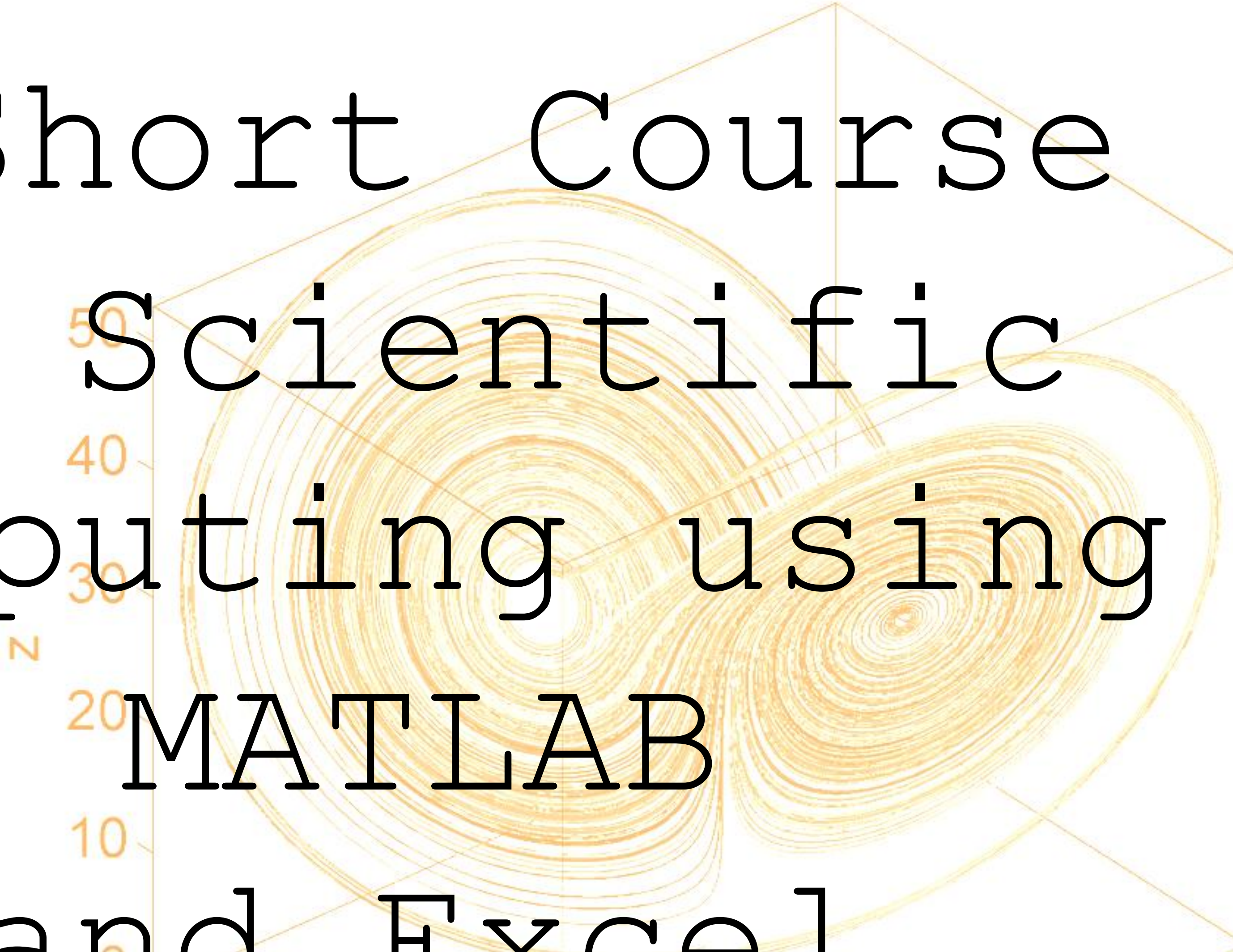
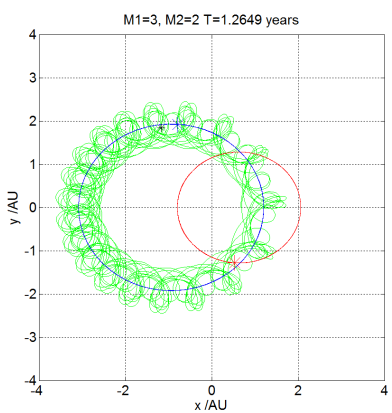
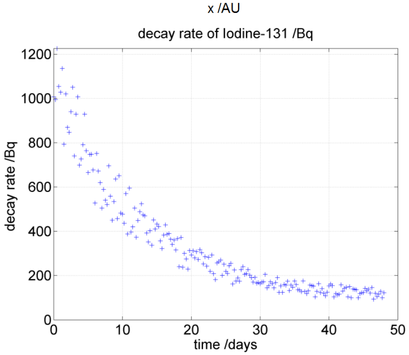
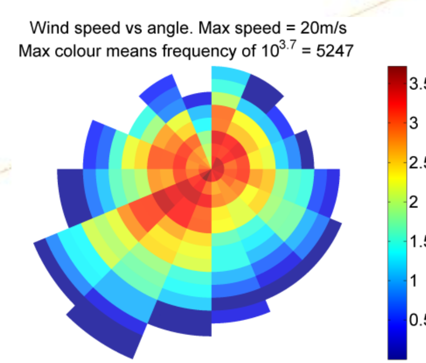
A Short Course in Scientific Computing using MATLAB and Excel







Dr Andrew French



Why are we doing this?

Think of a modern profession that *does not* involve the use of computers to create, record and store information, control machinery .... I'm not sure I can.

Experience of **data flow, data processing** and **information presentation** is a particularly vital element of scientific craft. But at the moment your Science experience is probably mostly theoretical problem solving, taking notes and performing lab experiments.

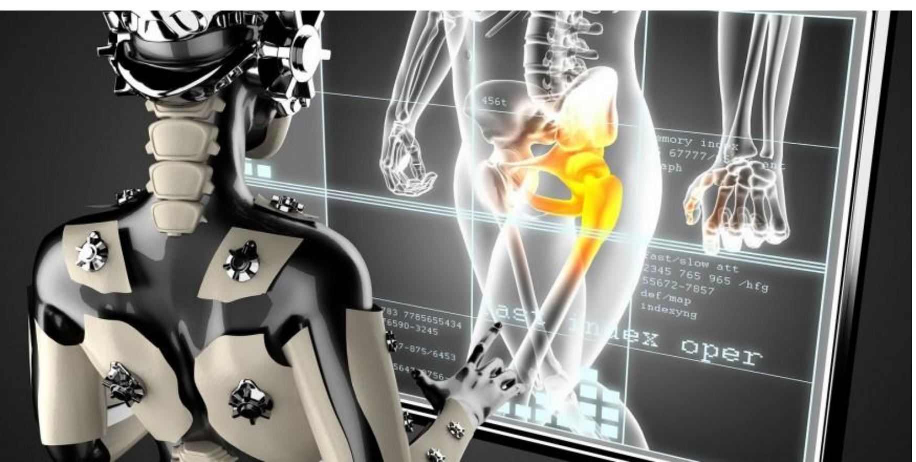
Real Scientists will spend most of their time on data flow, data processing and information presentation. **So start learning these skills and you are more likely to get a job.** The likes of Amazon, Google, Uber, Facebook will continue to 'disrupt' traditional industries. **If you don't have these skills, you will not have much to offer to the higher paid sectors of the economies of the future.**

But even if your horizon is merely "how can a get a Distinction at Pre-U and get into a top flight University?" skills in Scientific Computing are a great way to consolidate your subject, especially when you **begin to create projects and systems of your own design.**

***Making things yourself* is the BEST motivator for learning**

If most future jobs currently performed by humans will be done much more efficiently and safely by robots / artificial intelligence...

Wouldn’t you want to be the person programming this technology?



https://robohub.org/envisioning-the-future-of-robotics/

*Check out Nate Silver's excellent book of the same name

A Short Course in Scientific Computing

You are not going to become an expert in two weeks, so this is a taster. However, it is possible to experience a few things in a fairly complete way. These can be your templates for your own designs.

Project 1: The Signal and the Noise*

You're a soviet nuclear physicist sent to Chernobyl in 1986. You need to determine the presence of an isotope from its half life, but background levels are huge.... All you have is an Excel file of count rates. Your military commander demands results as soon as possible....

Project 2: The Weather with you

Investigate patterns in local measurements of temperature, humidity, pressure using our meteorological system on the roof of Science School.

Project 3: A Random Walk

Learn about loops and simple simulations using random numbers. And how gases and liquids diffuse and mix.

Project 4: The Planets

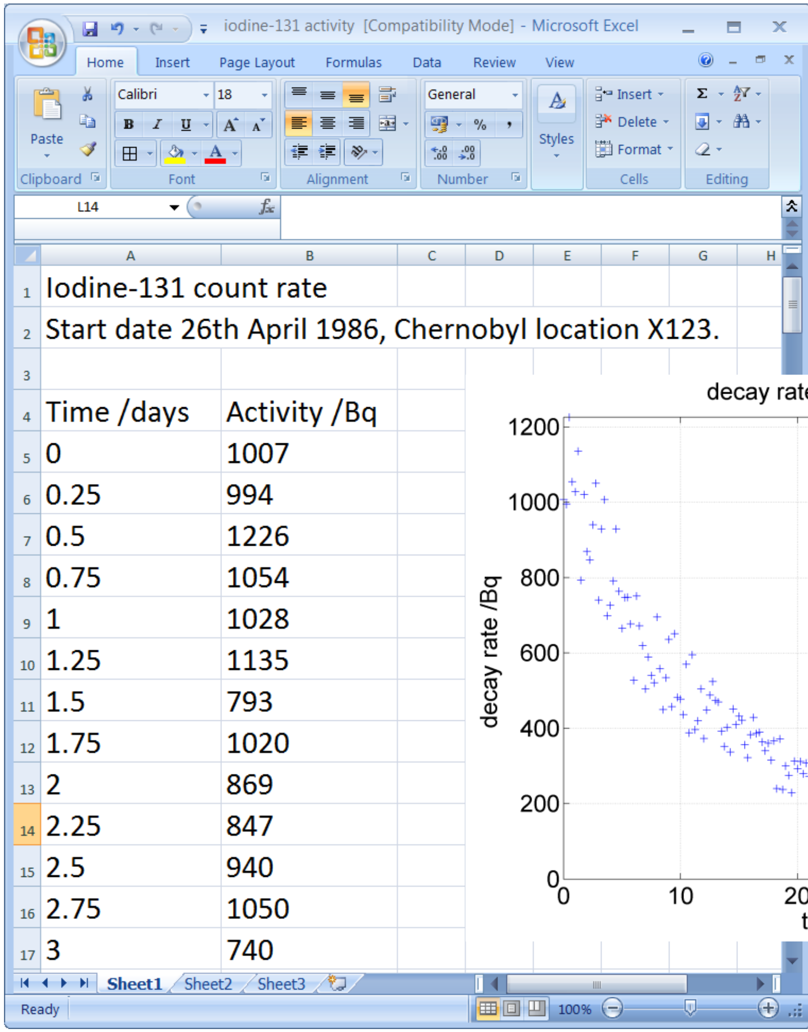
Make a gravity simulation based upon one the solar system planets and its moons (or the Solar System itself). Learn how to make animated plots.

Project 5: Visions of Chaos

Explore iconic mathematically generated imagery, that looks just like organic and geological forms . May's Bifurcations, The Mandlebrot Set and the Lorenz attractor.

Project 1: The Signal and the Noise

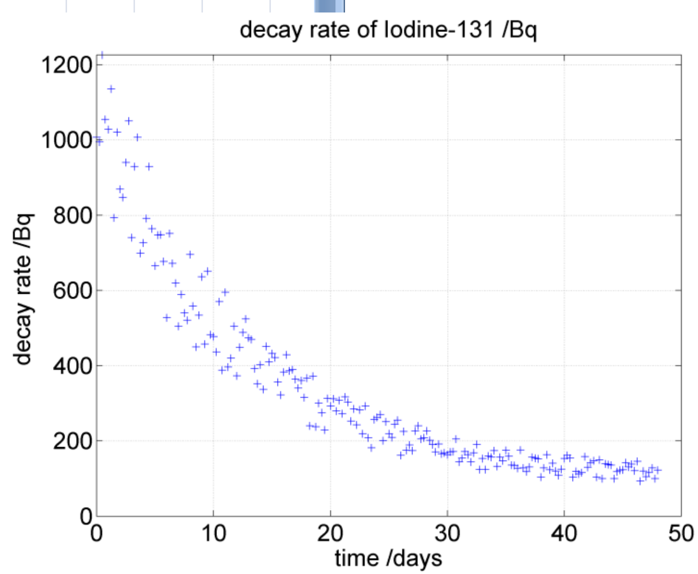
You are a soviet nuclear physicist sent to help with the Chernobyl disaster in 1986. You need to determine the presence of an isotope from its half life, but background levels are huge.... All you have is a text file of count rates. Your military commander demands results as soon as possible.

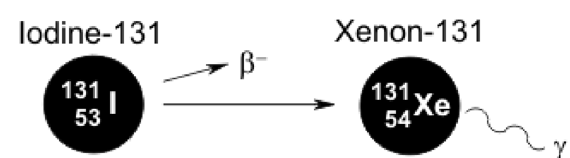


Iodine-131 count rate

Start date 26th April 1986, Chernobyl location X123.

Time /days	Activity /Bq
0	1007
0.25	994
0.5	1226
0.75	1054
1	1028
1.25	1135
1.5	793
1.75	1020
2	869
2.25	847
2.5	940
2.75	1050
3	740





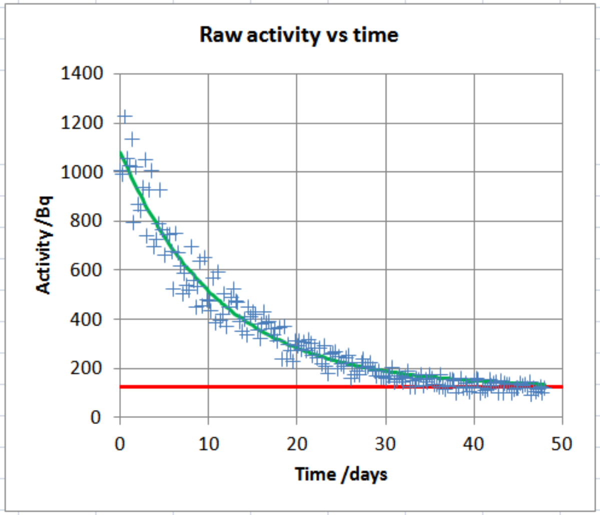
PERFORM ANALYSIS IN EXCEL

Iodine-131 count rate

Start date 26th April 1986, Chernobyl location X123.

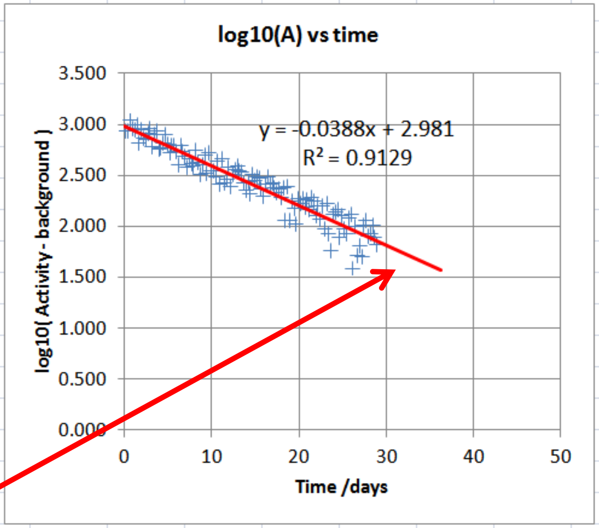
Time /days	Activity /Bq	A = Activity - Background	log10(A)	Model Activity /Bq
0	1007	884	2.946	1080.2
0.25	994	871	2.940	1059.1
0.5	1226	1103	3.043	1038.4
0.75	1054	931	2.969	1018.2
1	1028	905	2.957	998.39
1.25	1135	1012	3.005	979.05
1.5	793	670	2.826	960.14
1.75	1020	897	2.953	941.65
2	869	746	2.873	923.57
2.25	847	724	2.860	905.89
2.5	940	817	2.912	888.6
2.75	1050	927	2.967	871.69
3	740	617	2.790	855.15
3.25	929	806	2.906	838.98
3.5	1007	884	2.946	823.16
3.75	699	576	2.760	807.7
4	726	603	2.780	792.58
4.25	791	668	2.825	777.79
4.5	929	806	2.906	763.32
4.75	764	641	2.807	749.18
5	665	542	2.734	735.35
5.25	747	624	2.795	721.83
5.5	748	625	2.796	708.6
5.75	677	554	2.744	695.66
6	527	404	2.606	683.02
6.25	751	628	2.798	670.65
6.5	672	549	2.740	658.55
6.75	619	496	2.695	646.72
7	505	382	2.582	635.15
7.25	589	466	2.668	623.84
7.5	540	417	2.620	612.78
7.75	521	398		
8	695	572		
8.25	558	435		
8.5	450	327		
8.75	534	411		

NOTE IGNORE DATA AFTER 28.75 DAYS FOR BEST FIT



m	-0.039
c	2.981

Half life /days	7.759
A0 /Bq	957.2



Note this estimate is slightly different to the 100Bq used in the subsequent MATLAB analysis

Estimate background level /Bq

123

Time /days	Activity /Bq
0	123
50	123

$$A = \frac{A_0}{2^{t/t_{1/2}}}$$

$$\log_{10} A = \log_{10} A_0 - \log_{10}\left(2^{t/t_{1/2}}\right)$$

$$\log_{10} A = \log_{10} A_0 - \frac{t}{t_{1/2}} \log_{10} 2$$

$$y = \log_{10} A$$

$$x = t$$

$$y = mx + c$$

$$m = -\frac{\log_{10} 2}{t_{1/2}} \qquad c = \log_{10} A_0$$

$$t_{1/2} = -\frac{\log_{10} 2}{m} \qquad A_0 = 10^c$$

Data flow
Data processing and
Information
Presentation

is often best achieved by ***writing code*****.**

In other words
a **text file** which
is interpreted
by a *programming language* like
MATLAB or Python

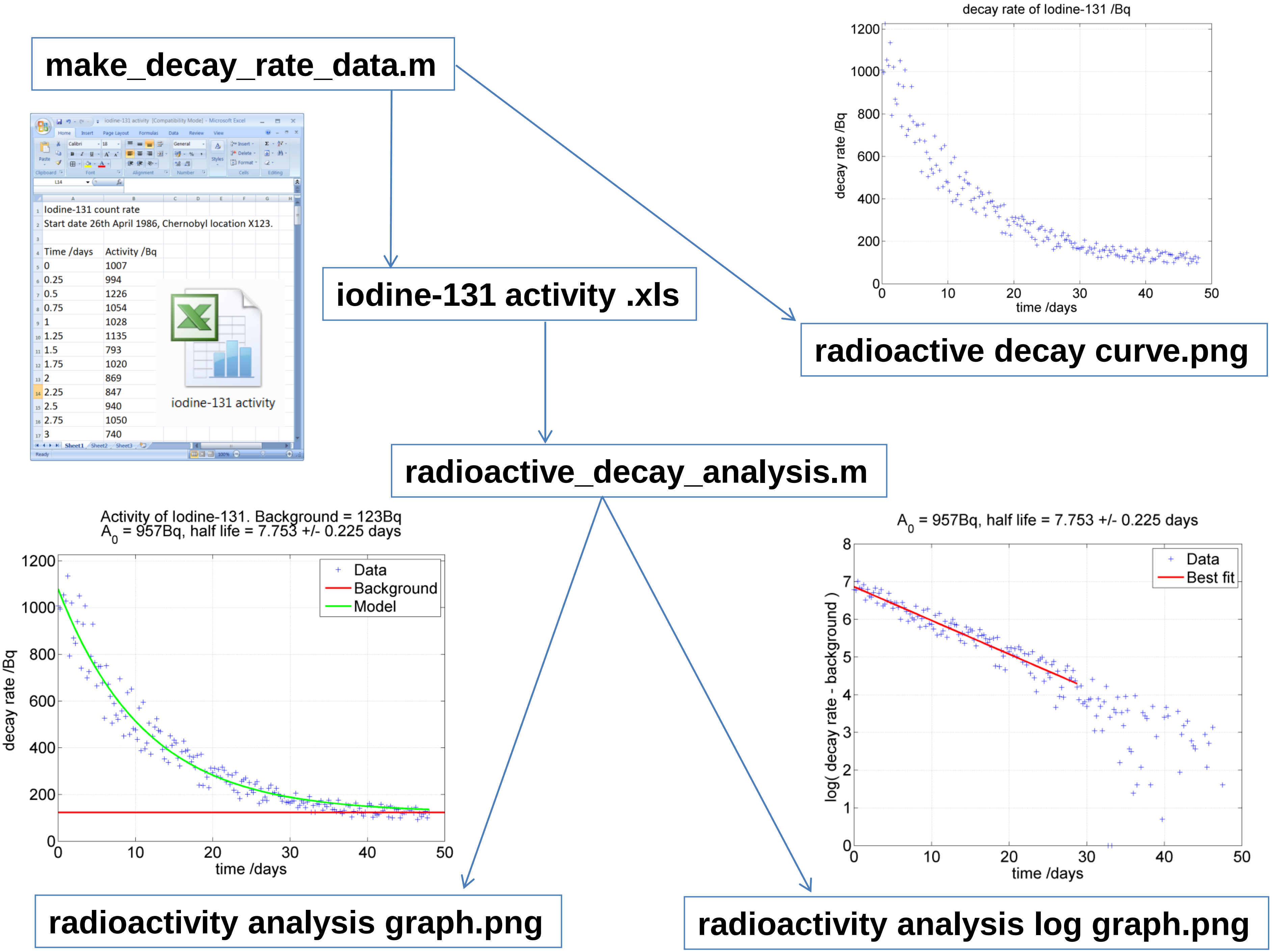
E:\Programming\A Course in Coding\2. MATLAB\Short Scientific Computing Course\1. The Signal and the Noise\radioactive_decay_analysis.r

File Edit Text Go Cell Tools Debug Desktop Window Help

Stack: Base

radioactive_decay_analysis.m

```
%radioactive_decay_analysis
% Analysis of Iodine-131 decay rate vs time data.
%
% LAST UPDATED by Andy French June 2019

function radioactive_decay_analysis

%Estimated background rate /Bq
B = 100;

%Fontsize for graphs
fsize = 18;

%

%Ingest Excel file of activity vs time
[num,txt,raw] = xlsread( 'iodine-131 activity.xls' );

%Extract vectors for time /days and activity /Bq
t = num(:,1); A = num(:,2);

%Plot activity vs time
fig1 = figure('color',[1 1 1],'name','radioactive decay curve');
plot(t,A,'+');
xlabel('time /days','fontsize',fsize);
ylabel('decay rate /Bq ','fontsize',fsize);
set(gca,'fontsize',fsize);
grid on; ylim([0,max(A)]);

%Overlay background level
xlimits = get( gca, 'xlim' ); hold on; plot( xlimits,[B,B],'r-' );

```


make_decay_rate_data.m
iodine-131 activity .xls
radioactive decay curve.png
radioactive_decay_analysis.m
radioactivity analysis graph.png
radioactivity analysis log graph.png
Iodine-131 count rate
Start date 26th April 1986, Chernobyl location X123.
Time /days
Activity /Bq
0 1007
0.25 994
0.5 1226
0.75 1054
1 1028
1.25 1135
1.5 793
1.75 1020
2 869
2.25 847
2.5 940
2.75 1050
3 740
iodine-131 activity
decay rate of Iodine-131 /Bq
decay rate /Bq
time /days
Activity of Iodine-131. Background = 123Bq
A0 = 957Bq, half life = 7.753 +/- 0.225 days
Data
Background
Model
decay rate /Bq
time /days
A0 = 957Bq, half life = 7.753 +/- 0.225 days
Data
Best fit
log(decay rate - background)
time /days

Project 2: The Weather with you

Investigate patterns in local measurements of temperature, humidity, pressure using our meteorological system on the roof of Science School.

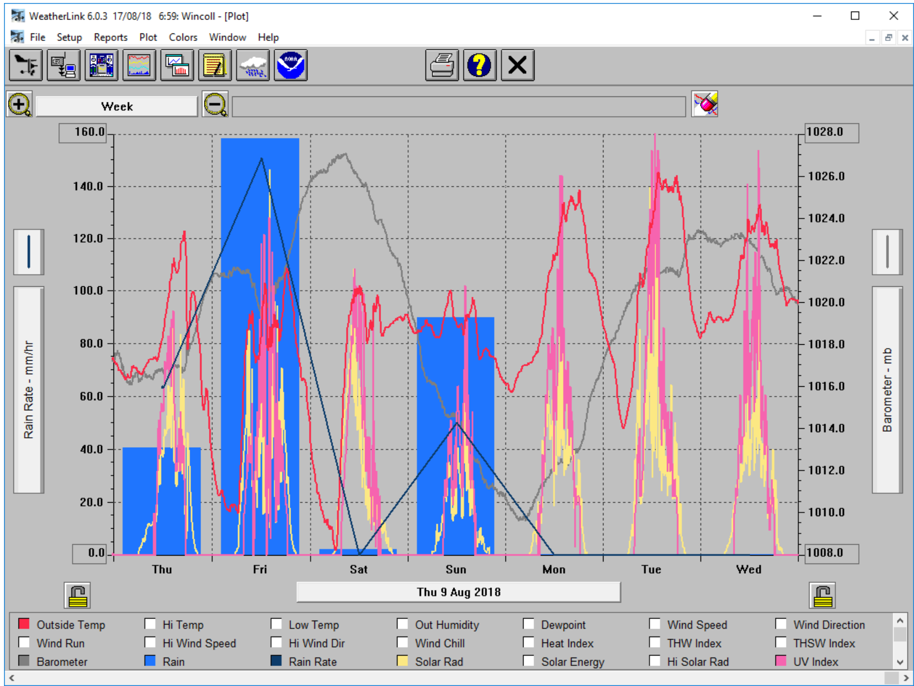
- Run **plot_met_data.m** and generate graphs from the files in the Met data directory Can you spot any trends?

