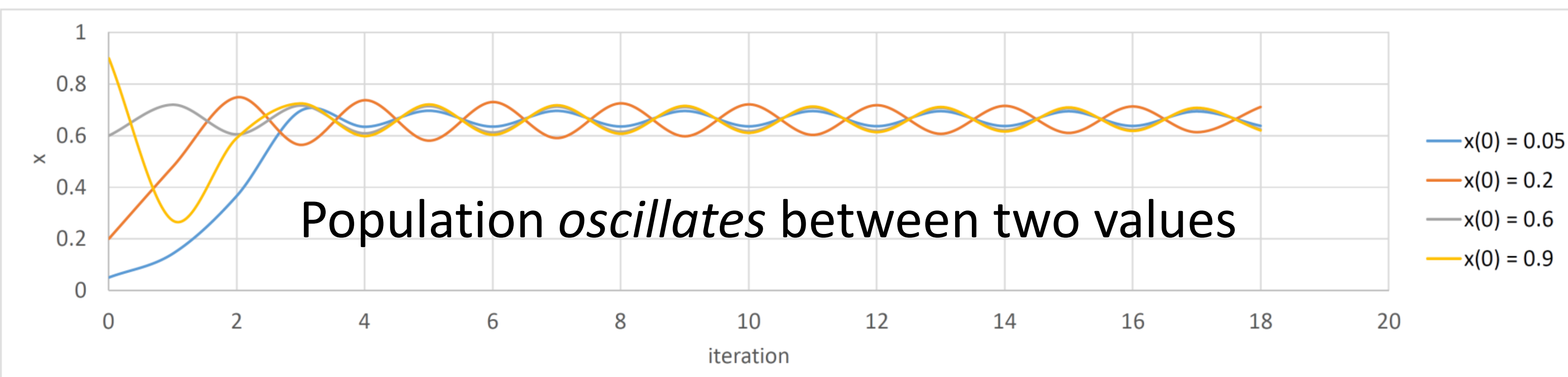


$$r = 3 \qquad x_{n+1} = r x_n (1 - x_n)$$

iteration number n

x(n)

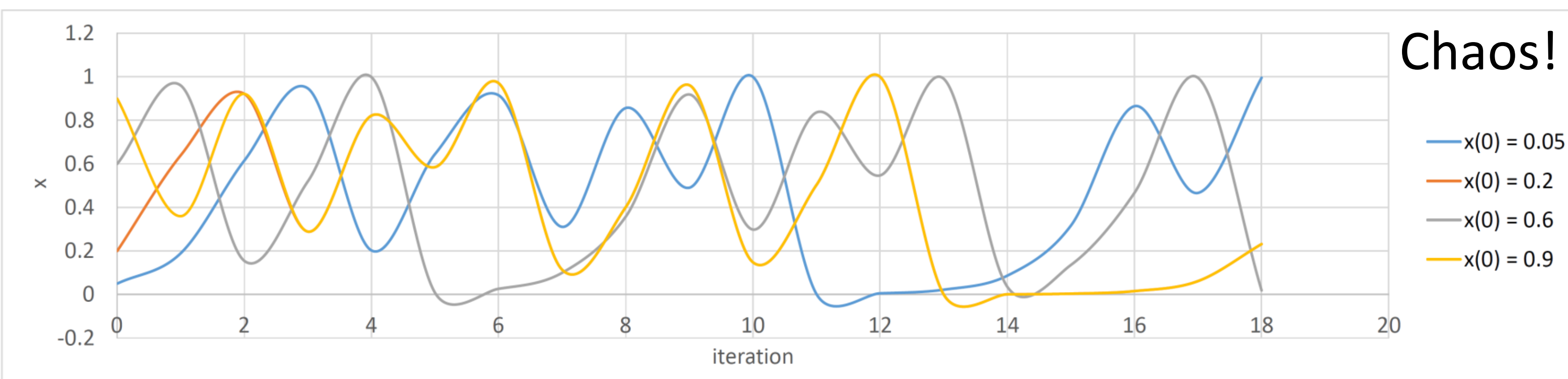
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.05	0.1425	0.366581	0.696598	0.634047	0.696094	0.634641	0.695615	0.635204	0.695159	0.635738	0.694725	0.636246	0.694311	0.63673	0.693915	0.637191	0.693536	0.637632
0.1	0.27	0.5913	0.724993	0.598135	0.721109	0.603333	0.717967	0.607471	0.71535	0.610873	0.713121	0.613738	0.711191	0.616195	0.709496	0.618334	0.707991	0.620219
0.15	0.3825	0.708581	0.619482	0.707172	0.621239	0.705904	0.622811	0.704752	0.62423	0.703701	0.625518	0.702736	0.626694	0.701846	0.627775	0.701021	0.628772	0.700253
0.2	0.48	0.7488	0.564296	0.737598	0.580641	0.730491	0.590622	0.725363	0.597634	0.721403	0.602943	0.718208	0.607155	0.715553	0.61061	0.713296	0.613514	0.711343
0.25	0.5625	0.738281	0.579666	0.73096	0.589973	0.725715	0.597158	0.721681	0.602573	0.718436	0.606857	0.715745	0.610362	0.71346	0.613304	0.711487	0.61582	0.709757
0.3	0.63	0.6993	0.630839	0.698644	0.631622	0.698027	0.632356	0.697446	0.633046	0.696897	0.633695	0.696377	0.634308	0.695884	0.634889	0.695415	0.635439	0.694969
0.35	0.6825	0.650081	0.682427	0.650161	0.682355	0.65024	0.682284	0.650318	0.682213	0.650395	0.682144	0.65047	0.682076	0.650545	0.682009	0.650619	0.681942	0.650691
0.4	0.72	0.6048	0.717051	0.608667	0.714575	0.611873	0.712453	0.614591	0.710607	0.616934	0.708979	0.618983	0.707529	0.620795	0.706226	0.622413	0.705045	0.62387
0.45	0.7425	0.573581	0.733757	0.586072	0.727775	0.594356	0.723291	0.600424	0.719745	0.605136	0.716839	0.608942	0.714395	0.612105	0.712298	0.614789	0.71047	0.617107
0.5	0.75	0.5625	0.738281	0.579666	0.73096	0.589973	0.725715	0.597158	0.721681	0.602573	0.718436	0.606857	0.715745	0.610362	0.71346	0.613304	0.711487	0.61582
0.55	0.7425	0.573581	0.733757	0.586072	0.727775	0.594356	0.723291	0.600424	0.719745	0.605136	0.716839	0.608942	0.714395	0.612105	0.712298	0.614789	0.71047	0.617107
0.6	0.72	0.6048	0.717051	0.608667	0.714575	0.611873	0.712453	0.614591	0.710607	0.616934	0.708979	0.618983	0.707529	0.620795	0.706226	0.622413	0.705045	0.62387
0.65	0.6825	0.650081	0.682427	0.650161	0.682355	0.65024	0.682284	0.650318	0.682213	0.650395	0.682144	0.65047	0.682076	0.650545	0.682009	0.650619	0.681942	0.650691
0.7	0.63	0.6993	0.630839	0.698644	0.631622	0.698027	0.632356	0.697446	0.633046	0.696897	0.633695	0.696377	0.634308	0.695884	0.634889	0.695415	0.635439	0.694969
0.75	0.5625	0.738281	0.579666	0.73096	0.589973	0.725715	0.597158	0.721681	0.602573	0.718436	0.606857	0.715745	0.610362	0.71346	0.613304	0.711487	0.61582	0.709757
0.8	0.48	0.7488	0.564296	0.737598	0.580641	0.730491	0.590622	0.725363	0.597634	0.721403	0.602943	0.718208	0.607155	0.715553	0.61061	0.713296	0.613514	0.711343
0.85	0.3825	0.708581	0.619482	0.707172	0.621239	0.705904	0.622811	0.704752	0.62423	0.703701	0.625518	0.702736	0.626694	0.701846	0.627775	0.701021	0.628772	0.700253
0.9	0.27	0.5913	0.724993	0.598135	0.721109	0.603333	0.717967	0.607471	0.71535	0.610873	0.713121	0.613738	0.711191	0.616195	0.709496	0.618334	0.707991	0.620219
0.95	0.1425	0.366581	0.696598	0.634047	0.696094	0.634641	0.695615	0.635204	0.695159	0.635738	0.694725	0.636246	0.694311	0.63673	0.693915	0.637191	0.693536	0.637632
1	-6.7E-16	-2E-15	-6E-15	-1.8E-14	-5.4E-14	-1.6E-13	-4.9E-13	-1.5E-12	-4.4E-12	-1.3E-11	-3.9E-11	-1.2E-10	-3.5E-10	-1.1E-09	-3.2E-09	-9.6E-09	-2.9E-08	-8.6E-08

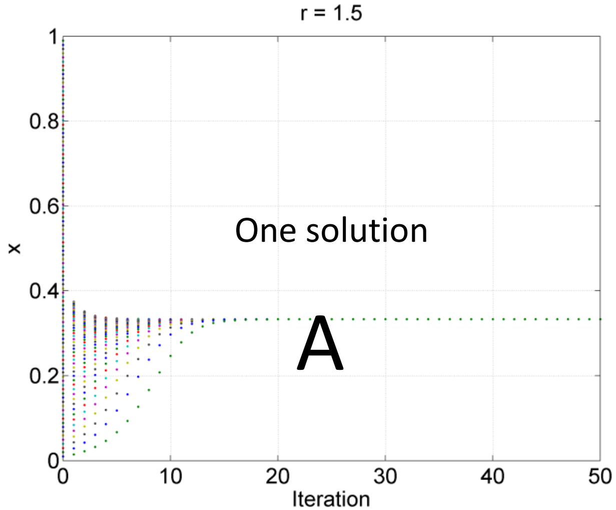


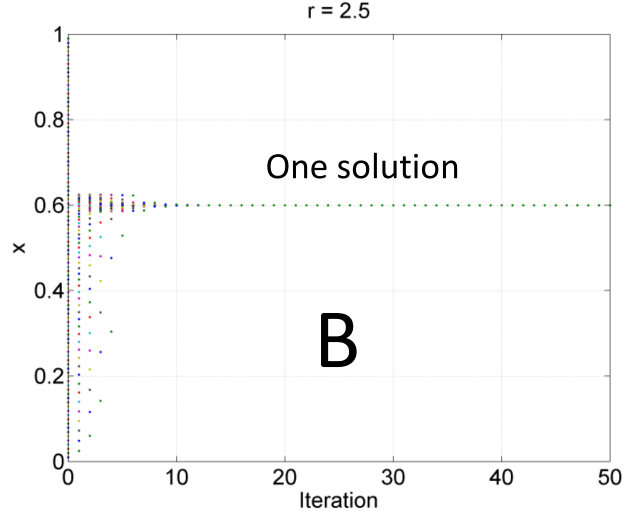
$$r = 4 \qquad x_{n+1} = r x_n \left(1 - x_n\right)$$

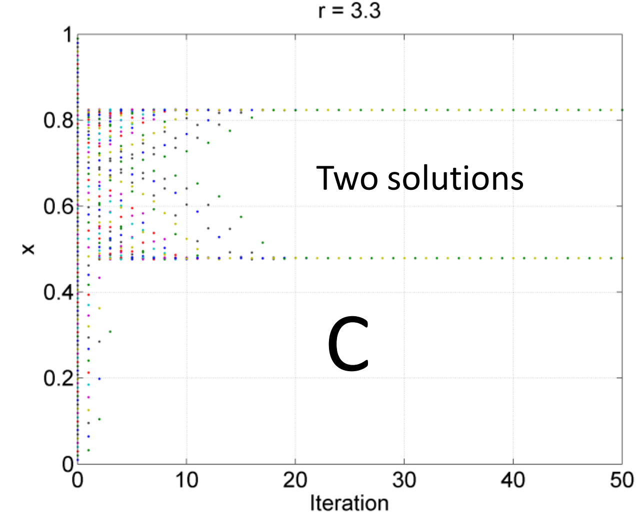
iteration number n

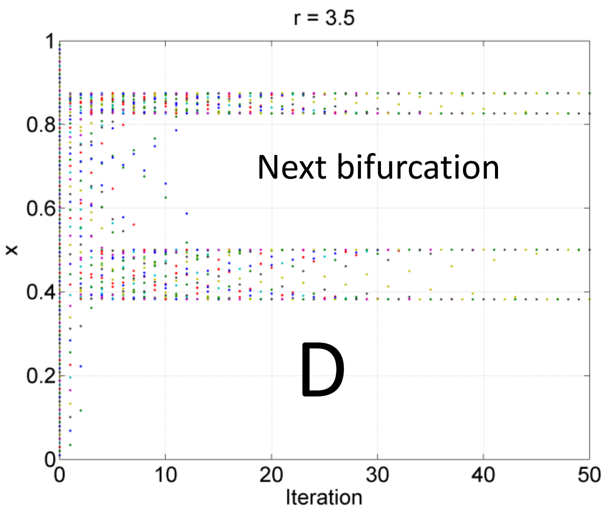
x(n)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0.05	0.19	0.6156	0.946547	0.202385	0.6457	0.915085	0.310816	0.856838	0.490667	0.999652	0.001393	0.005565	0.022137	0.086589	0.316366	0.865114	0.466766	0.995582
	0.1	0.36	0.9216	0.289014	0.821939	0.585421	0.970813	0.113339	0.401974	0.961563	0.147837	0.503924	0.999938	0.000246	0.000985	0.003936	0.015682	0.061745	0.23173
	0.15	0.51	0.9996	0.001599	0.006387	0.025386	0.098965	0.356683	0.917841	0.301635	0.842605	0.530488	0.996282	0.014817	0.058389	0.219918	0.686217	0.861293	0.47787
	0.2	0.64	0.9216	0.289014	0.821939	0.585421	0.970813	0.113339	0.401974	0.961563	0.147837	0.503924	0.999938	0.000246	0.000985	0.003936	0.015682	0.061745	0.23173
	0.25	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75
	0.3	0.84	0.5376	0.994345	0.022492	0.087945	0.320844	0.871612	0.447617	0.989024	0.043422	0.166146	0.554165	0.988265	0.046391	0.176954	0.582565	0.972732	0.106097
	0.35	0.91	0.3276	0.881113	0.419012	0.973764	0.102192	0.366996	0.92924	0.263011	0.775345	0.69674	0.845174	0.523421	0.997806	0.008757	0.034722	0.134065	0.464367
	0.4	0.96	0.1536	0.520028	0.998395	0.006408	0.025467	0.099273	0.35767	0.918969	0.29786	0.836557	0.546917	0.991195	0.034909	0.134761	0.466403	0.995485	0.017978
	0.45	0.99	0.0396	0.152127	0.515939	0.998984	0.00406	0.016176	0.063657	0.238418	0.7263	0.795154	0.651537	0.908147	0.333665	0.889331	0.393686	0.954789	0.172666
	0.5	1	4.44E-16	1.78E-15	7.11E-15	2.84E-14	1.14E-13	4.55E-13	1.82E-12	7.28E-12	2.91E-11	1.16E-10	4.66E-10	1.86E-09	7.45E-09	2.98E-08	1.19E-07	4.77E-07	1.91E-06
	0.55	0.99	0.0396	0.152127	0.515939	0.998984	0.00406	0.016176	0.063657	0.238418	0.7263	0.795154	0.651537	0.908147	0.333665	0.889331	0.393686	0.954789	0.172666
	0.6	0.96	0.1536	0.520028	0.998395	0.006408	0.025467	0.099273	0.35767	0.918969	0.29786	0.836557	0.546917	0.991195	0.034909	0.134761	0.466403	0.995485	0.017978
	0.65	0.91	0.3276	0.881113	0.419012	0.973764	0.102192	0.366996	0.92924	0.263011	0.775345	0.69674	0.845174	0.523421	0.997806	0.008757	0.034722	0.134065	0.464367
	0.7	0.84	0.5376	0.994345	0.022492	0.087945	0.320844	0.871612	0.447617	0.989024	0.043422	0.166146	0.554165	0.988265	0.046391	0.176954	0.582565	0.972732	0.106097
	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75
	0.8	0.64	0.9216	0.289014	0.821939	0.585421	0.970813	0.113339	0.401974	0.961563	0.147837	0.503924	0.999938	0.000246	0.000985	0.003936	0.015682	0.061745	0.23173
	0.85	0.51	0.9996	0.001599	0.006387	0.025386	0.098965	0.356683	0.917841	0.301635	0.842605	0.530488	0.996282	0.014817	0.058389	0.219918	0.686217	0.861293	0.47787
	0.9	0.36	0.9216	0.289014	0.821939	0.585421	0.970813	0.113339	0.401974	0.961563	0.147837	0.503924	0.999938	0.000246	0.000985	0.003936	0.015682	0.061745	0.23173
	0.95	0.19	0.6156	0.946547	0.202385	0.6457	0.915085	0.310816	0.856838	0.490667	0.999652	0.001393	0.005565	0.022137	0.086589	0.316366	0.865114	0.466766	0.995582
	1	-8.9E-16	-3.6E-15	-1.4E-14	-5.7E-14	-2.3E-13	-9.1E-13	-3.6E-12	-1.5E-11	-5.8E-11	-2.3E-10	-9.3E-10	-3.7E-09	-1.5E-08	-6E-08	-2.4E-07	-9.5E-07	-3.8E-06	-1.5E-05

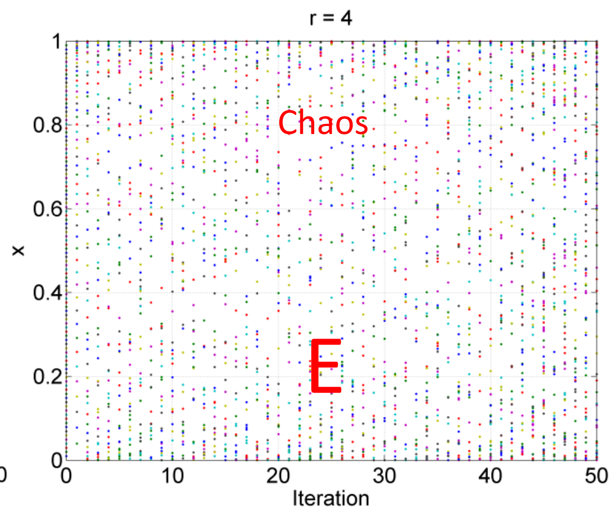




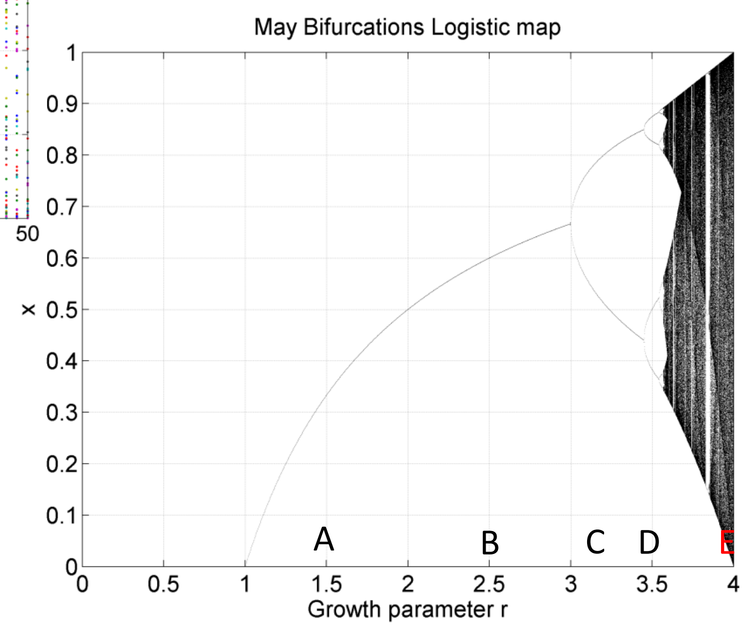








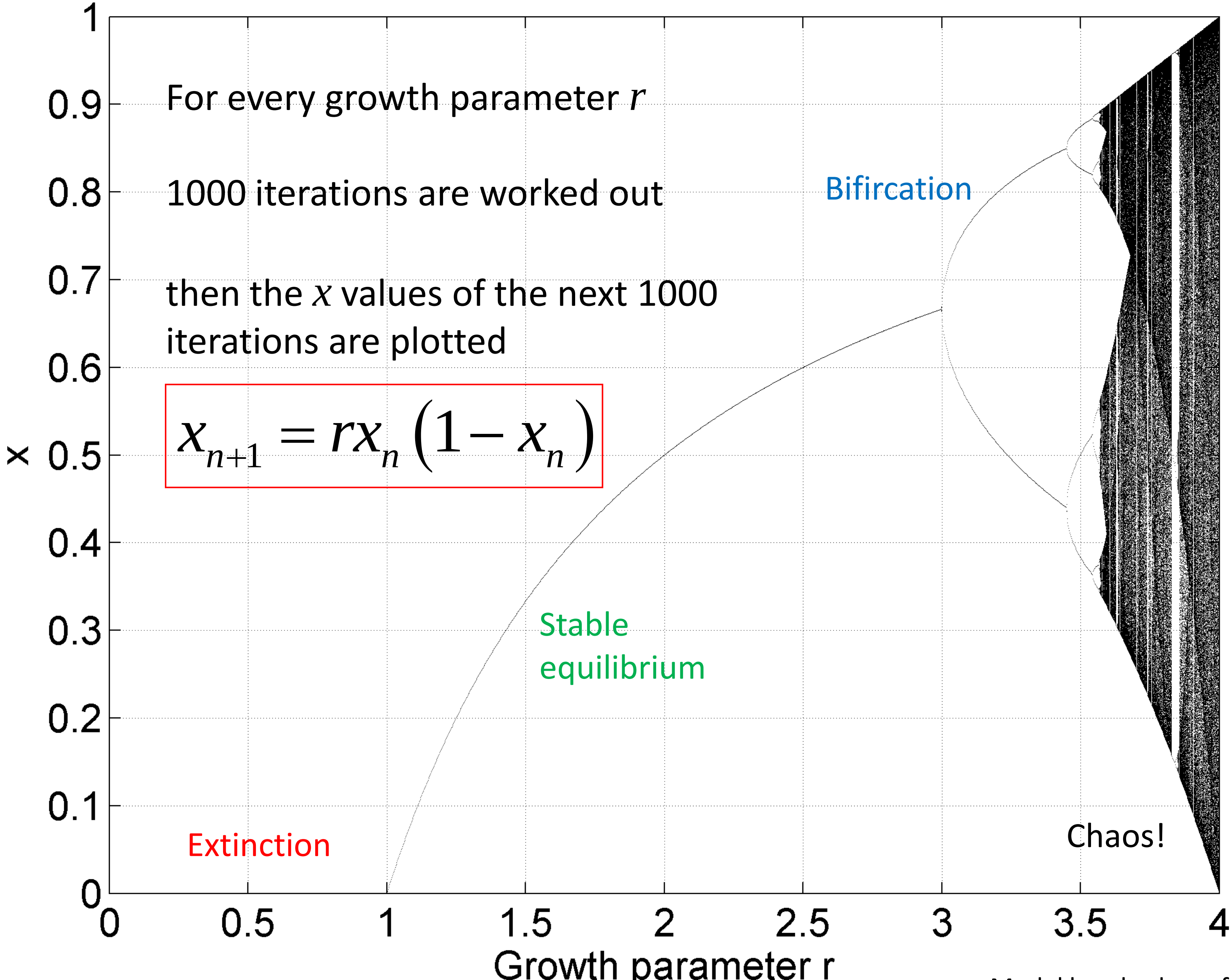
$$x_{n+1} = r x_n \left(1 - x_n\right)$$



Tracking the bifurcations maps the 'road to chaos'. The ratio of successive bifurcation intervals is a **universal constant!**

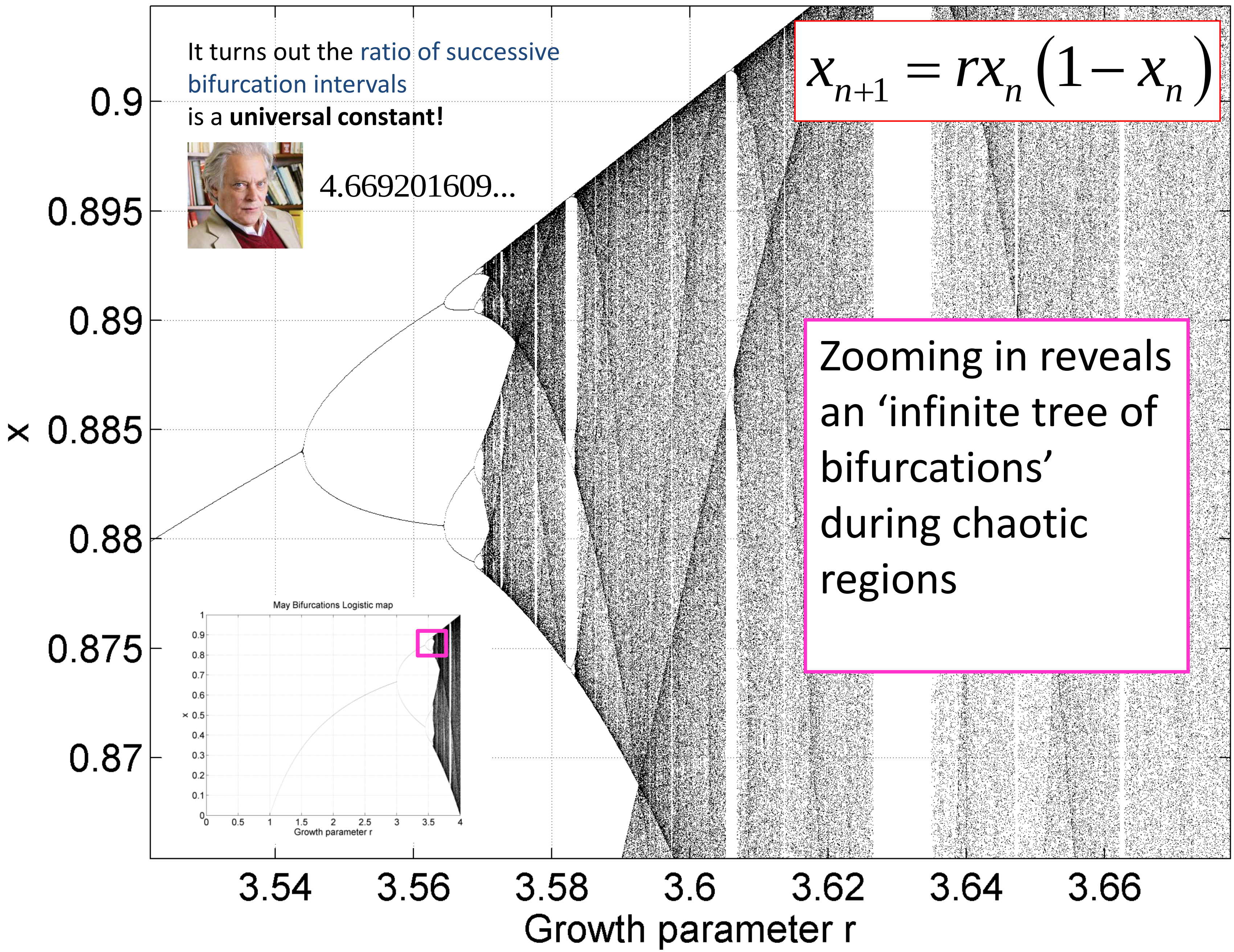
4.669201609...