- Load a met_data file into **MATLAB**. Investigate its structure.

- Adapt code from **plot_met_data.m** to make your own graphs.

- How about temperature vs pressure, or temperature vs humidity. Are there any correlations?

- Could you work out the *rate* of change of temperature, time etc? (And plot this).



Geostationary

EUMETCAST

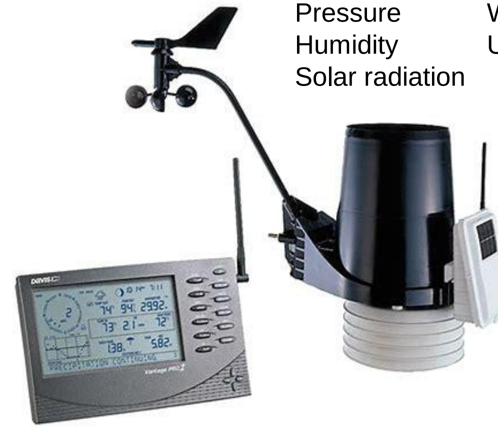
Earth Observation data

e.g. full hemisphere weather every **15 minutes at 1 pixel per km^2 resolution!**

Astronomical telescope

(+ spectrometer, digital camera etc)

Star track?



Temperature
Pressure
Humidity
Solar radiation

Wind speed
Wind direction
UV index

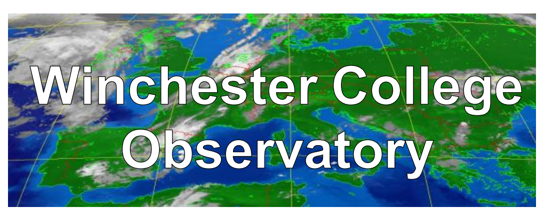
Davis Vantage Pro **automated weather station**

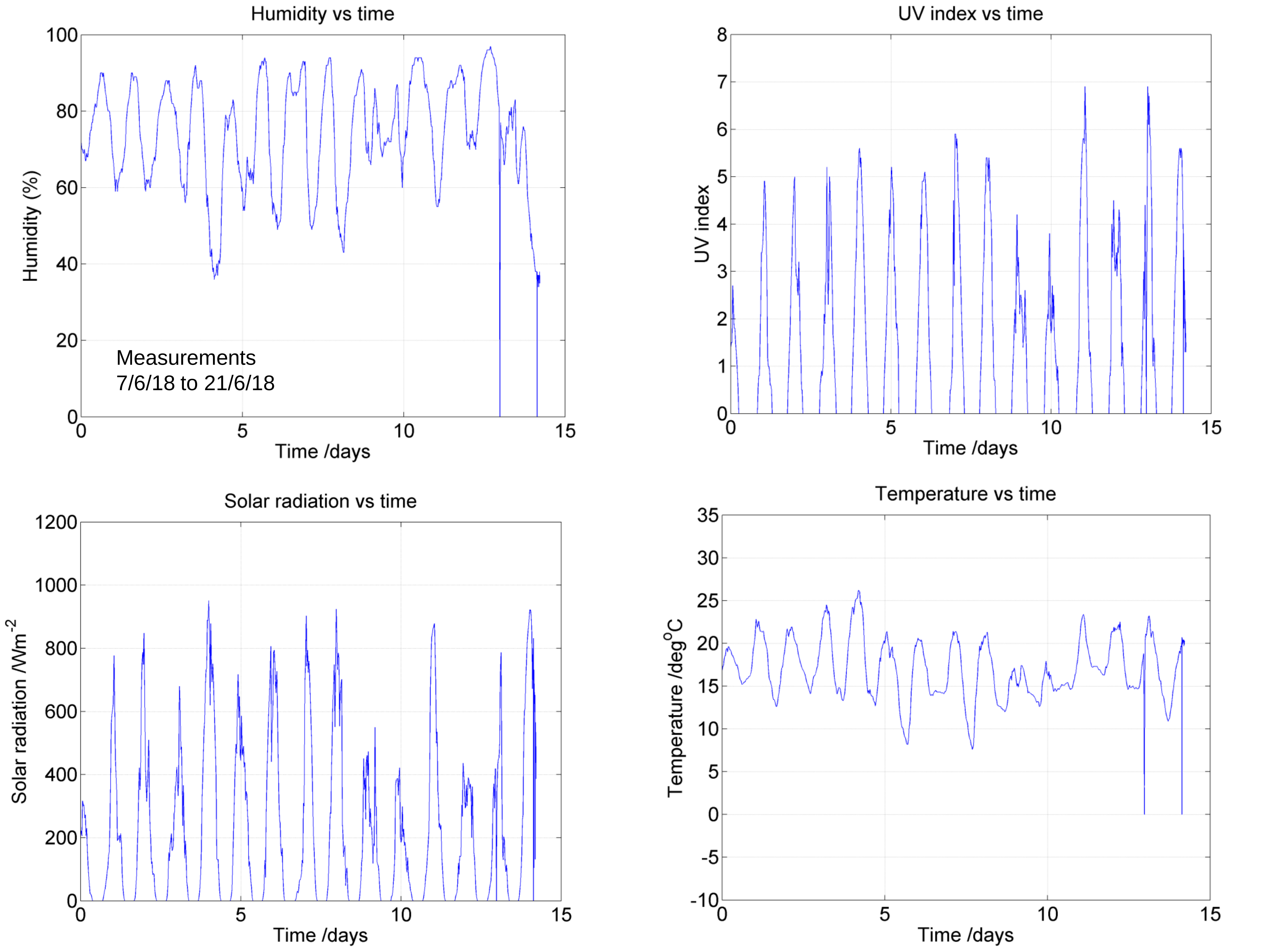
Dartcom PC based receiver system running software to ingest and process each data stream simultaneously

Processed data (e.g. temperature variation vs time Excel sheet, indexed images for plotting cloud cover over UK vs time etc)

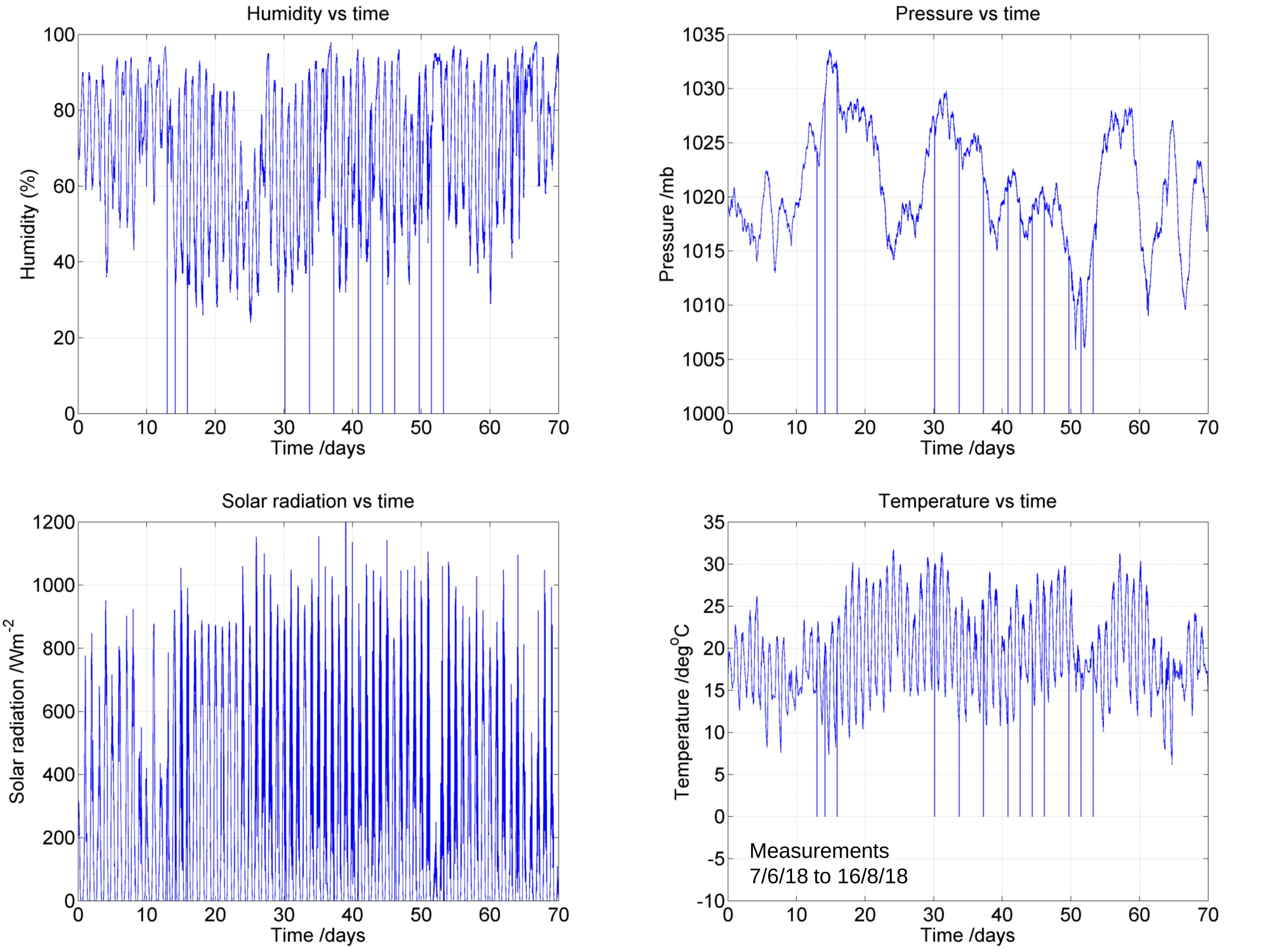
USB sticks/hard drives (possibly internal network) to Z drive / Firefly for general Wincoll access

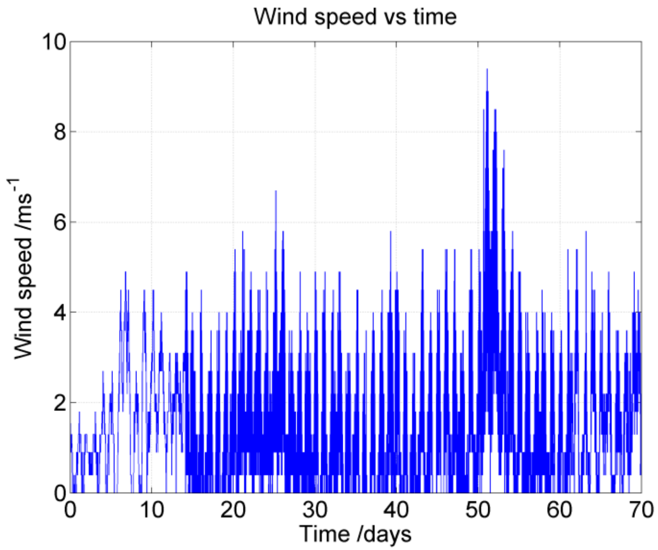
Workstation console in room beneath observatory

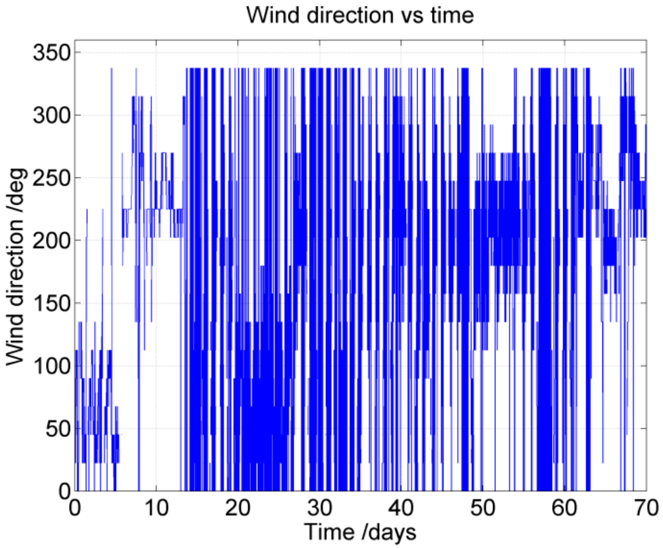




Humidity vs time
Humidity (%)
Time /days
Measurements
7/6/18 to 21/6/18
UV index vs time
UV index
Time /days
Solar radiation vs time
Solar radiation /Wm^{-2}
Time /days
Temperature vs time
Temperature /deg^{o}C
Time /days

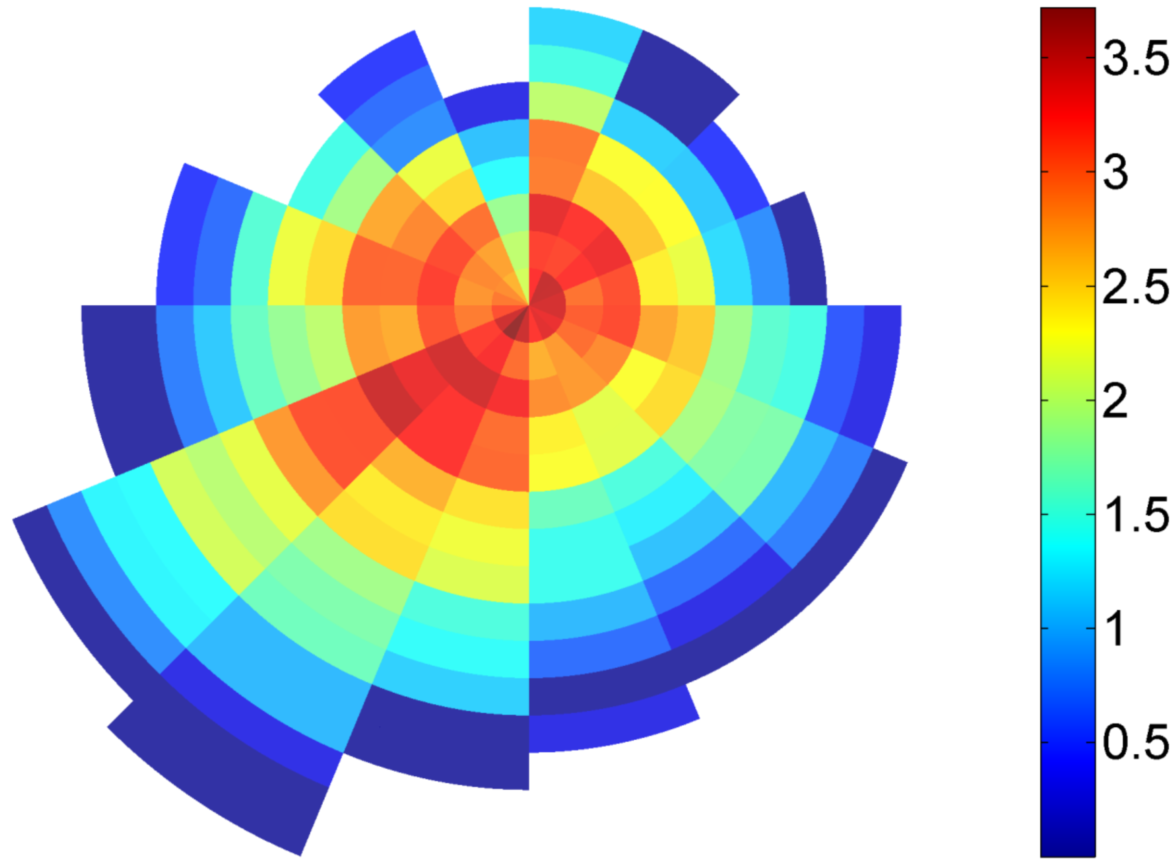

Humidity vs time
Humidity (%)
Time /days
Pressure vs time
Pressure /mb
Time /days
Solar radiation vs time
Solar radiation /Wm-2
Time /days
Temperature vs time
Temperature /degoC
Time /days
Measurements
7/6/18 to 16/8/18





Measurements
7/6/18 to 16/8/18

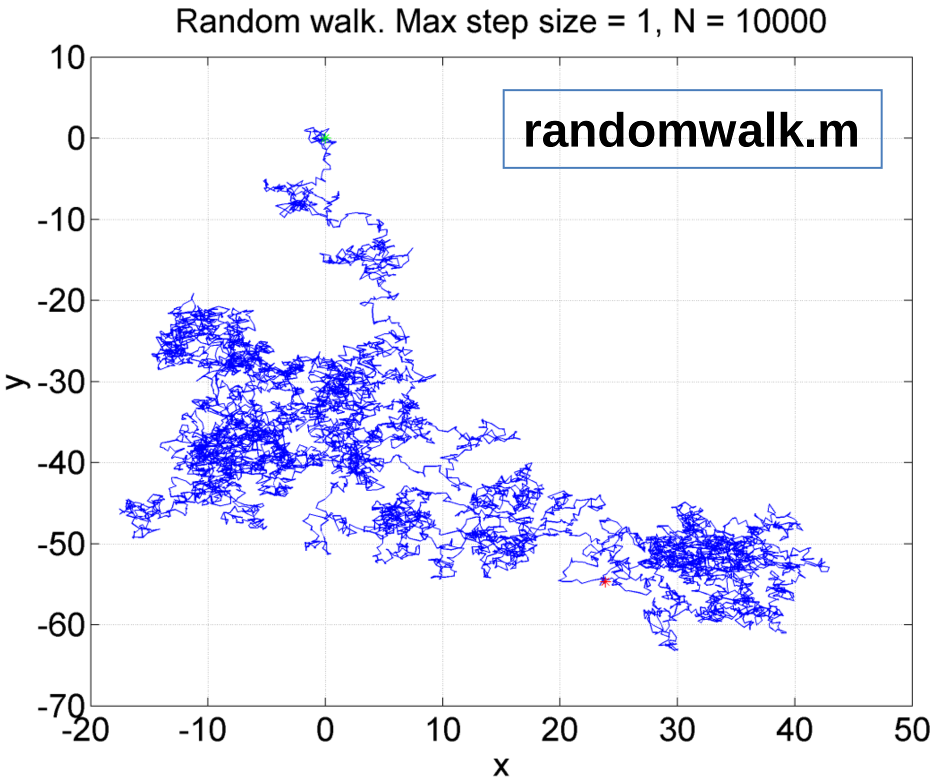
Wind speed vs angle. Max speed = 20m/s
Max colour means frequency of $10^{3.7}$ = 5247



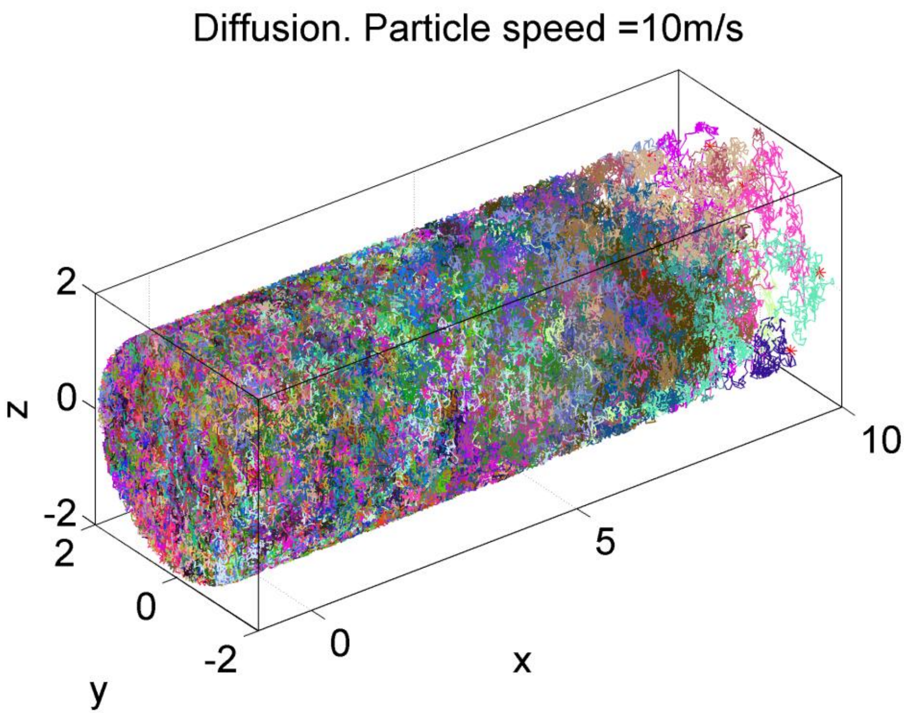
This 'wind rose' displays the frequency of wind measurements in circular sectors. Angle corresponds to 16 wind direction sectors (e.g. N, NNE etc) and range corresponds to wind speed. The colour scale is the *logarithm* of frequency.

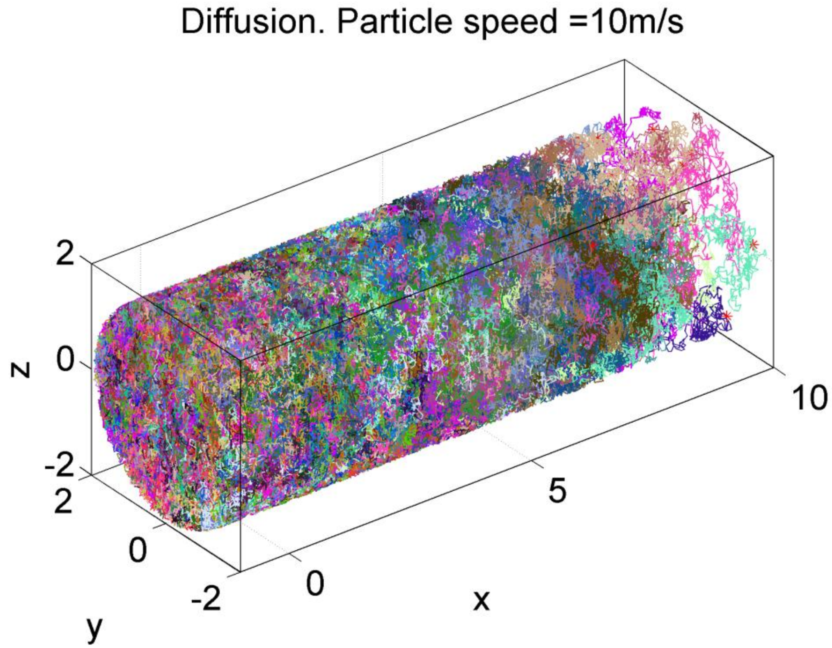
Project 3: A Random Walk

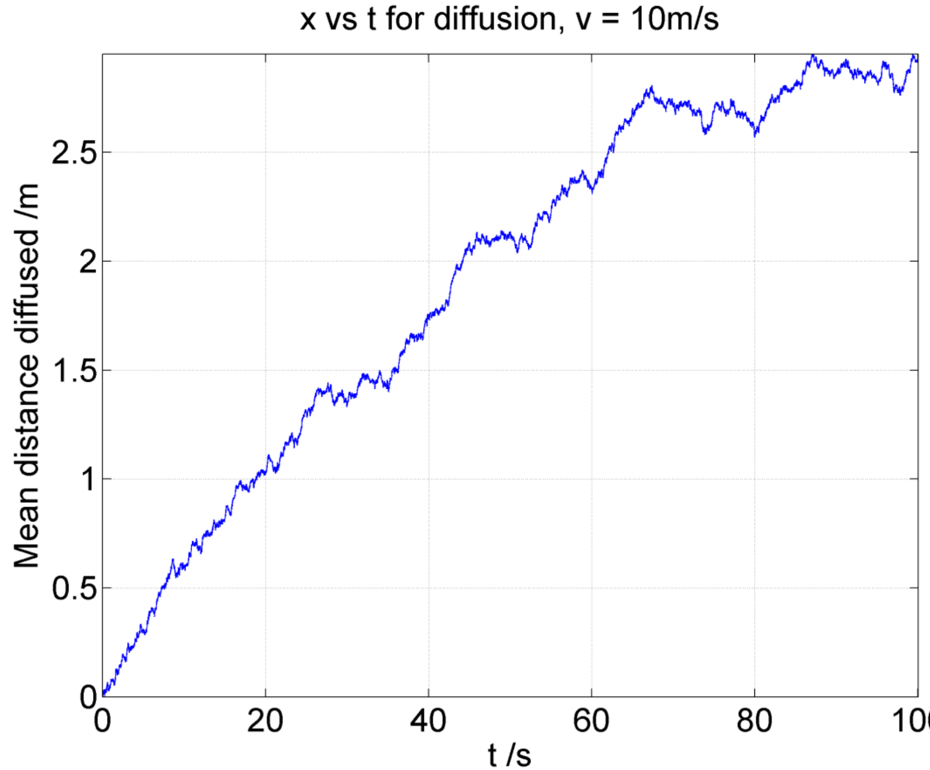
Learn about loops and simple simulations using random numbers. And how gases and liquids diffuse and mix.