May Bifurcations Logistic map



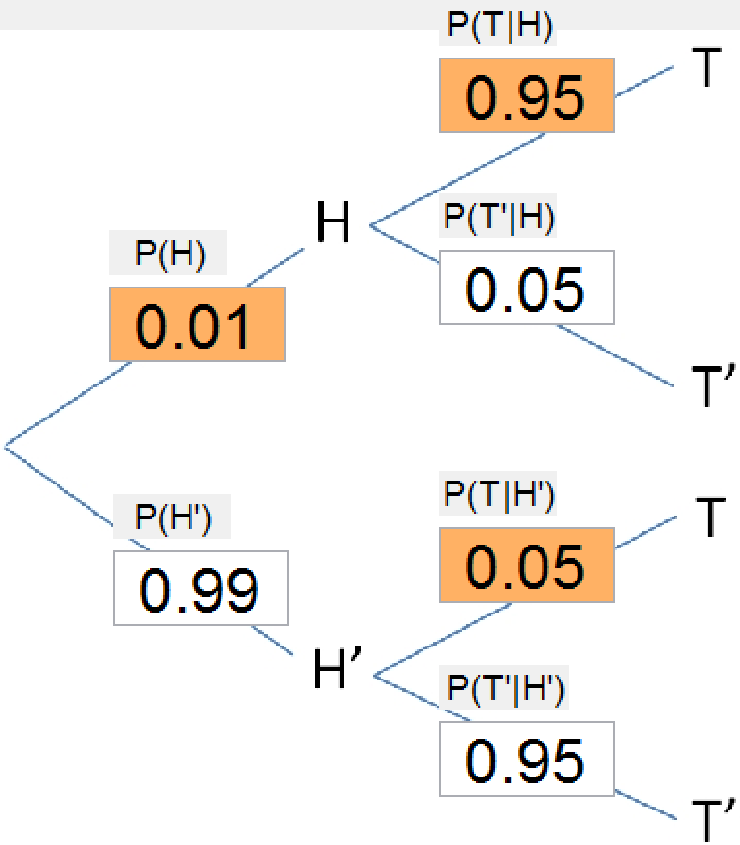
Model breaks down for $r > 4$

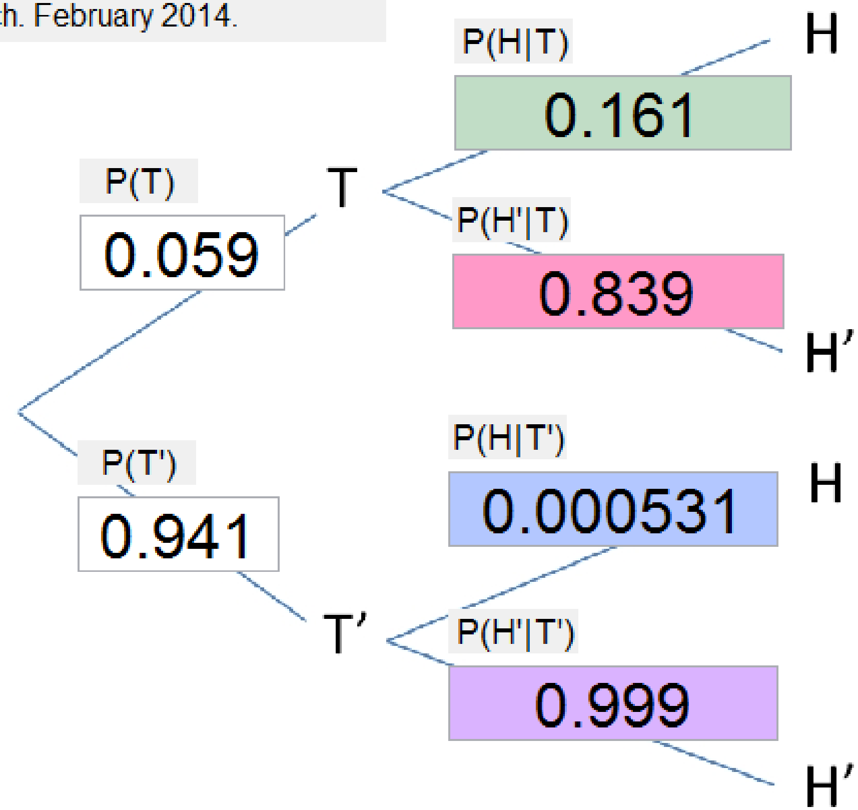
May Bifurcations Logistic map



BAYES-O-METER

A. French. February 2014.





P(H|T)
Probability of hypothesis true given pass of test: **0.161**

P(H'|T) (False positive)
Probability of hypothesis false given pass of test: **0.839**

P(H|T') (False negative)
Probability of hypothesis true given fail of test: **0.000531**

P(H'|T')
Probability of hypothesis false given fail of test: **0.999**

Thomas Bayes
1701-1761

Lorenz and Rössler **strange attractors**

Edward Lorenz was using a Royal McBee LGP-30 computer in 1961 to model weather patterns. He accidentally fed in 3 digit precision numbers into the model from a printout rather than the 6 digits used by the computer. These tiny errors created a hugely different weather forecast....

Lorenz's weather model was *very* sensitive to initial conditions.

His equations looked a bit like these:

$$\frac{dx}{dt} = s\left(y - x\right)$$

$$\frac{dy}{dt} = x\left(r - z\right) - y$$

$$\frac{dz}{dt} = xy - bz$$

$$s = 10$$

$$r = 28$$

$$b = \tfrac{8}{3}$$

Edward Lorenz
1917-2008

Although x,y,z trajectories are chaotic, they tend to *gravitate towards a particular region.*

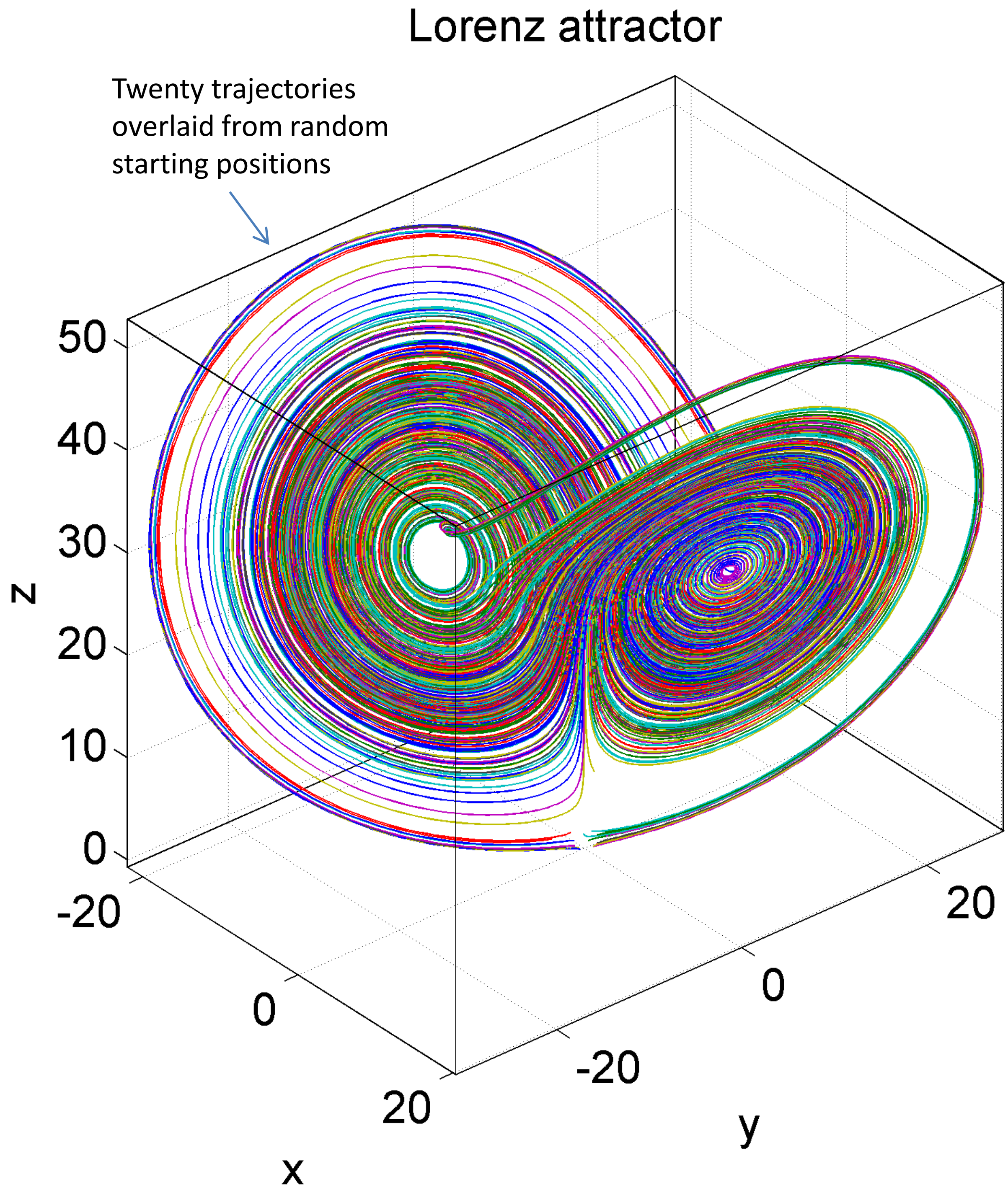
This region is called a **strange attractor.**

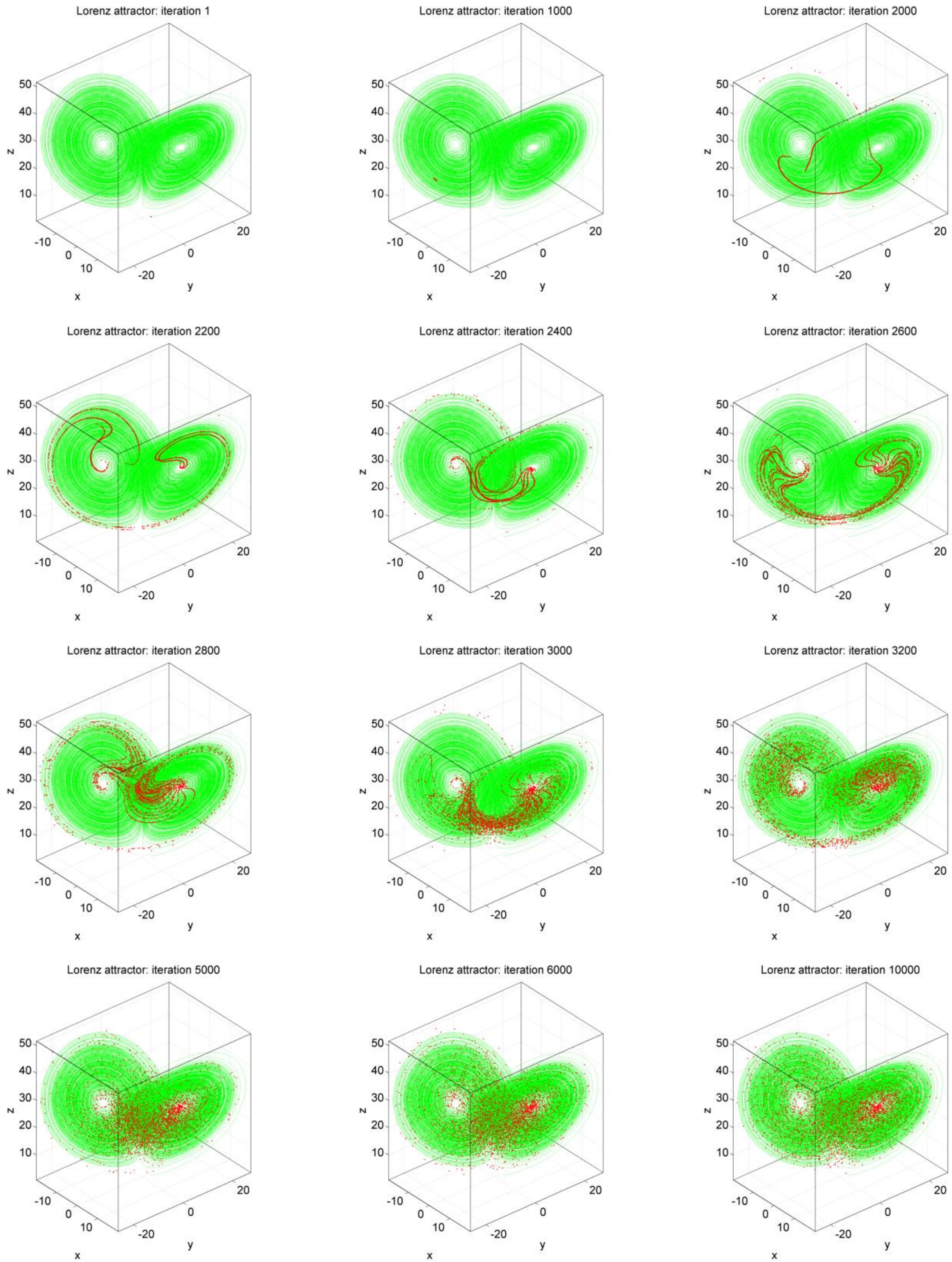
$$\frac{dx}{dt} = s(y - x)$$

$$\frac{dy}{dt} = x(r - z) - y$$

$$\frac{dz}{dt} = xy - bz$$

$$s = 10 \quad r = 28 \quad b = \tfrac{8}{3}$$





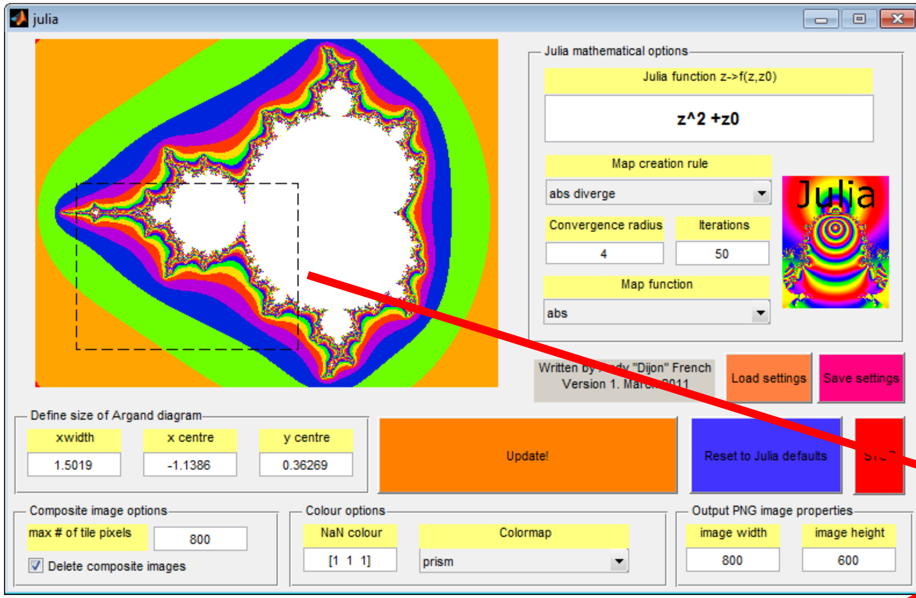
Applying the Lorenz equations, a cluster of initial x,y,z values separated by a *tiny* random deviation will eventually **spread out evenly throughout the strange attractor.**

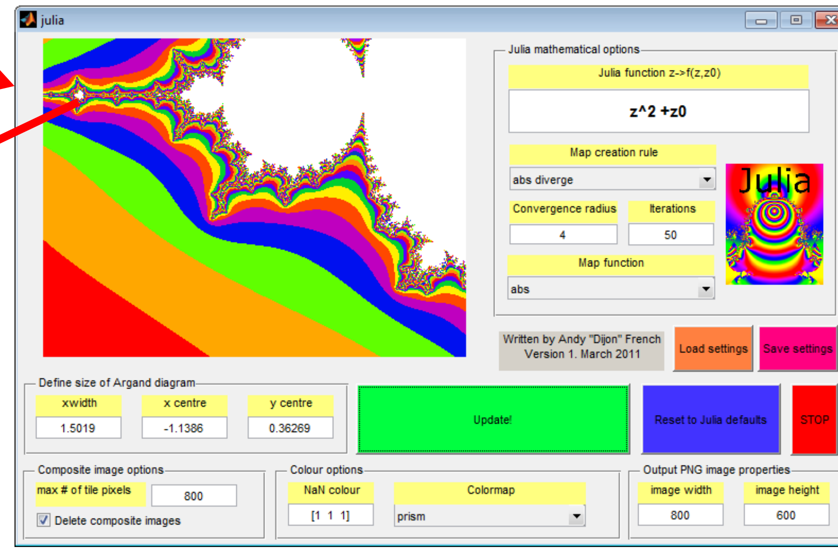
Based upon Shaw *et al*; "Chaos", Scientific American 54:12 (1986) 46-57

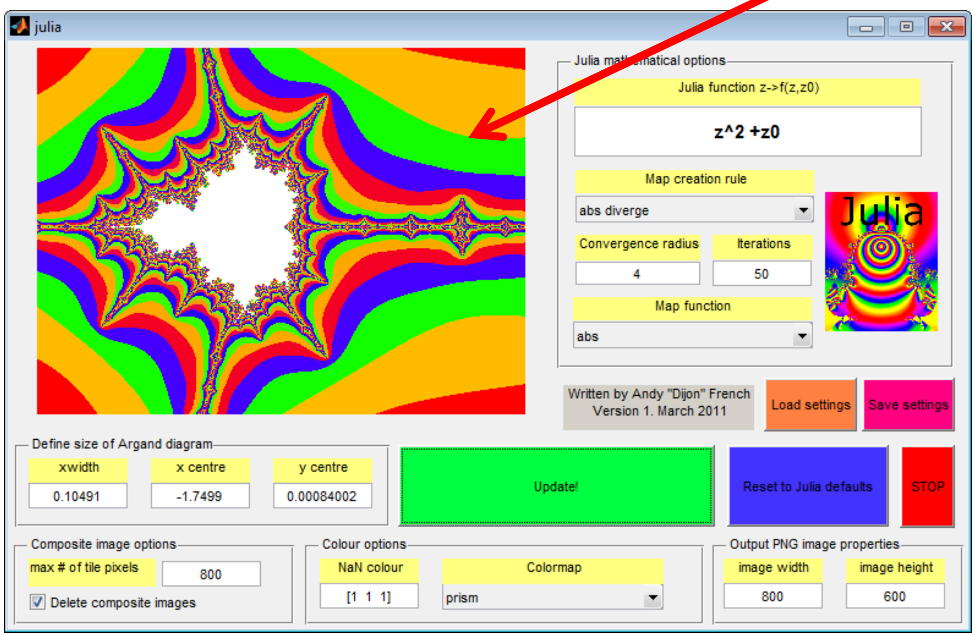
Mandlebrot, complex numbers and iteration

The *Mandlebrot Set* has infinite complexity!

... But a recursive *fractal* geometry







Benoit Mandlebrot (1924-2010)