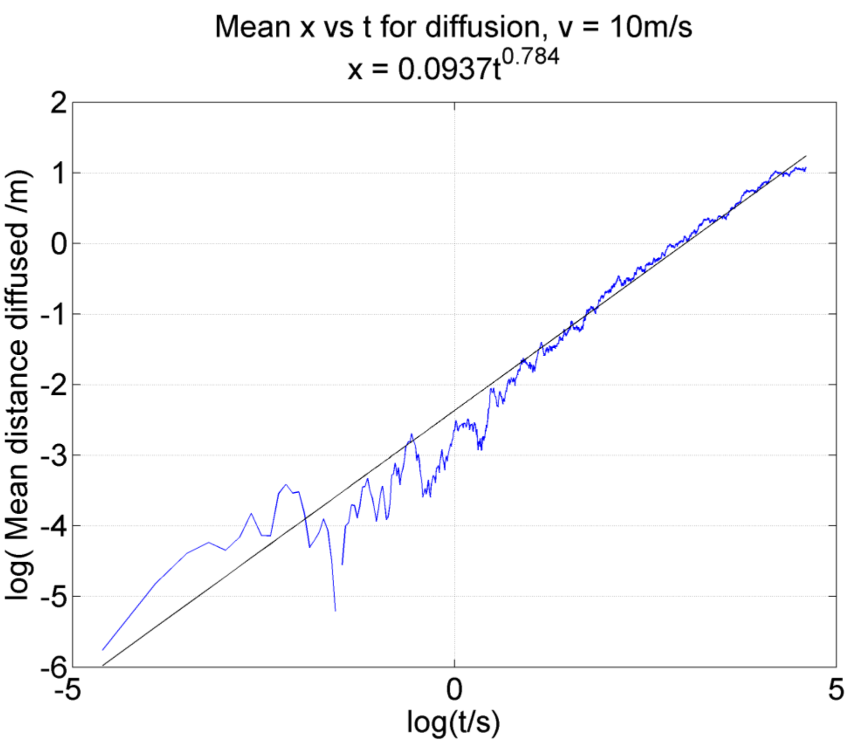


diffusion.m



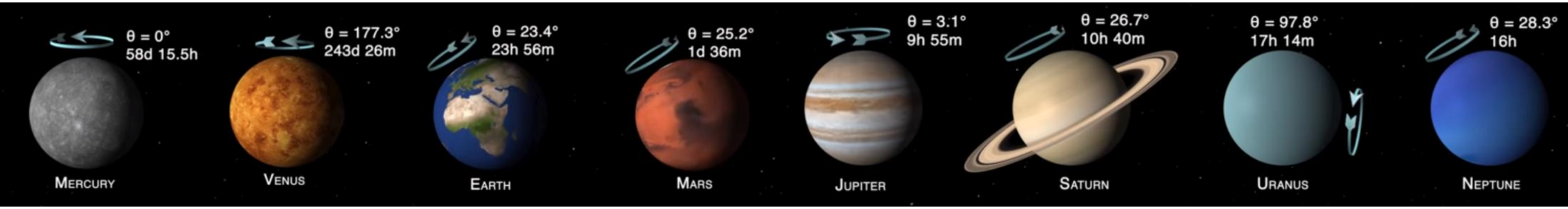

Diffusion. Particle speed =10m/s
z
x
y
2
0
-2
2
0
-2
0
5
10

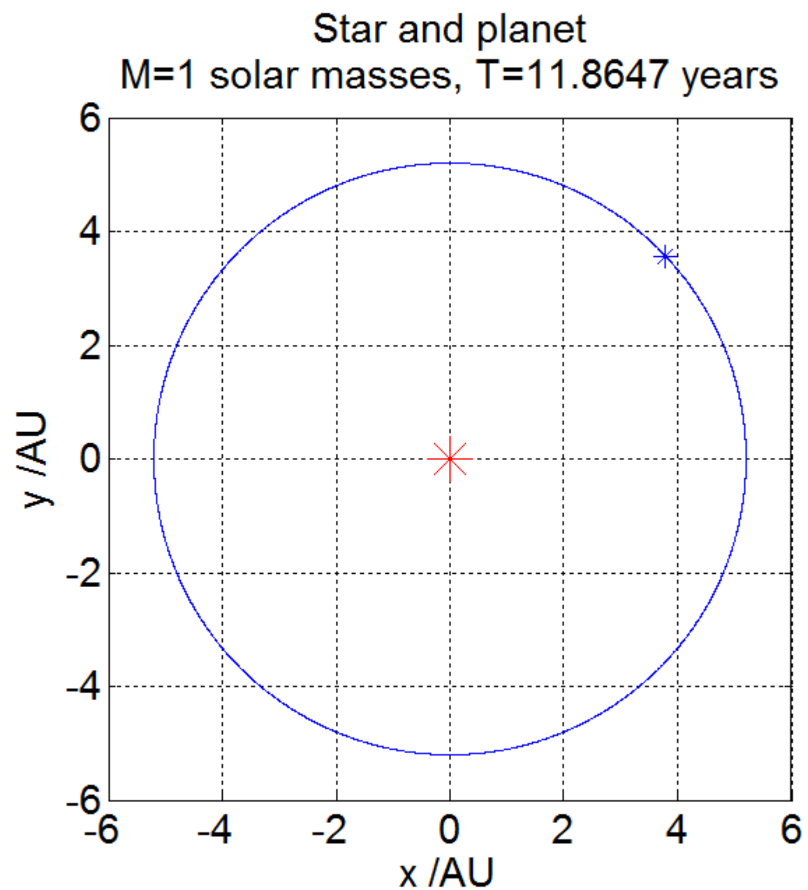

x vs t for diffusion, v = 10m/s
Mean distance diffused /m
t /s
0
0.5
1
1.5
2
2.5
0
20
40
60
80
100

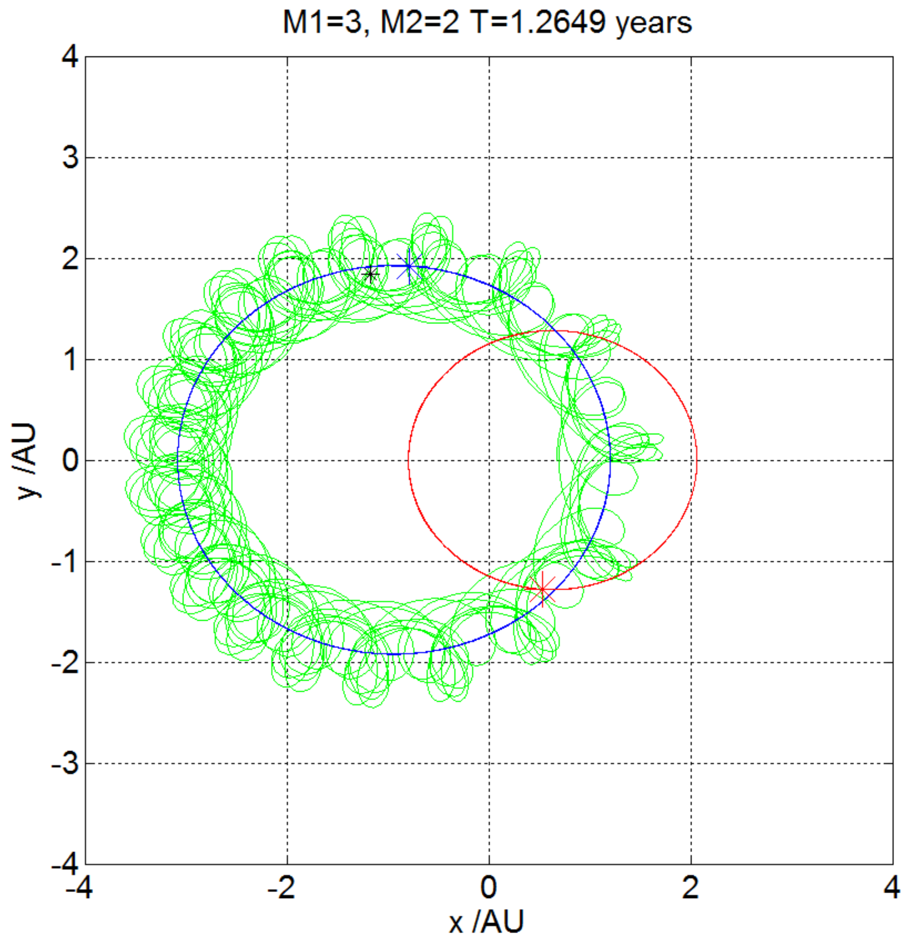

Mean x vs t for diffusion, v = 10m/s
x = 0.0937t^0.784
log(Mean distance diffused /m)
log(t/s)
2
1
0
-1
-2
-3
-4
-5
-6
-5
0
5

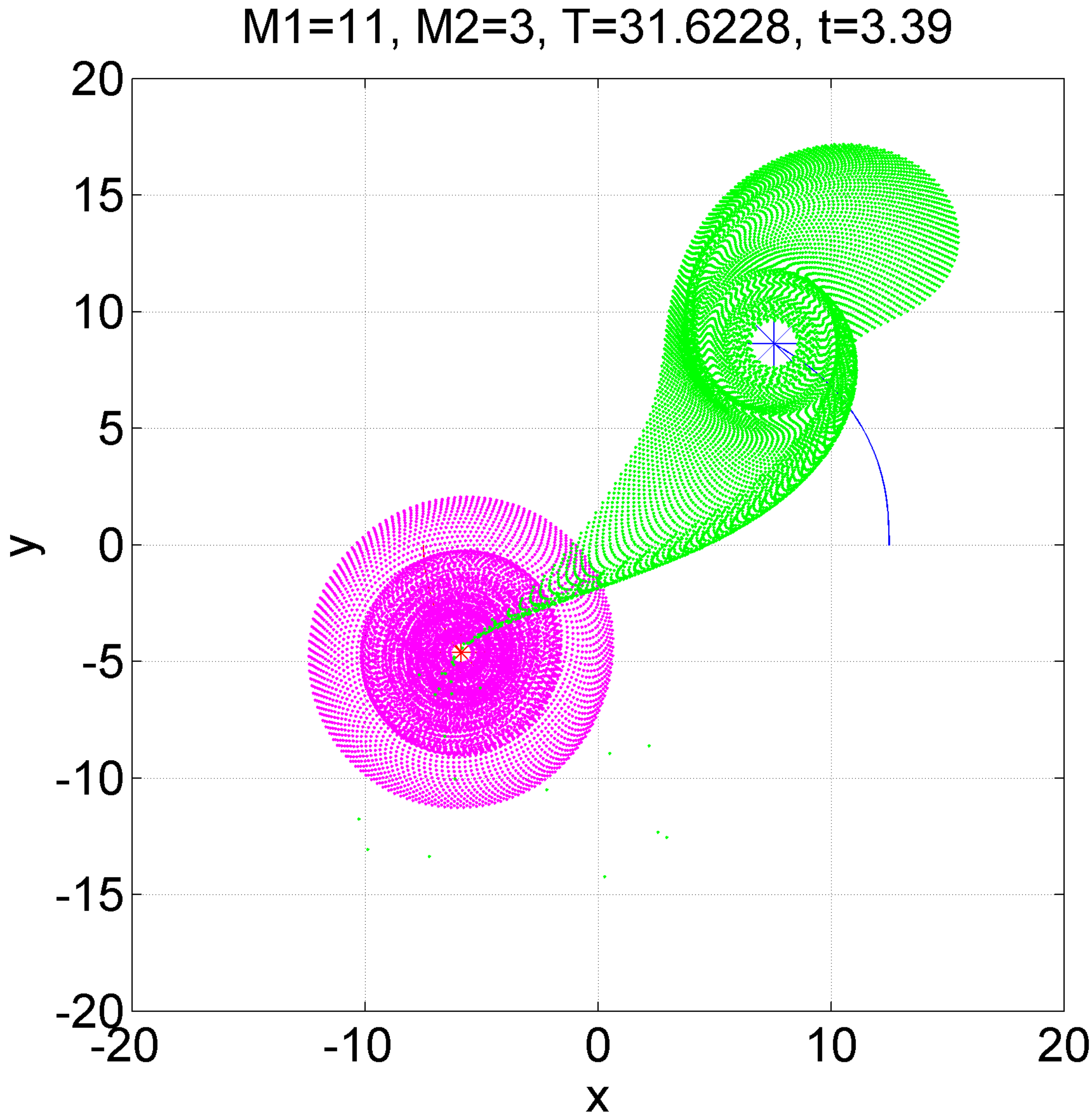
Project 4: The Planets

Make a gravity simulation based upon one the solar system planets and its moons (or the Solar System itself). Learn how to make animated plots.









M1=11, M2=3, T=31.6228, t=3.39
20
15
10
5
0
-5
-10
-15
-20
-20
-10
0
10
20
x
y

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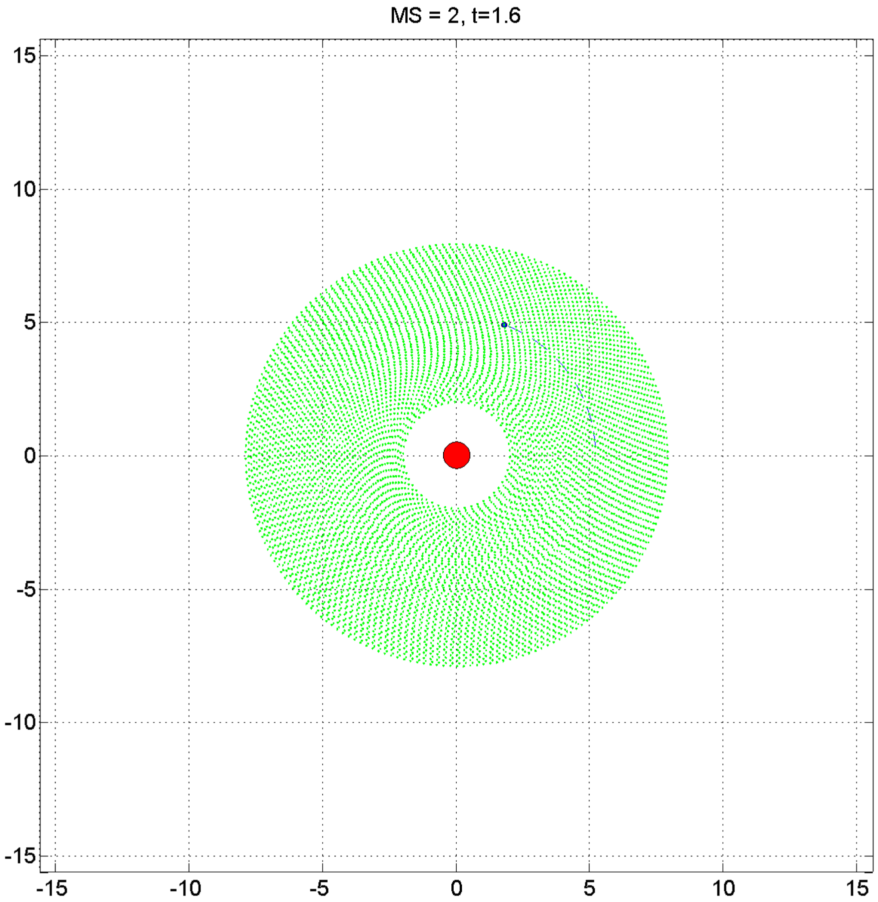
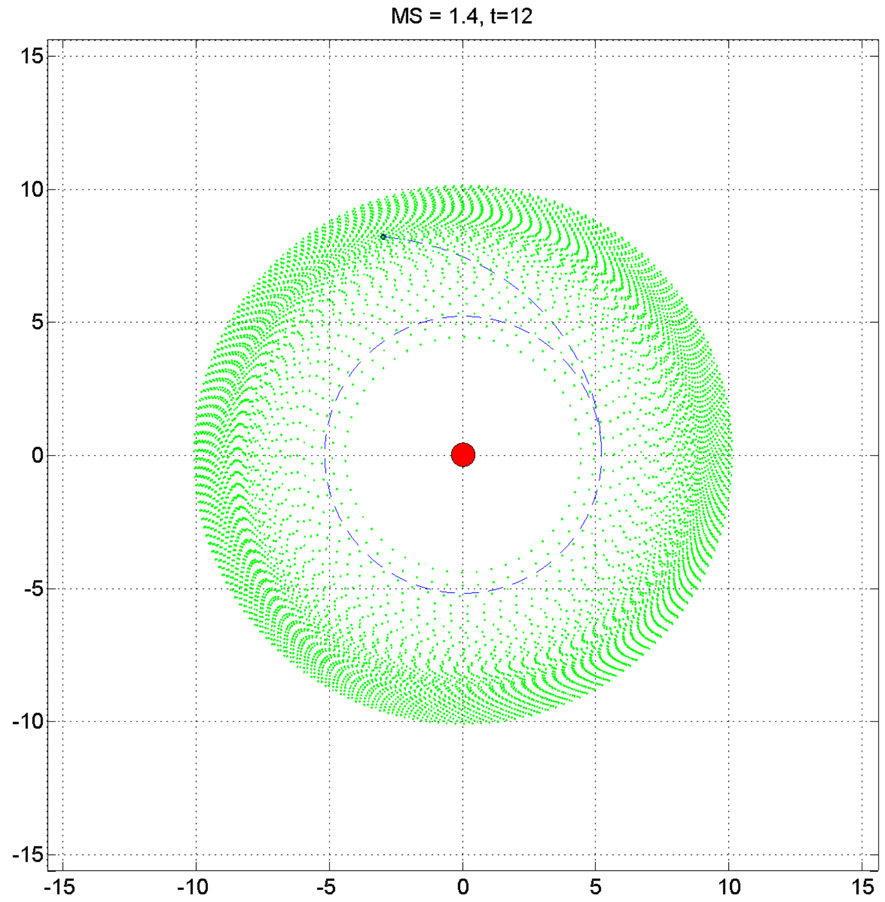
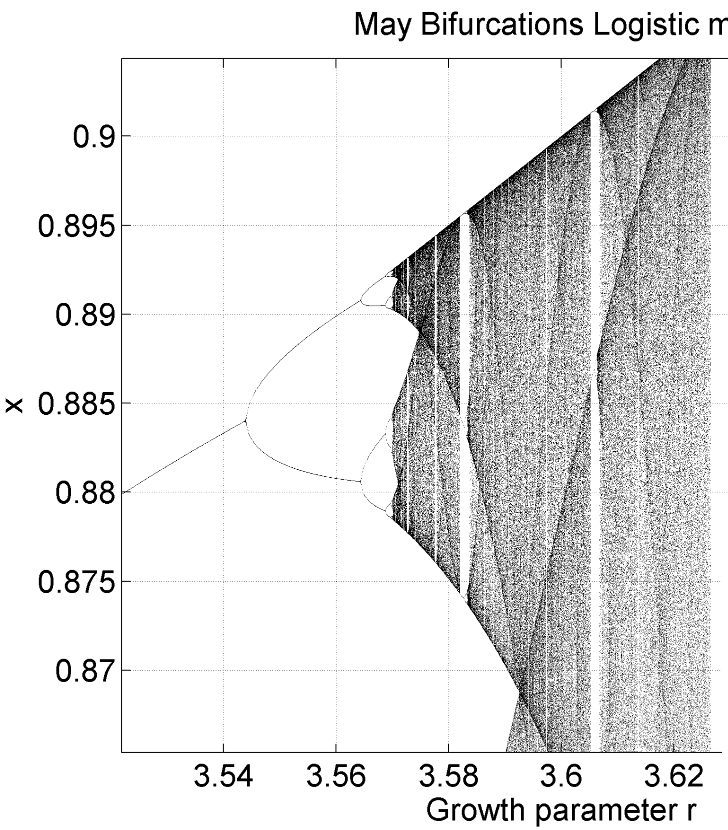
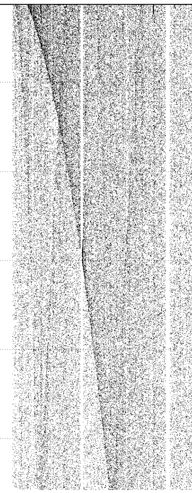


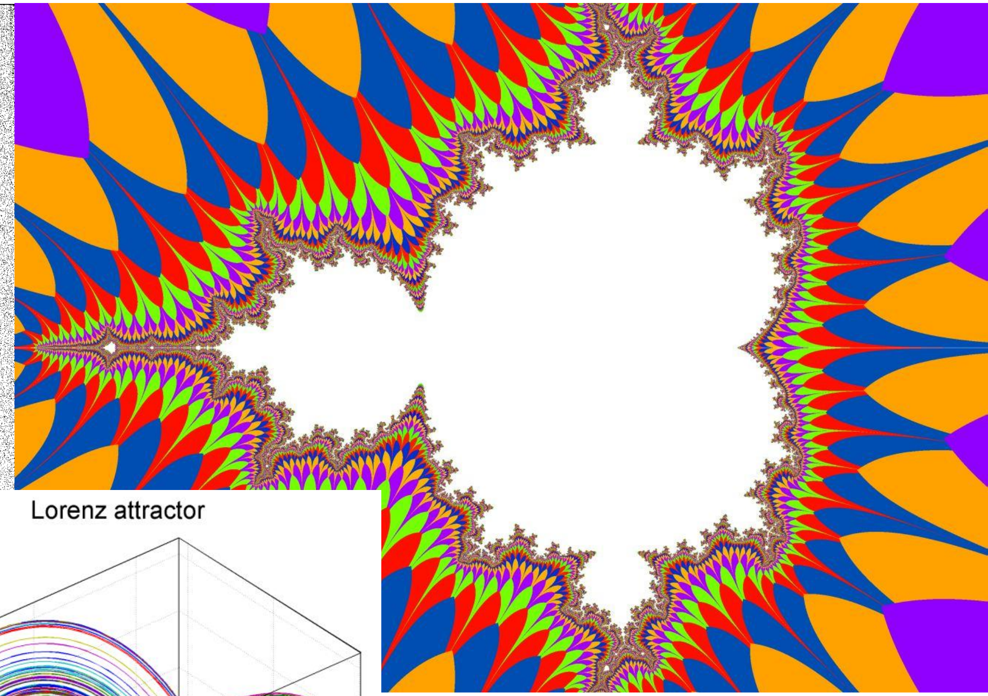


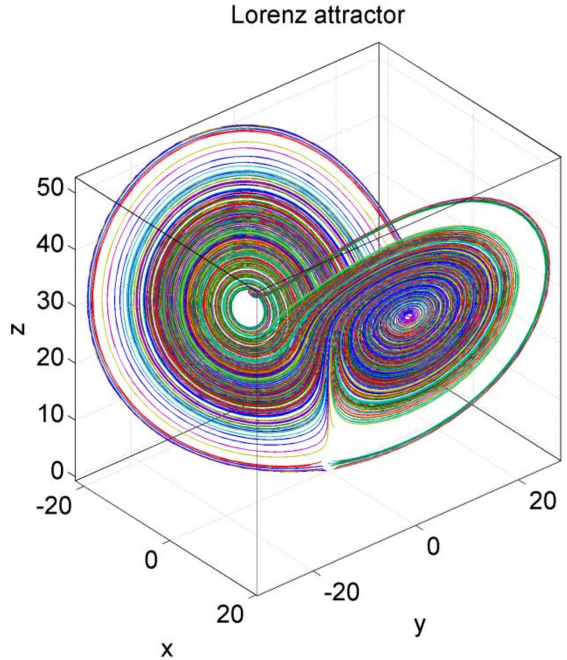
Project 5: Visions of Chaos

Explore iconic mathematically generated imagery. May's Bifurcations, The Mandlebrot Set and the Lorenz attractor.









The logistic map and population modelling

I published this model in 1976

Robert May
1936-

Assume an ecosystem can support a maximum number of rabbits. Let x be the fraction of this maximum at year n.

To account for **reproduction**, next year's population is proportional to the previous.

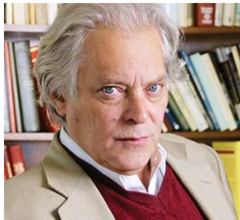
To account for **starvation**, next year's population is *also proportional* to the fraction of the maximum population as yet unfilled.

$$x_{n+1} = r x_n (1 - x_n)$$

Growth parameter

The population next year is predicted using this **iterative equation** called a **logistic map**

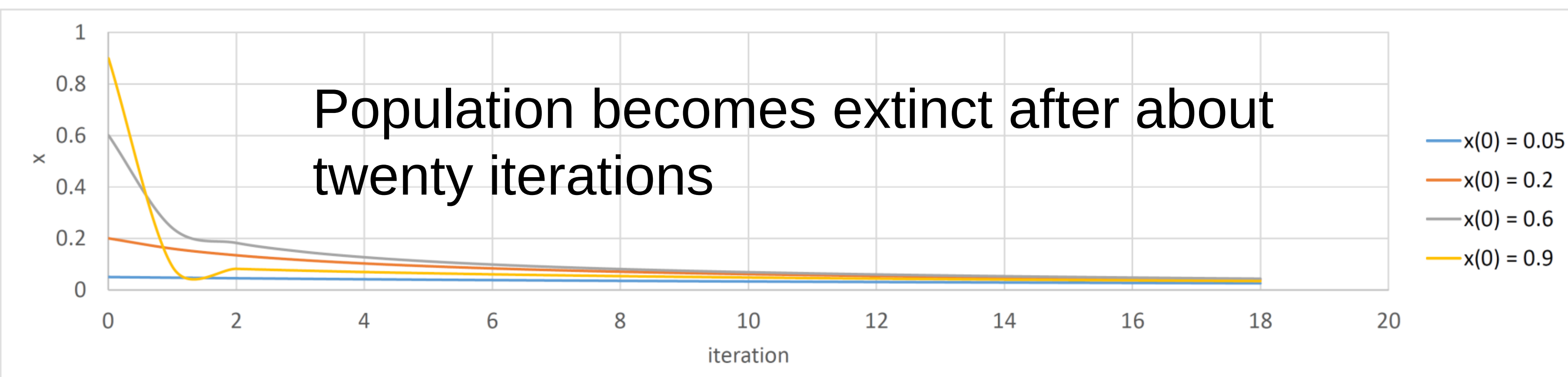
The pattern of x values with n is not always simple .....



$$r = 1 \qquad x_{n+1} = r x_n (1 - x_n)$$

iteration number n

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
x(n)	**0**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0.05	0.0475	0.045244	0.043197	0.041331	0.039623	0.038053	0.036605	0.035265	0.034021	0.032864	0.031784	0.030773	0.029826	0.028937	0.028099	0.02731	0.026564	0.025858
	0.1	0.09	0.0819	0.075192	0.069538	0.064703	0.060516	0.056854	0.053622	0.050746	0.048171	0.045851	0.043749	0.041835	0.040084	0.038478	0.036997	0.035628	0.034359
	0.15	0.1275	0.111244	0.098869	0.089094	0.081156	0.07457	0.069009	0.064247	0.060119	0.056505	0.053312	0.05047	0.047923	0.045626	0.043544	0.041648	0.039914	0.038321
	0.2	0.16	0.1344	0.116337	0.102802	0.092234	0.083727	0.076717	0.070831	0.065814	0.061483	0.057703	0.054373	0.051417	0.048773	0.046394	0.044242	0.042284	0.040496
	0.25	0.1875	0.152344	0.129135	0.112459	0.099812	0.08985	0.081777	0.075089	0.069451	0.064627	0.060451	0.056796	0.053571	0.050701	0.04813	0.045814	0.043715	0.041804
	0.3	0.21	0.1659	0.138377	0.119229	0.105013	0.093986	0.085152	0.077901	0.071833	0.066673	0.062228	0.058355	0.05495	0.05193	0.049234	0.04681	0.044619	0.042628
	0.35	0.2275	0.175744	0.144858	0.123874	0.108529	0.096751	0.08739	0.079753	0.073392	0.068006	0.063381	0.059364	0.05584	0.052722	0.049942	0.047448	0.045197	0.043154
	0.4	0.24	0.1824	0.14913	0.12689	0.110789	0.098515	0.08881	0.080923	0.074374	0.068843	0.064103	0.059994	0.056395	0.053214	0.050383	0.047844	0.045555	0.04348
	0.45	0.2475	0.186244	0.151557	0.128587	0.112053	0.099497	0.089597	0.08157	0.074916	0.069304	0.064501	0.06034	0.056699	0.053485	0.050624	0.048061	0.045751	0.043658
	0.5	0.25	0.1875	0.152344	0.129135	0.112459	0.099812	0.08985	0.081777	0.075089	0.069451	0.064627	0.060451	0.056796	0.053571	0.050701	0.04813	0.045814	0.043715
	0.55	0.2475	0.186244	0.151557	0.128587	0.112053	0.099497	0.089597	0.08157	0.074916	0.069304	0.064501	0.06034	0.056699	0.053485	0.050624	0.048061	0.045751	0.043658
	0.6	0.24	0.1824	0.14913	0.12689	0.110789	0.098515	0.08881	0.080923	0.074374	0.068843	0.064103	0.059994	0.056395	0.053214	0.050383	0.047844	0.045555	0.04348
	0.65	0.2275	0.175744	0.144858	0.123874	0.108529	0.096751	0.08739	0.079753	0.073392	0.068006	0.063381	0.059364	0.05584	0.052722	0.049942	0.047448	0.045197	0.043154
	0.7	0.21	0.1659	0.138377	0.119229	0.105013	0.093986	0.085152	0.077901	0.071833	0.066673	0.062228	0.058355	0.05495	0.05193	0.049234	0.04681	0.044619	0.042628
	0.75	0.1875	0.152344	0.129135	0.112459	0.099812	0.08985	0.081777	0.075089	0.069451	0.064627	0.060451	0.056796	0.053571	0.050701	0.04813	0.045814	0.043715	0.041804
	0.8	0.16	0.1344	0.116337	0.102802	0.092234	0.083727	0.076717	0.070831	0.065814	0.061483	0.057703	0.054373	0.051417	0.048773	0.046394	0.044242	0.042284	0.040496
	0.85	0.1275	0.111244	0.098869	0.089094	0.081156	0.07457	0.069009	0.064247	0.060119	0.056505	0.053312	0.05047	0.047923	0.045626	0.043544	0.041648	0.039914	0.038321
	0.9	0.09	0.0819	0.075192	0.069538	0.064703	0.060516	0.056854	0.053622	0.050746	0.048171	0.045851	0.043749	0.041835	0.040084	0.038478	0.036997	0.035628	0.034359
	0.95	0.0475	0.045244	0.043197	0.041331	0.039623	0.038053	0.036605	0.035265	0.034021	0.032864	0.031784	0.030773	0.029826	0.028937	0.028099	0.02731	0.026564	0.025858
	1	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16

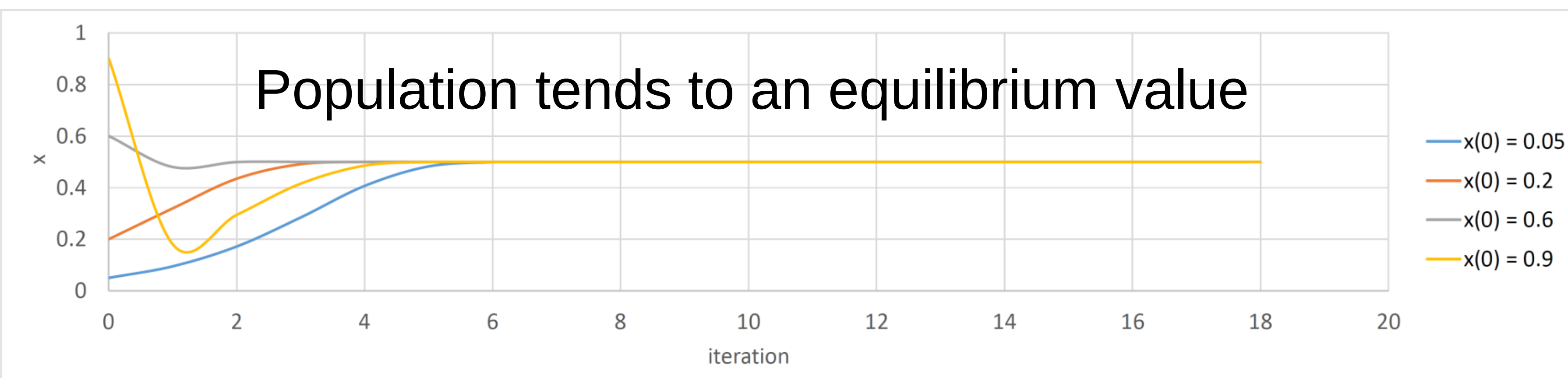


$$r = 2 \qquad x_{n+1} = r x_n (1 - x_n)$$

iteration number n

x(n)

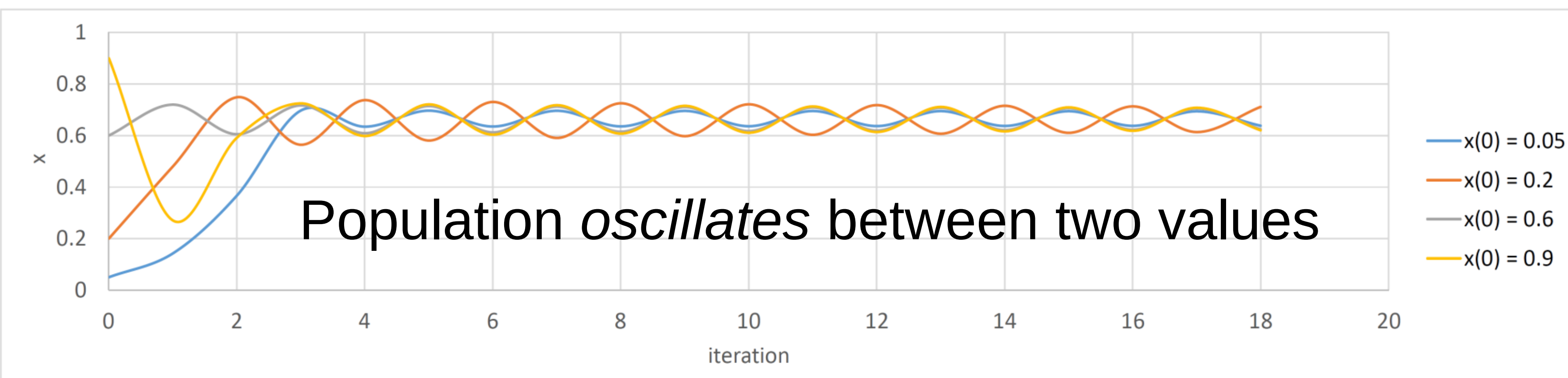
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.05	0.095	0.17195	0.284766	0.407349	0.482832	0.49941	0.499999	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.1	0.18	0.2952	0.416114	0.485926	0.499604	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.15	0.255	0.37995	0.471176	0.498338	0.499994	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.2	0.32	0.4352	0.491602	0.499859	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.25	0.375	0.46875	0.498047	0.499992	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.3	0.42	0.4872	0.499672	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.35	0.455	0.49595	0.499967	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.4	0.48	0.4992	0.499999	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.45	0.495	0.49995	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.55	0.495	0.49995	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.6	0.48	0.4992	0.499999	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.65	0.455	0.49595	0.499967	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.7	0.42	0.4872	0.499672	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.75	0.375	0.46875	0.498047	0.499992	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.8	0.32	0.4352	0.491602	0.499859	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.85	0.255	0.37995	0.471176	0.498338	0.499994	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.9	0.18	0.2952	0.416114	0.485926	0.499604	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.95	0.095	0.17195	0.284766	0.407349	0.482832	0.49941	0.499999	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
1	-4.4E-16	-8.9E-16	-1.8E-15	-3.6E-15	-7.1E-15	-1.4E-14	-2.8E-14	-5.7E-14	-1.1E-13	-2.3E-13	-4.5E-13	-9.1E-13	-1.8E-12	-3.6E-12	-7.3E-12	-1.5E-11	-2.9E-11	-5.8E-11



$$r = 3 \qquad x_{n+1} = r x_n \left(1 - x_n\right)$$

iteration number n

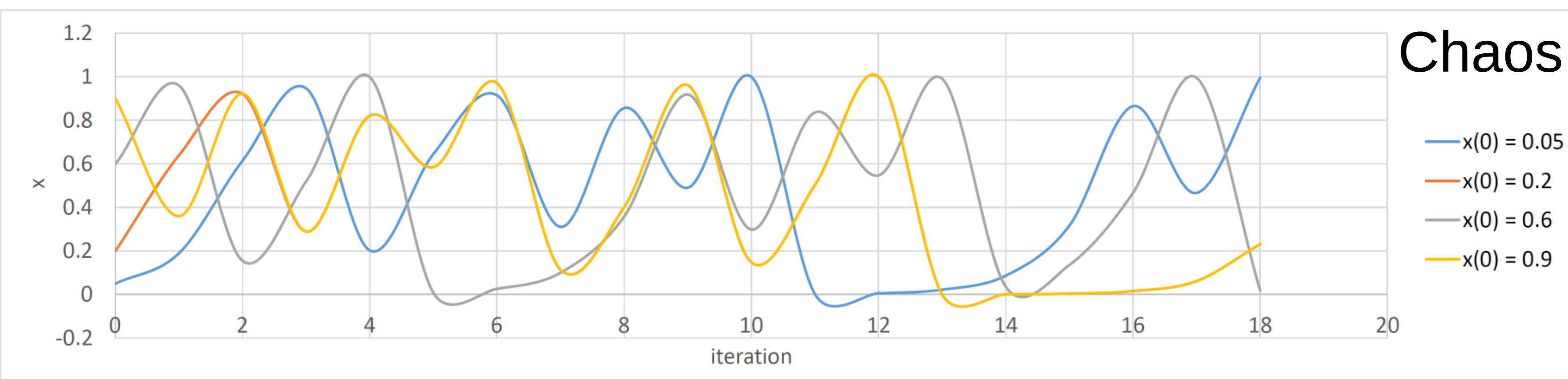
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
x(n)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0.05	0.1425	0.366581	0.696598	0.634047	0.696094	0.634641	0.695615	0.635204	0.695159	0.635738	0.694725	0.636246	0.694311	0.63673	0.693915	0.637191	0.693536	0.637632
	0.1	0.27	0.5913	0.724993	0.598135	0.721109	0.603333	0.717967	0.607471	0.71535	0.610873	0.713121	0.613738	0.711191	0.616195	0.709496	0.618334	0.707991	0.620219
	0.15	0.3825	0.708581	0.619482	0.707172	0.621239	0.705904	0.622811	0.704752	0.62423	0.703701	0.625518	0.702736	0.626694	0.701846	0.627775	0.701021	0.628772	0.700253
	0.2	0.48	0.7488	0.564296	0.737598	0.580641	0.730491	0.590622	0.725363	0.597634	0.721403	0.602943	0.718208	0.607155	0.715553	0.61061	0.713296	0.613514	0.711343
	0.25	0.5625	0.738281	0.579666	0.73096	0.589973	0.725715	0.597158	0.721681	0.602573	0.718436	0.606857	0.715745	0.610362	0.71346	0.613304	0.711487	0.61582	0.709757
	0.3	0.63	0.6993	0.630839	0.698644	0.631622	0.698027	0.632356	0.697446	0.633046	0.696897	0.633695	0.696377	0.634308	0.695884	0.634889	0.695415	0.635439	0.694969
	0.35	0.6825	0.650081	0.682427	0.650161	0.682355	0.65024	0.682284	0.650318	0.682213	0.650395	0.682144	0.65047	0.682076	0.650545	0.682009	0.650619	0.681942	0.650691
	0.4	0.72	0.6048	0.717051	0.608667	0.714575	0.611873	0.712453	0.614591	0.710607	0.616934	0.708979	0.618983	0.707529	0.620795	0.706226	0.622413	0.705045	0.62387
	0.45	0.7425	0.573581	0.733757	0.586072	0.727775	0.594356	0.723291	0.600424	0.719745	0.605136	0.716839	0.608942	0.714395	0.612105	0.712298	0.614789	0.71047	0.617107
	0.5	0.75	0.5625	0.738281	0.579666	0.73096	0.589973	0.725715	0.597158	0.721681	0.602573	0.718436	0.606857	0.715745	0.610362	0.71346	0.613304	0.711487	0.61582
	0.55	0.7425	0.573581	0.733757	0.586072	0.727775	0.594356	0.723291	0.600424	0.719745	0.605136	0.716839	0.608942	0.714395	0.612105	0.712298	0.614789	0.71047	0.617107
	0.6	0.72	0.6048	0.717051	0.608667	0.714575	0.611873	0.712453	0.614591	0.710607	0.616934	0.708979	0.618983	0.707529	0.620795	0.706226	0.622413	0.705045	0.62387
	0.65	0.6825	0.650081	0.682427	0.650161	0.682355	0.65024	0.682284	0.650318	0.682213	0.650395	0.682144	0.65047	0.682076	0.650545	0.682009	0.650619	0.681942	0.650691
	0.7	0.63	0.6993	0.630839	0.698644	0.631622	0.698027	0.632356	0.697446	0.633046	0.696897	0.633695	0.696377	0.634308	0.695884	0.634889	0.695415	0.635439	0.694969
	0.75	0.5625	0.738281	0.579666	0.73096	0.589973	0.725715	0.597158	0.721681	0.602573	0.718436	0.606857	0.715745	0.610362	0.71346	0.613304	0.711487	0.61582	0.709757
	0.8	0.48	0.7488	0.564296	0.737598	0.580641	0.730491	0.590622	0.725363	0.597634	0.721403	0.602943	0.718208	0.607155	0.715553	0.61061	0.713296	0.613514	0.711343
	0.85	0.3825	0.708581	0.619482	0.707172	0.621239	0.705904	0.622811	0.704752	0.62423	0.703701	0.625518	0.702736	0.626694	0.701846	0.627775	0.701021	0.628772	0.700253
	0.9	0.27	0.5913	0.724993	0.598135	0.721109	0.603333	0.717967	0.607471	0.71535	0.610873	0.713121	0.613738	0.711191	0.616195	0.709496	0.618334	0.707991	0.620219
	0.95	0.1425	0.366581	0.696598	0.634047	0.696094	0.634641	0.695615	0.635204	0.695159	0.635738	0.694725	0.636246	0.694311	0.63673	0.693915	0.637191	0.693536	0.637632
	1	-6.7E-16	-2E-15	-6E-15	-1.8E-14	-5.4E-14	-1.6E-13	-4.9E-13	-1.5E-12	-4.4E-12	-1.3E-11	-3.9E-11	-1.2E-10	-3.5E-10	-1.1E-09	-3.2E-09	-9.6E-09	-2.9E-08	-8.6E-08



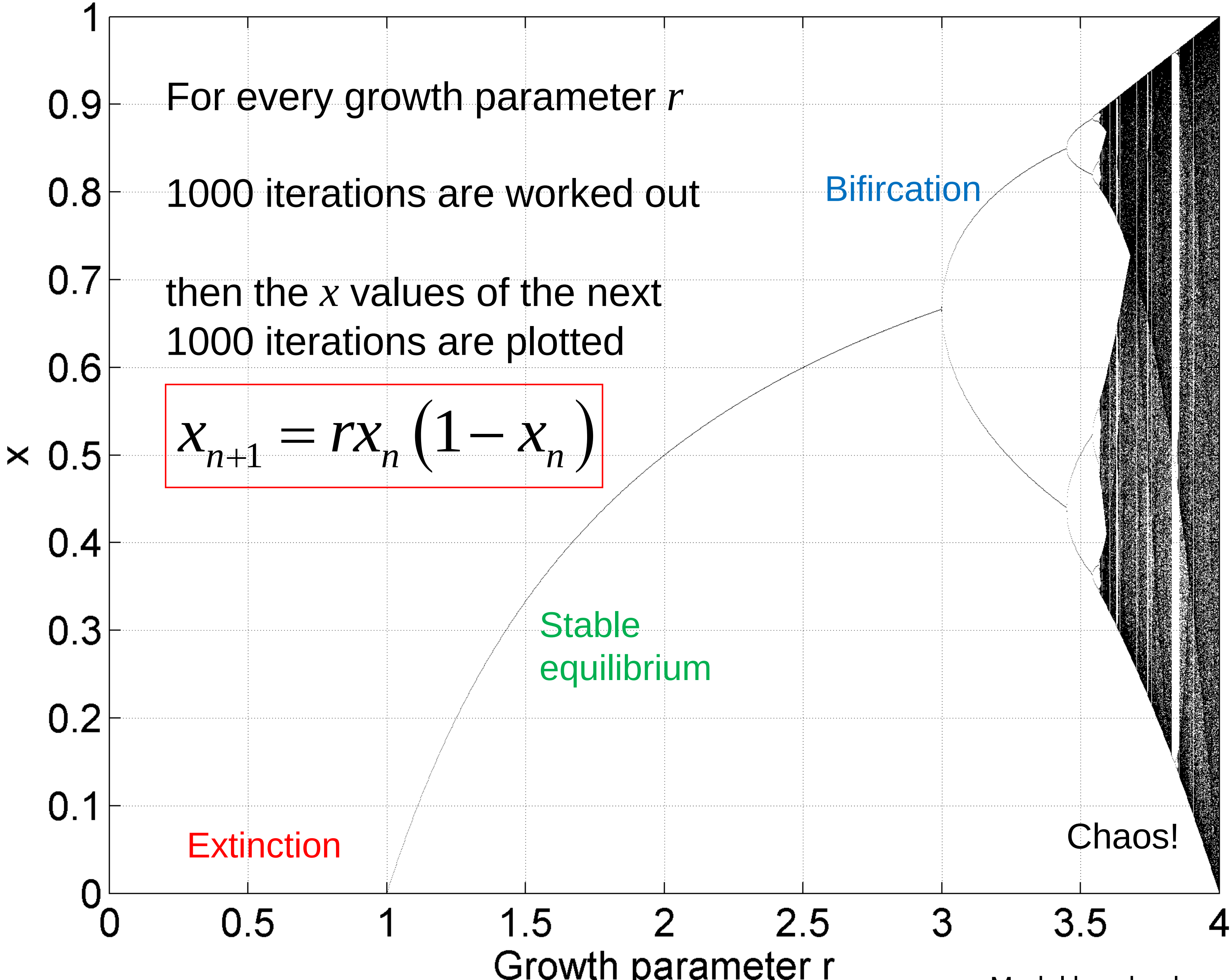
$$r = 4 \qquad x_{n+1} = r x_n \left(1 - x_n\right)$$

iteration number n

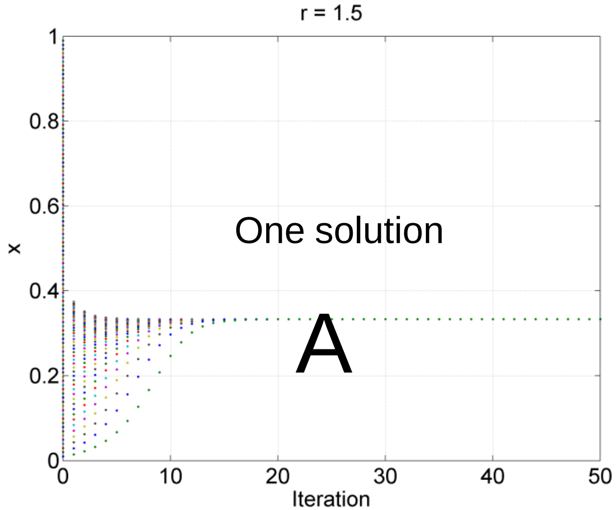
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
x(n)	**0**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0.05	0.19	0.6156	0.946547	0.202385	0.6457	0.915085	0.310816	0.856838	0.490667	0.999652	0.001393	0.005565	0.022137	0.086589	0.316366	0.865114	0.466766	0.995582
	0.1	0.36	0.9216	0.289014	0.821939	0.585421	0.970813	0.113339	0.401974	0.961563	0.147837	0.503924	0.999938	0.000246	0.000985	0.003936	0.015682	0.061745	0.23173
	0.15	0.51	0.9996	0.001599	0.006387	0.025386	0.098965	0.356683	0.917841	0.301635	0.842605	0.530488	0.996282	0.014817	0.058389	0.219918	0.686217	0.861293	0.47787
	0.2	0.64	0.9216	0.289014	0.821939	0.585421	0.970813	0.113339	0.401974	0.961563	0.147837	0.503924	0.999938	0.000246	0.000985	0.003936	0.015682	0.061745	0.23173
	0.25	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75
	0.3	0.84	0.5376	0.994345	0.022492	0.087945	0.320844	0.871612	0.447617	0.989024	0.043422	0.166146	0.554165	0.988265	0.046391	0.176954	0.582565	0.972732	0.106097
	0.35	0.91	0.3276	0.881113	0.419012	0.973764	0.102192	0.366996	0.92924	0.263011	0.775345	0.69674	0.845174	0.523421	0.997806	0.008757	0.034722	0.134065	0.464367
	0.4	0.96	0.1536	0.520028	0.998395	0.006408	0.025467	0.099273	0.35767	0.918969	0.29786	0.836557	0.546917	0.991195	0.034909	0.134761	0.466403	0.995485	0.017978
	0.45	0.99	0.0396	0.152127	0.515939	0.998984	0.00406	0.016176	0.063657	0.238418	0.7263	0.795154	0.651537	0.908147	0.333665	0.889331	0.393686	0.954789	0.172666
	0.5	1	4.44E-16	1.78E-15	7.11E-15	2.84E-14	1.14E-13	4.55E-13	1.82E-12	7.28E-12	2.91E-11	1.16E-10	4.66E-10	1.86E-09	7.45E-09	2.98E-08	1.19E-07	4.77E-07	1.91E-06
	0.55	0.99	0.0396	0.152127	0.515939	0.998984	0.00406	0.016176	0.063657	0.238418	0.7263	0.795154	0.651537	0.908147	0.333665	0.889331	0.393686	0.954789	0.172666
	0.6	0.96	0.1536	0.520028	0.998395	0.006408	0.025467	0.099273	0.35767	0.918969	0.29786	0.836557	0.546917	0.991195	0.034909	0.134761	0.466403	0.995485	0.017978
	0.65	0.91	0.3276	0.881113	0.419012	0.973764	0.102192	0.366996	0.92924	0.263011	0.775345	0.69674	0.845174	0.523421	0.997806	0.008757	0.034722	0.134065	0.464367
	0.7	0.84	0.5376	0.994345	0.022492	0.087945	0.320844	0.871612	0.447617	0.989024	0.043422	0.166146	0.554165	0.988265	0.046391	0.176954	0.582565	0.972732	0.106097
	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75
	0.8	0.64	0.9216	0.289014	0.821939	0.585421	0.970813	0.113339	0.401974	0.961563	0.147837	0.503924	0.999938	0.000246	0.000985	0.003936	0.015682	0.061745	0.23173
	0.85	0.51	0.9996	0.001599	0.006387	0.025386	0.098965	0.356683	0.917841	0.301635	0.842605	0.530488	0.996282	0.014817	0.058389	0.219918	0.686217	0.861293	0.47787
	0.9	0.36	0.9216	0.289014	0.821939	0.585421	0.970813	0.113339	0.401974	0.961563	0.147837	0.503924	0.999938	0.000246	0.000985	0.003936	0.015682	0.061745	0.23173
	0.95	0.19	0.6156	0.946547	0.202385	0.6457	0.915085	0.310816	0.856838	0.490667	0.999652	0.001393	0.005565	0.022137	0.086589	0.316366	0.865114	0.466766	0.995582
	1	-8.9E-16	-3.6E-15	-1.4E-14	-5.7E-14	-2.3E-13	-9.1E-13	-3.6E-12	-1.5E-11	-5.8E-11	-2.3E-10	-9.3E-10	-3.7E-09	-1.5E-08	-6E-08	-2.4E-07	-9.5E-07	-3.8E-06	-1.5E-05