Mandlebrot transformations of **complex numbers**

$$i^2 = -1$$

$$z = x + iy$$

$$x = \mathrm{Re}(z)$$

$$y = \mathrm{Im}(z)$$

$$|z| = \sqrt{x^2 + y^2}$$

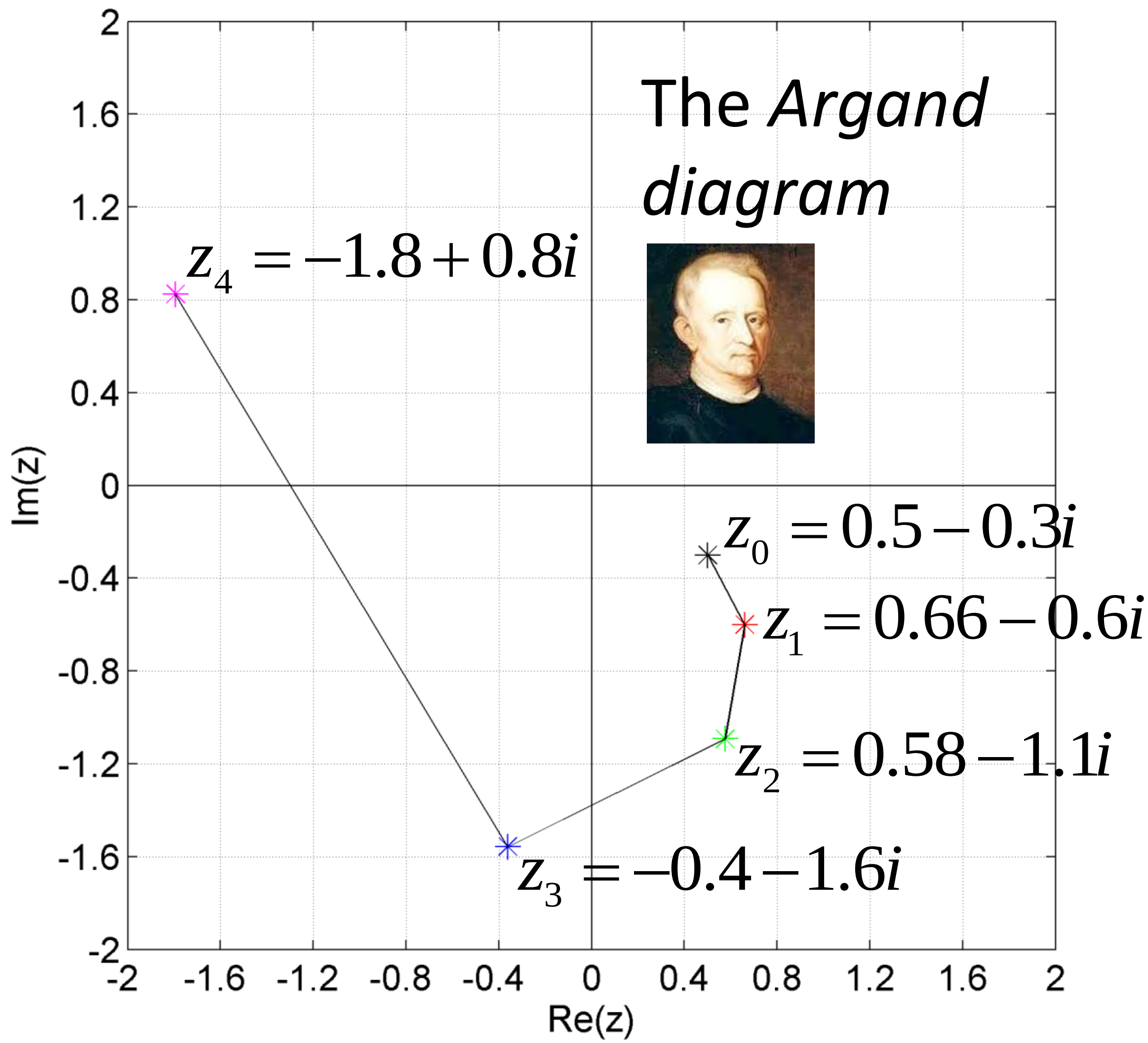
$$(1+i)(1+i)$$

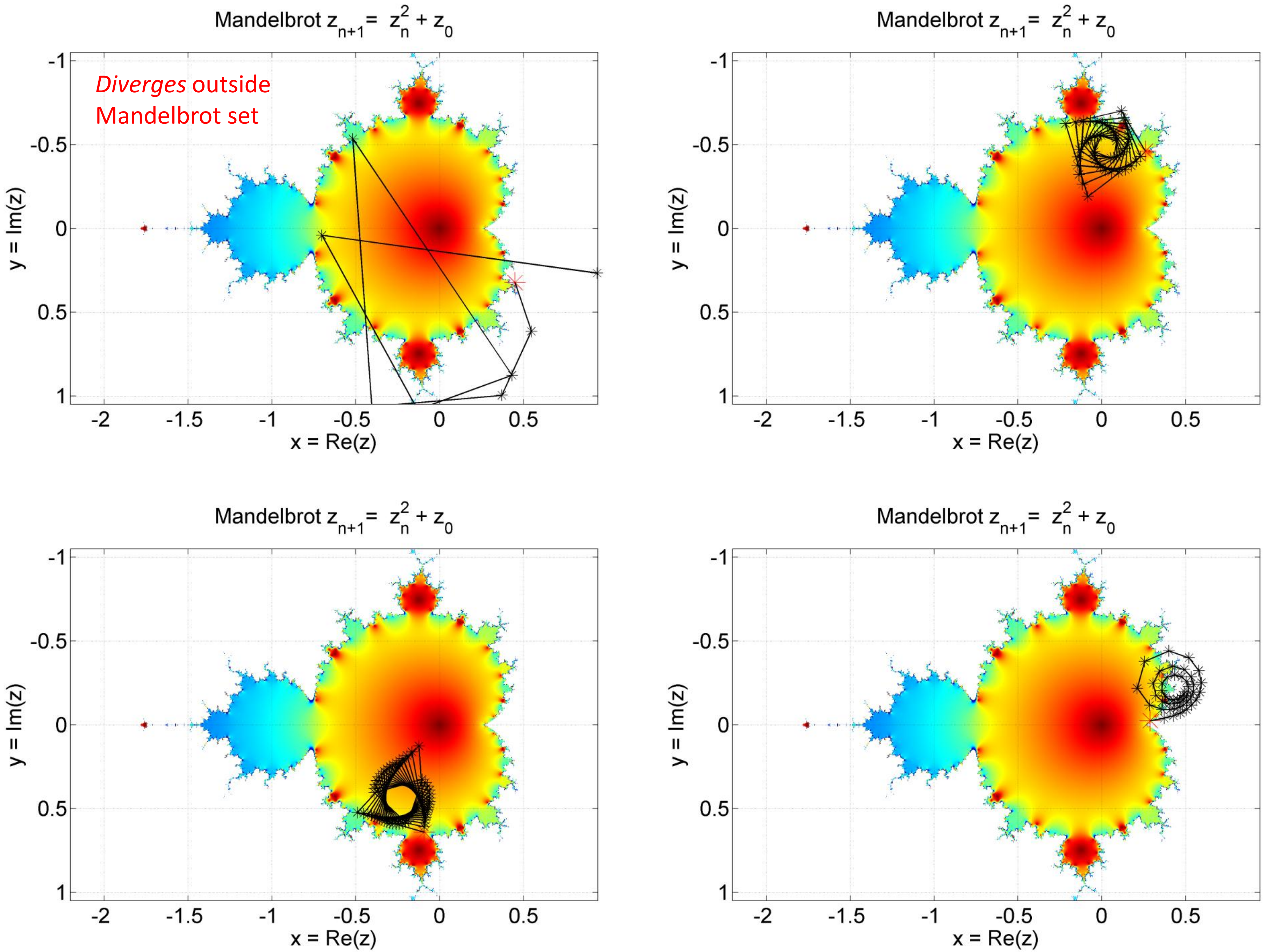
$$= 1 + 2i + i^2$$

$$= 1 + 2i - 1$$

$$= 2i$$

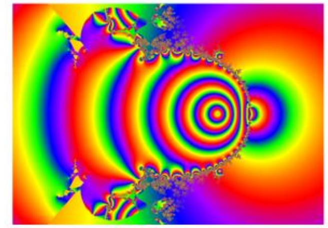
$$z_{n+1} = z_n^2 + z_0$$

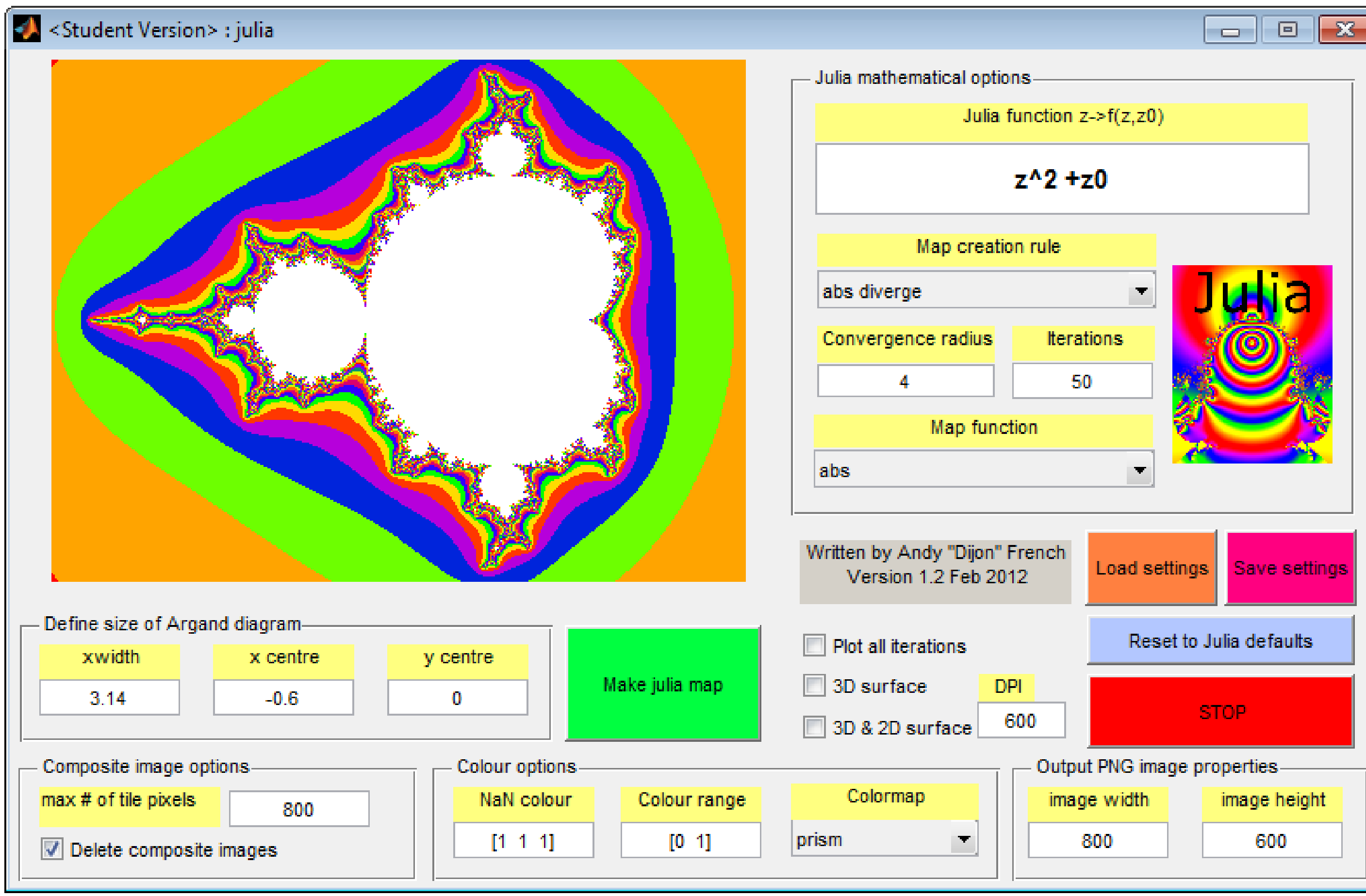


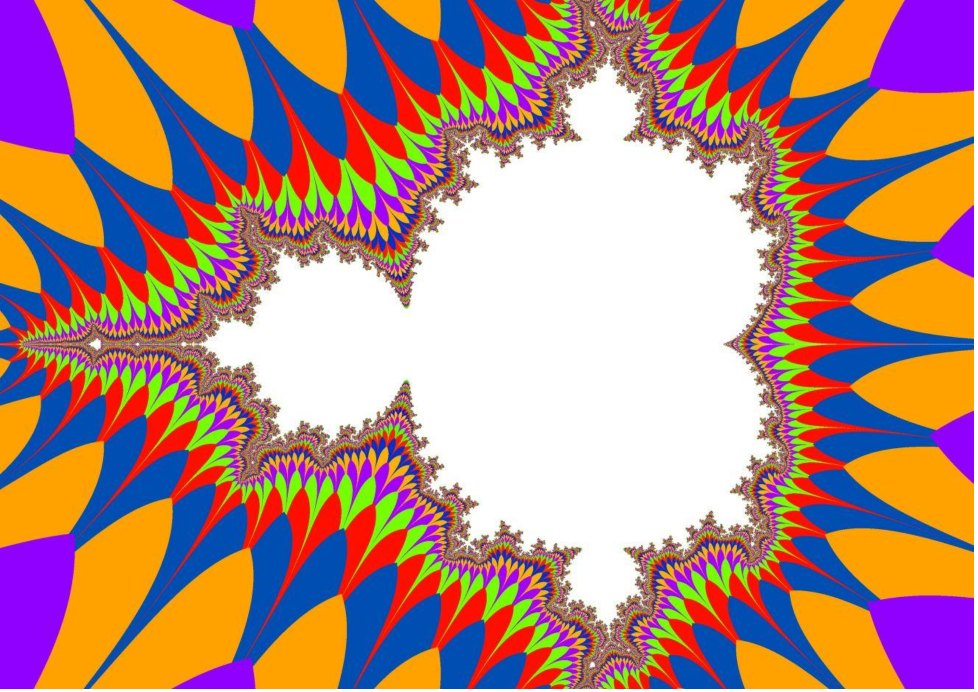

Mandelbrot $z_{n+1} = z_n^2 + z_0$
Diverges outside
Mandelbrot set
-1
-0.5
0
0.5
1
-2
-1.5
-1
-0.5
0
0.5
x = Re(z)
y = Im(z)
Mandelbrot $z_{n+1} = z_n^2 + z_0$
Mandelbrot $z_{n+1} = z_n^2 + z_0$
Mandelbrot $z_{n+1} = z_n^2 + z_0$

Gaston Julia
(1893-1978)

julia



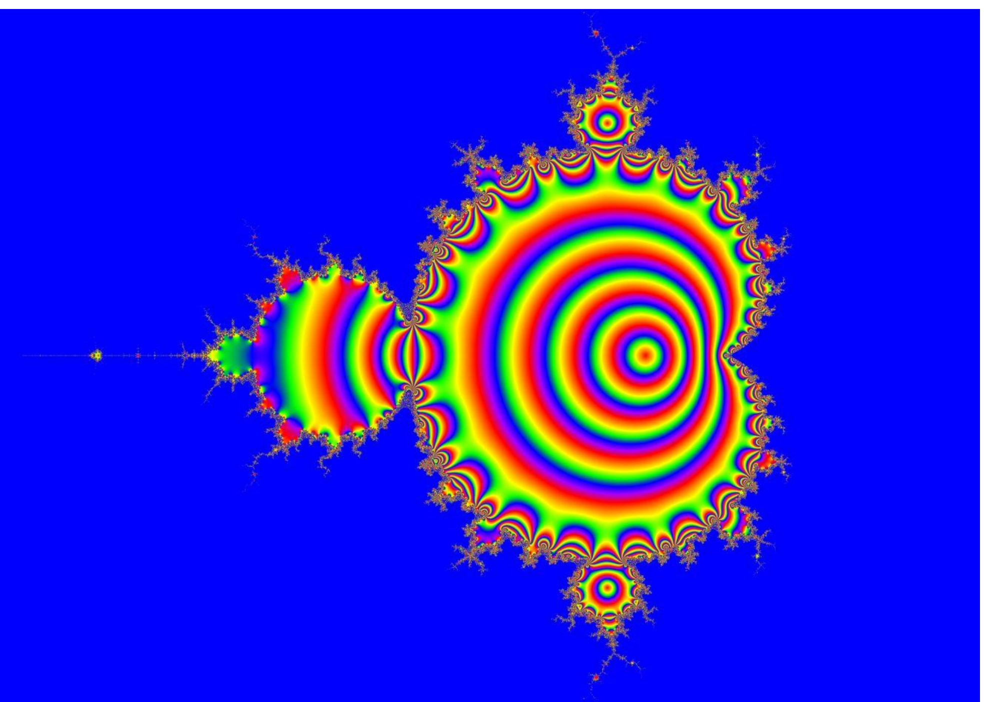




`julia.m` plot option `abs diverge`

Plot a surface with height *h(x,y)*. This is the *iteration number* when $|z|$ exceeds a certain value e.g. 4

In this case *colours* indicate height *h(x,y)*. It is a 'colour-map'.

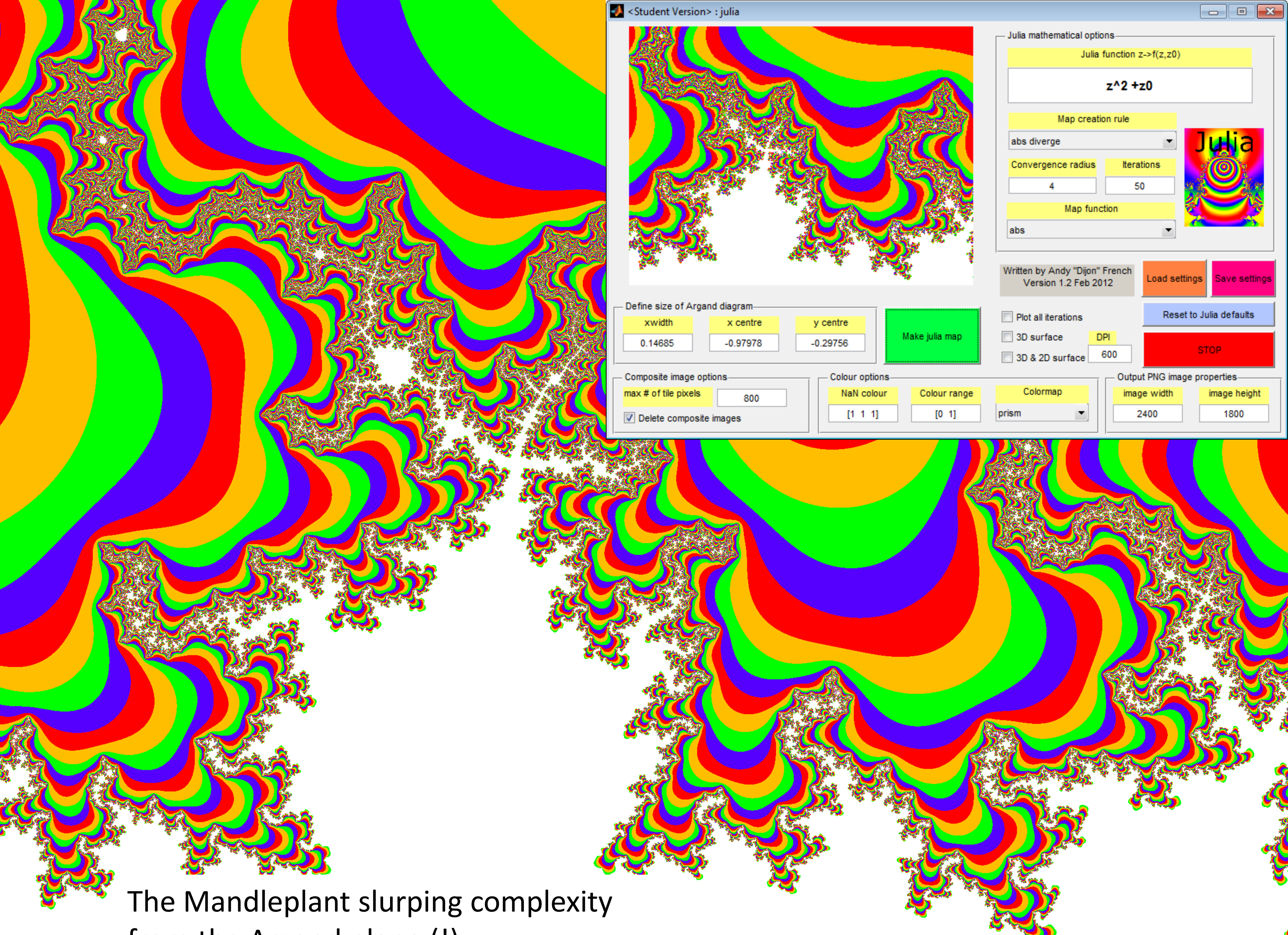


`julia.m` plot option `plot z`

Plot a surface with height *h(x,y)*

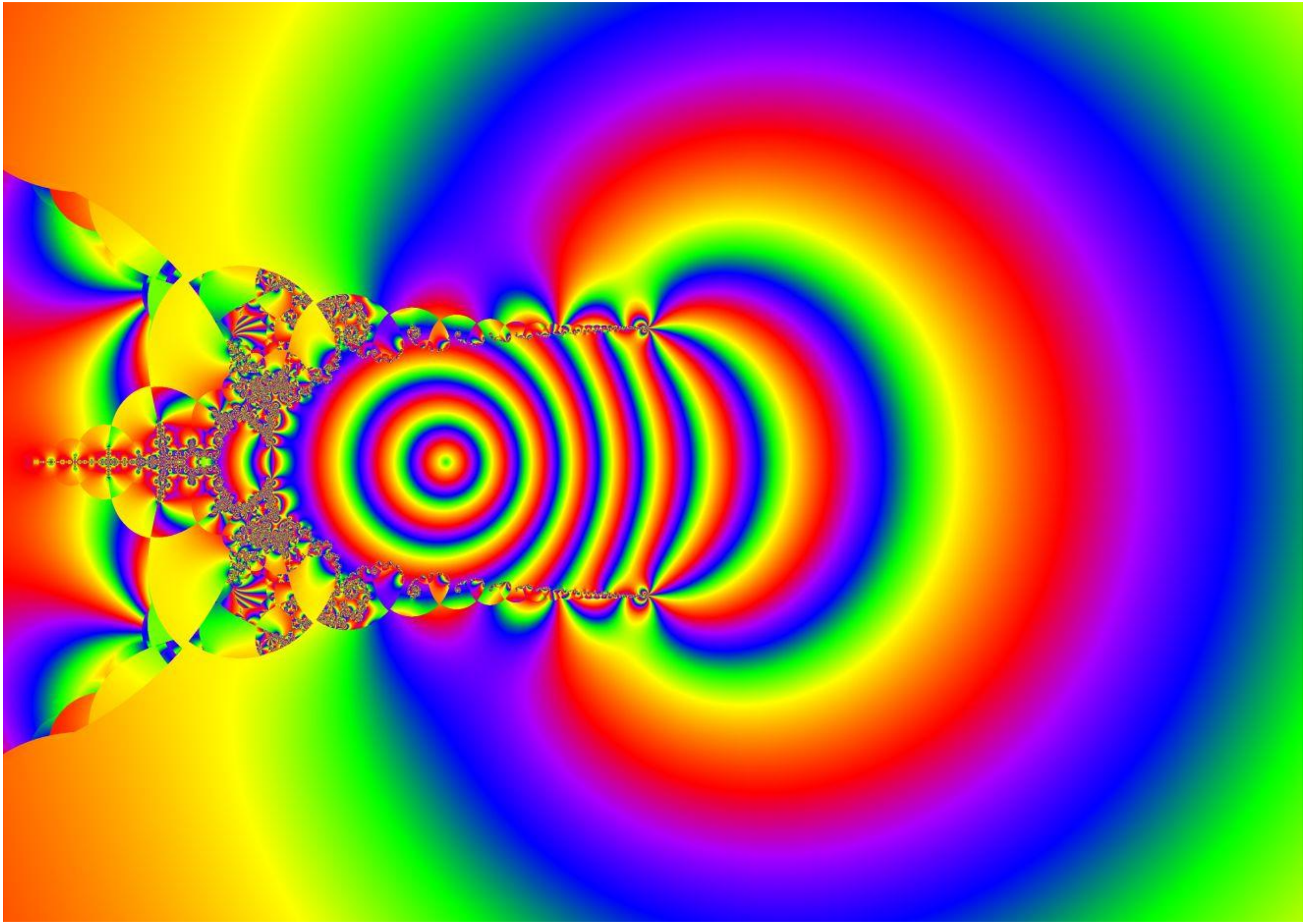
$$x = \mathrm{Re}(z), \quad y = \mathrm{Im}(z)$$

$$h(x,y) = e^{-\sqrt{x^2+y^2}}$$



The Mandleplant slurping complexity
from the Argand plane (!)


The
Mandlebrot
Variations

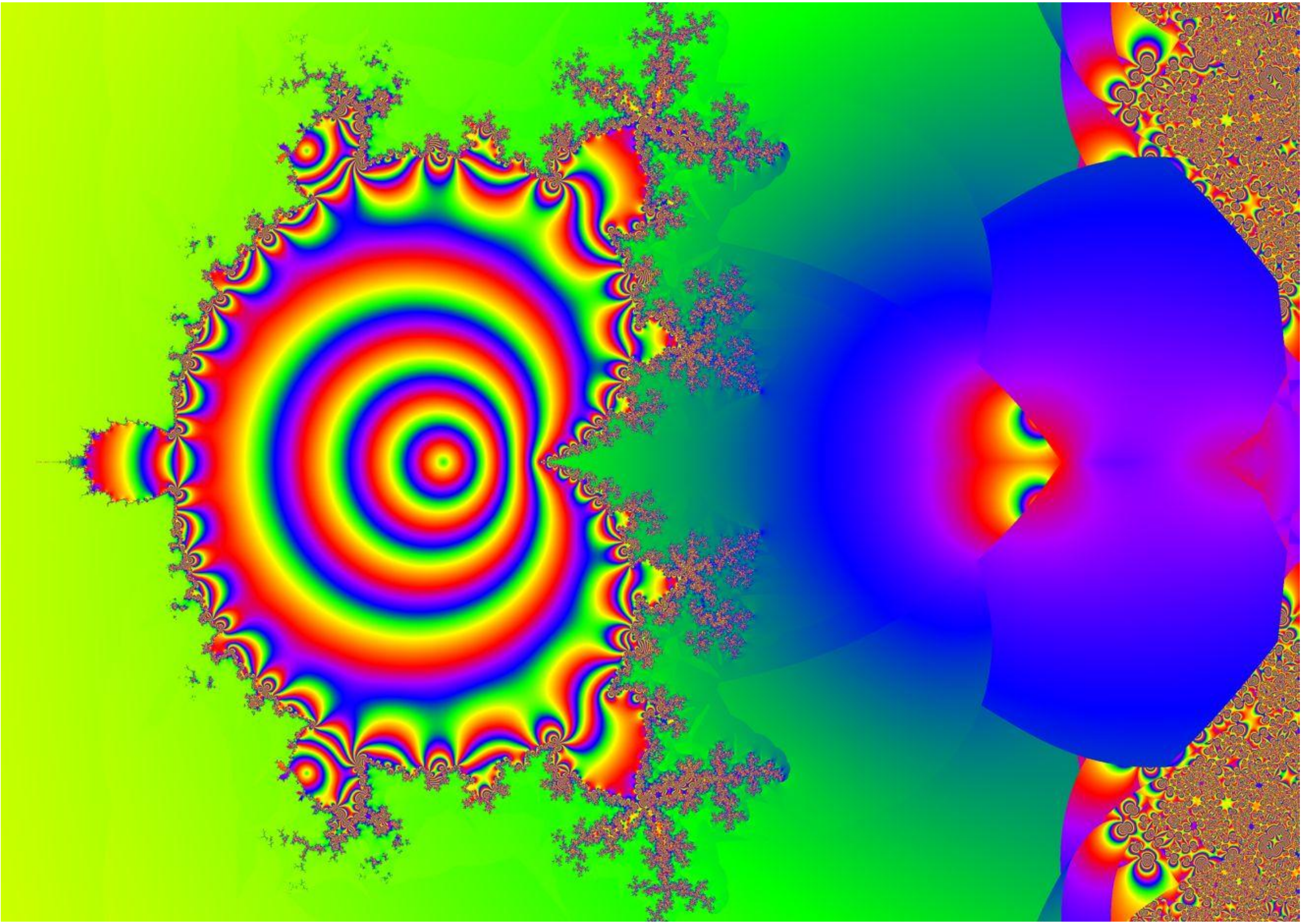


The light bulb

$$z_{n+1} = \log\left(z_n^2 + z_0\right)$$

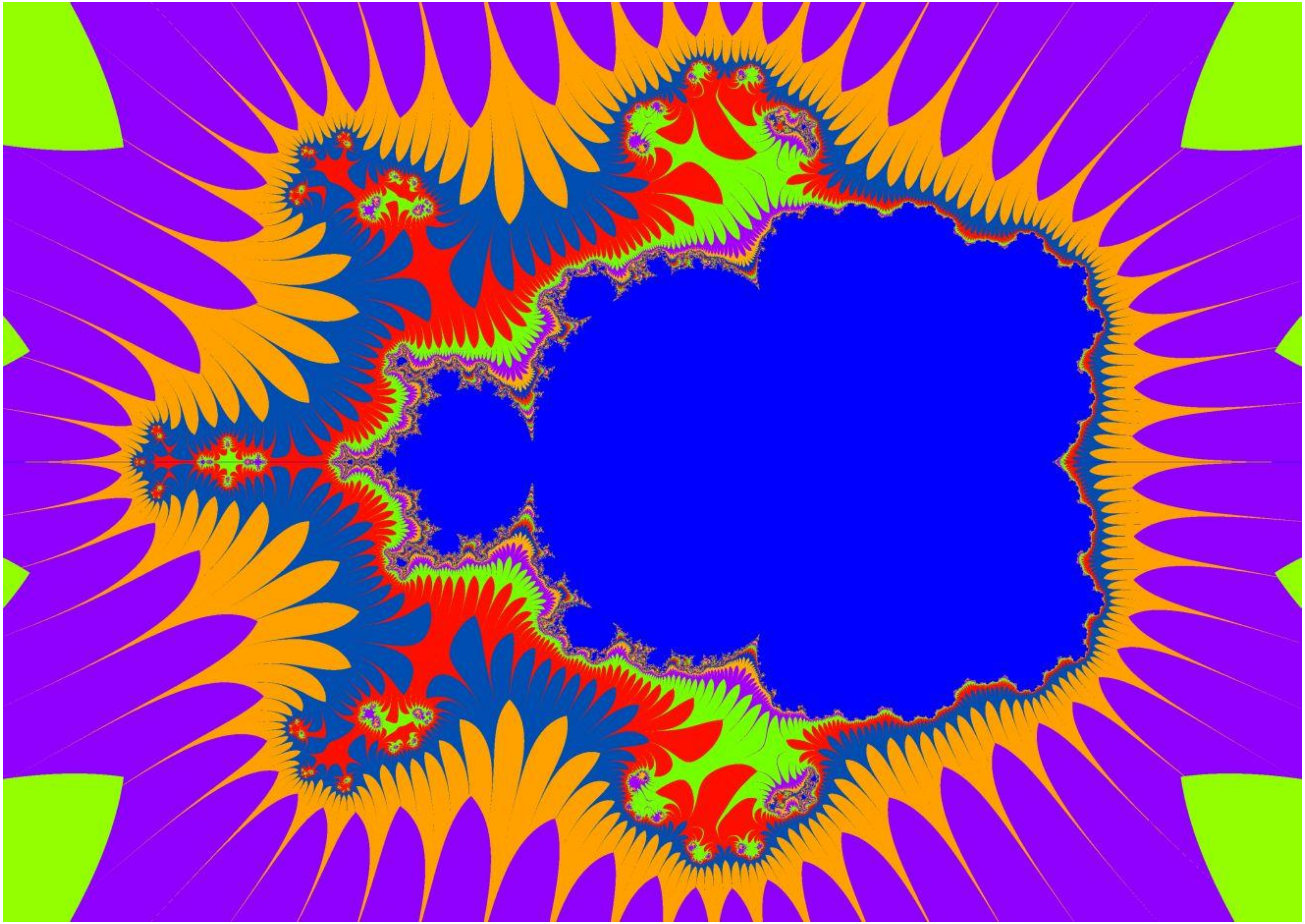
7 steps to enlightenment

$$z_{n+1} = \tan^{-1}\left(z_n^2 + z_0\right)$$



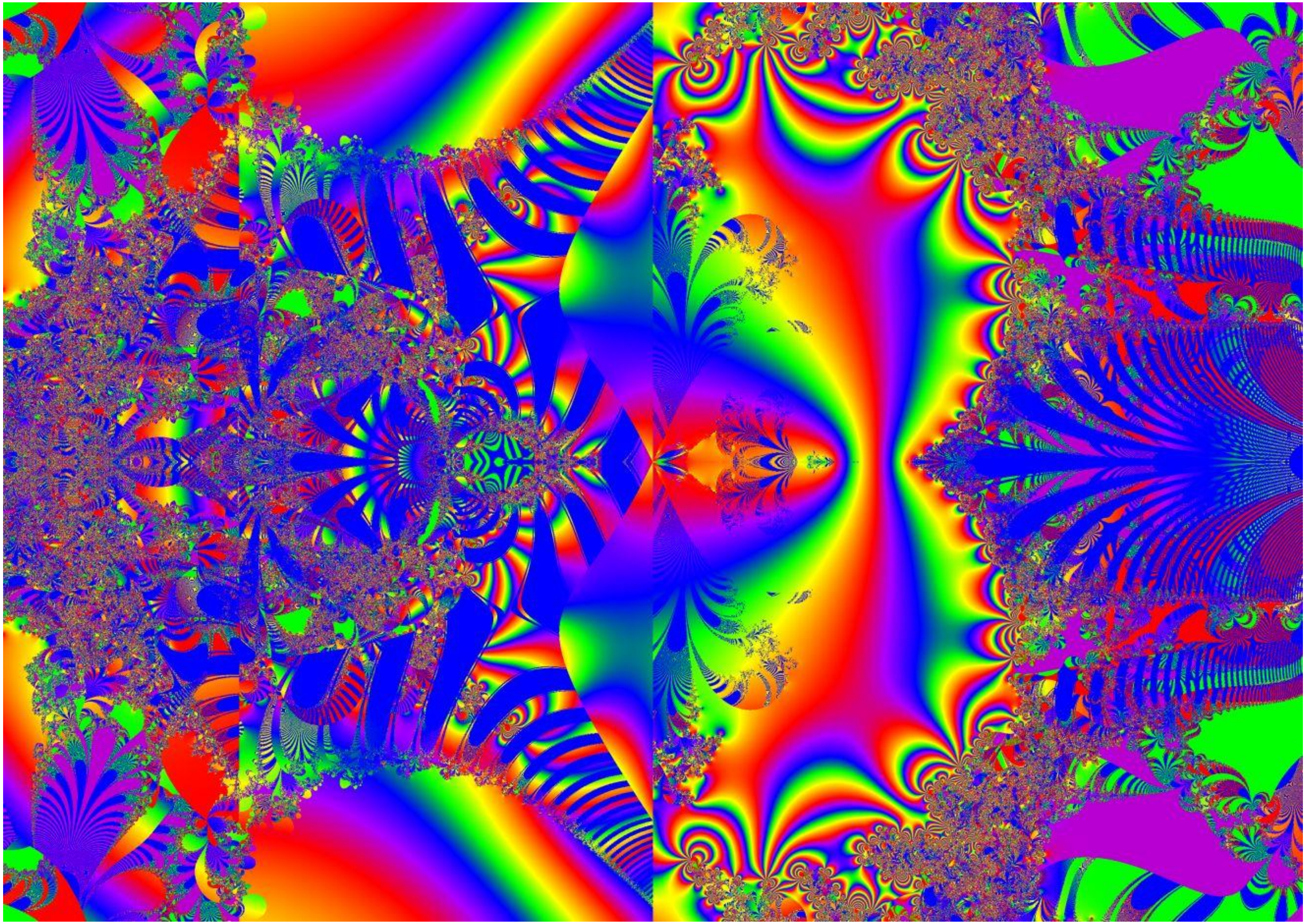
The Mandlerocket!