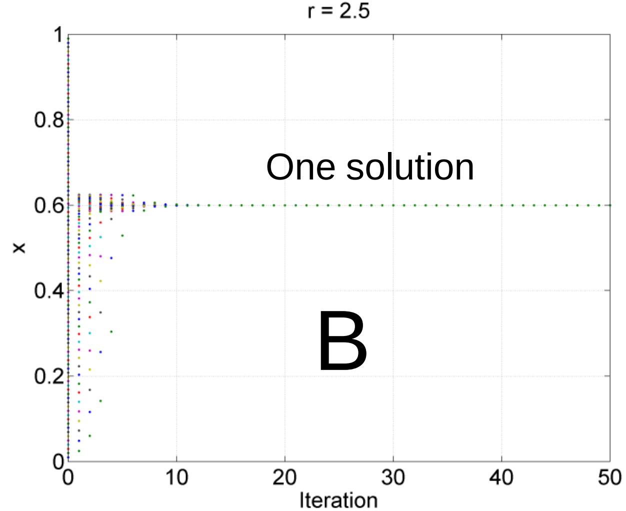


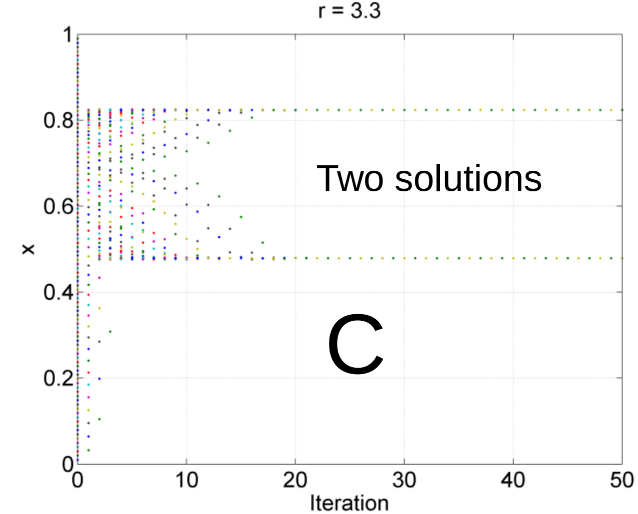
May Bifurcations Logistic map

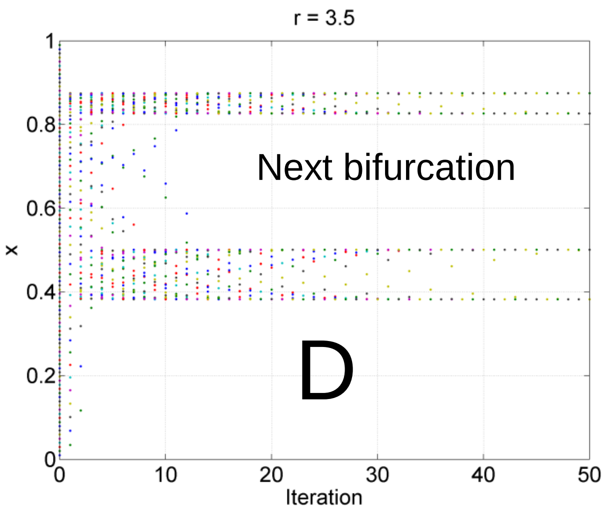


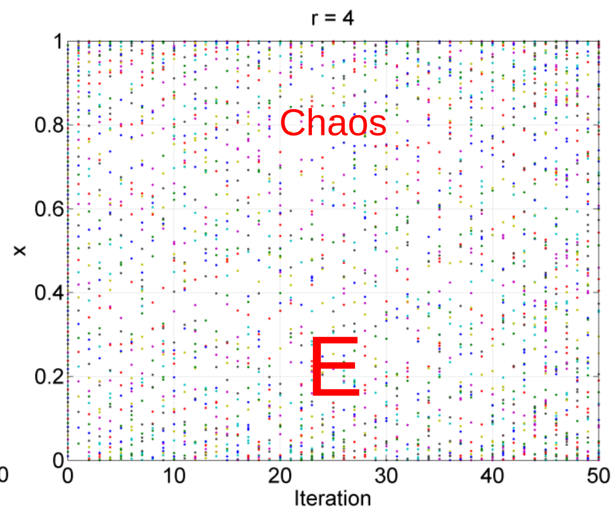
Model breaks down for *r* > 4



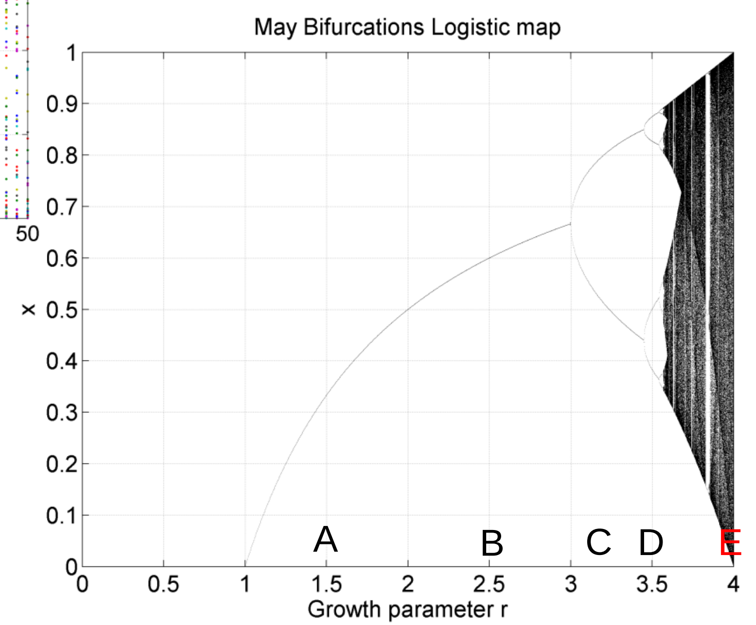






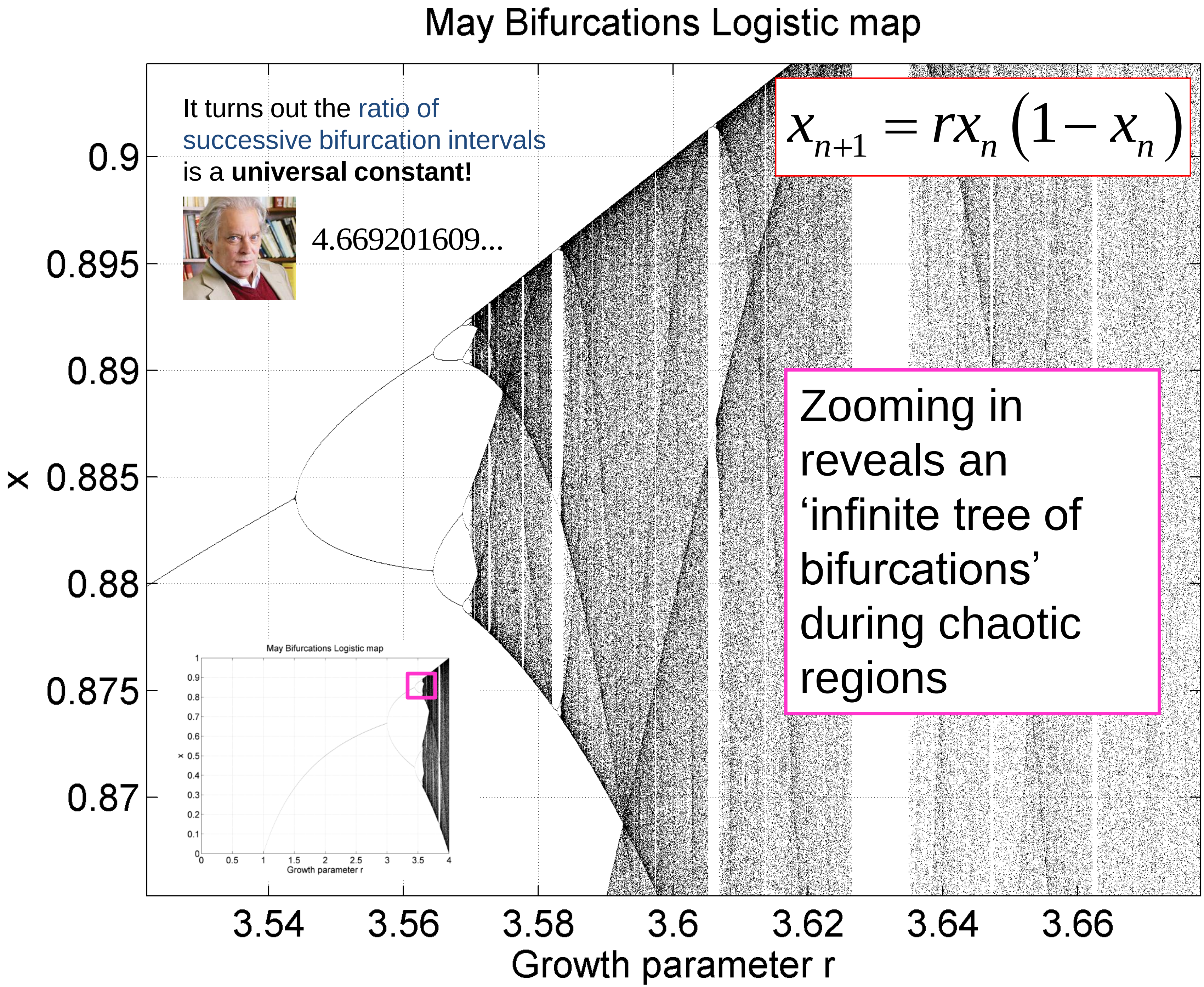


$$x_{n+1} = rx_n(1 - x_n)$$

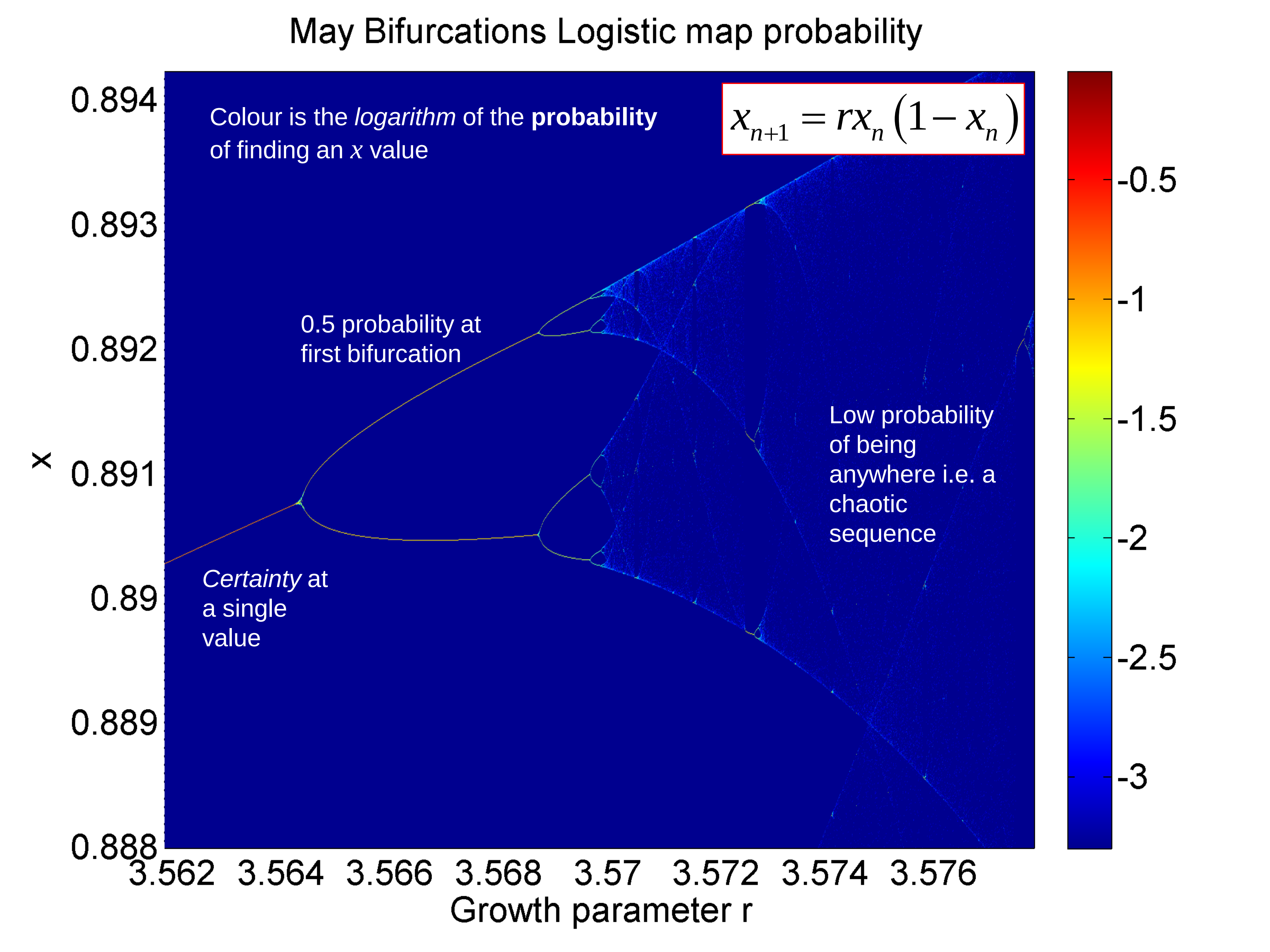


Tracking the bifurcations maps the 'road to chaos'. The ratio of successive bifurcation intervals is a **universal constant!**

4.669201609...



May Bifurcations Logistic map
It turns out the ratio of successive bifurcation intervals is a **universal constant!**
4.669201609...
$x_{n+1} = rx_n(1 - x_n)$
Zooming in reveals an ‘infinite tree of bifurcations’ during chaotic regions
x
0.9
0.895
0.89
0.885
0.88
0.875
0.87
3.54
3.56
3.58
3.6
3.62
3.64
3.66
Growth parameter r


May Bifurcations Logistic map probability
Colour is the *logarithm* of the **probability** of finding an *x* value
$x_{n+1} = r x_n (1 - x_n)$
0.5 probability at first bifurcation
Low probability of being anywhere i.e. a chaotic sequence
Certainty at a single value
0.894
0.893
0.892
0.891
0.89
0.889
0.888
x
3.562
3.564
3.566
3.568
3.57
3.572
3.574
3.576
Growth parameter r
-0.5
-1
-1.5
-2
-2.5
-3

Lorenz and Rössler **strange attractors**

Edward Lorenz was using a Royal McBee LGP-30 computer in 1961 to model weather patterns. He accidentally fed in 3 digit precision numbers into the model from a printout rather than the 6 digits used by the computer. These tiny errors created a hugely different weather forecast....

Lorenz's weather model was *very* sensitive to initial conditions.

His equations looked a bit like these:

$$\frac{dx}{dt} = s(y - x)$$

$$\frac{dy}{dt} = x(r - z) - y$$

$$\frac{dz}{dt} = xy - bz$$

$$s = 10$$

$$r = 28$$

$$b = \tfrac{8}{3}$$

Edward Lorenz
1917-2008

Although x,y,z trajectories are chaotic, they tend to *gravitate towards a particular region.*

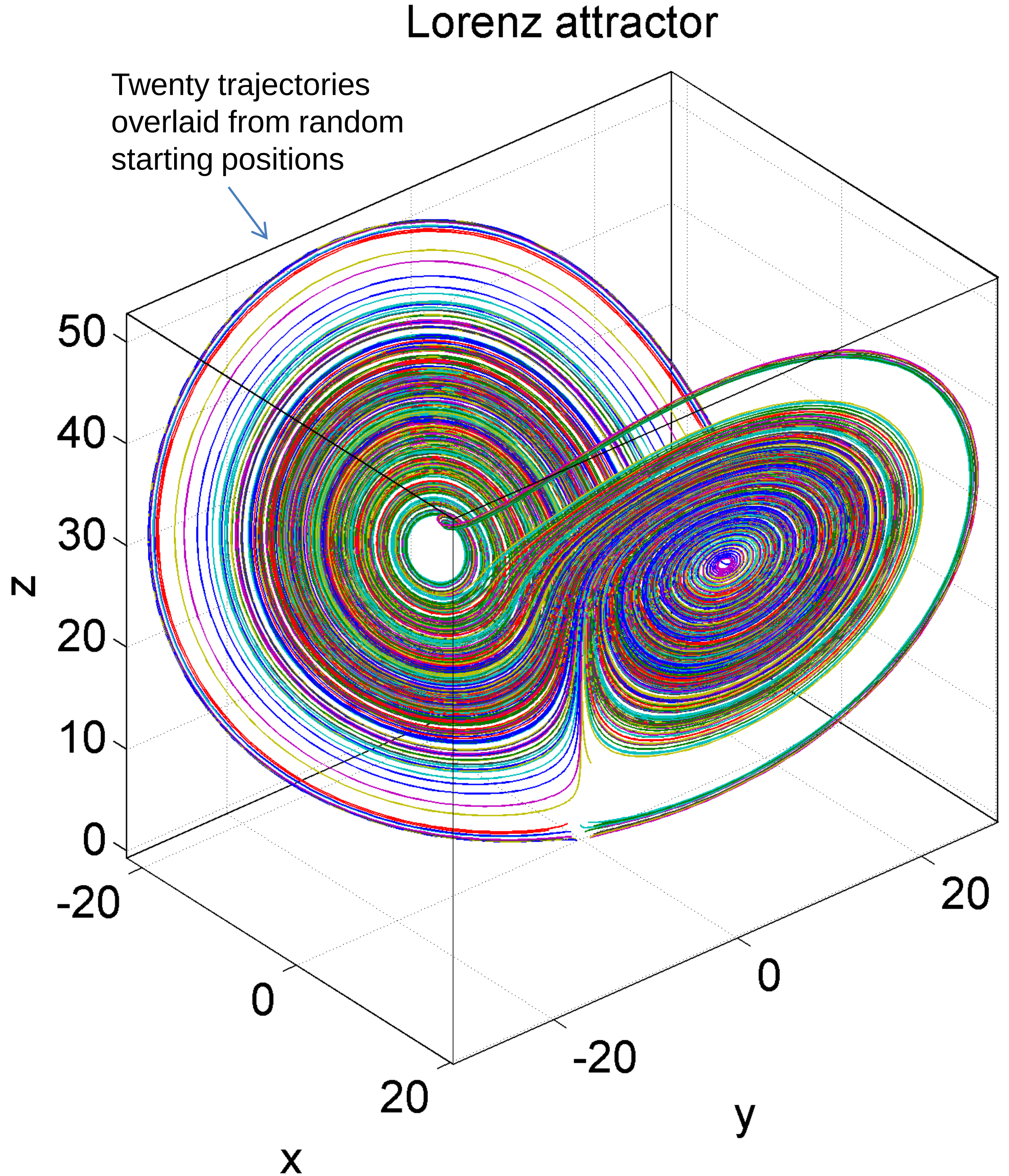
This region is called a **strange attractor.**

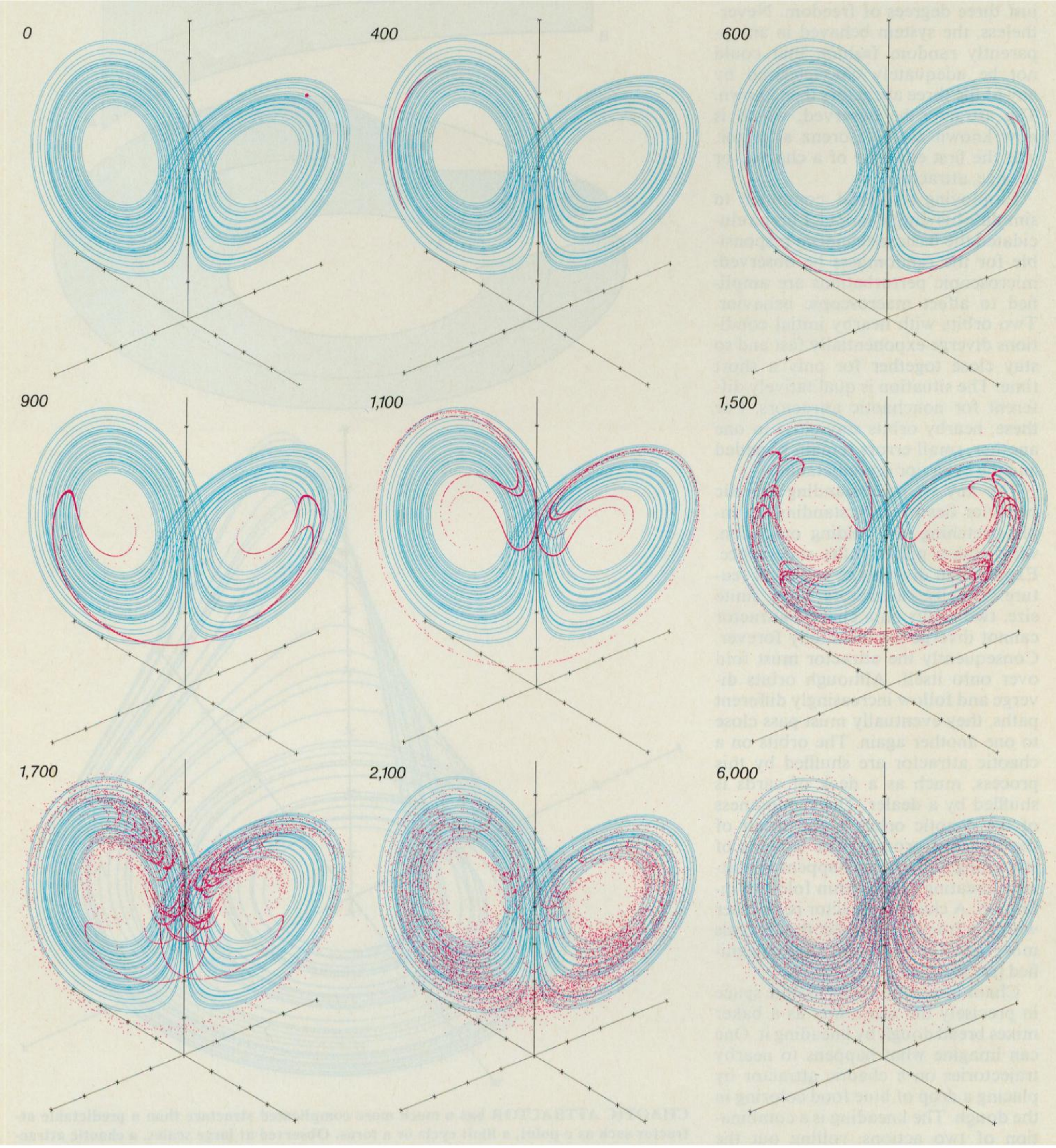
$$\frac{dx}{dt} = s(y-x)$$

$$\frac{dy}{dt} = x(r-z)-y$$

$$\frac{dz}{dt} = xy-bz$$

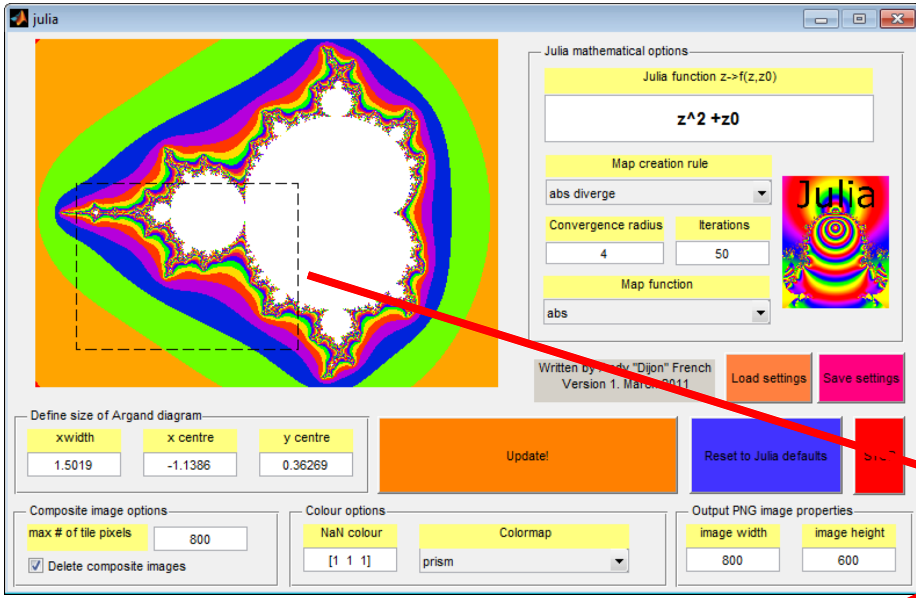
$$s=10 \quad r=28 \quad b=\tfrac{8}{3}$$





Applying the Lorenz equations, a cluster of initial x,y,z values separated by a *tiny* random deviation will eventually **spread out evenly throughout the strange attractor.**

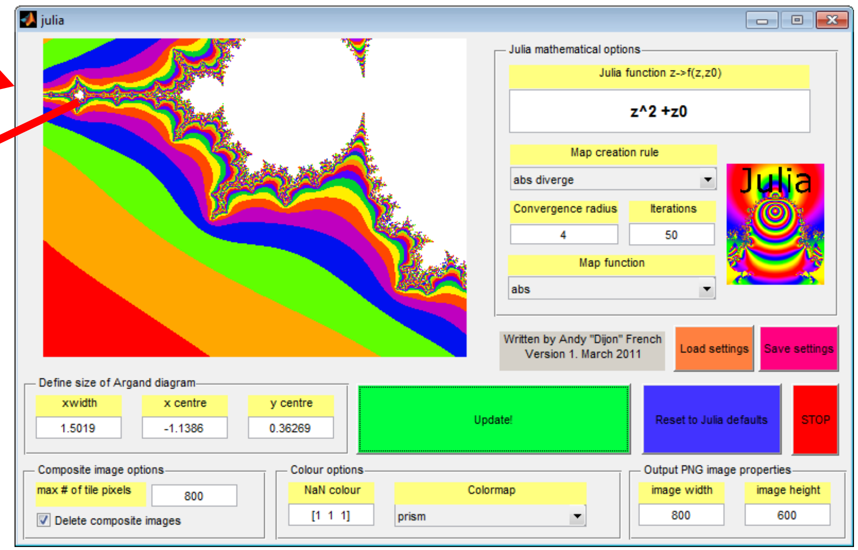
Shaw *et al*; “Chaos”, Scientific American 54:12 (1986) 46-57

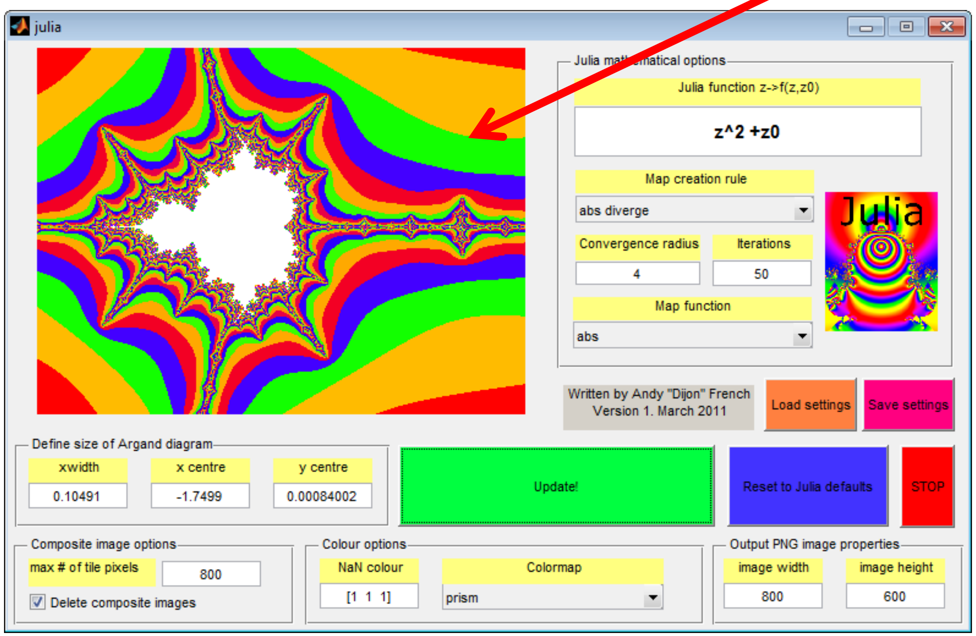


Mandlebrot, complex numbers and iteration

The *Mandlebrot Set* has infinite complexity!