$$z_{n+1} = \sin^{-1}\left(z_n^2 + z_0\right)$$



Micro mandlebeast

$$z_{n+1} = \left(z_n^2 + z_0 \right)^2$$

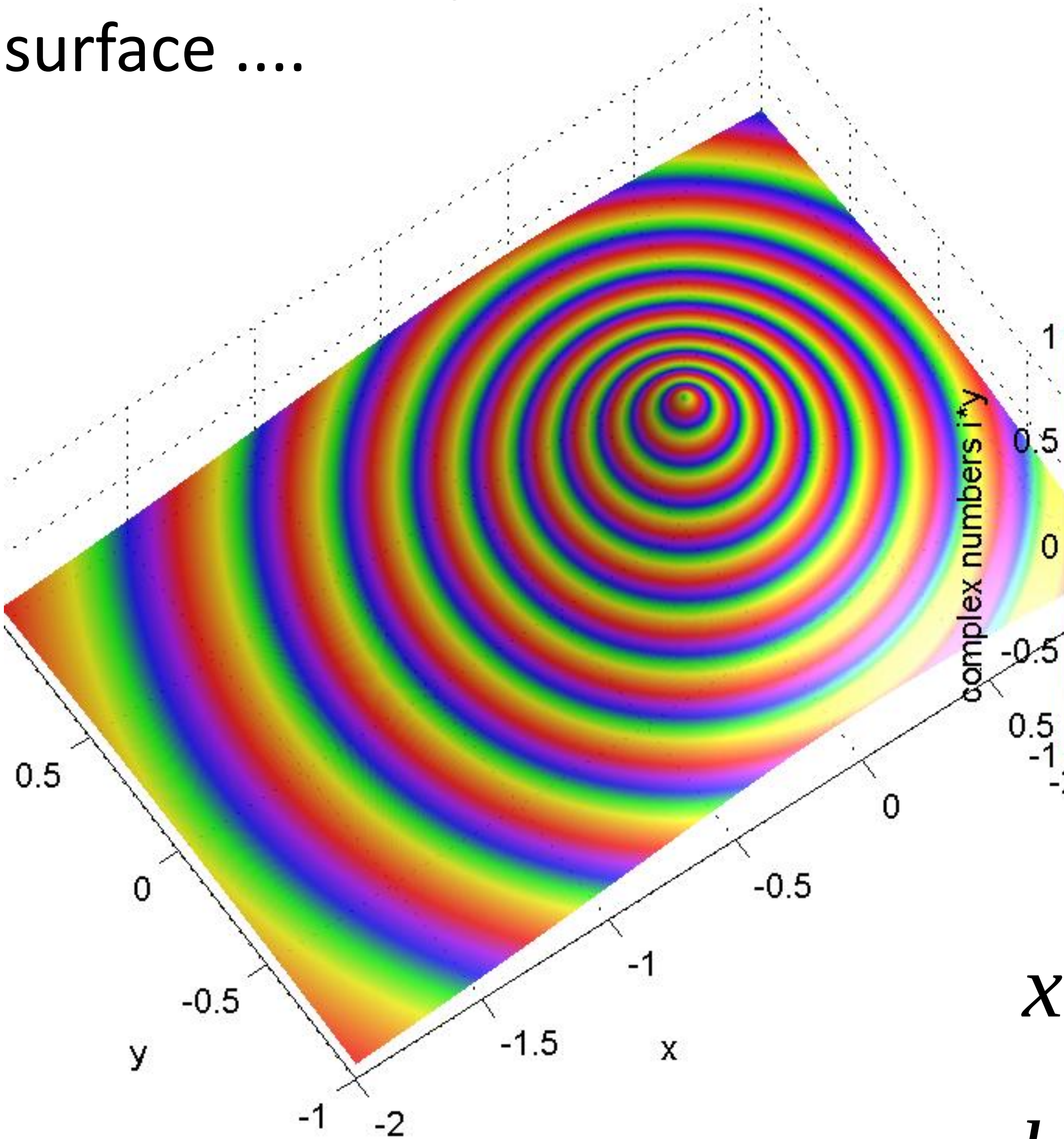


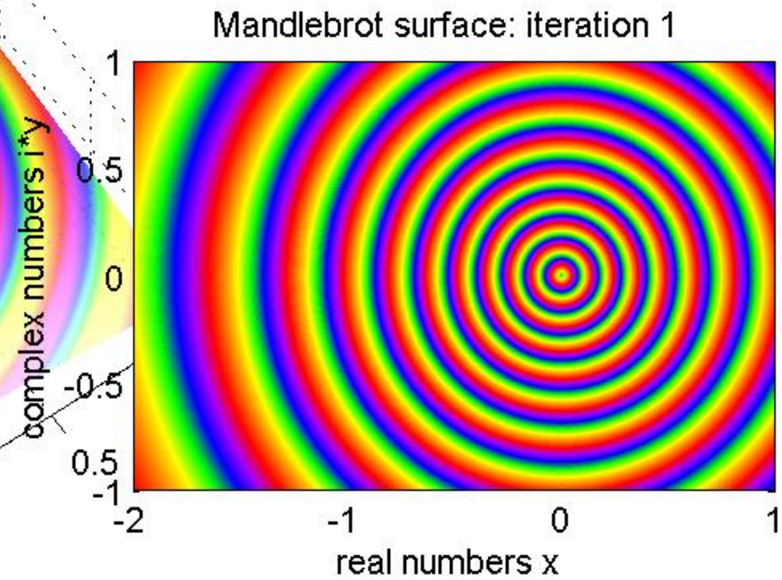
The profusion of power

$$z_{n+1} = \left(z_n^2 + z_0 \right)^{z_n}$$

Remember *h(x,y)* is a surface ....

$$z_{n+1} = z_n^2 + z_0$$

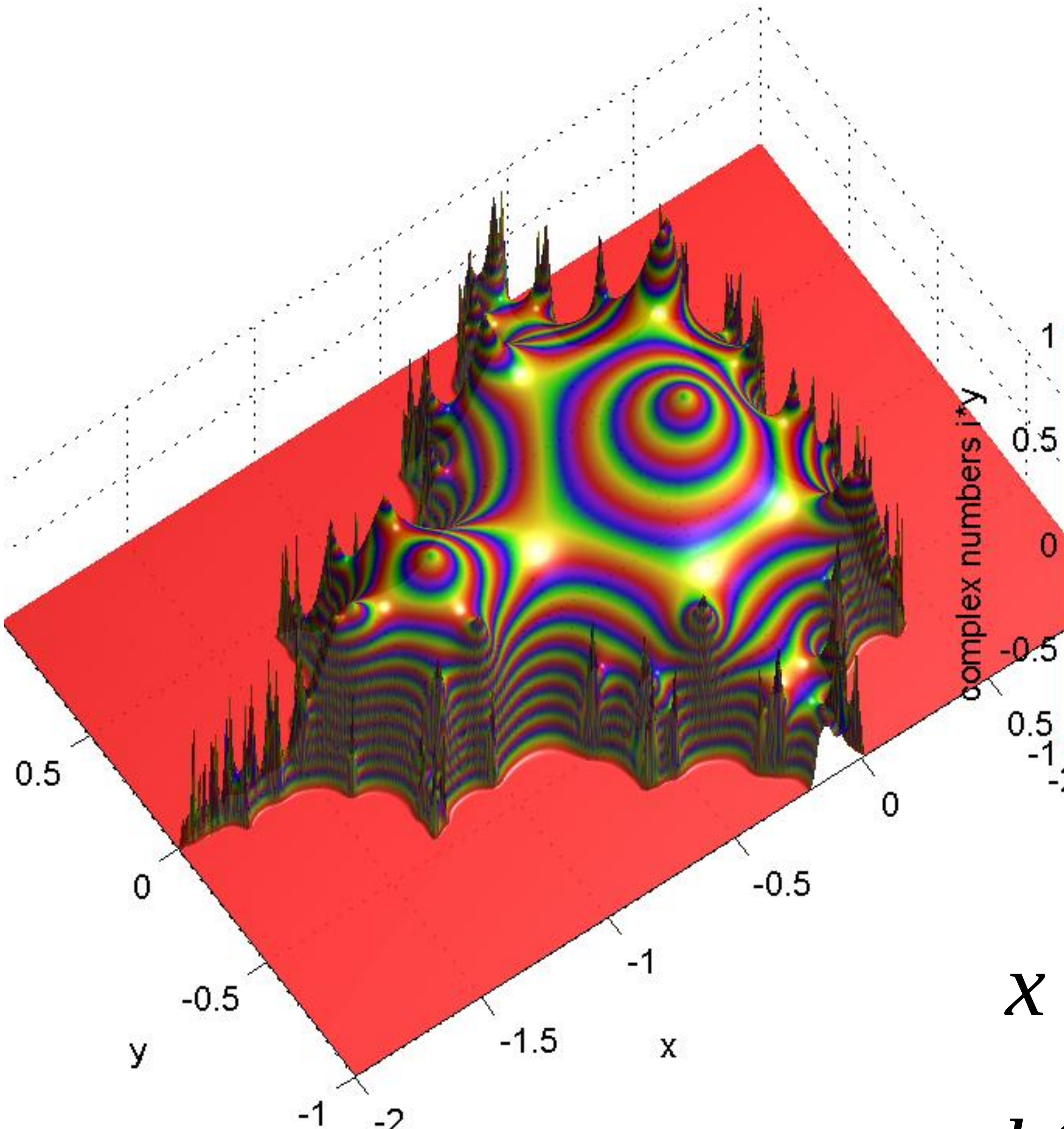


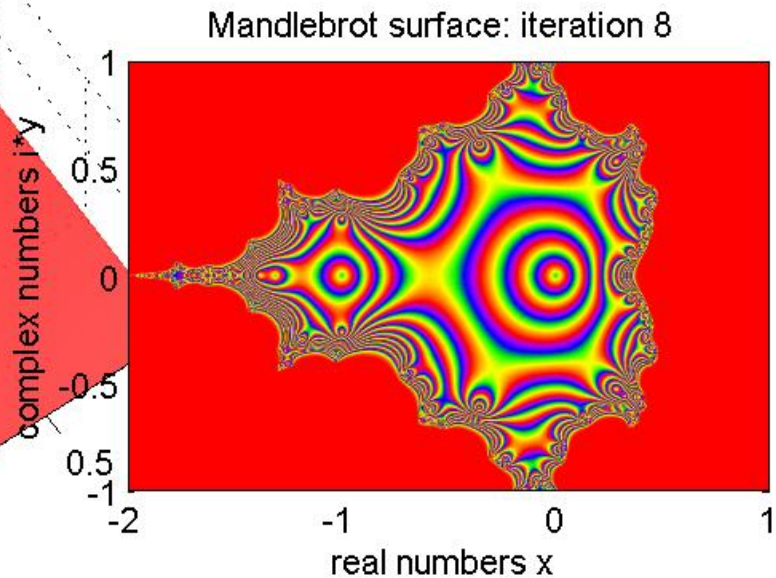


$$x = \mathrm{Re}(z), \quad y = \mathrm{Im}(z)$$

$$h(x,y) = e^{-\sqrt{x^2+y^2}}$$

$$z_{n+1} = z_n^2 + z_0$$

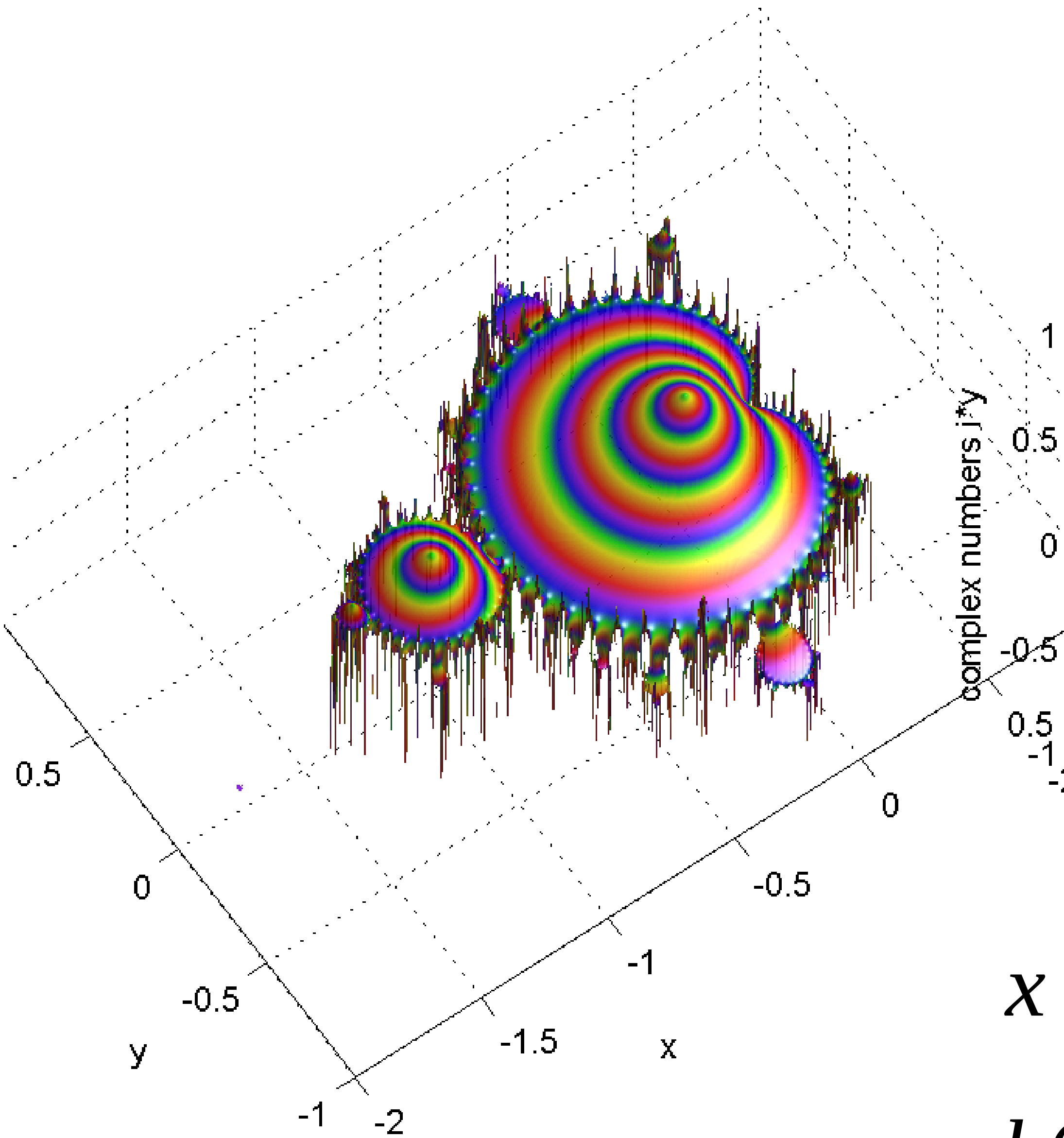


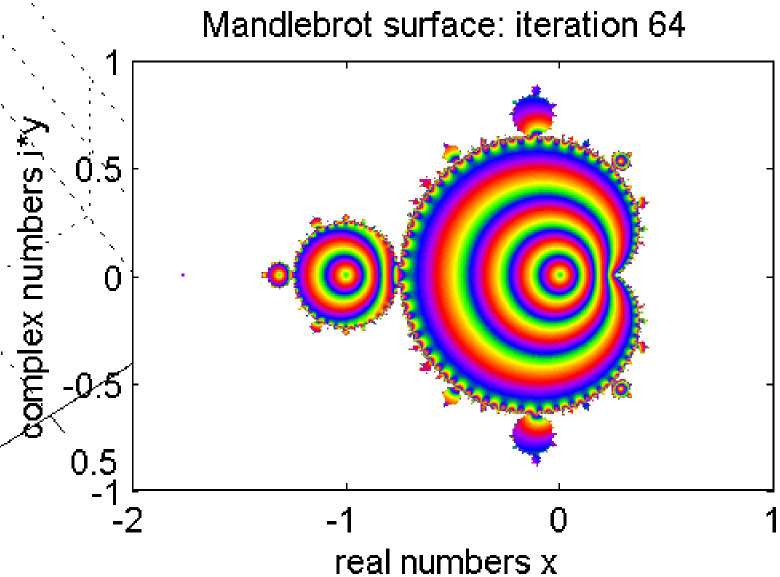


$$x = \mathrm{Re}(z), \quad y = \mathrm{Im}(z)$$

$$h(x, y) = e^{-\sqrt{x^2 + y^2}}$$

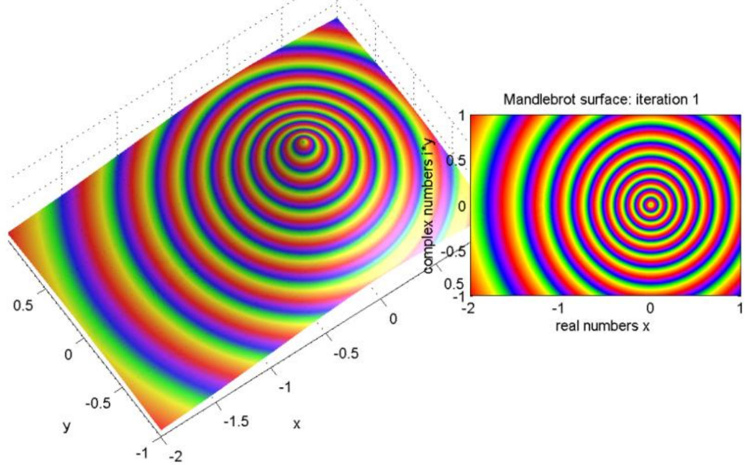
$$z_{n+1} = z_n^2 + z_0$$

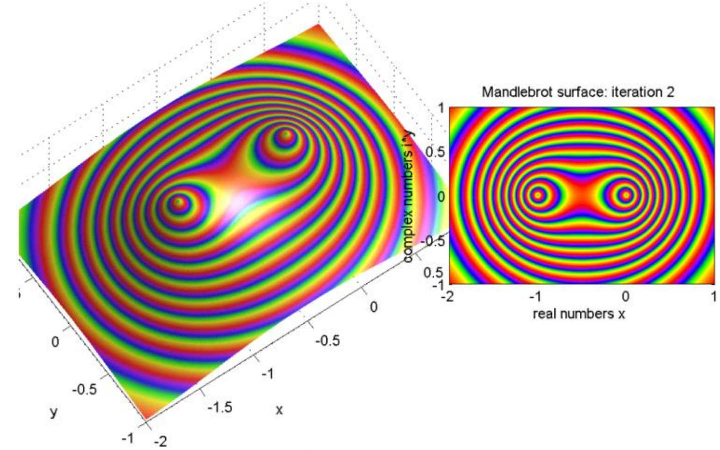


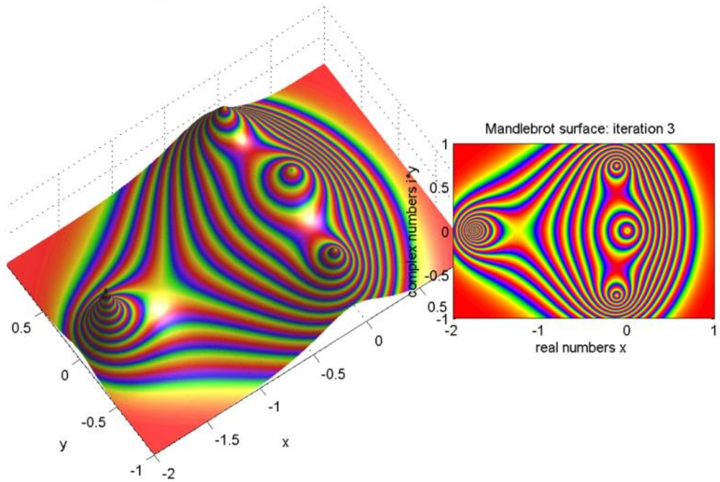


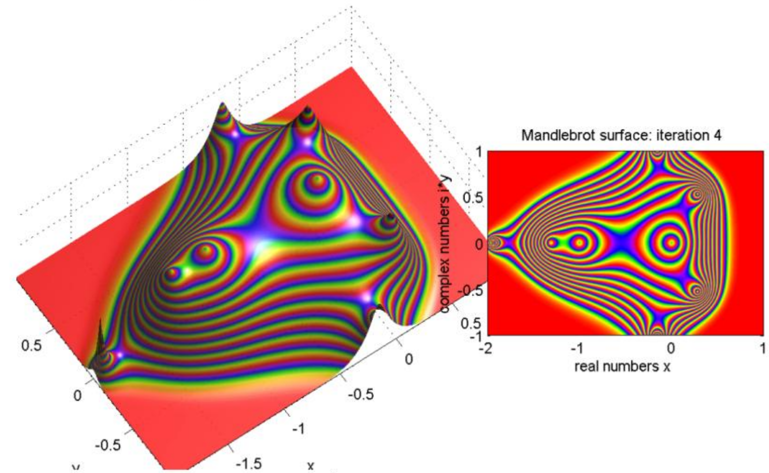
$$x = \mathrm{Re}(z), \quad y = \mathrm{Im}(z)$$

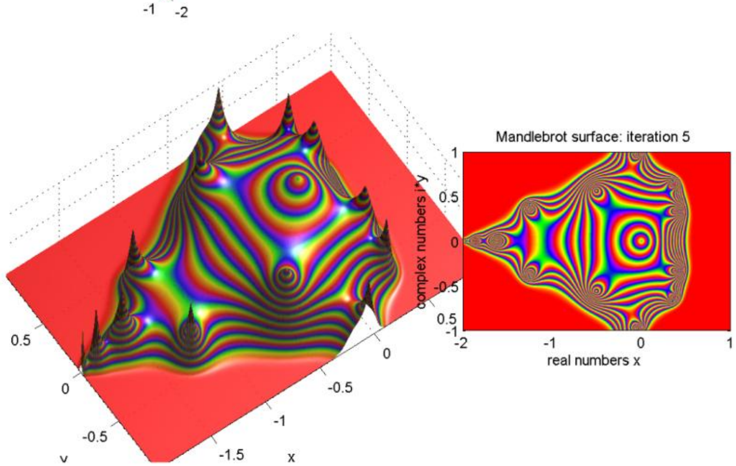
$$h(x,y) = e^{-\sqrt{x^2+y^2}}$$

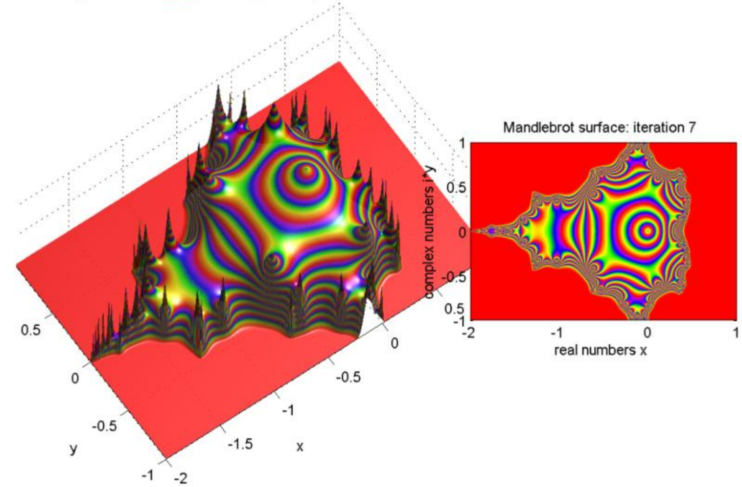

Mandlebrot surface: iteration 1
complex numbers i*y
real numbers x
x
y

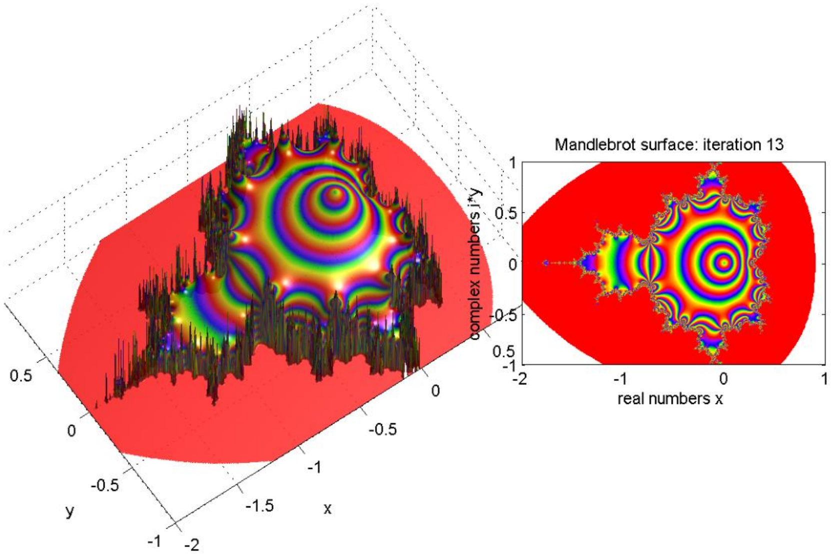

Mandlebrot surface: iteration 2
complex numbers i*y
real numbers x
x
y

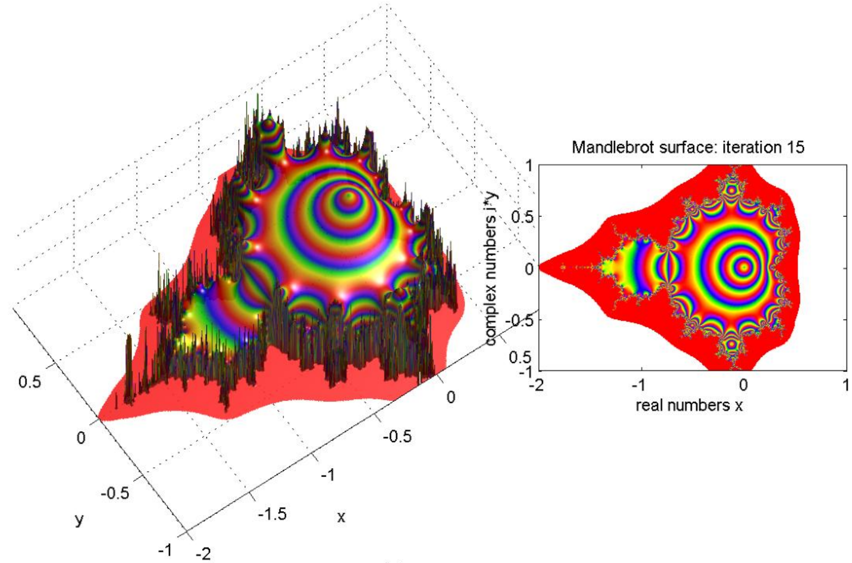

Mandlebrot surface: iteration 3
complex numbers i*y
real numbers x
x
y

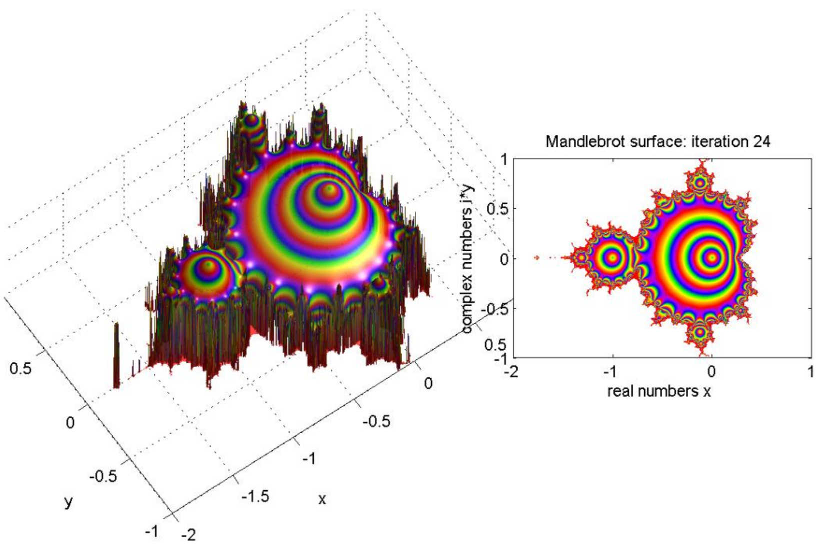

Mandlebrot surface: iteration 4
complex numbers i*y
real numbers x
x

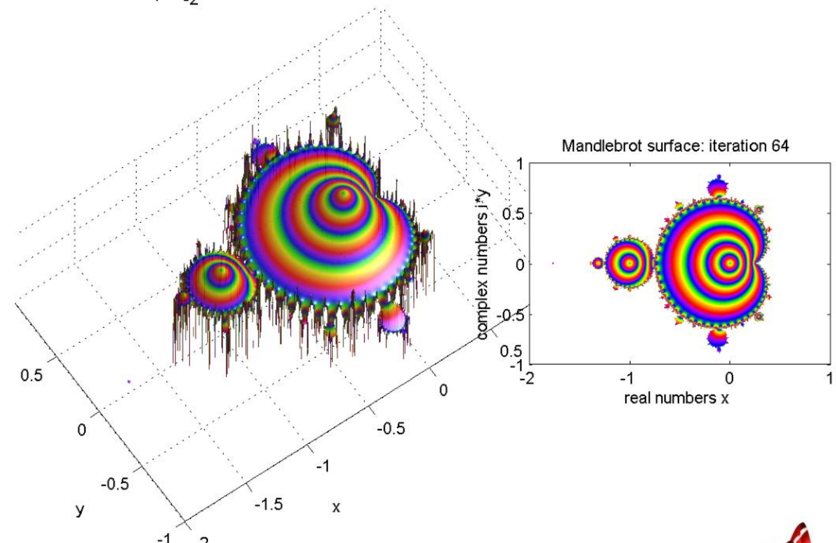

Mandlebrot surface: iteration 5
complex numbers i*y
real numbers x
x


Mandlebrot surface: iteration 7
complex numbers i*y
real numbers x
x
y







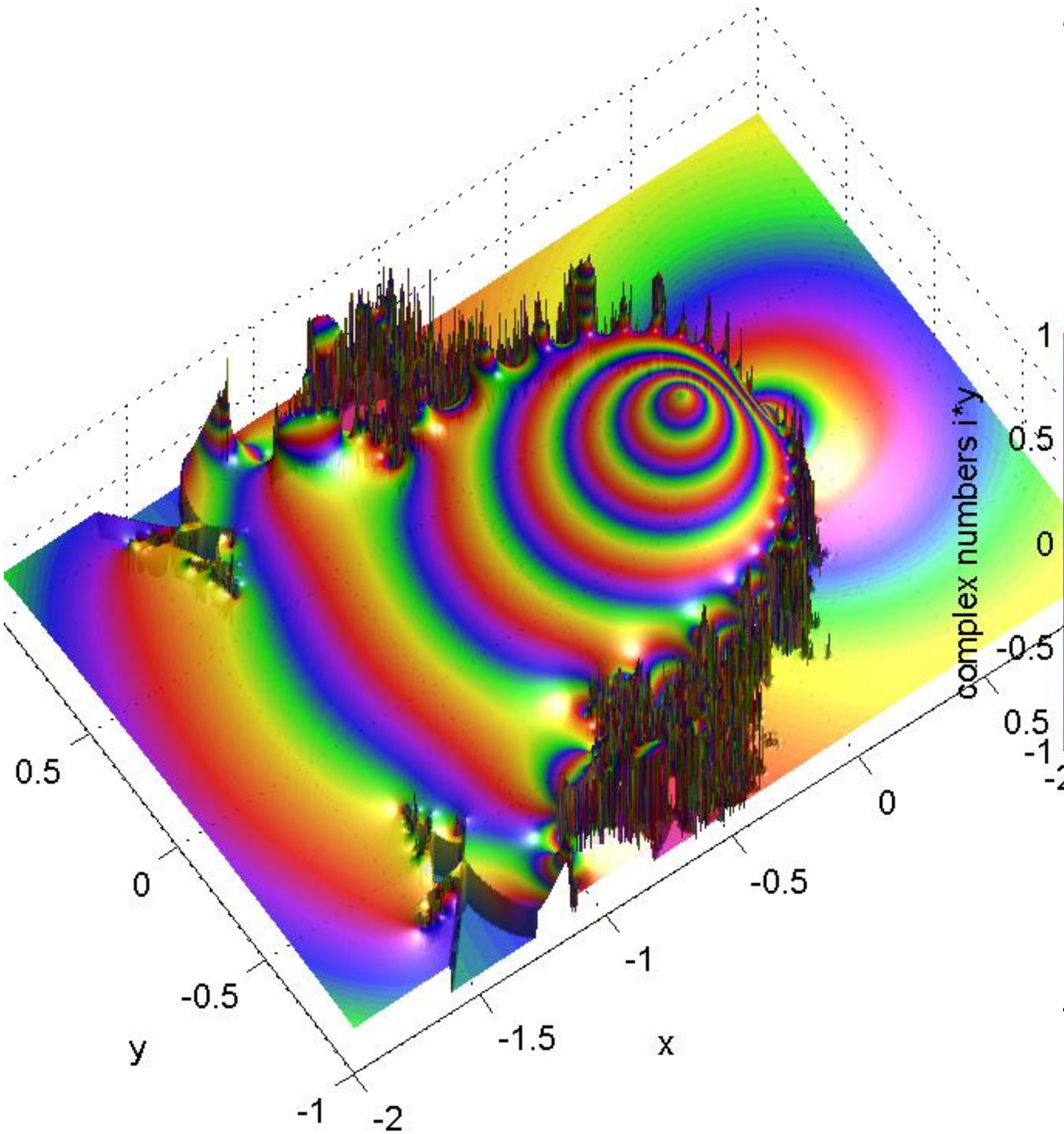


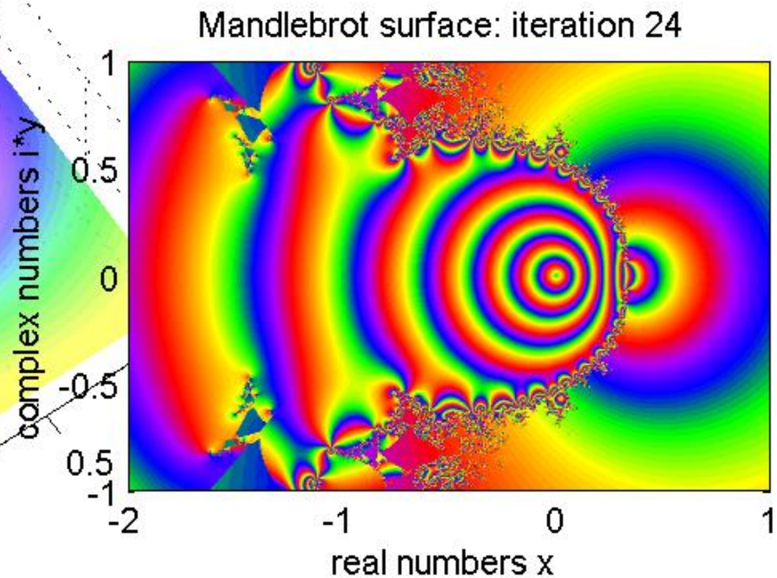
Selection from *Day of Julia.*
Mathematicon Exhibition, 2014

μathematicon

7 steps to enlightenment

$$z_{n+1} = \tan^{-1}\left(z_n^2 + z_0\right)$$



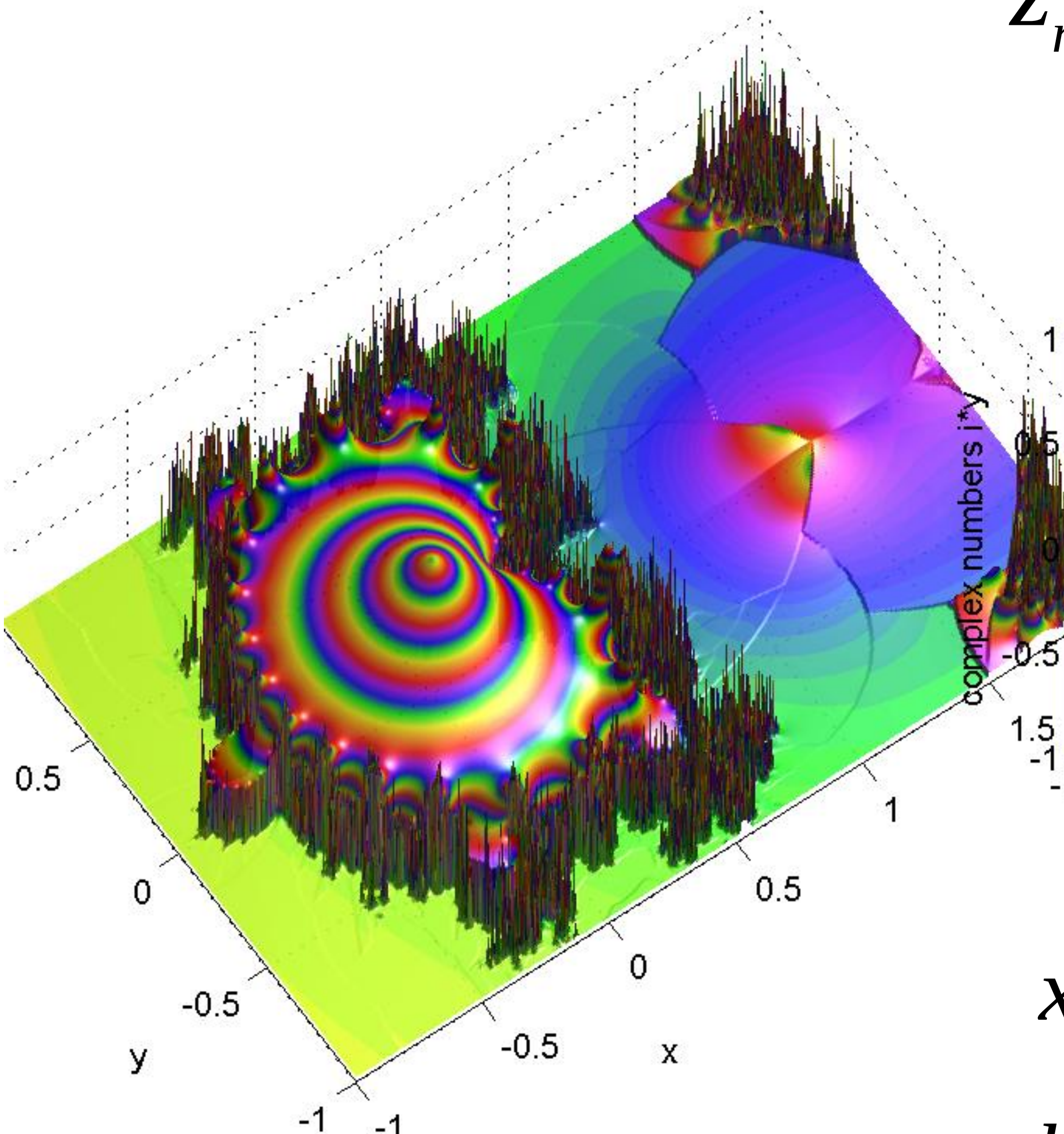


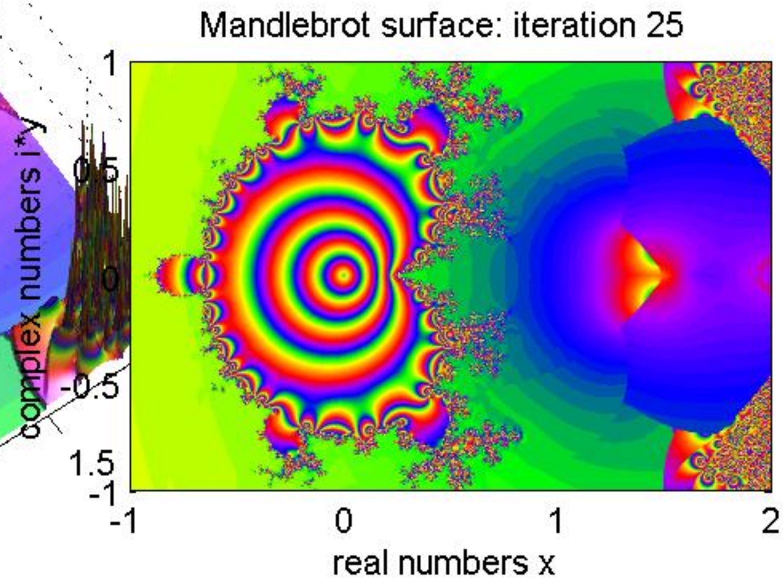
$$x = \mathrm{Re}(z), \quad y = \mathrm{Im}(z)$$

$$h(x, y) = e^{-\sqrt{x^2+y^2}}$$

The Mandlerocket

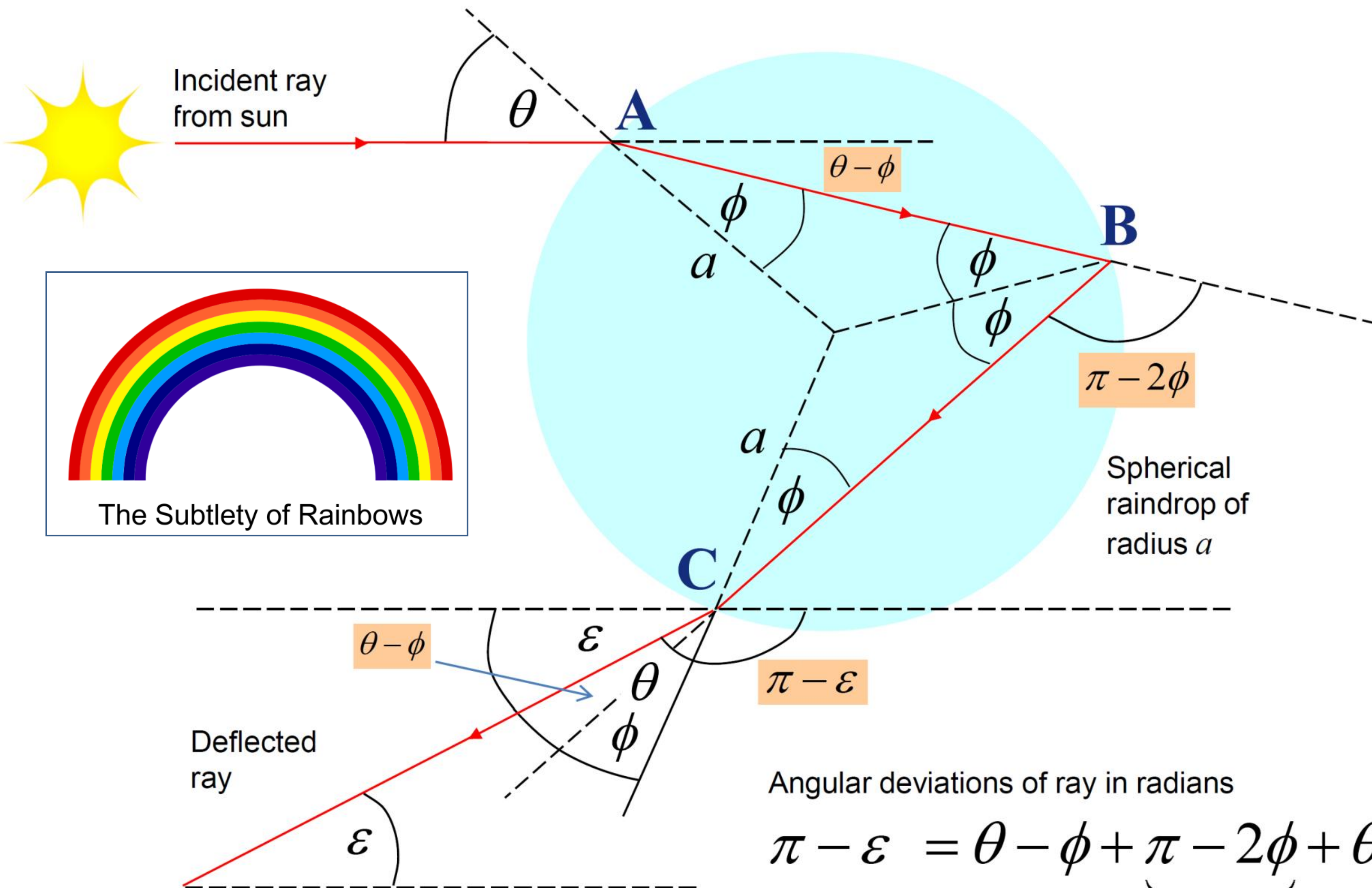
$$z_{n+1} = \sin^{-1}\left(z_n^2 + z_0\right)$$





$$x = \mathrm{Re}(z), \quad y = \mathrm{Im}(z)$$

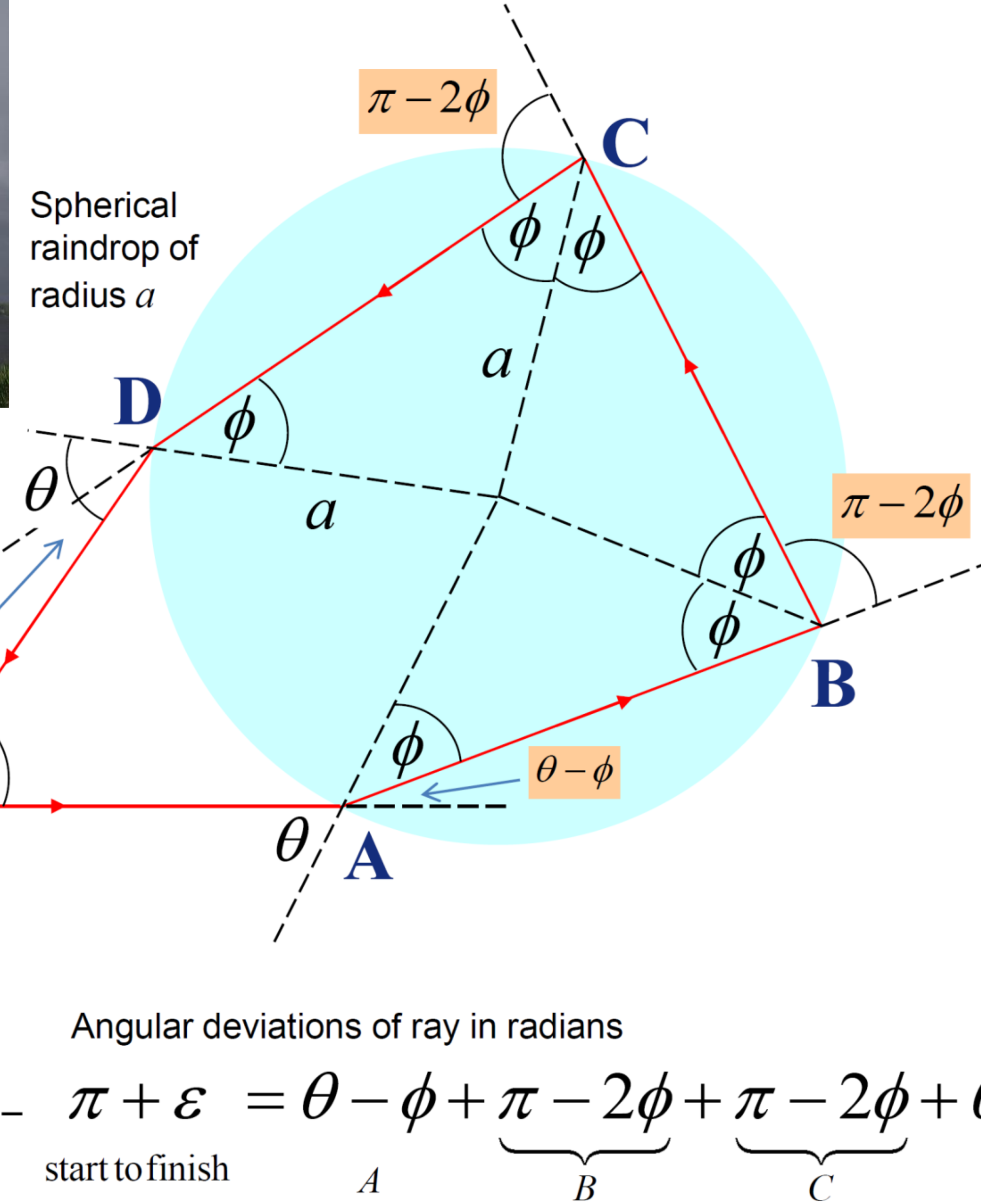
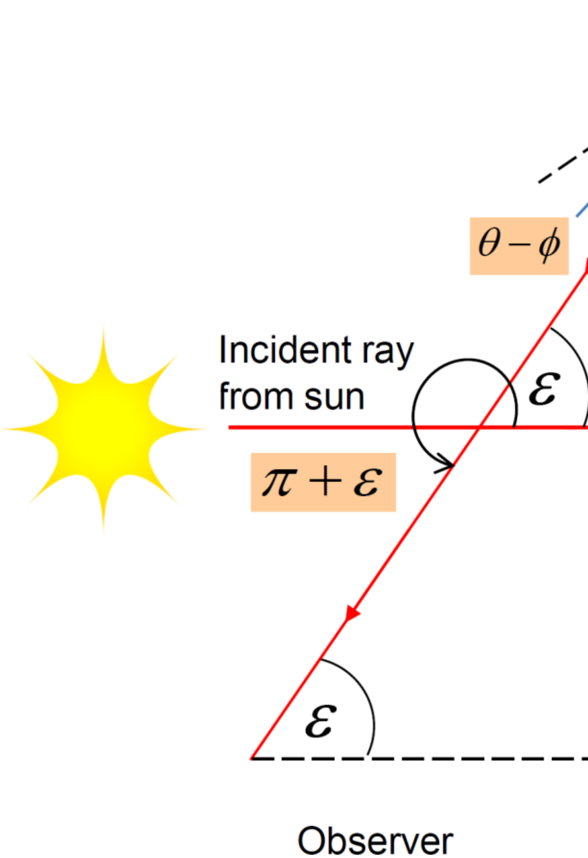
$$h(x, y) = e^{-\sqrt{x^2 + y^2}}$$



Angular deviations of ray in radians

$$\underset{\text{start to finish}}{\pi-\varepsilon} = \underset{A}{\theta-\phi} + \underbrace{\pi-2\phi}_{B} + \underset{C}{\theta-\phi}$$

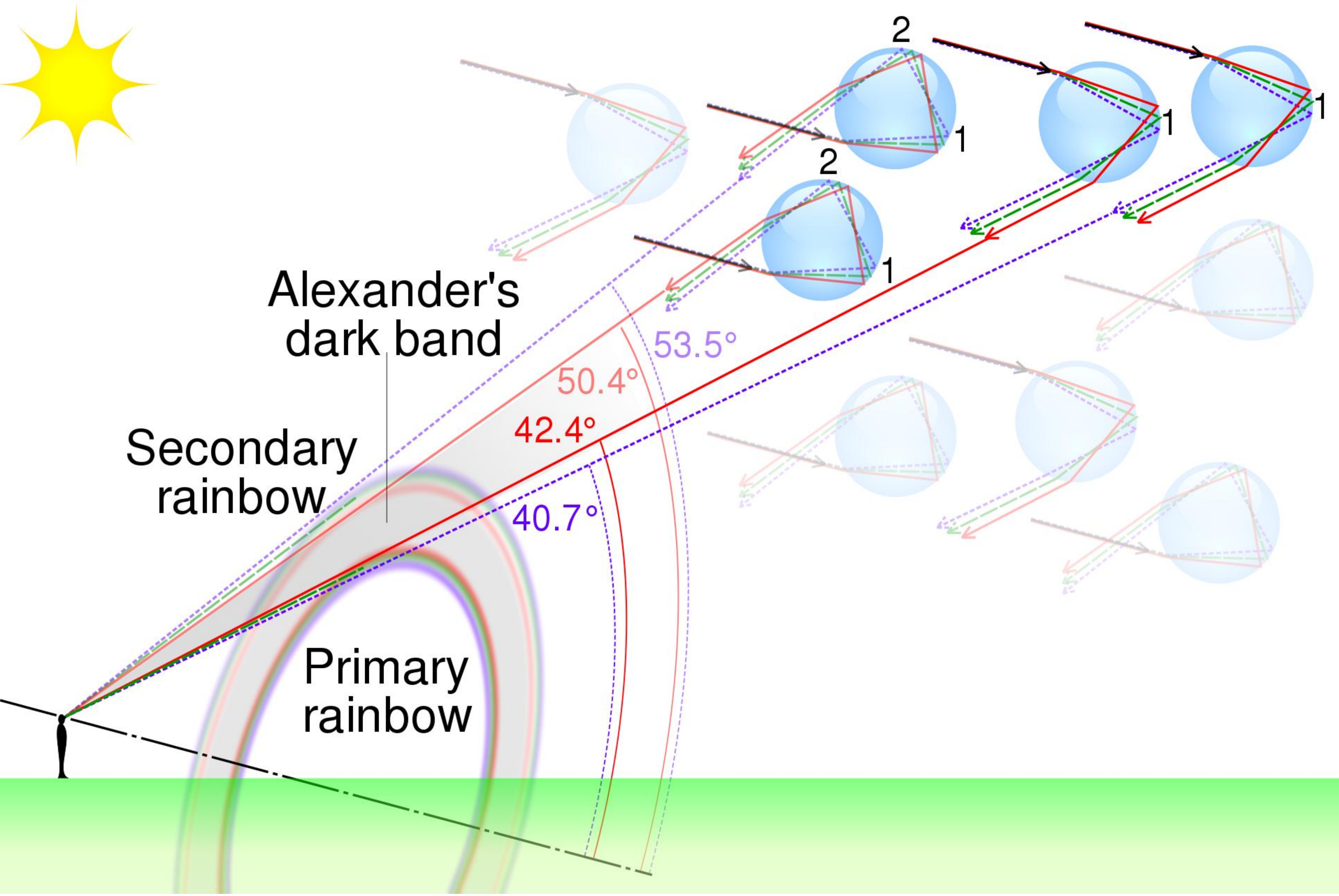
$$\therefore \varepsilon = 4\phi - 2\theta$$