... But a recursive *fractal* geometry





Benoit Mandlebrot (1924-2010)

Mandlebrot transformations of **complex numbers**

$$i^2 = -1$$

$$z = x + iy$$

$$x = \mathrm{Re}(z)$$

$$y = \mathrm{Im}(z)$$

$$|z| = \sqrt{x^2 + y^2}$$

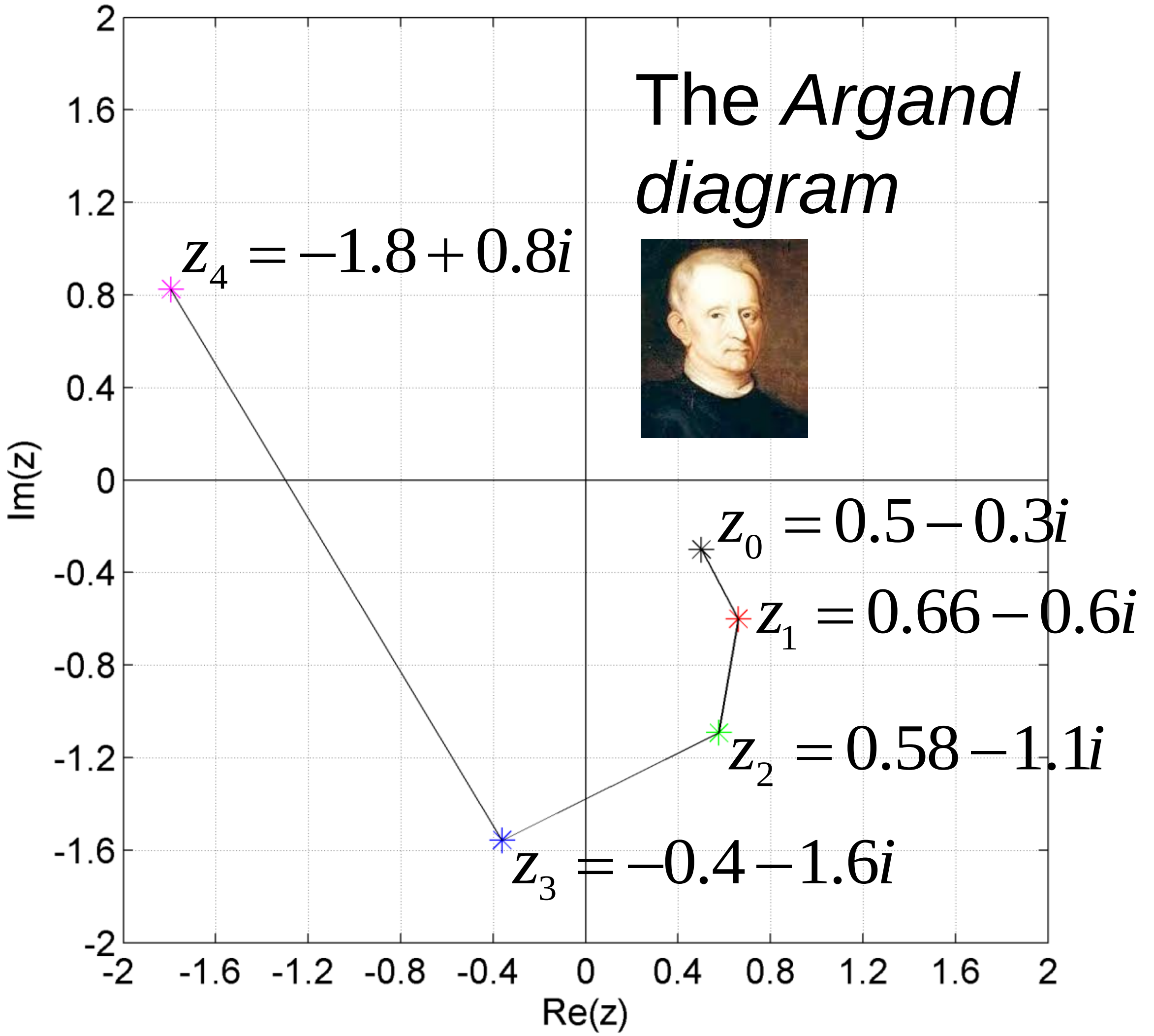
$$(1+i)(1+i)$$

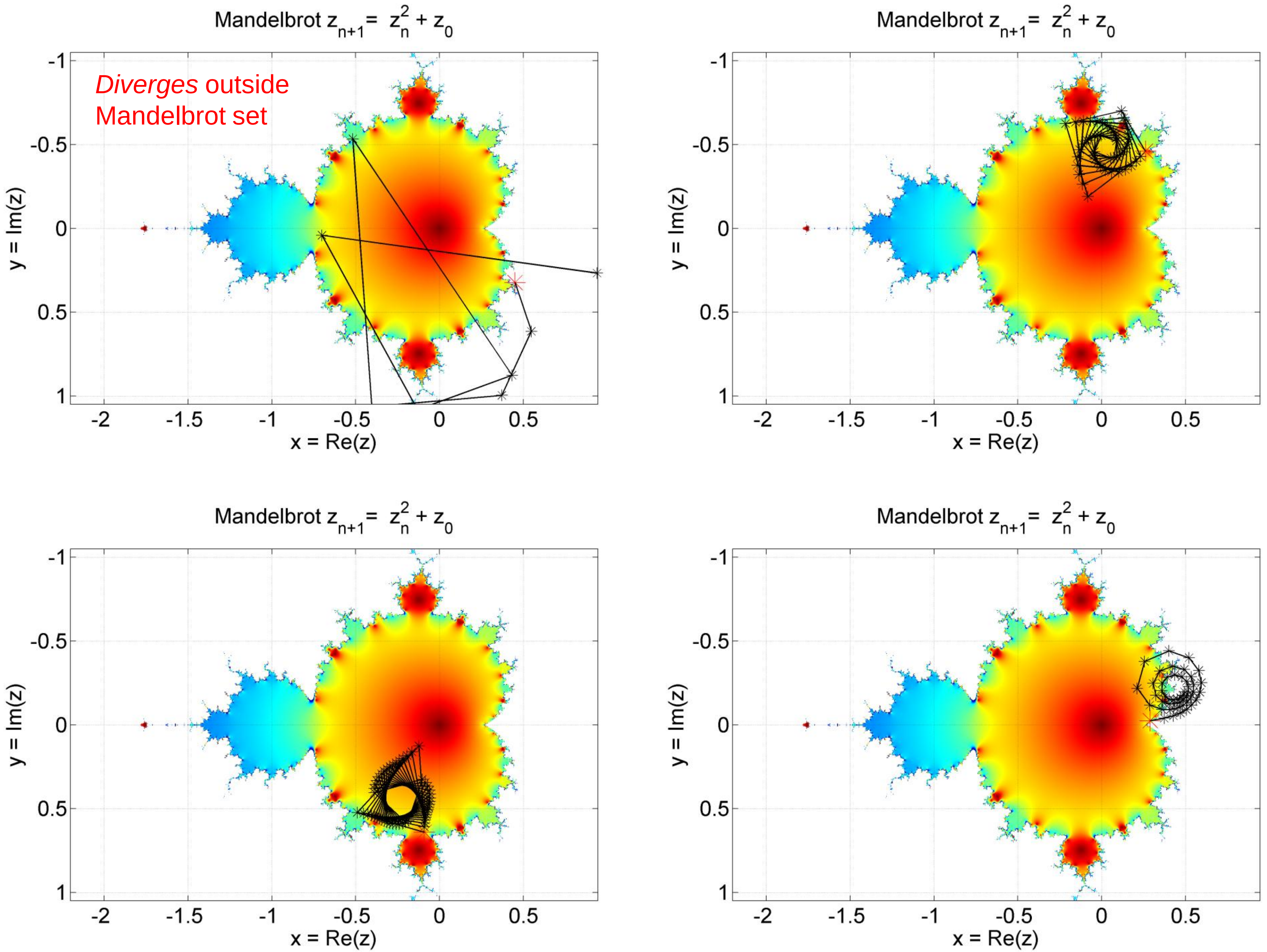
$$= 1 + 2i + i^2$$

$$= 1 + 2i - 1$$

$$= 2i$$

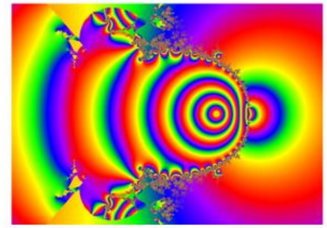
$$z_{n+1} = z_n^2 + z_0$$

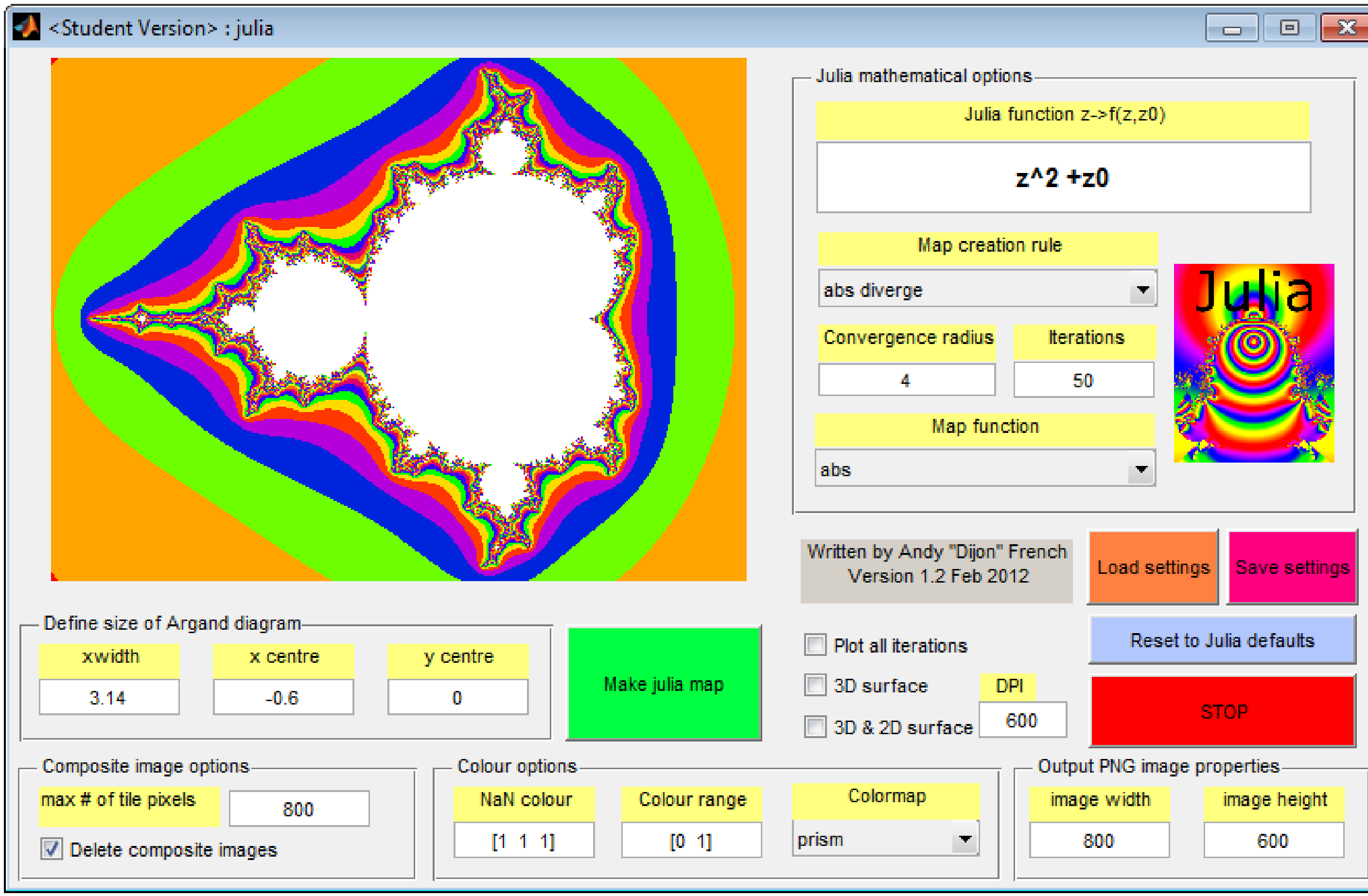


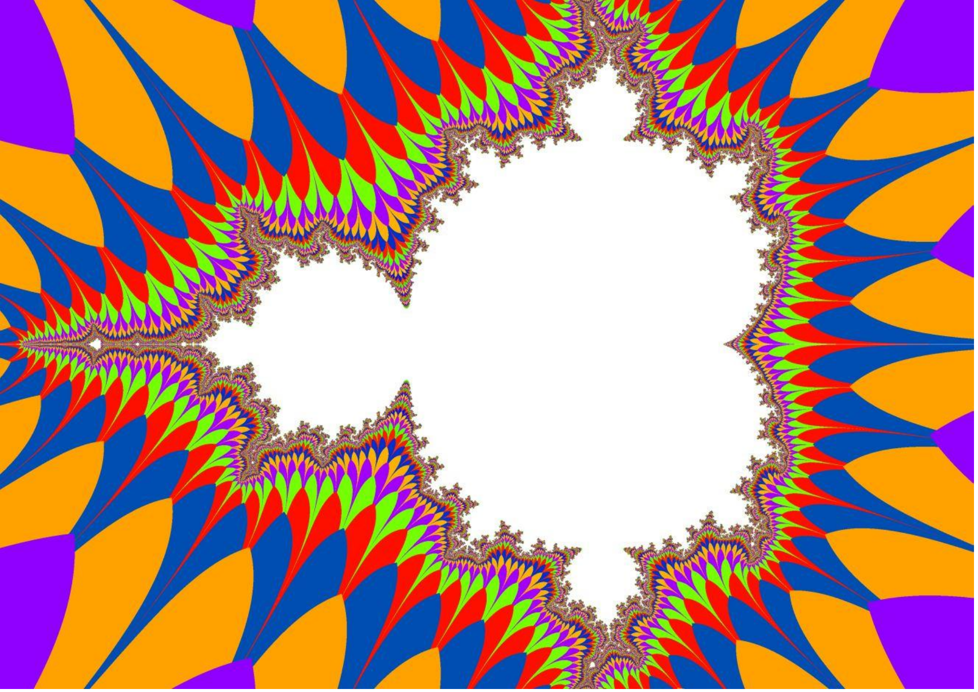

Mandelbrot $z_{n+1} = z_n^2 + z_0$
Diverges outside
Mandelbrot set
y = Im(z)
x = Re(z)
-1
-0.5
0
0.5
1
-2
-1.5
-1
-0.5
0
0.5
Mandelbrot $z_{n+1} = z_n^2 + z_0$
y = Im(z)
x = Re(z)
Mandelbrot $z_{n+1} = z_n^2 + z_0$
y = Im(z)
x = Re(z)
Mandelbrot $z_{n+1} = z_n^2 + z_0$
y = Im(z)
x = Re(z)

Gaston Julia
(1893-1978)

julia



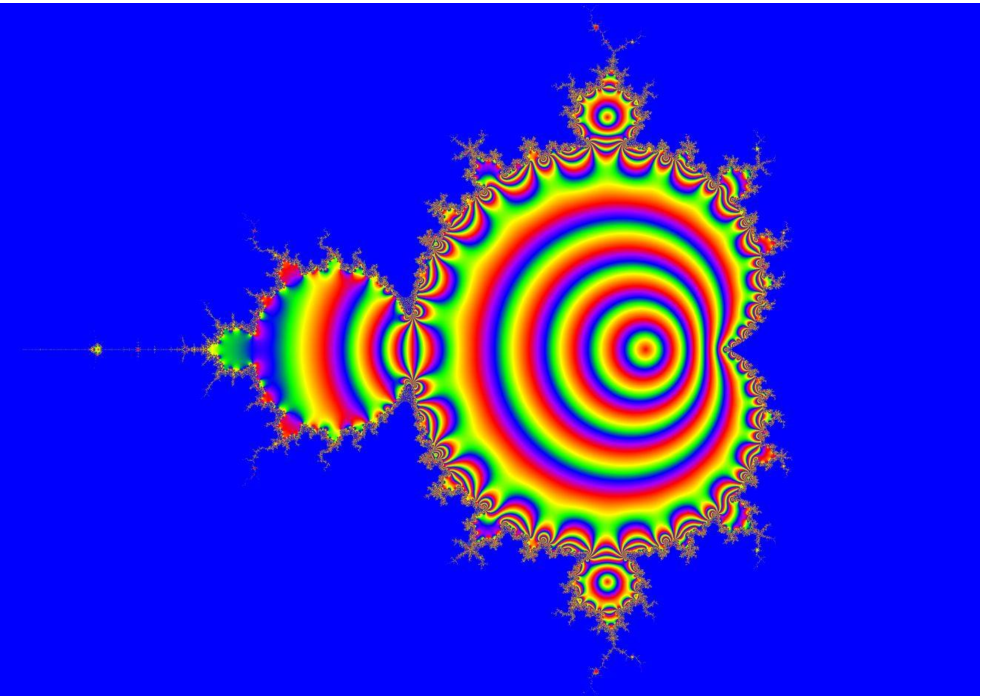




`julia.m` plot option `abs diverge`

Plot a surface with height $h(x,y)$. This is the *iteration number* when $|z|$ exceeds a certain value e.g. 4

In this case *colours* indicate height $h(x,y)$. It is a ‘colour-map’.

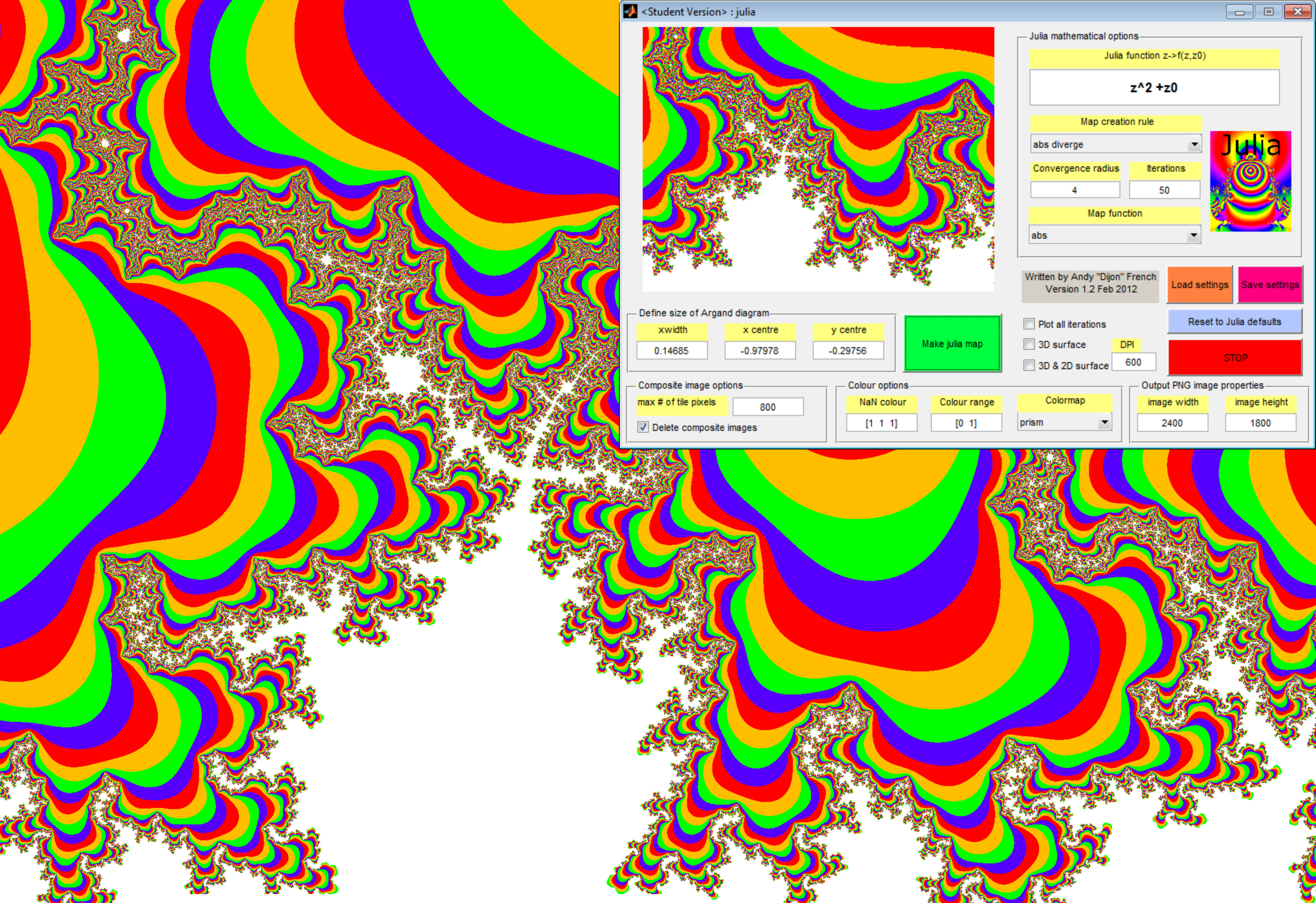


`julia.m` plot option `plot z`

Plot a surface with height $h(x,y)$

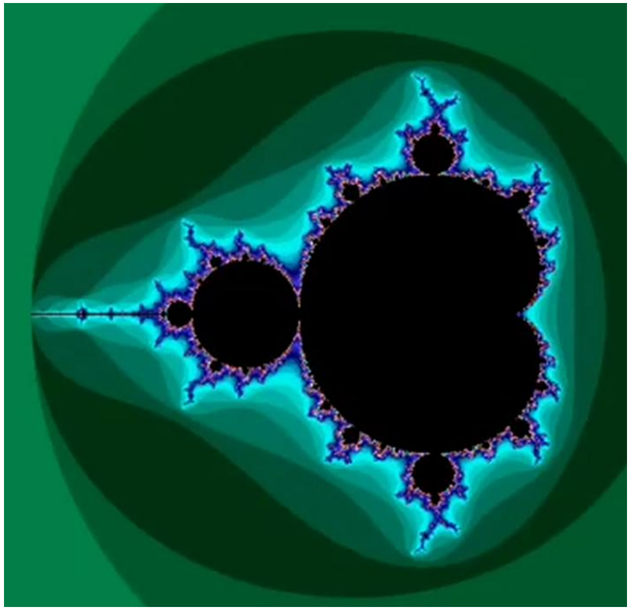
$$x = \mathrm{Re}(z), \quad y = \mathrm{Im}(z)$$

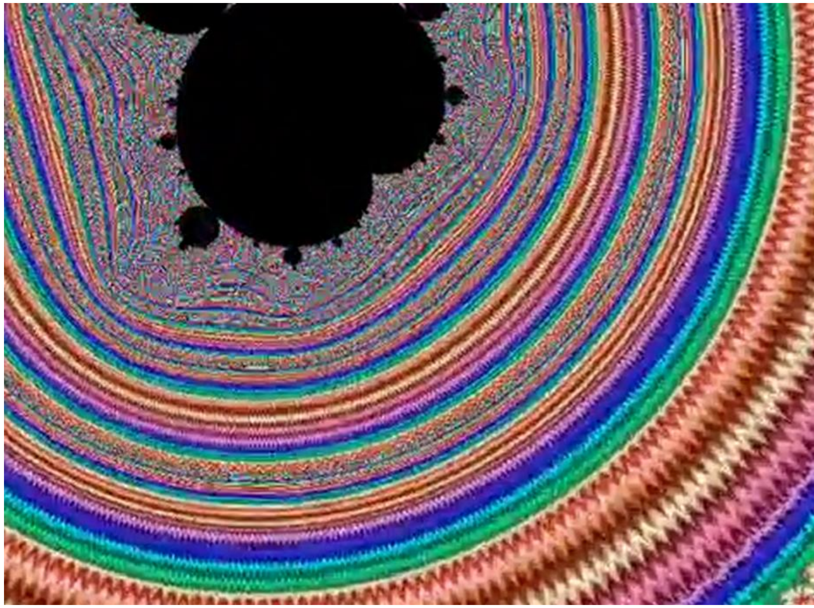
$$h(x,y) = e^{-\sqrt{x^2+y^2}}$$

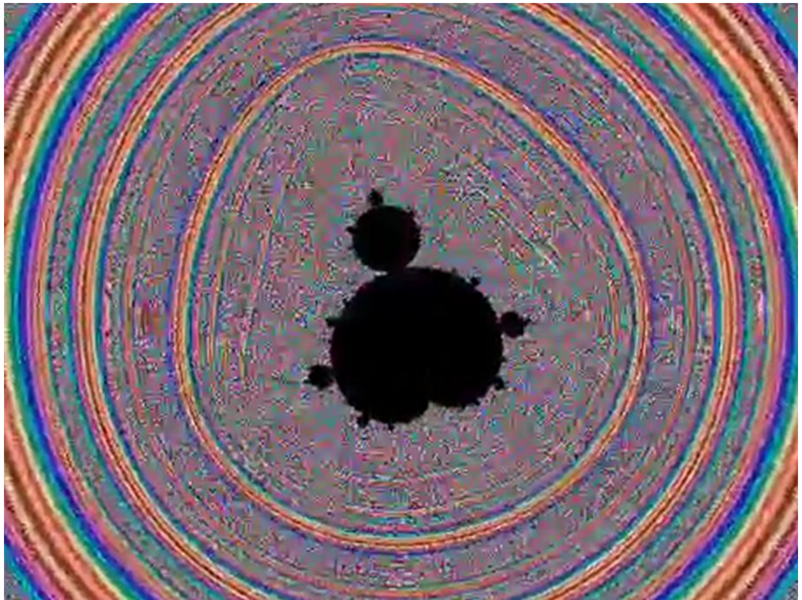


The Mandleplant slurping complexity
from the Argand plane (!)