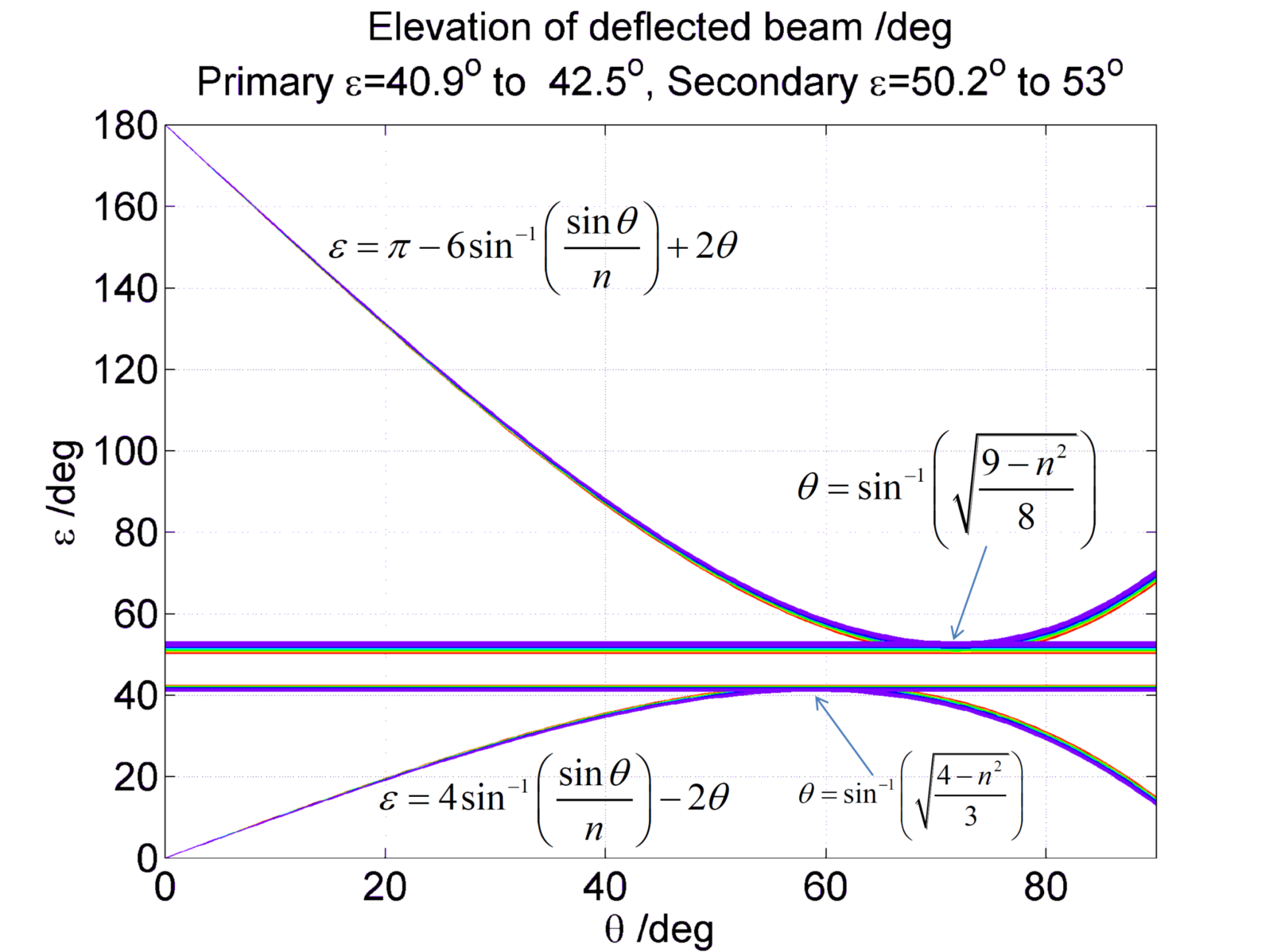
Angular deviations of ray in radians

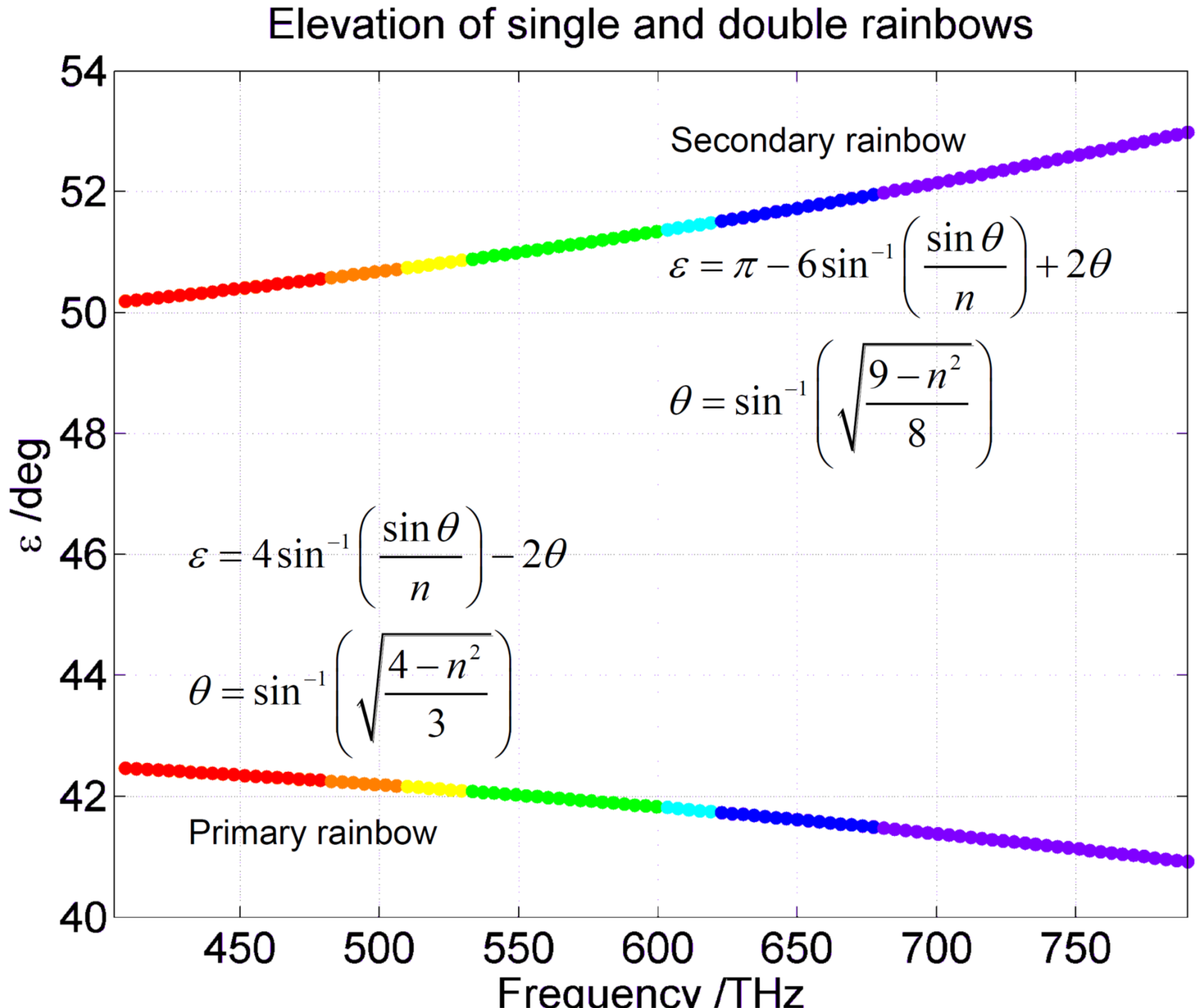
$$\underset{\text{start to finish}}{\pi + \varepsilon} = \underset{A}{\theta - \phi} + \underbrace{\pi - 2\phi}_{B} + \underbrace{\pi - 2\phi}_{C} + \underset{D}{\theta - \phi}$$

$$\therefore \varepsilon = \pi - 6\phi + 2\theta$$

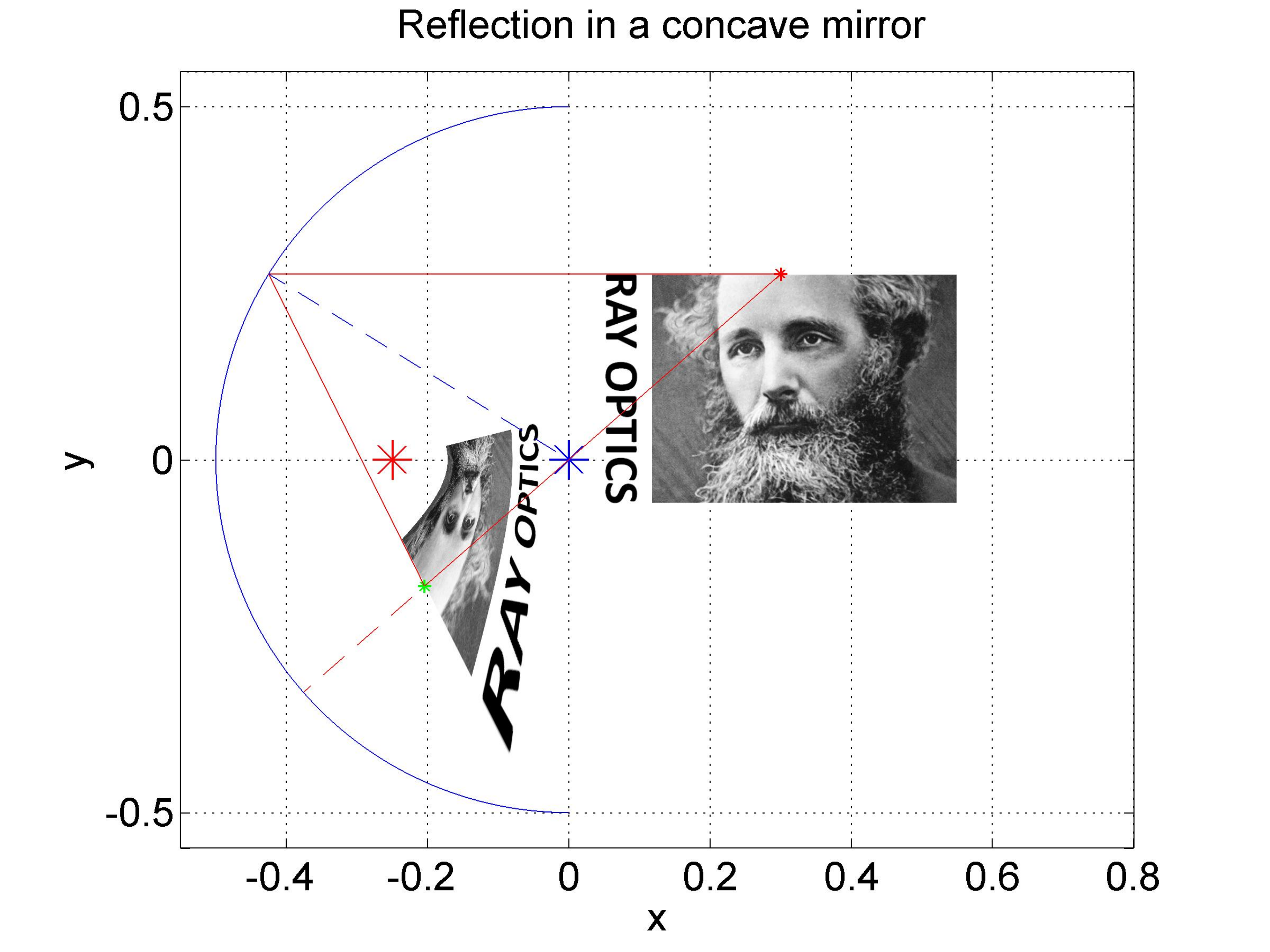


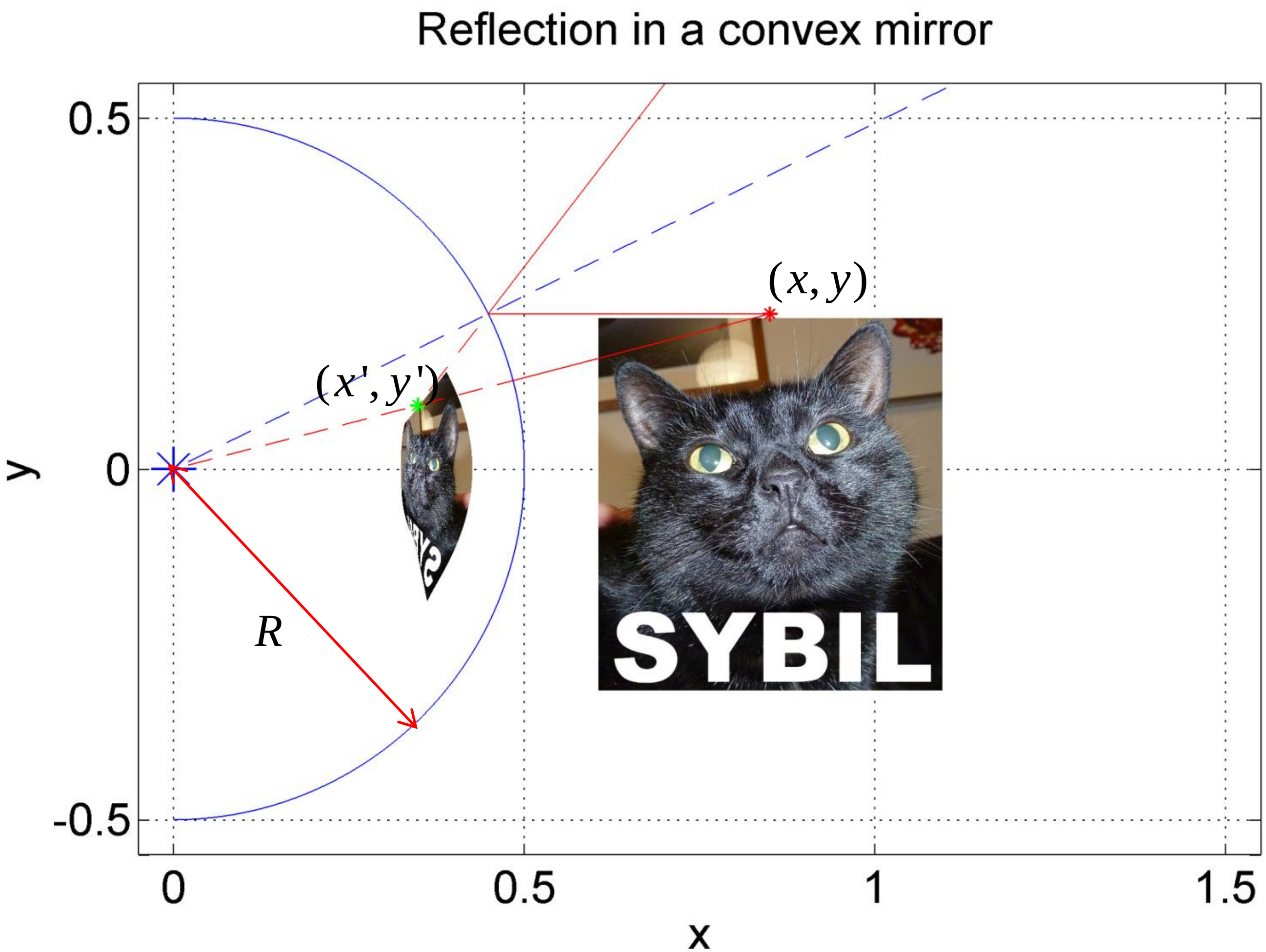
https://en.wikipedia.org/wiki/Rainbow


Elevation of deflected beam /deg
Primary ε=40.9° to 42.5°, Secondary ε=50.2° to 53°
$\varepsilon = \pi - 6\sin^{-1}\left(\frac{\sin\theta}{n}\right) + 2\theta$
$\theta = \sin^{-1}\left(\sqrt{\frac{9-n^2}{8}}\right)$
$\varepsilon = 4\sin^{-1}\left(\frac{\sin\theta}{n}\right) - 2\theta$
$\theta = \sin^{-1}\left(\sqrt{\frac{4-n^2}{3}}\right)$
ε /deg
θ /deg
180
160
140
120
100
80
60
40
20
0
0
20
40
60
80



Elevation of single and double rainbows
Secondary rainbow
$\varepsilon = \pi - 6\sin^{-1}\left(\frac{\sin\theta}{n}\right) + 2\theta$
$\theta = \sin^{-1}\left(\sqrt{\frac{9-n^2}{8}}\right)$
$\varepsilon = 4\sin^{-1}\left(\frac{\sin\theta}{n}\right) - 2\theta$
$\theta = \sin^{-1}\left(\sqrt{\frac{4-n^2}{3}}\right)$
Primary rainbow
ε /deg
54
52
50
48
46
44
42
40
450
500
550
600
650
700
750
Frequency /THz


Reflection in a concave mirror
0.5
0
-0.5
y
-0.4
-0.2
0
0.2
0.4
0.6
0.8
x
RAY OPTICS
RAY OPTICS



$$\alpha = \tfrac{1}{2}\tan^{-1}\left(\tfrac{y}{x}\right)$$

$$k = \frac{x}{\cos(2\alpha)}$$

$$y' = \frac{k\sin\alpha}{\tfrac{k}{R} - \cos\alpha + \tfrac{x}{y}\sin\alpha}$$

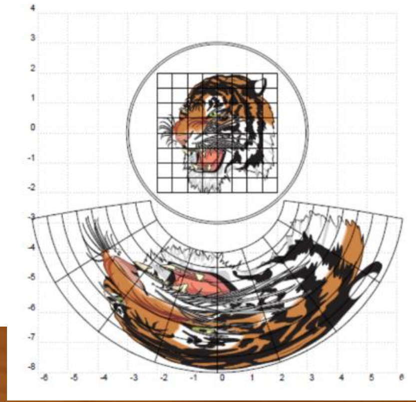
$$x' = x\frac{y'}{y}$$

Virtual image from object coordinates

We see an upright, distorted *virtual image* in a cylindrical mirror.

i.e. the *apparent source* of (diverging) light rays from the mirror

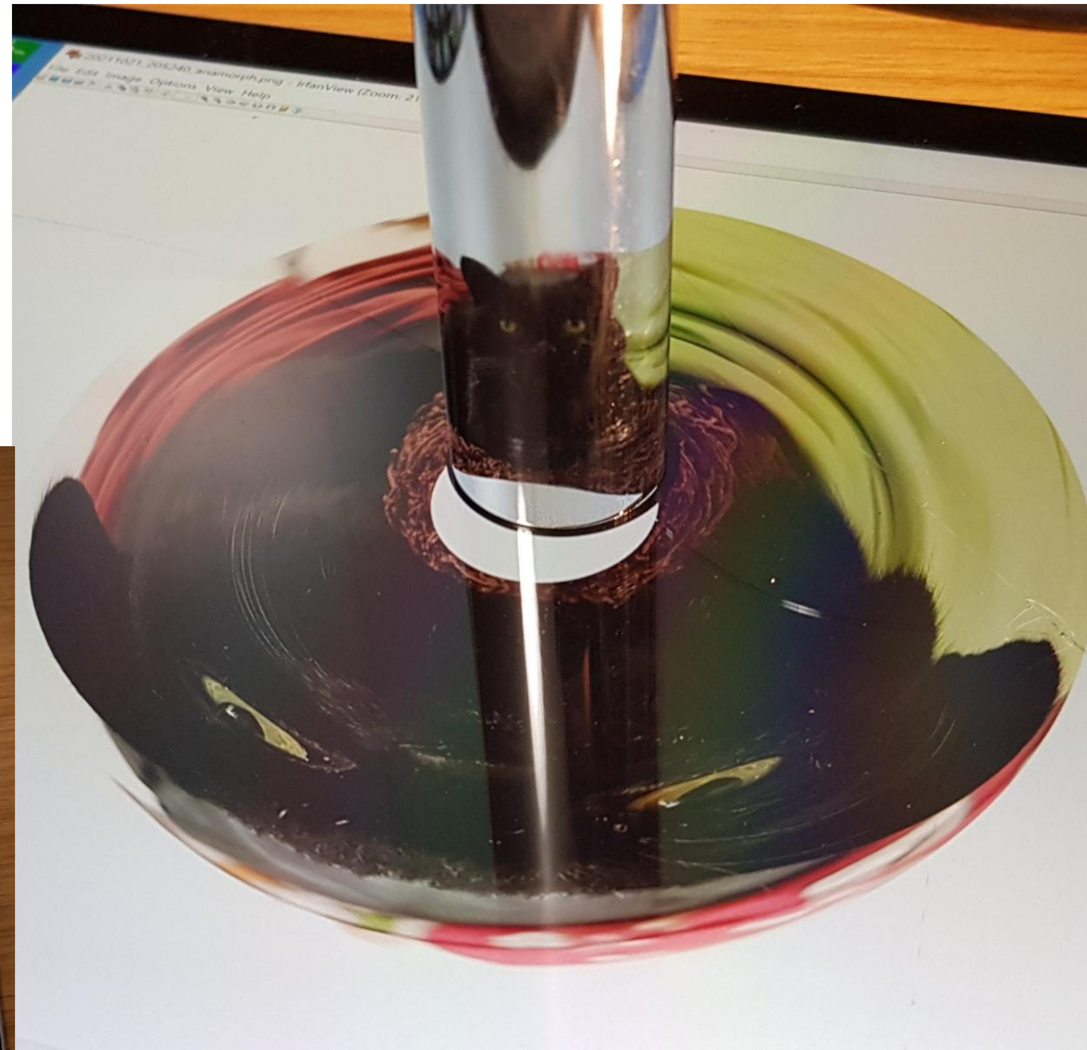
A MATLAB program `anamorph.m` fits *any* bitmap image into a unit circle, and then calculates an anamorphic projection based upon a mapping of a rectangular grid inside the black circle to a circle sector beyond.

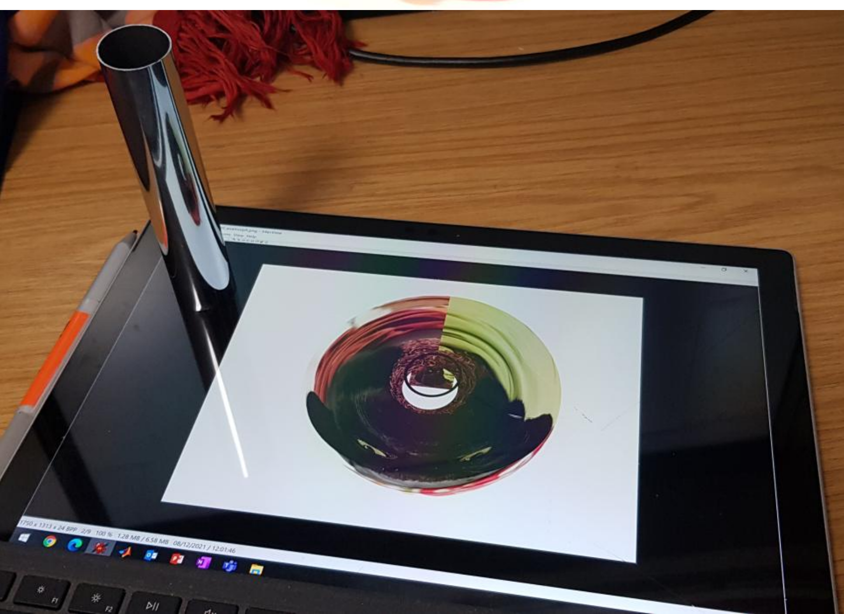


A 2.5cm diameter curtain rail section works as a mirror.

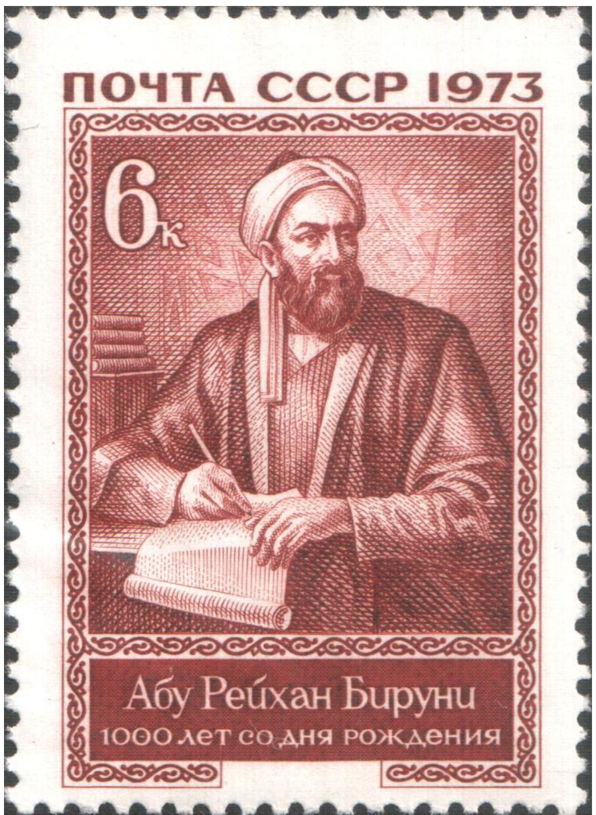
If you load the image onto a flat tablet screen and zoom until a polished cylinder fits into the black circle, the virtual image is of the correct proportions! You also don't have to print...

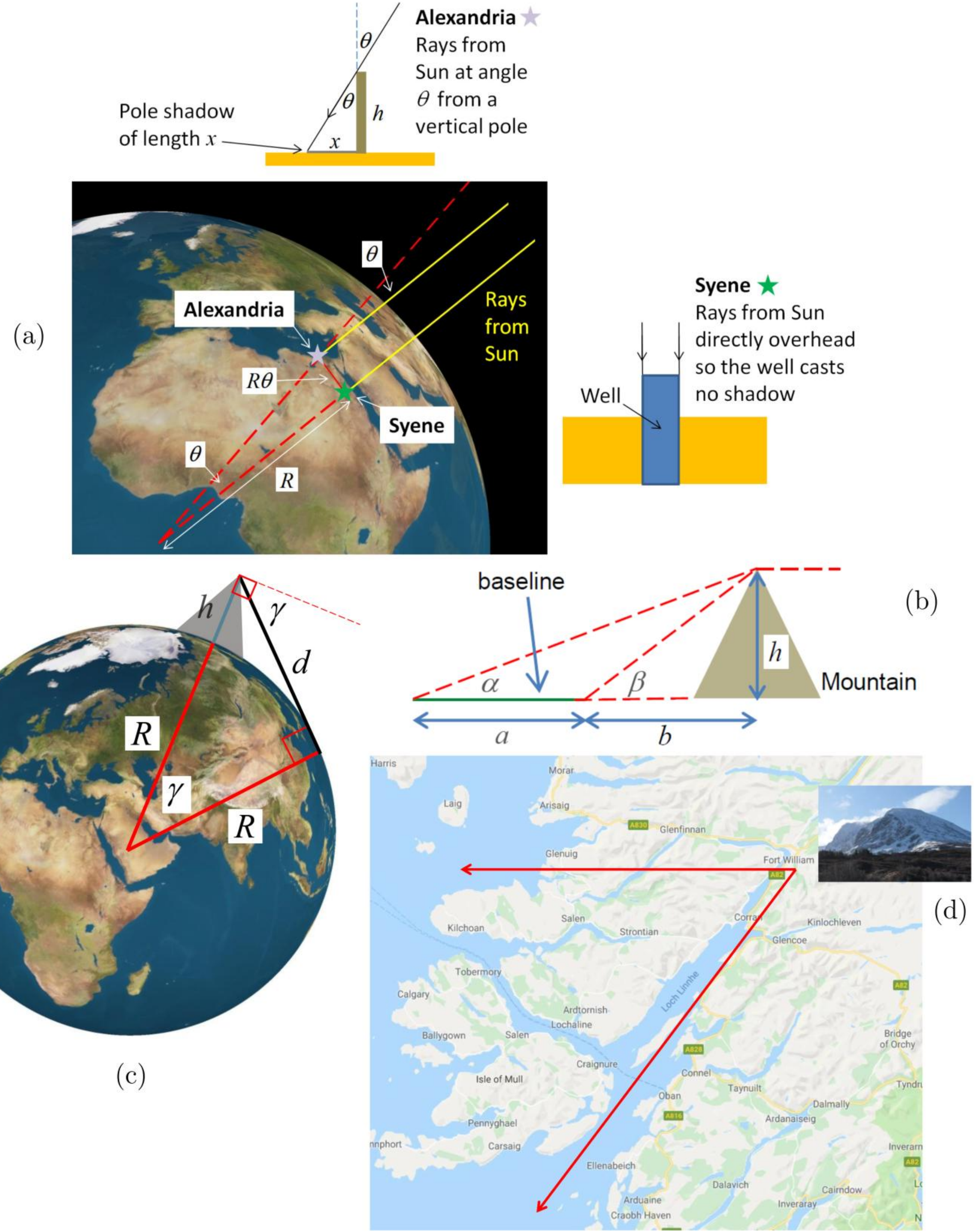
Sybil the cat was unperturbed by this anamorphic transformation.



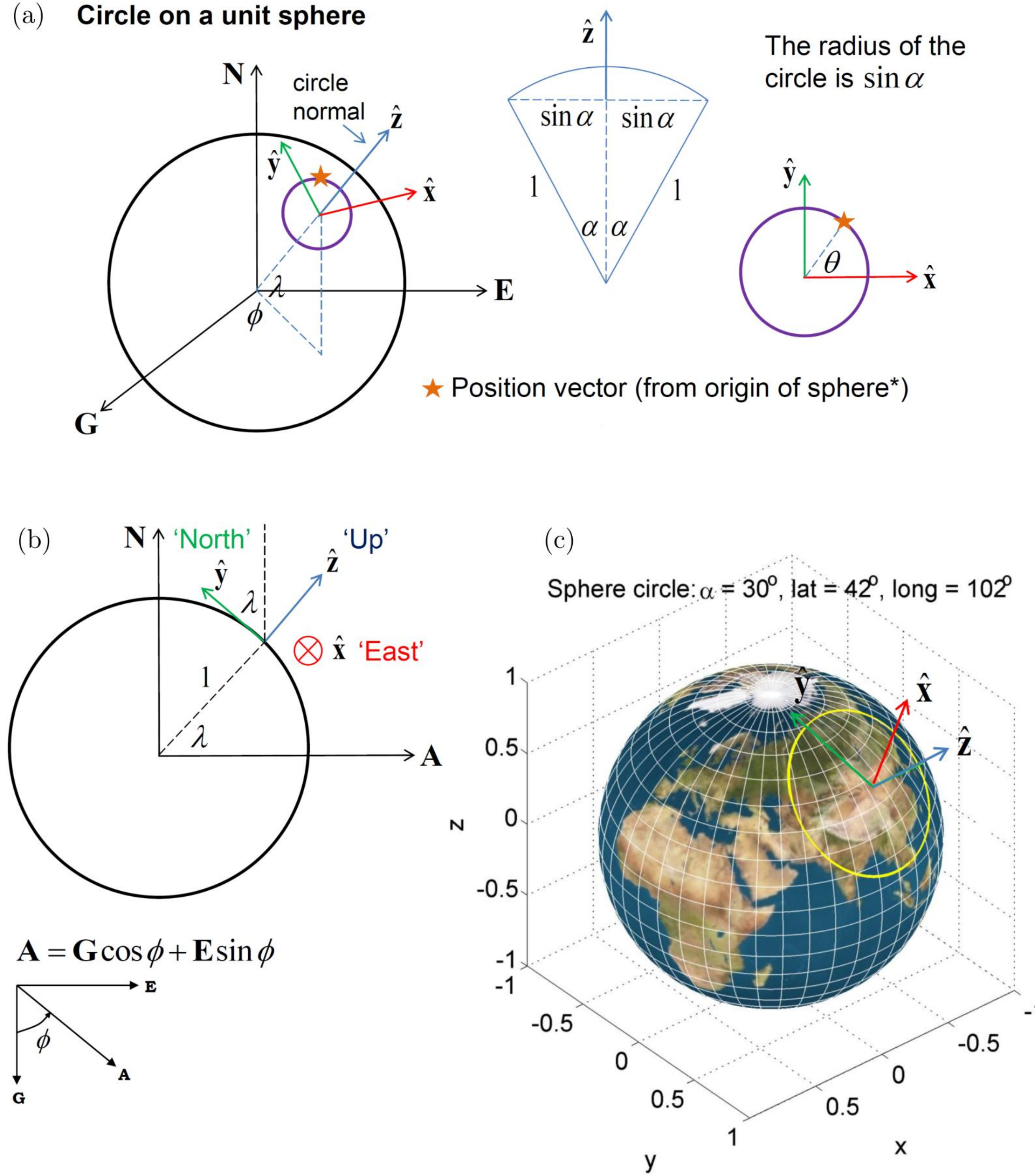


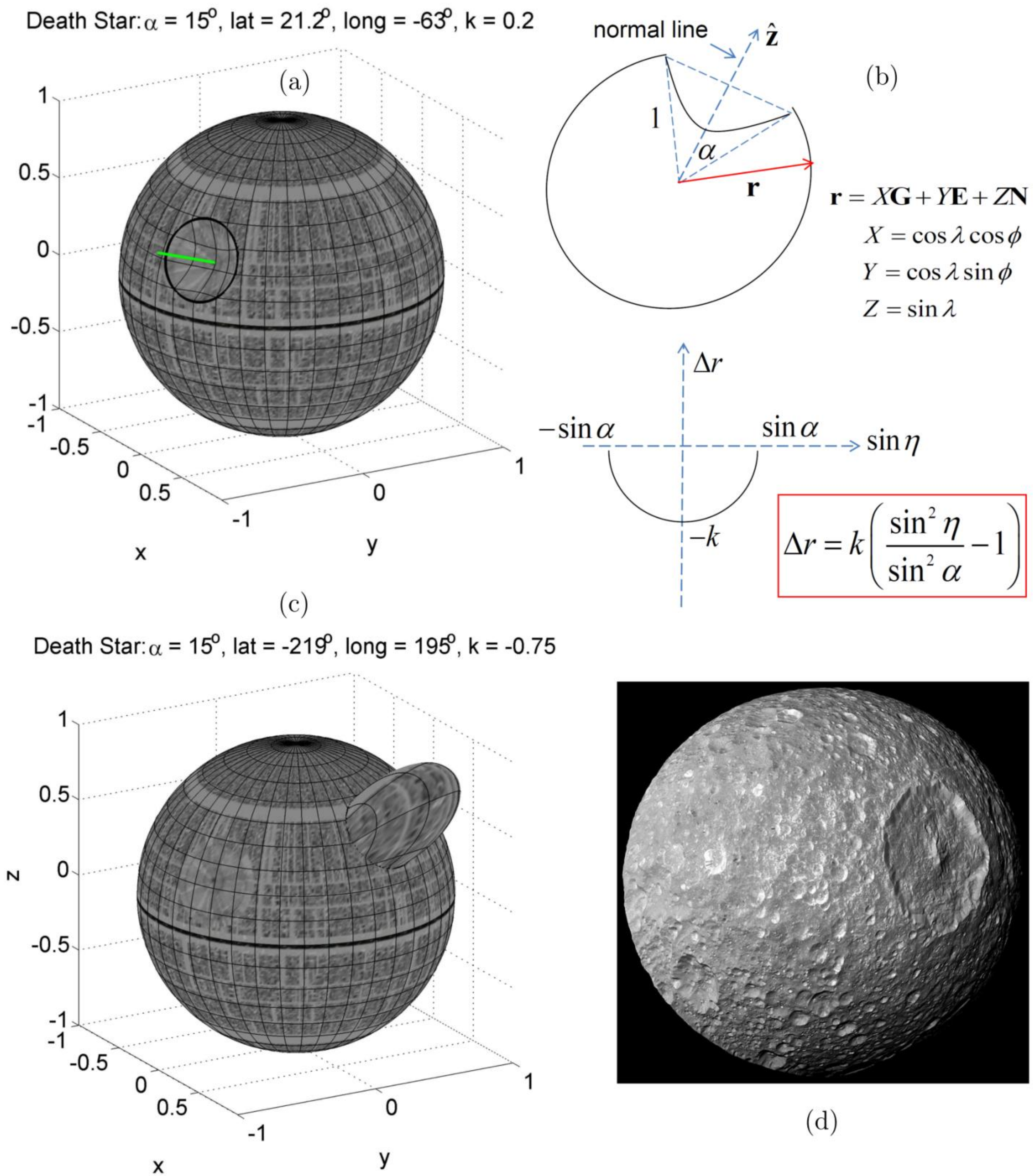
Eratosthenes of Cyrene



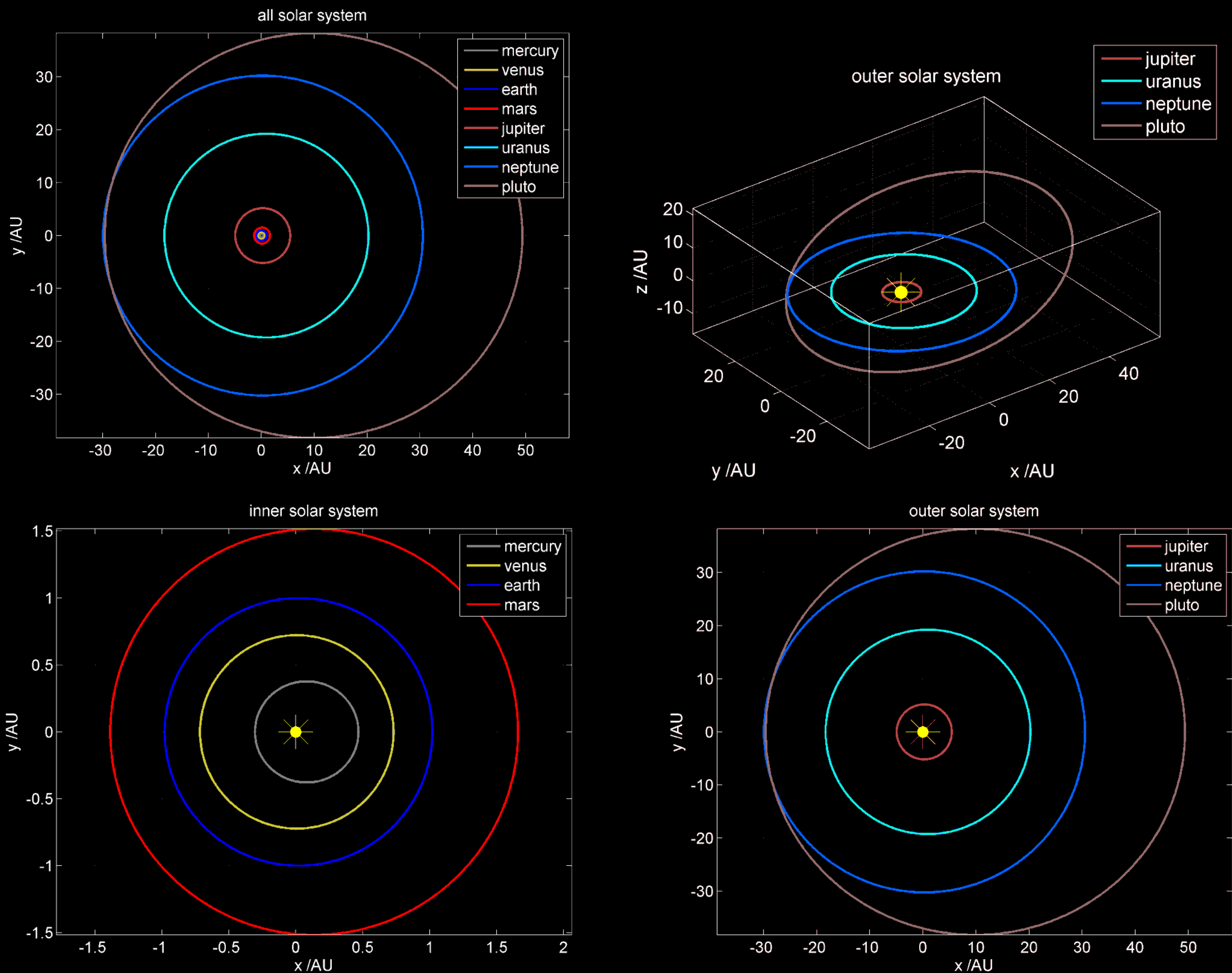


Navigating the Sphere

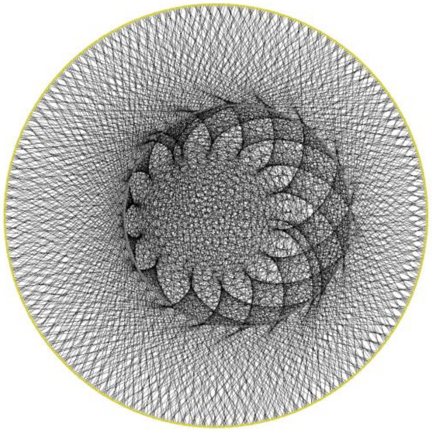




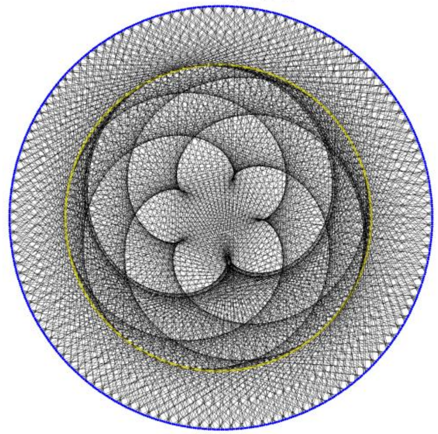
Death Star: α = 15°, lat = 21.2°, long = -63°, k = 0.2
(a)
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
x
-1
0
1
y
normal line
$\hat{\mathbf{z}}$
(b)
1
α
$\mathbf{r}$
$\mathbf{r} = X\mathbf{G} + Y\mathbf{E} + Z\mathbf{N}$
$X = \cos\lambda\cos\phi$
$Y = \cos\lambda\sin\phi$
$Z = \sin\lambda$
Δr
$-\sin\alpha$
$\sin\alpha$
$\sin\eta$
$-k$
$\Delta r = k\left(\frac{\sin^2\eta}{\sin^2\alpha} - 1\right)$
(c)
Death Star: α = 15°, lat = -219°, long = 195°, k = -0.75
1
0.5
z
0
-0.5
-1
-1
-0.5
0
0.5
x
-1
0
1
y
(d)


all solar system
mercury
venus
earth
mars
jupiter
uranus
neptune
pluto
y /AU
x /AU
outer solar system
jupiter
uranus
neptune
pluto
z /AU
y /AU
x /AU
inner solar system
mercury
venus
earth
mars
y /AU
x /AU
outer solar system
jupiter
uranus
neptune
pluto
y /AU
x /AU

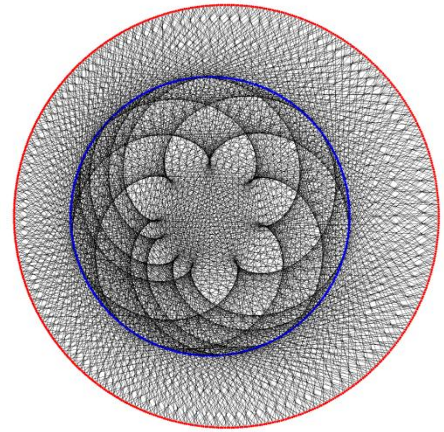
mercury venus spirograph



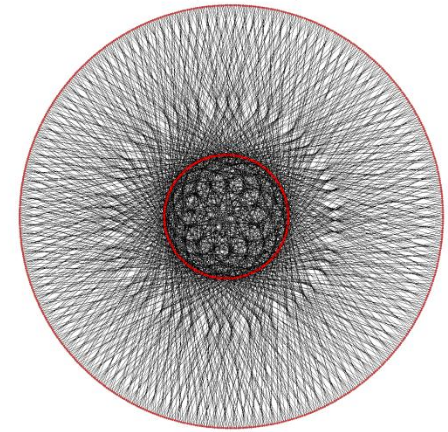
venus earth spirograph



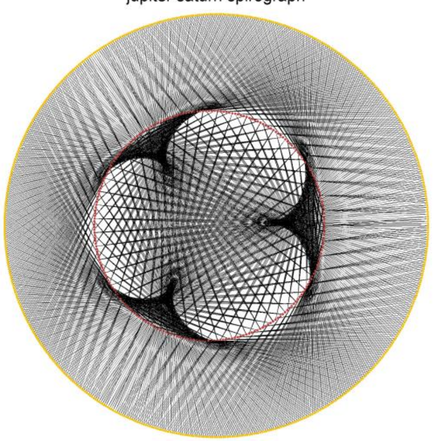
earth mars spirograph



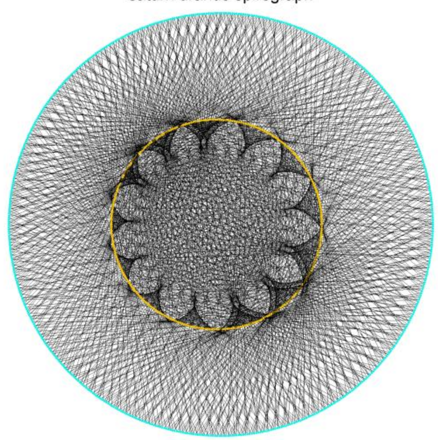
mars jupiter spirograph



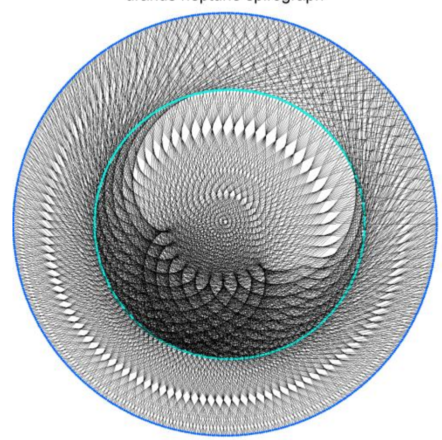
jupiter saturn spirograph



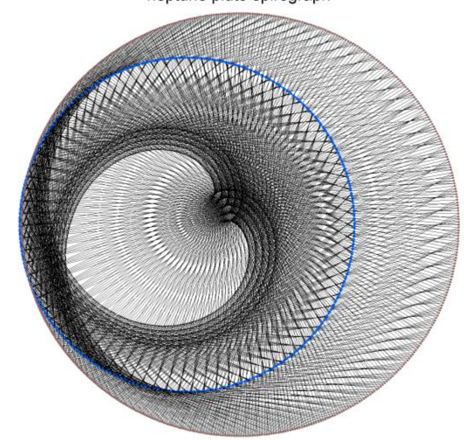
saturn uranus spirograph



uranus neptune spirograph



neptune pluto spirograph

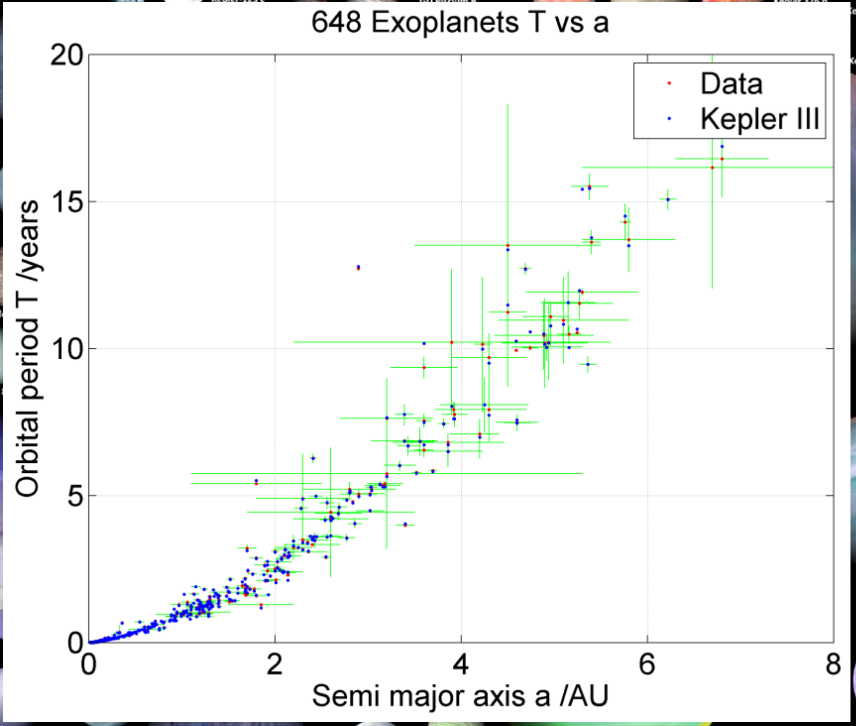


Solar system spirograph!

Image by Martin Vargic

Extrasolar Planets and Kepler's Third Law

$$T^2 = \frac{4\pi^2}{G(M+m)}a^3$$



LINE OF BEST FIT CALCULATOR y = mx

Dr Andy French. March 2019

N	660

paste as values x,y data here

x	y	x^2	y^2	xy	xfit	yfit	(y-fit)^2	ylower	yupper
0.111	0.336	0.012	0.113	0.037	0.111	0.331	0.000	0.330	0.331
0.188	0.558	0.035	0.312	0.105	0.188	0.561	0.000	0.560	0.562
0.442	1.329	0.196	1.766	0.588	0.442	1.324	0.000	1.322	1.326
0.225	0.685	0.051	0.470	0.154	0.225	0.674	0.000	0.673	0.675
0.591	1.980	0.349	3.922	1.171	0.591	1.768	0.045	1.766	1.771
0.125	0.375	0.016	0.140	0.047	0.125	0.373	0.000	0.373	0.374
0.318	0.955	0.101	0.911	0.304	0.318	0.952	0.000	0.950	0.953
0.462	1.380	0.213	1.904	0.637	0.462	1.382	0.000	1.380	1.384
-0.036	-0.108	0.001	0.012	0.004	-0.036	-0.108	0.000	-0.108	-0.109
0.548	1.602	0.300	2.567	0.878	0.548	1.639	0.001	1.637	1.641
0.651	1.953	0.424	3.814	1.272	0.651	1.949	0.000	1.946	1.951
-0.002	0.015	0.000	0.000	0.000	-0.002	-0.007	0.000	-0.007	-0.007
-0.060	-0.171	0.004	0.029	0.010	-0.060	-0.181	0.000	-0.181	-0.181
0.076	0.229	0.006	0.052	0.017	0.076	0.226	0.000	0.226	0.226
0.322	0.954	0.104	0.910	0.307	0.322	0.964	0.000	0.963	0.965
0.556	1.645	0.309	2.706	0.915	0.556	1.664	0.000	1.662	1.667
1.064	3.181	1.133	10.117	3.386	1.064	3.185	0.000	3.180	3.189
-0.945	-2.836	0.894	8.045	2.682	-0.945	-2.829	0.000	-2.824	-2.833
-0.619	-1.875	0.383	3.515	1.161	-0.619	-1.853	0.000	-1.850	-1.855
0.760	2.268	0.578	5.145	1.725	0.760	2.275	0.000	2.272	2.278
-1.807	-5.434	3.265	29.530	9.819	-1.807	-5.406	0.001	-5.398	-5.414
-0.107	-0.336	0.012	0.113	0.036	-0.107	-0.321	0.000	-0.321	-0.322
-1.299	-3.898	1.688	15.193	5.064	-1.299	-3.887	0.000	-3.882	-3.893
-0.663	-1.987	0.439	3.950	1.317	-0.663	-1.982	0.000	-1.979	-1.985
-0.322	-0.968	0.104	0.936	0.312	-0.322	-0.965	0.000	-0.963	-0.966
0.279	0.822	0.078	0.676	0.229	0.279	0.834	0.000	0.833	0.835
0.623	1.884	0.388	3.548	1.174	0.623	1.865	0.000	1.862	1.867
-1.369	-4.104	1.873	16.843	5.617	-1.369	-4.095	0.000	-4.089	-4.101
-0.362	-1.080	0.131	1.166	0.390	-0.362	-1.082	0.000	-1.080	-1.083
-1.252	-3.760	1.567	14.134	4.706	-1.252	-3.745	0.000	-3.740	-3.751
-0.167	-0.504	0.028	0.254	0.084	-0.167	-0.501	0.000	-0.500	-0.502
0.358	1.076	0.128	1.157	0.385	0.358	1.071	0.000	1.069	1.072
0.127	0.380	0.016	0.145	0.048	0.127	0.380	0.000	0.380	0.381
0.415	1.684	0.172	2.835	0.699	0.415	1.242	0.195	1.240	1.243
-1.595	-4.790	2.545	22.941	7.640	-1.595	-4.773	0.000	-4.766	-4.780
-0.977	-2.931	0.954	8.589	2.863	-0.977	-2.922	0.000	-2.918	-2.927
-1.361	-3.979	1.853	15.831	5.417	-1.361	-4.073	0.009	-4.067	-4.079
-1.396	-4.189	1.949	17.550	5.849	-1.396	-4.177	0.000	-4.171	-4.183
-1.292	-3.876	1.670	15.020	5.009	-1.292	-3.867	0.000	-3.861	-3.873
-1.569	-4.710	2.461	22.185	7.388	-1.569	-4.693	0.000	-4.686	-4.700
-1.209	-3.627	1.462	13.157	4.385	-1.209	-3.617	0.000	-3.612	-3.623
-1.336	-3.955	1.786	15.641	5.285	-1.336	-3.998	0.002	-3.992	-4.004
-1.530	-4.588	2.341	21.053	7.021	-1.530	-4.578	0.000	-4.572	-4.585
-1.286	-3.861	1.653	14.904	4.963	-1.286	-3.847	0.000	-3.841	-3.852
-1.551	-4.654	2.407	21.663	7.220	-1.551	-4.641	0.000	-4.635	-4.648

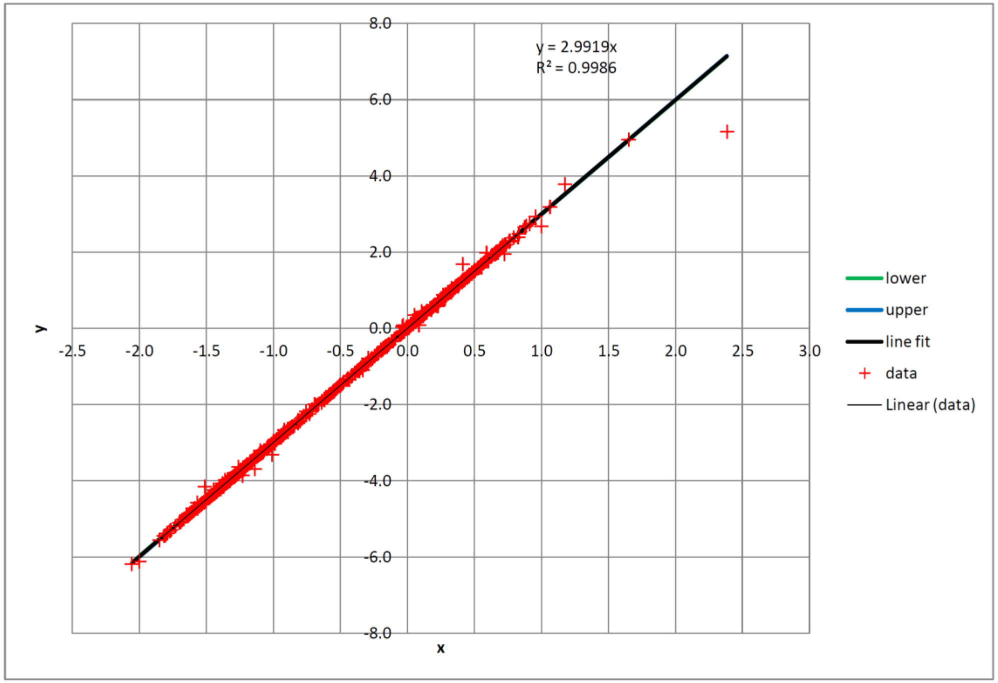
xbar	ybar	x^2 bar	y^2 bar	xy bar
-0.572	-1.716	0.950	8.508	2.841