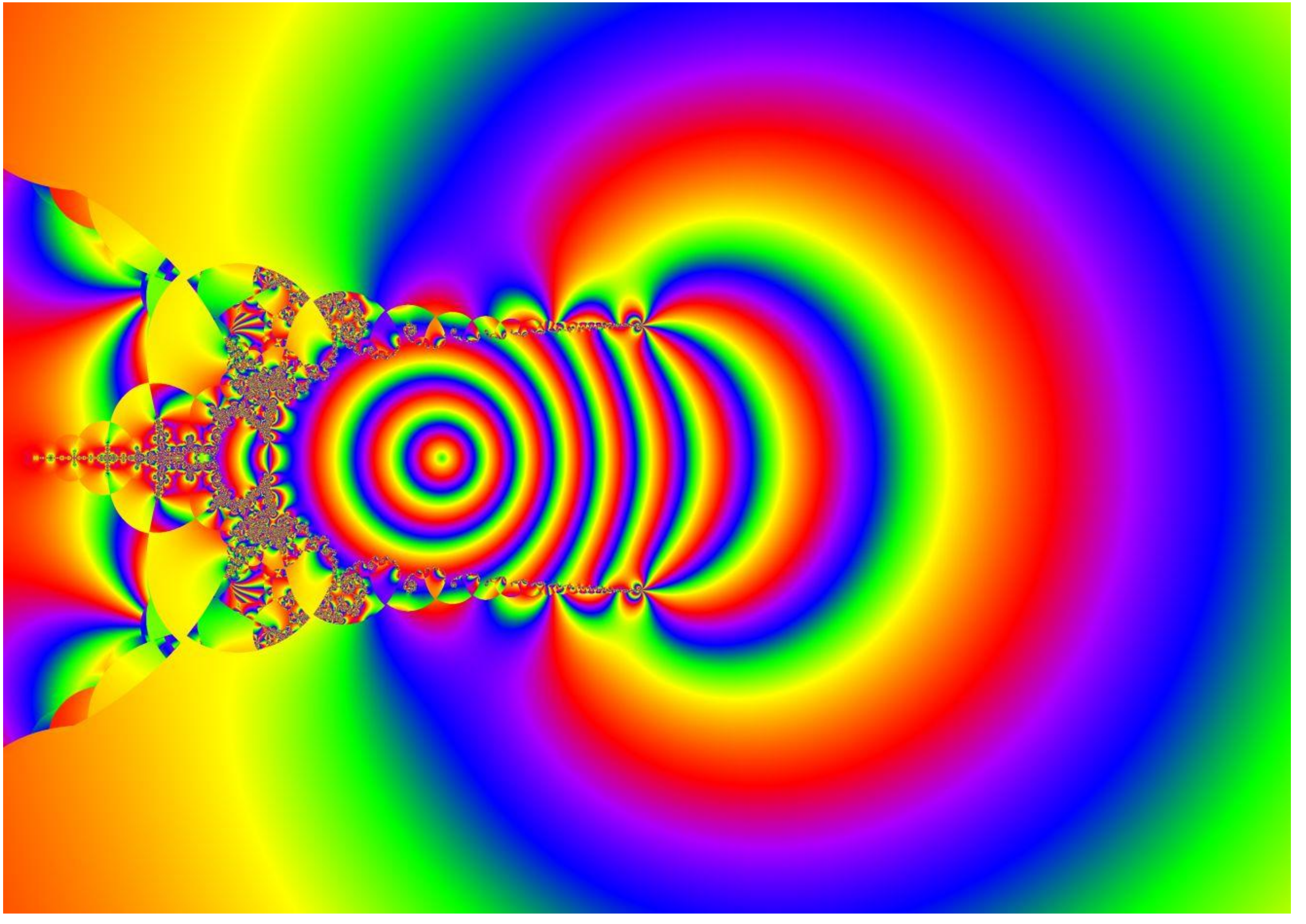
Mandelbrot Deep Zoom







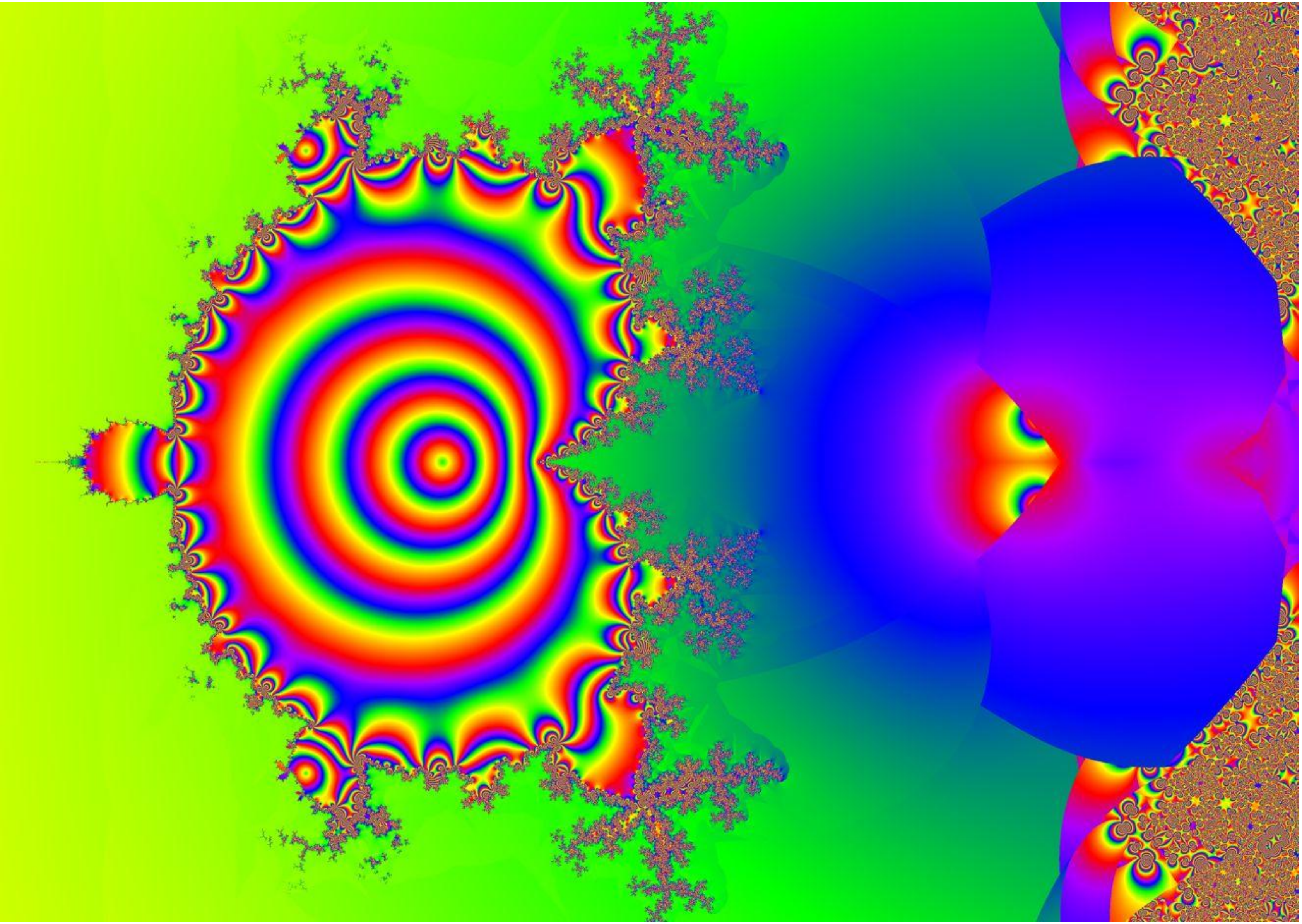

The
Mandlebrot
Variations



The light bulb

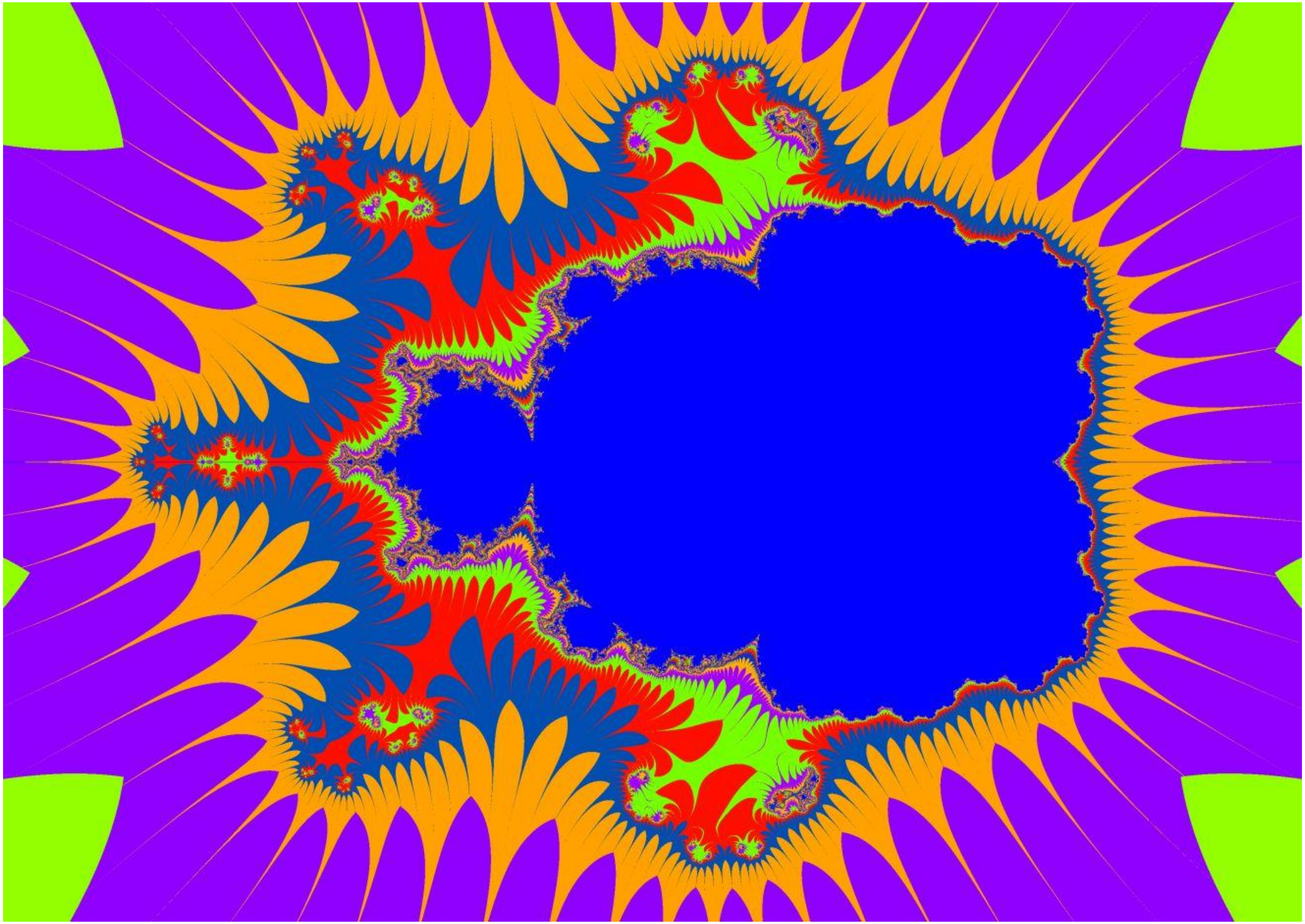
$$z_{n+1} = \log\left(z_n^2 + z_0\right)$$

7 steps to enlightenment $z_{n+1} = \tan^{-1}\left(z_n^2 + z_0\right)$



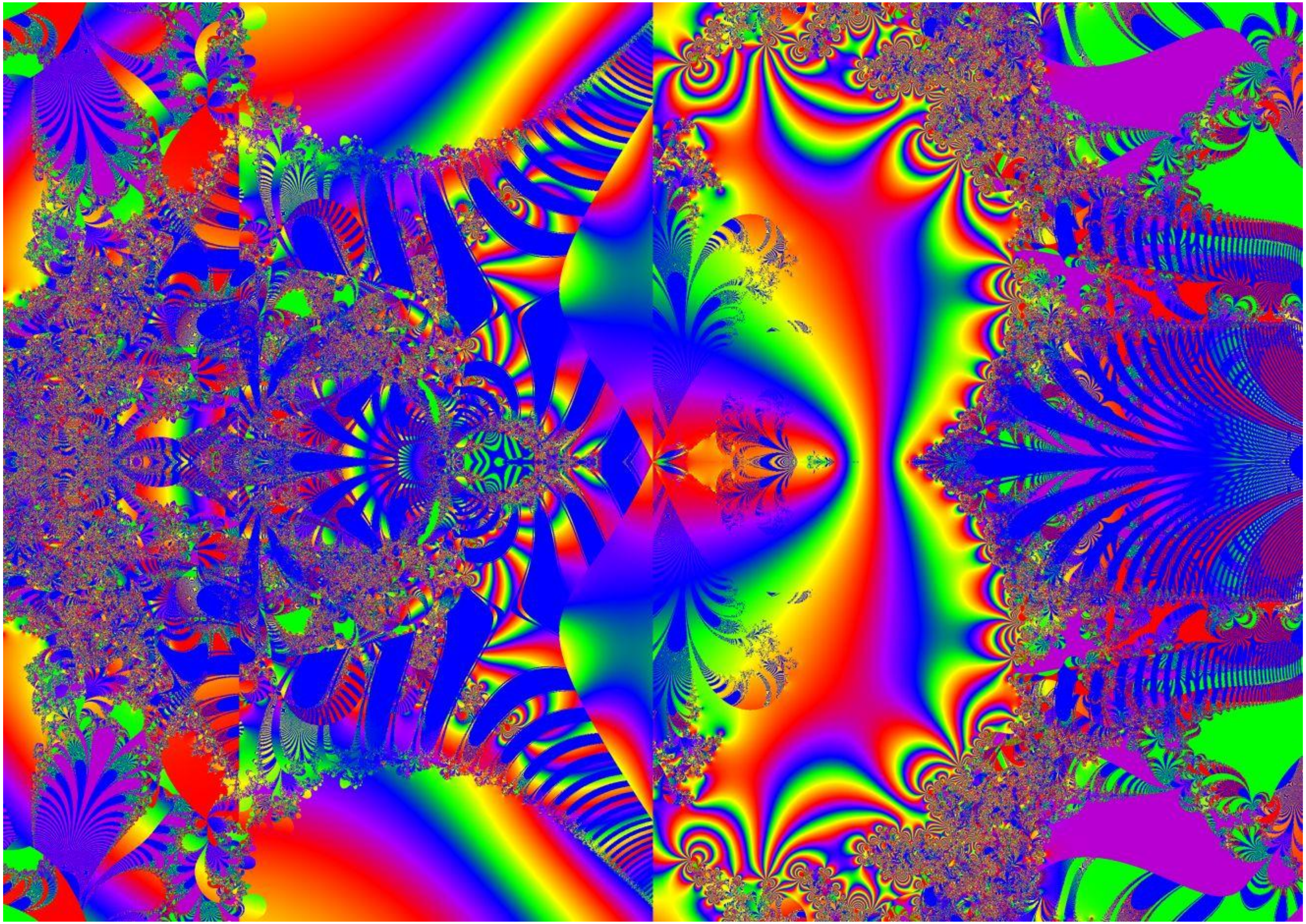
The Mandlerocket!

$$z_{n+1} = \sin^{-1}\left(z_n^2 + z_0\right)$$



Micro mandlebeast

$$z_{n+1} = \left(z_n^2 + z_0 \right)^2$$

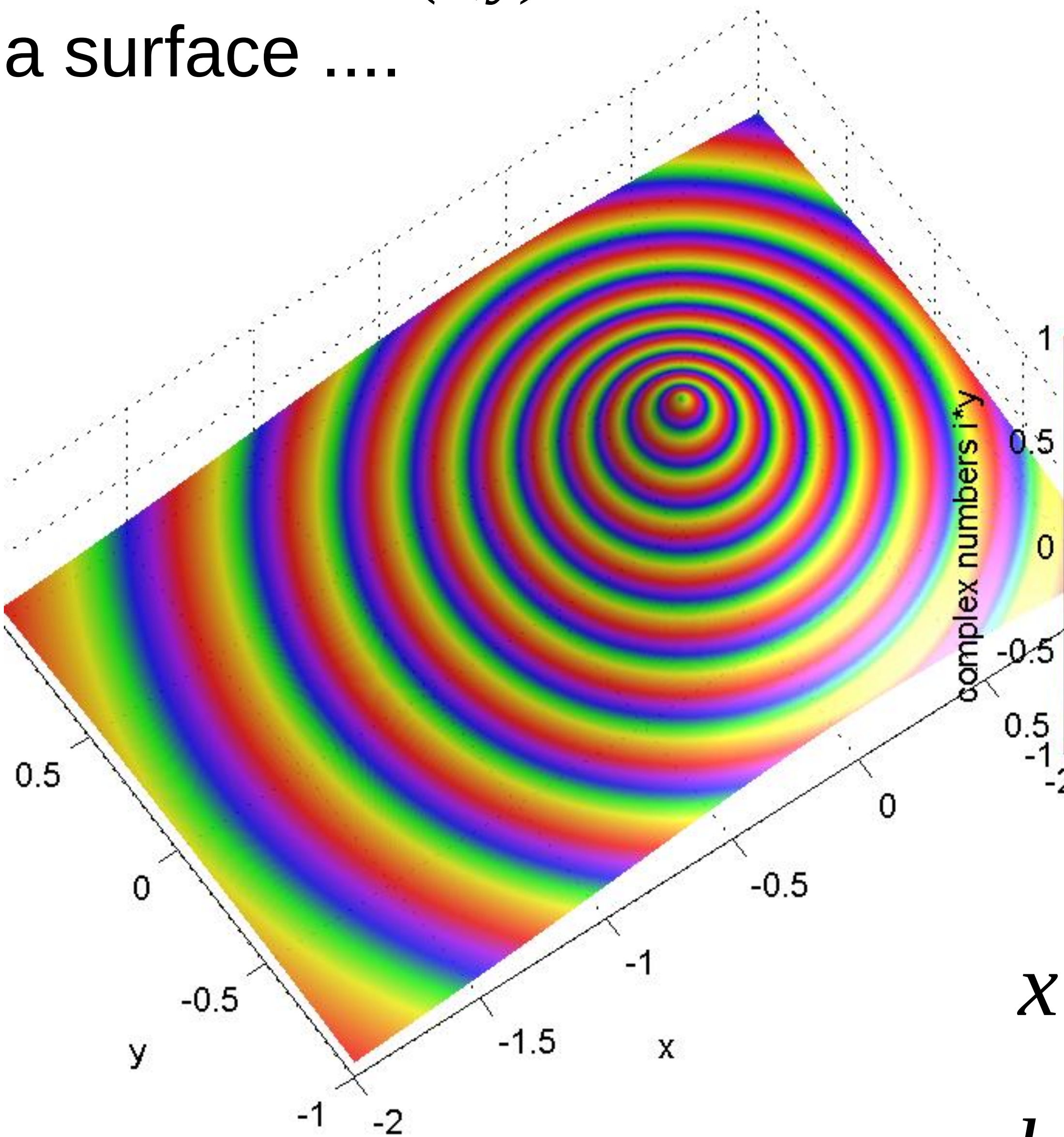


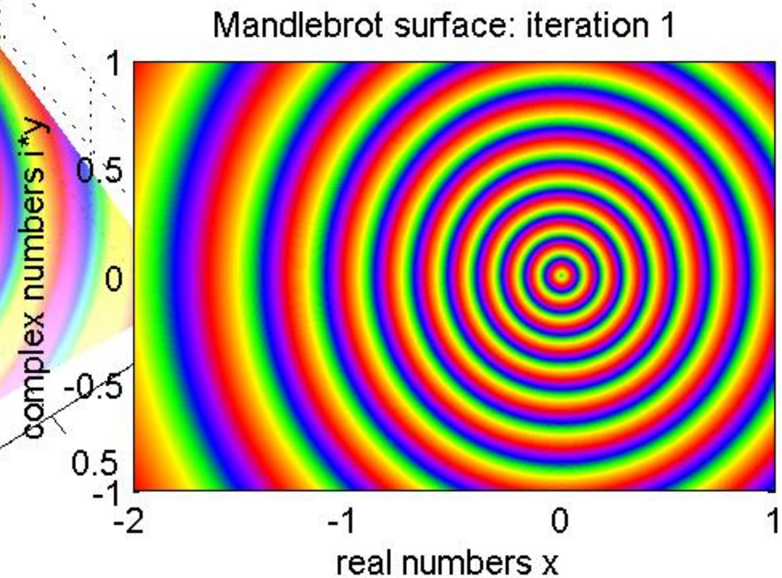
The profusion of power

$$z_{n+1} = \left(z_n^2 + z_0\right)^{z_n}$$

Remember *h(x,y)* is a surface ....