Vx	Vy	Cov[x,y]	s
0.623	5.563	1.860	0.089

r	r^2
0.999	0.999

m
2.992

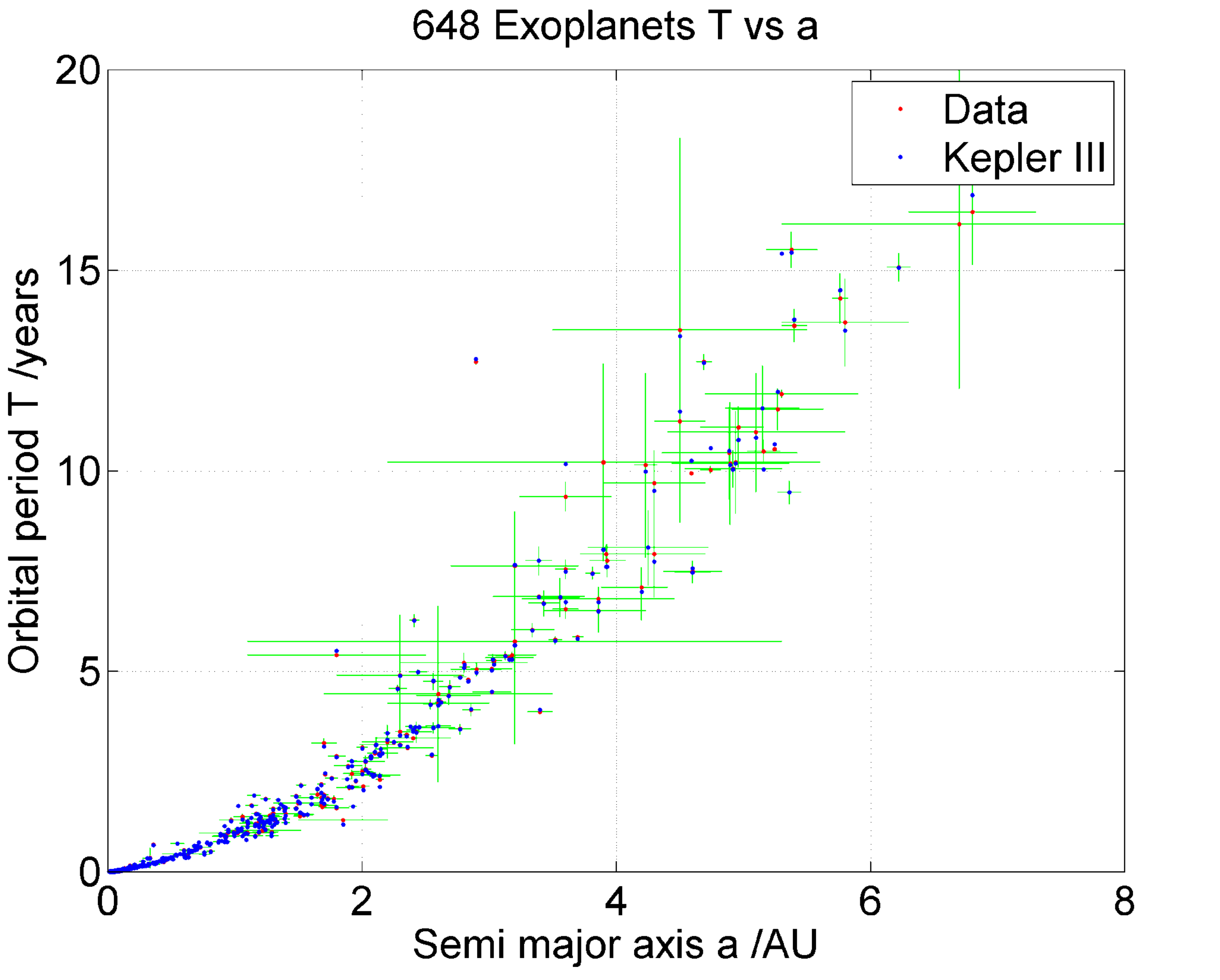
dm
0.004



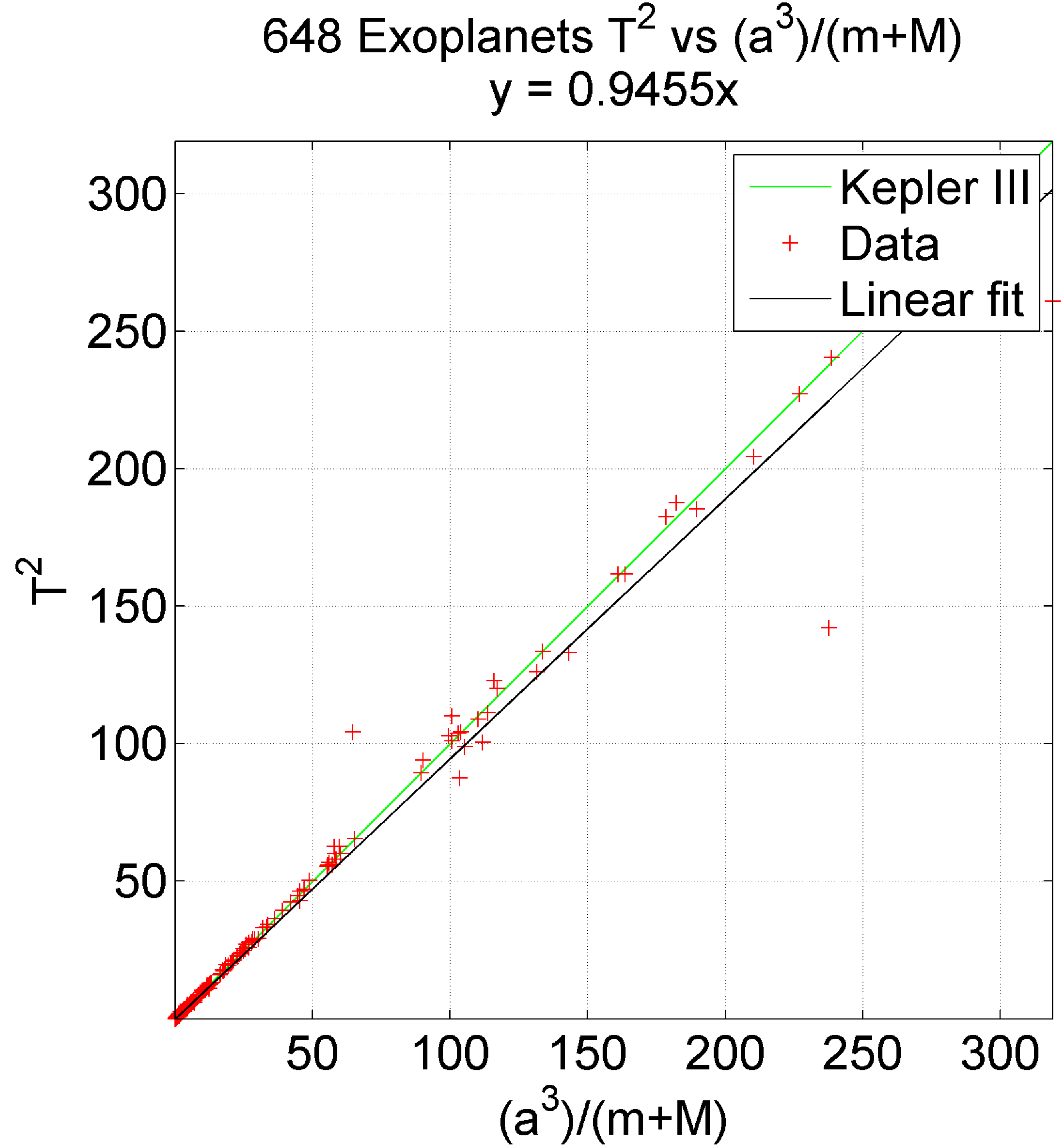
Note bug in old versions fo Excel (<2003), that will give an incorrect R^2 value for the built-in trend line function when 'set intercept at 0,0' is chosen

$$y = 2\log\left(\frac{T}{\mathrm{Yr}}\right) + \log\left(\frac{M}{M_\odot} + \frac{m}{M_\odot}\right), \quad x = \log\left(\frac{a}{\mathrm{AU}}\right)$$

$$y = \left(2.992 \pm 0.004\right)x$$



648 Exoplanets T vs a
Data
Kepler III
Orbital period T /years
Semi major axis a /AU
0
2
4
6
8
0
5
10
15
20

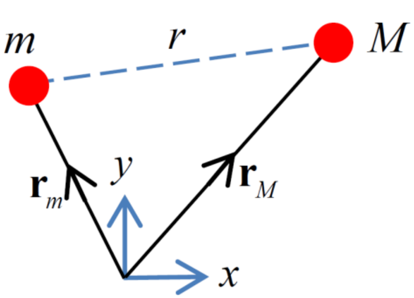

648 Exoplanets T^2 vs (a^3)/(m+M)
y = 0.9455x
Kepler III
Data
Linear fit
T^2
(a^3)/(m+M)
300
250
200
150
100
50
50
100
150
200
250
300

Verlet method

$$\mathbf{a}_n = f(t_n, \mathbf{r}_n, \mathbf{v}_n)$$
$$t_{n+1} = t_n + \Delta t$$
$$\mathbf{r}_{n+1} = \mathbf{r}_n + \mathbf{v}_n \Delta t + \tfrac{1}{2}\mathbf{a}_n \Delta t^2$$
$$\mathbf{V} = \mathbf{v}_n + \mathbf{a}_n \Delta t$$
$$\mathbf{A} = f(t_{n+1}, \mathbf{r}_{n+1}, \mathbf{V})$$
$$\mathbf{v}_{n+1} = \mathbf{v}_n + \tfrac{1}{2}(\mathbf{a}_n + \mathbf{A})\Delta t$$

Newton's Law of Gravitation

$$\mathbf{a}_{n,i} = -G \sum_{j \neq i}^{N} M_j \frac{\mathbf{r}_i - \mathbf{r}_j}{\left|\mathbf{r}_i - \mathbf{r}_j\right|^3}$$

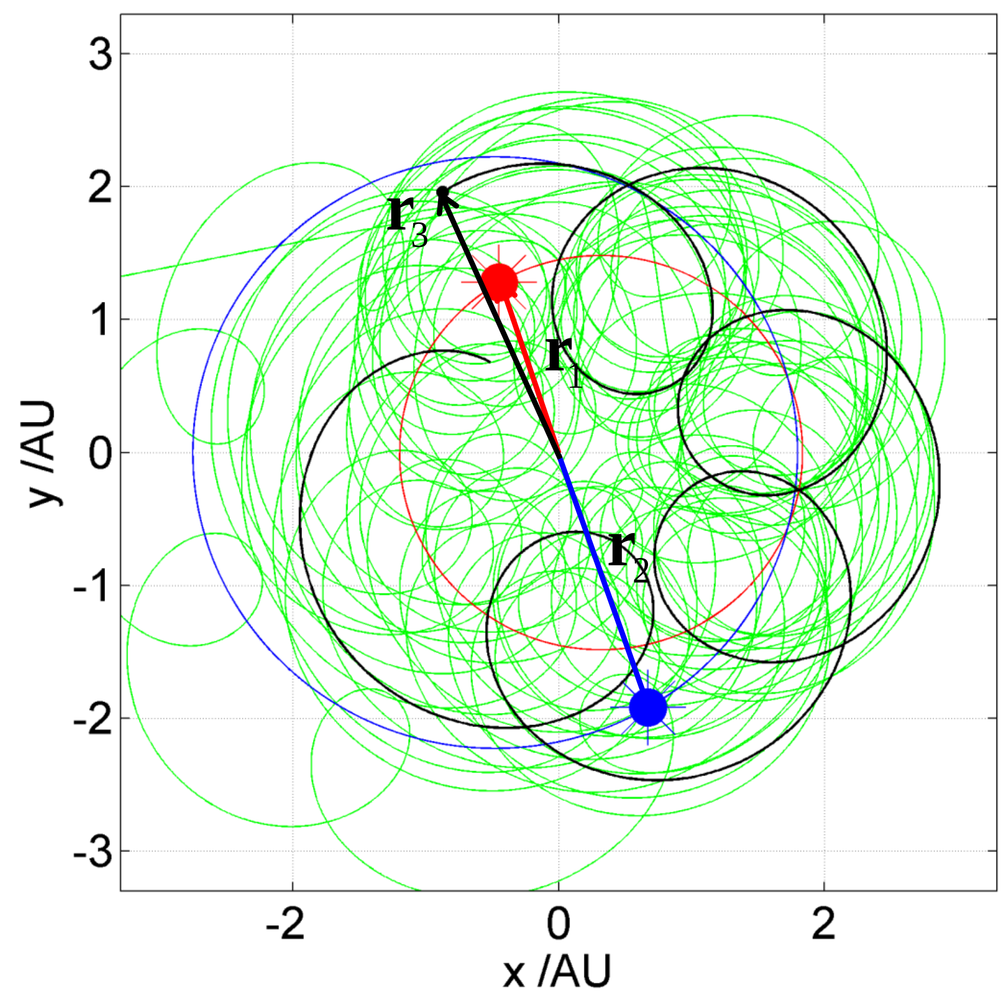


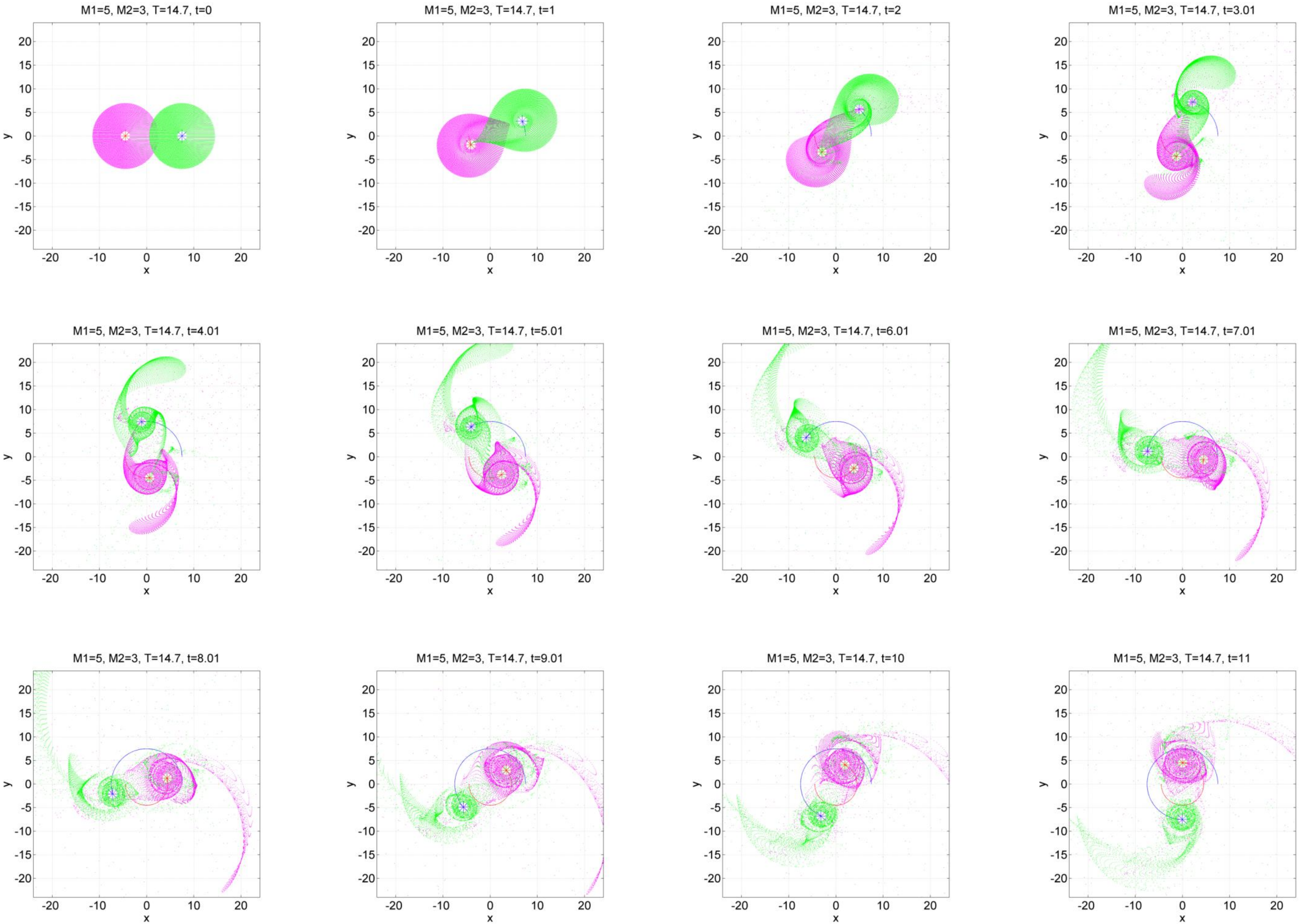
$$\mathbf{r} = \mathbf{r}_M - \mathbf{r}_m$$
$$r = |\mathbf{r}|$$

In this simulation:

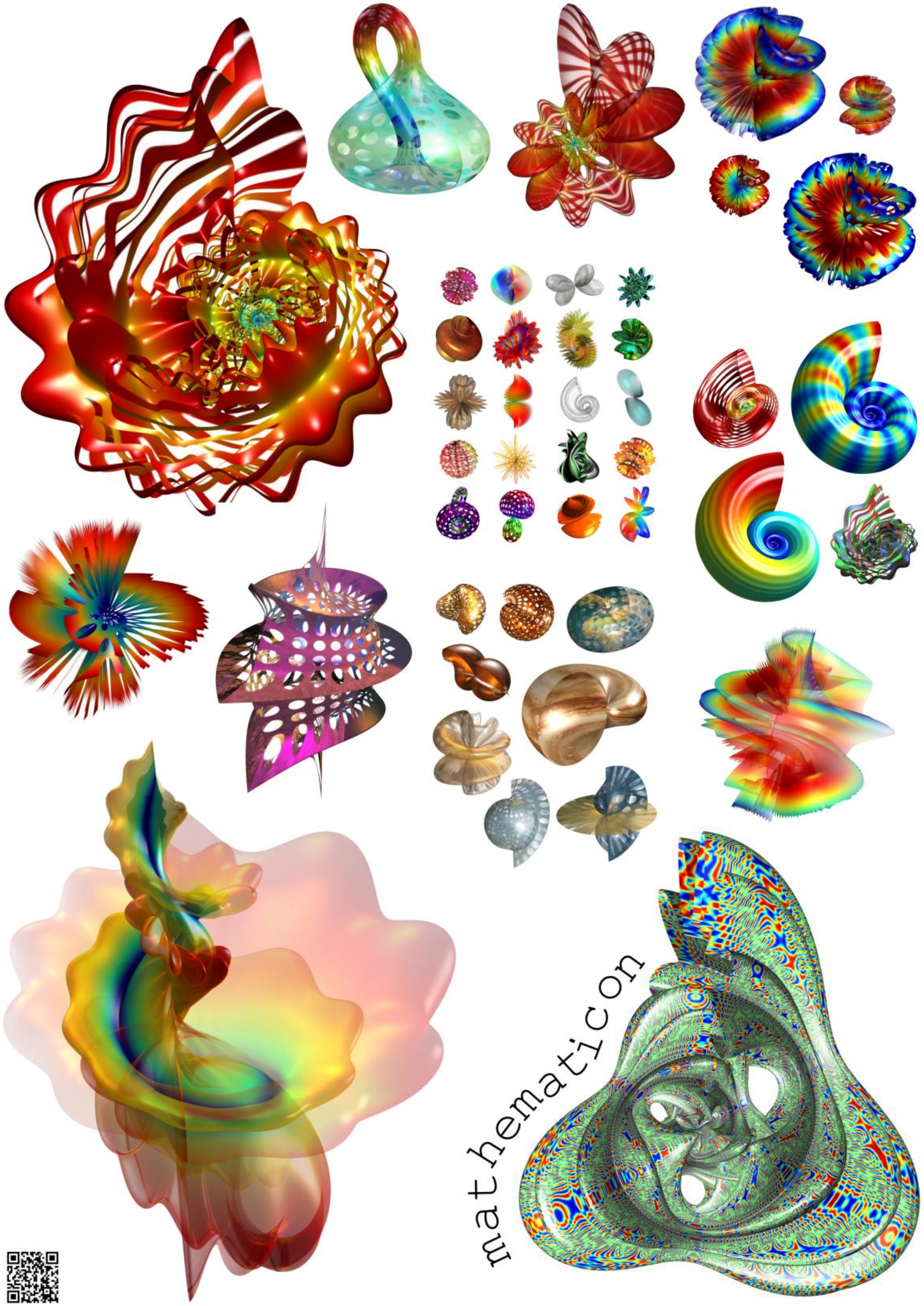
$$M_1 = 3M_\odot$$
$$M_2 = 2M_\odot$$
$$M_3 \ll M_\odot$$

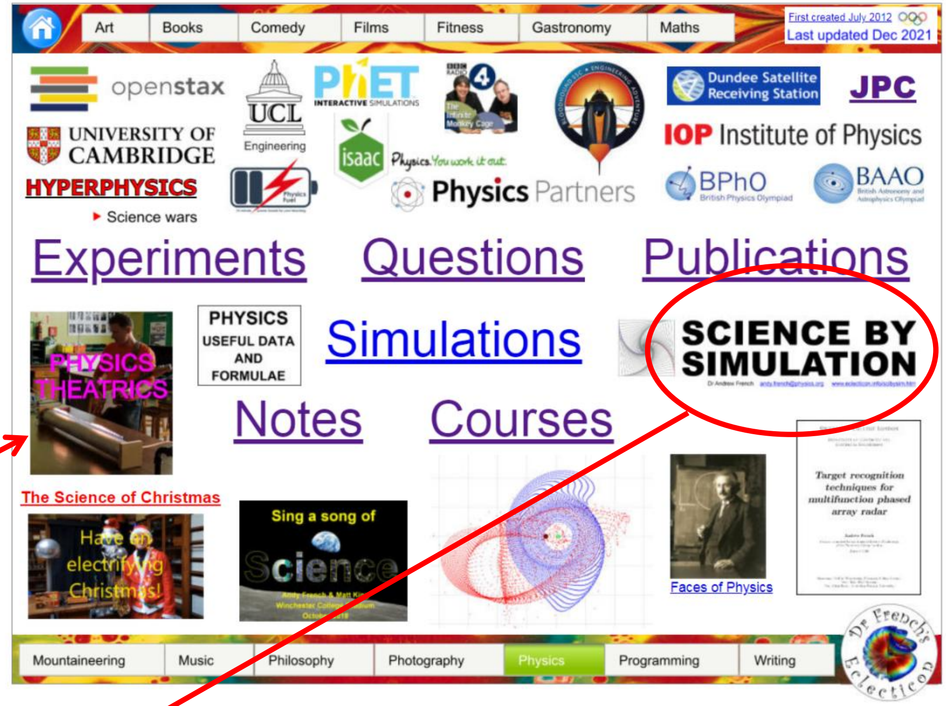
M1=3, M2=2 T=2.32 years, a=3AU, k=1.1, a_p=0.965AU.

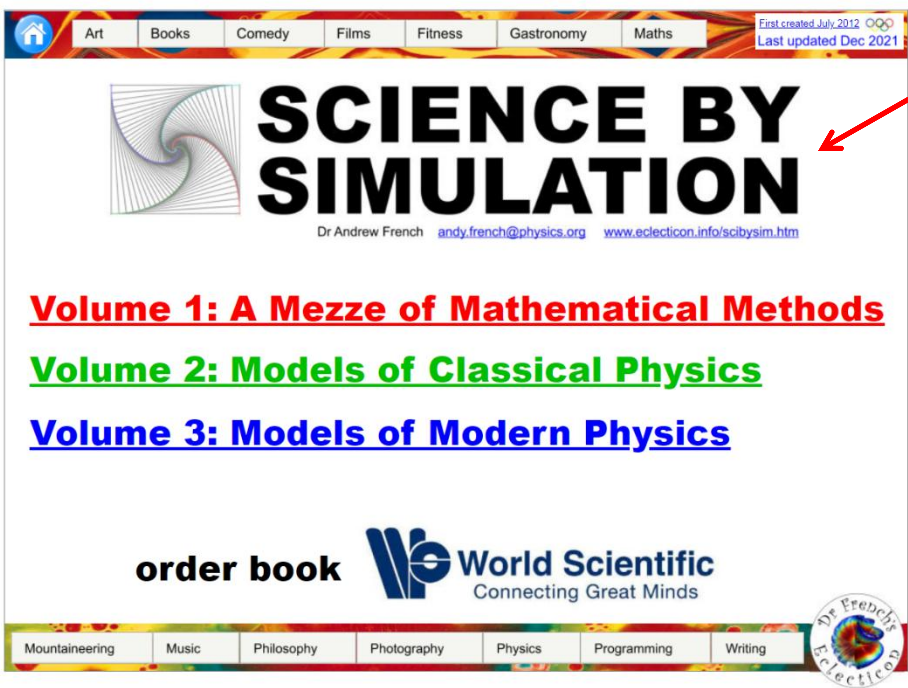




A possible explanation for common spiral galactic forms


mathematicon





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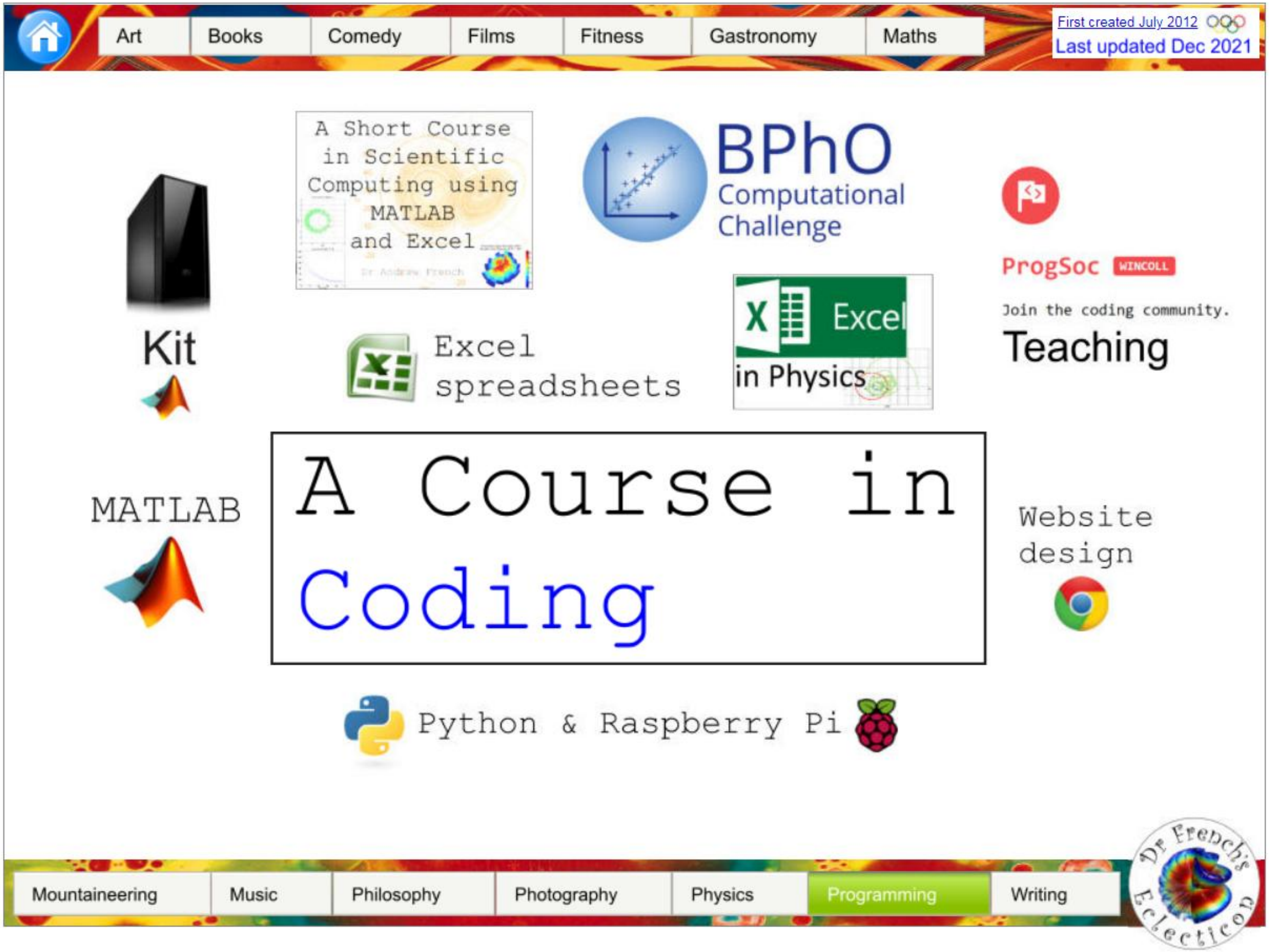

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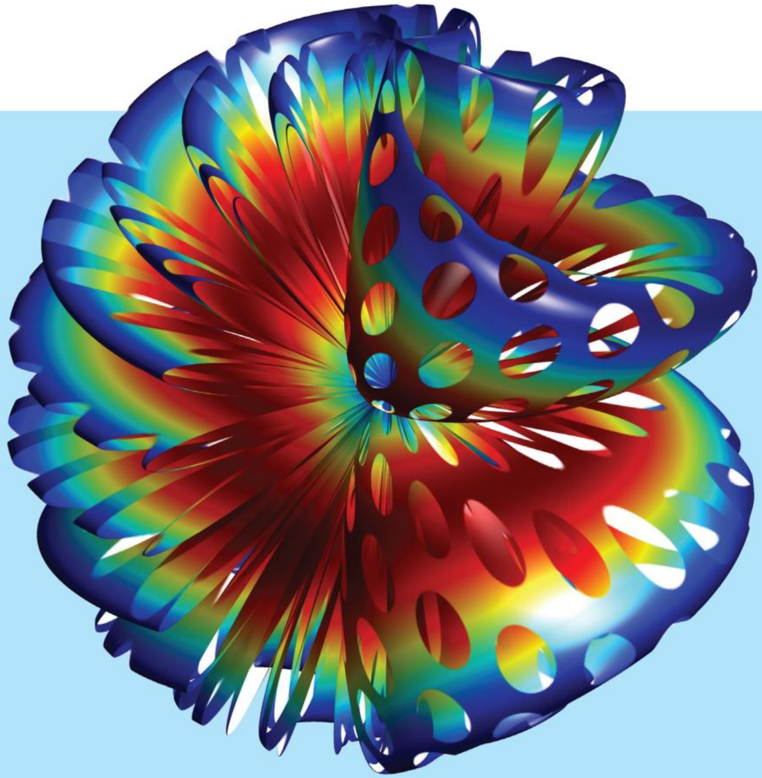

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