$$z_{n+1} = z_n^2 + z_0$$

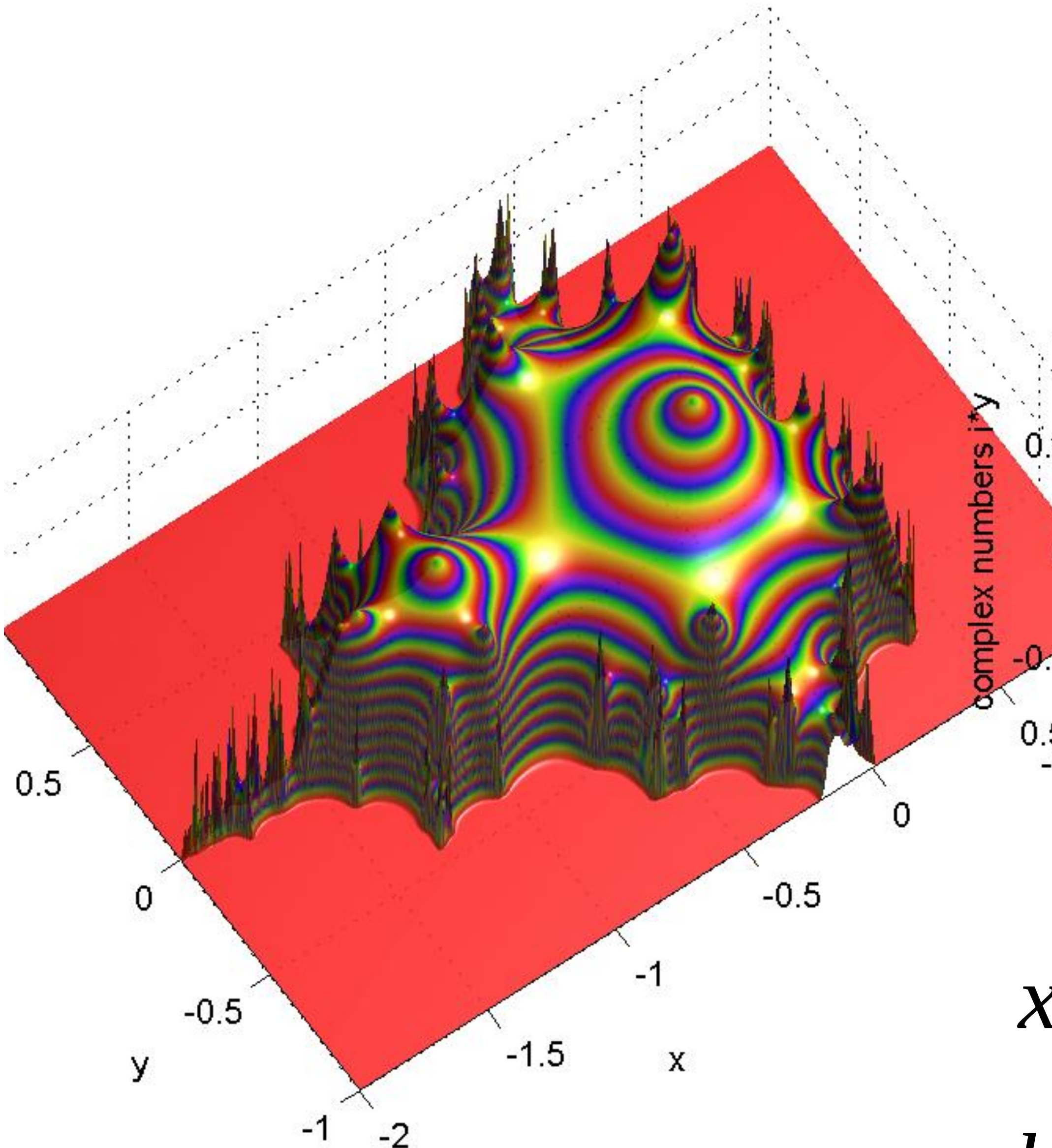


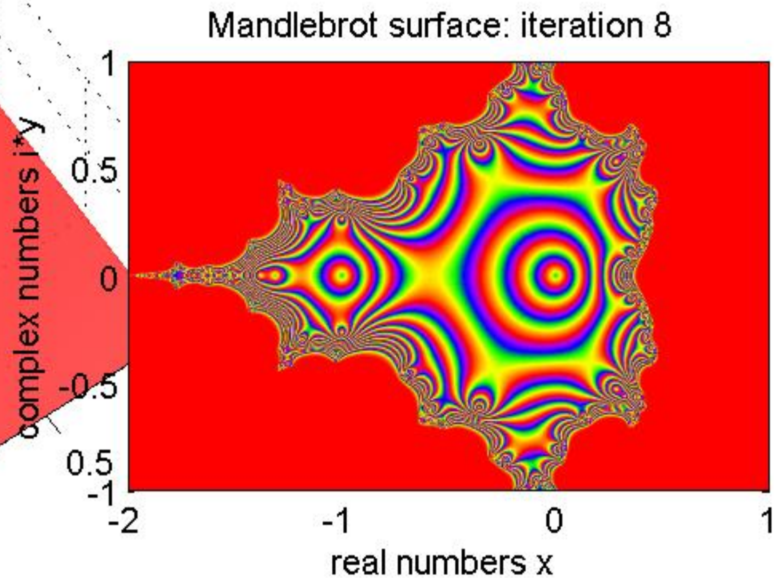


$$x = \mathrm{Re}(z), \quad y = \mathrm{Im}(z)$$

$$h(x, y) = e^{-\sqrt{x^2 + y^2}}$$

$$z_{n+1} = z_n^2 + z_0$$





$$x = \mathrm{Re}(z), \quad y = \mathrm{Im}(z)$$

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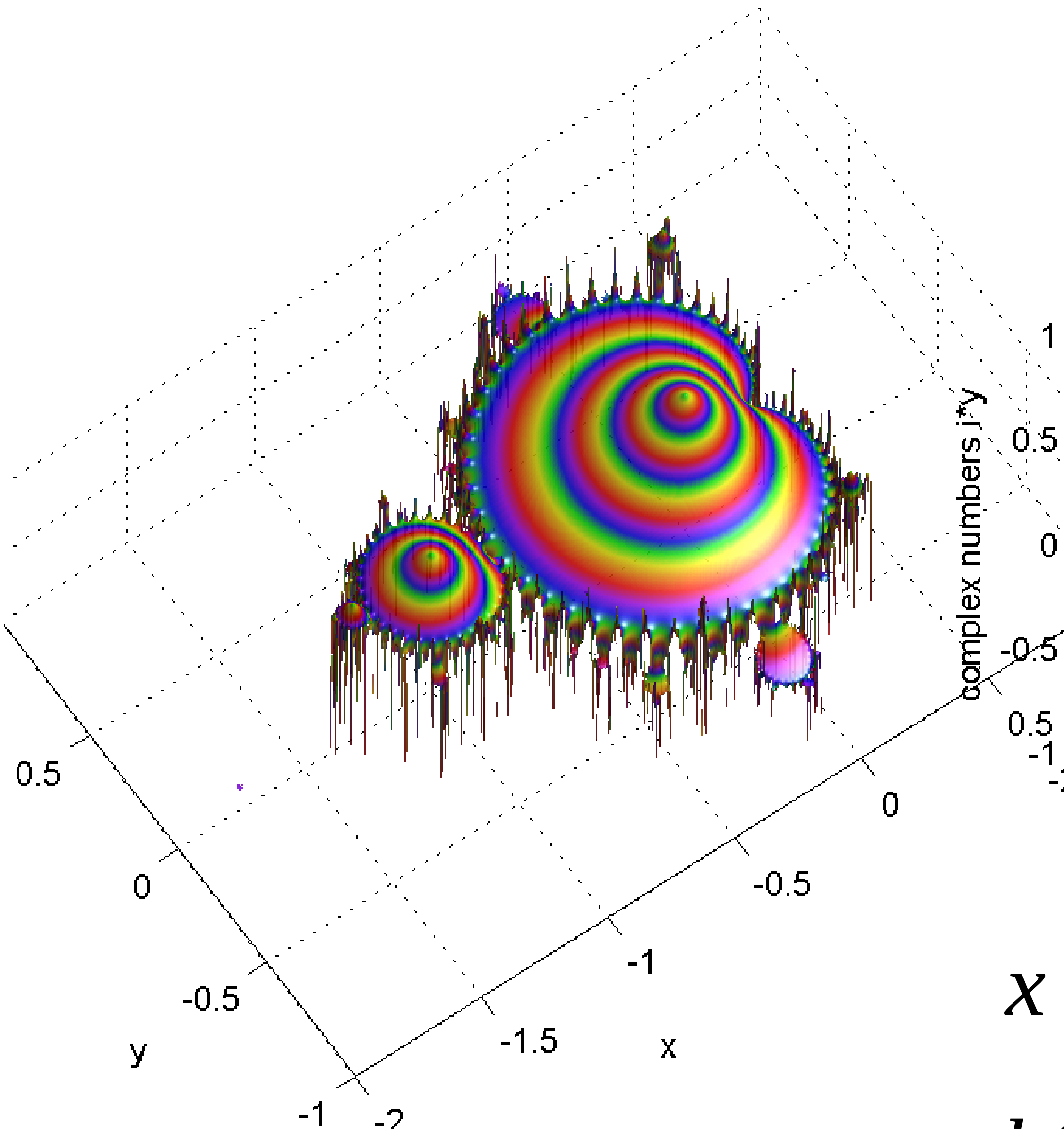
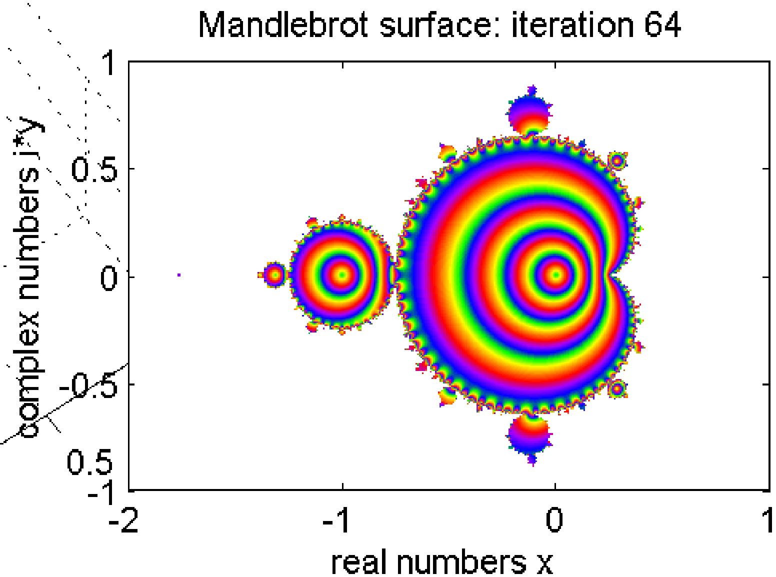
$$z_{n+1} = z_n^2 + z_0$$





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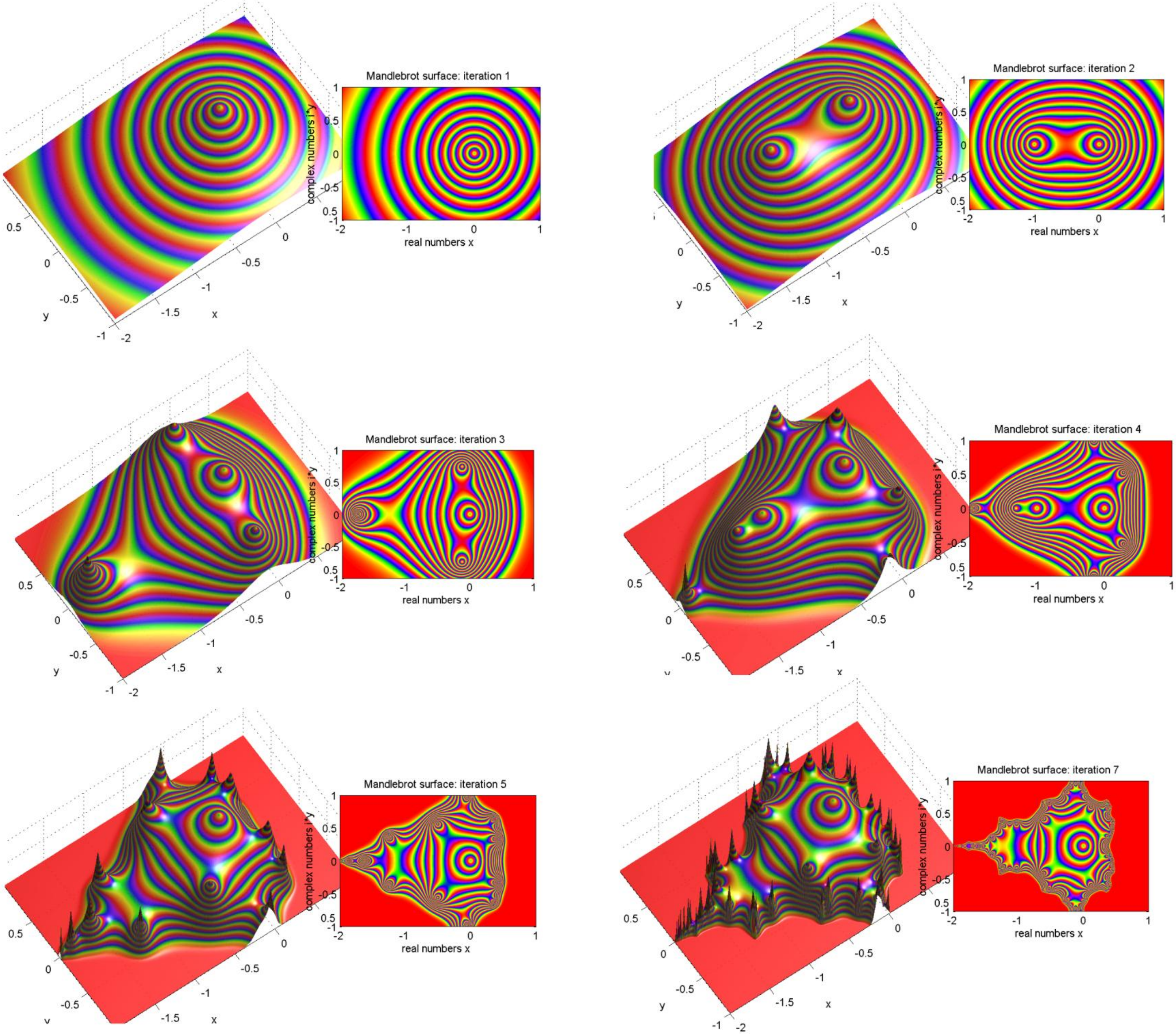
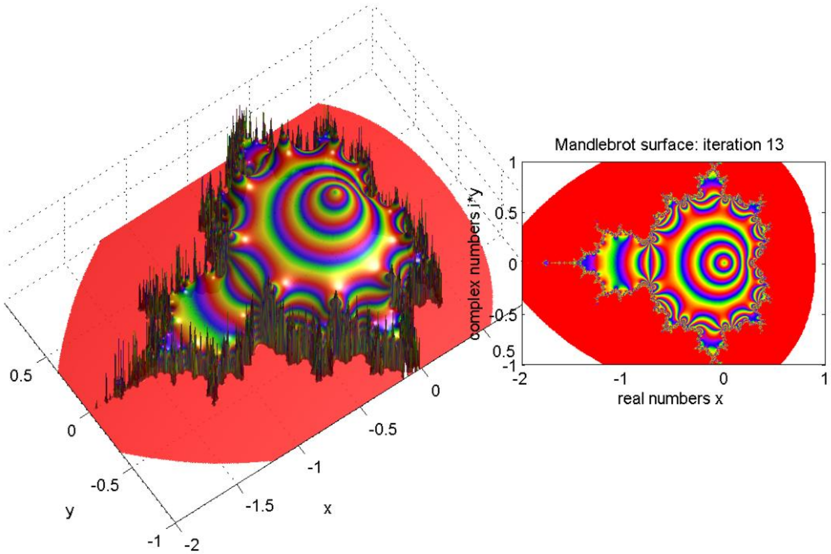
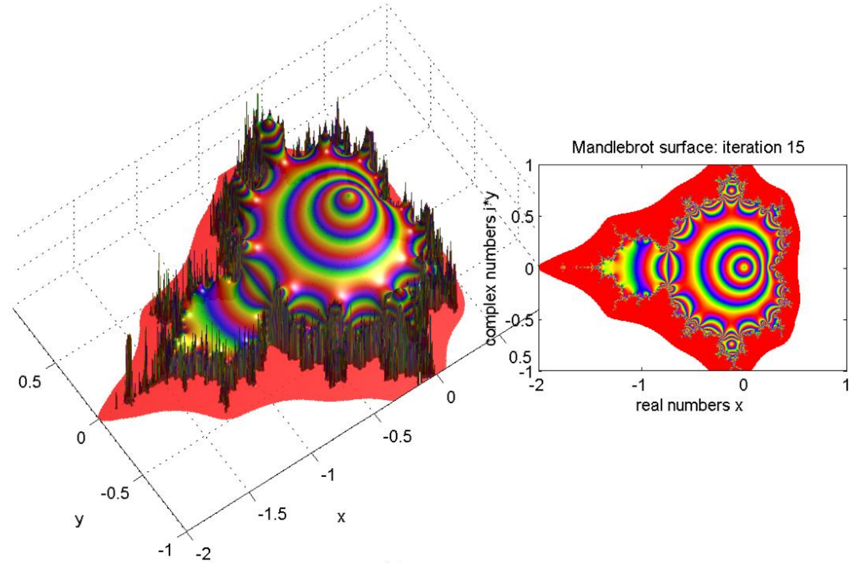
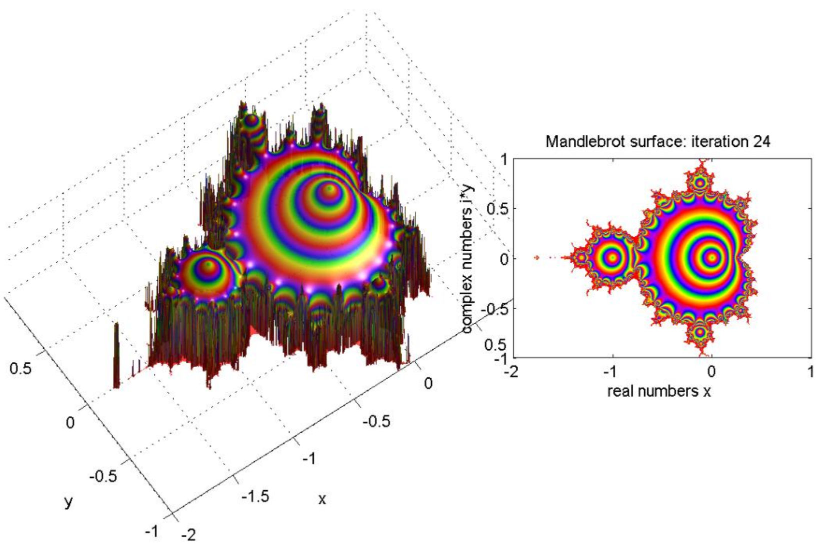
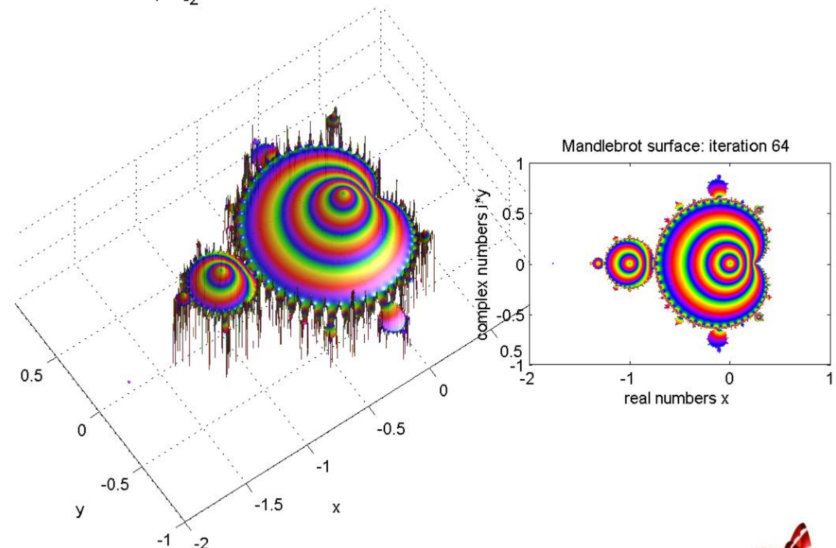

Mandlebrot surface: iteration 1
Mandlebrot surface: iteration 2
Mandlebrot surface: iteration 3
Mandlebrot surface: iteration 4
Mandlebrot surface: iteration 5
Mandlebrot surface: iteration 7
real numbers x
complex numbers i*y
x
y







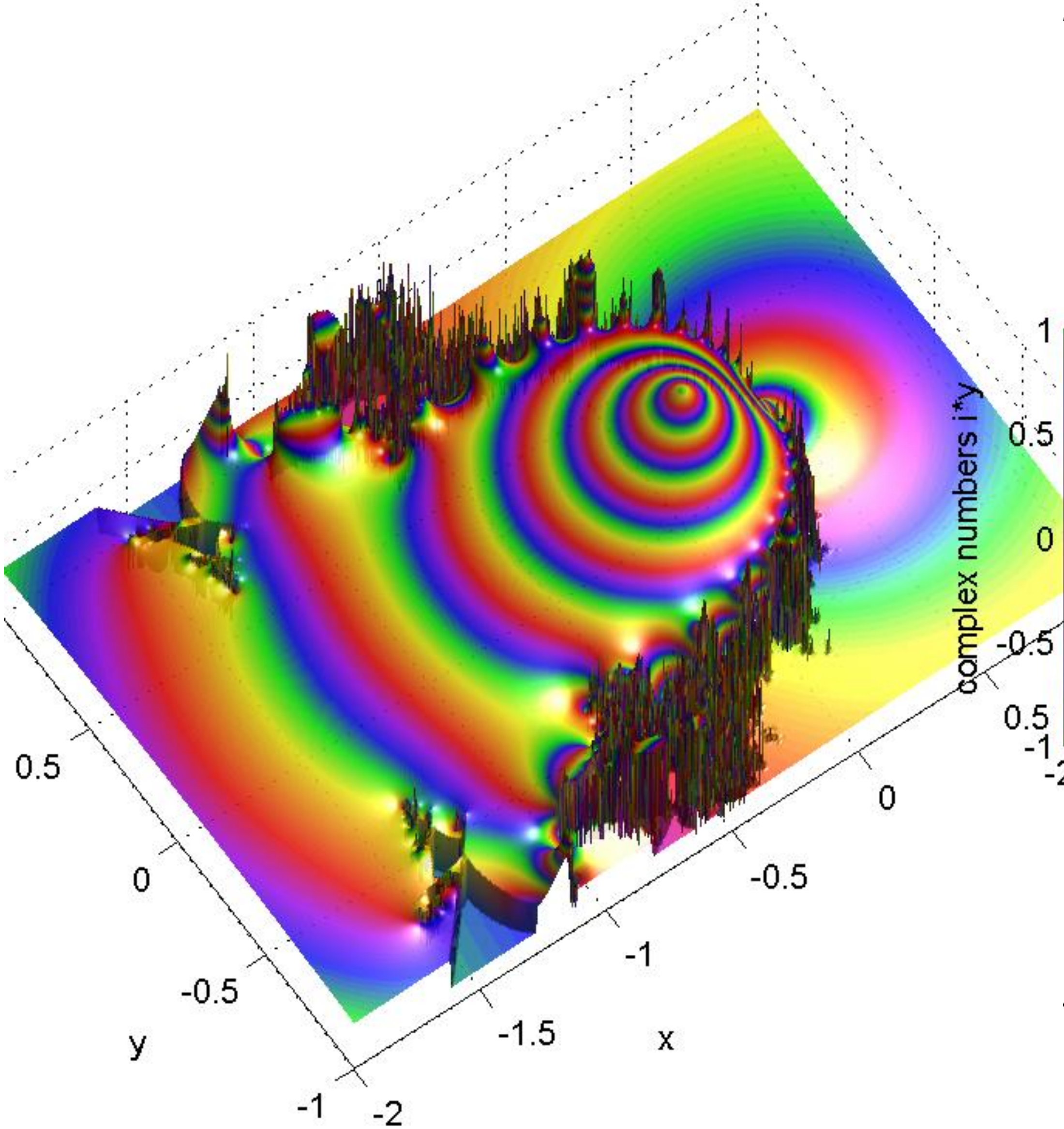


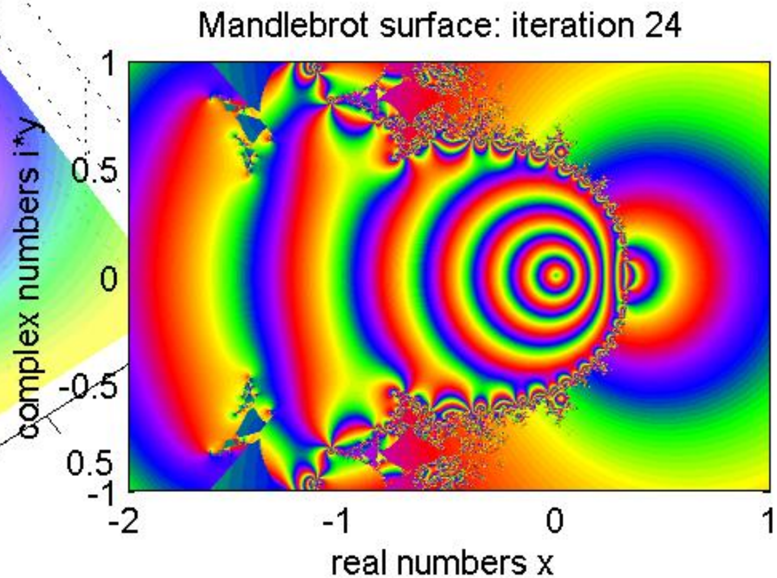
Selection from *Day of Julia.*
Mathematicon Exhibition, 2014

μathematicon

7 steps to enlightenment

$$z_{n+1} = \tan^{-1}\left(z_n^2 + z_0\right)$$





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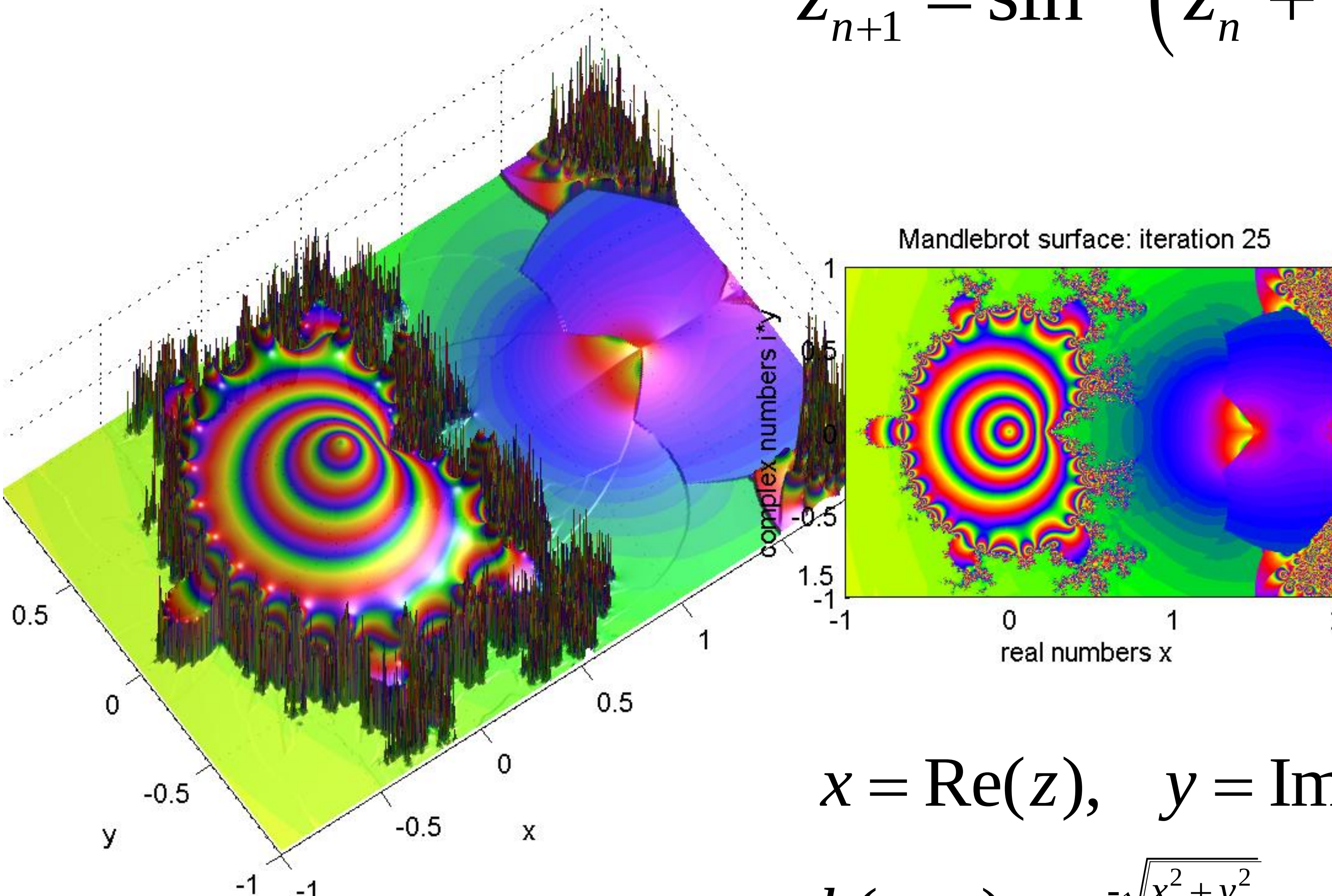
The Mandlerocket

$$z_{n+1} = \sin^{-1}\left(z_n^2 + z_0\right)$$



$$x = \mathrm{Re}(z), \quad y = \mathrm{Im}(z)$$

$$h(x, y) = e^{-\sqrt{x^2 + y^2}}$$