Slinky Physics

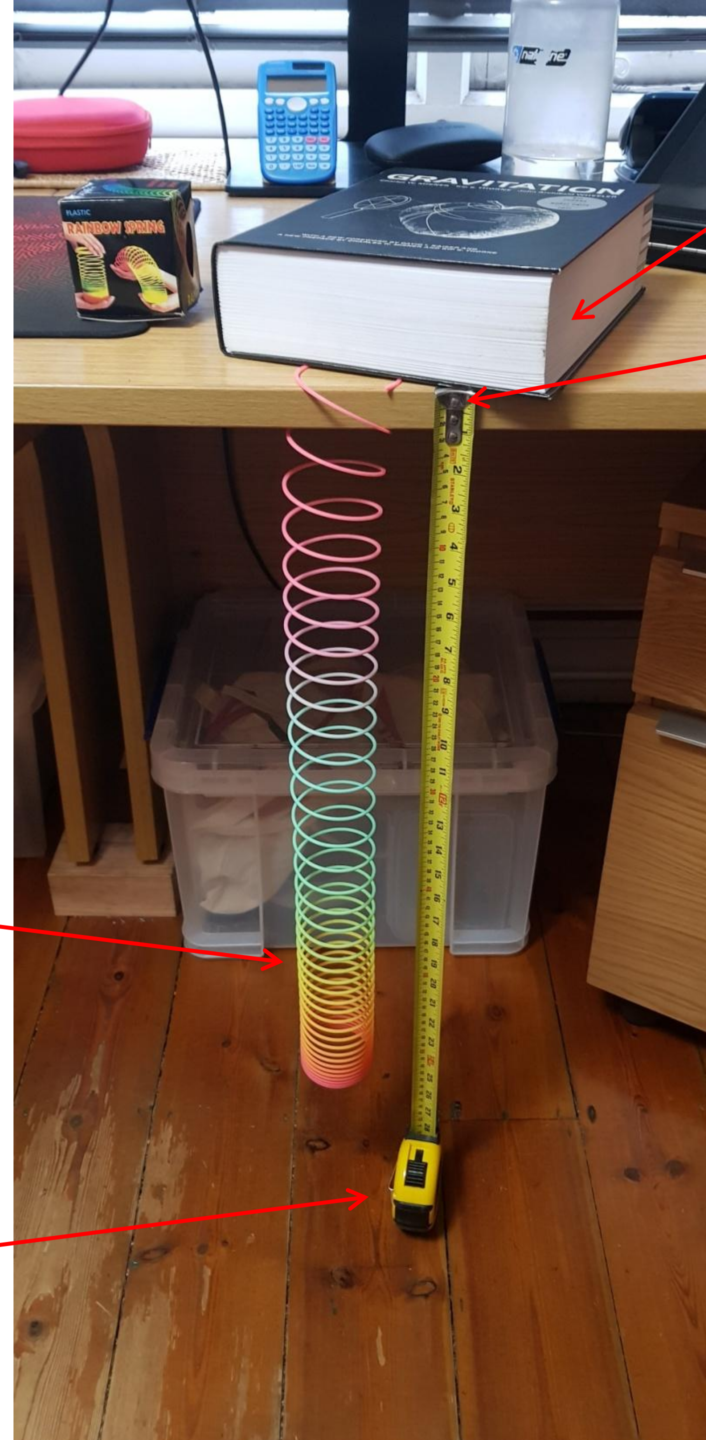
[Youtube video]

Dr Andrew French. June 2022.

Equipment setup

38 (approx) coil ‘[Rainbow Spring]’

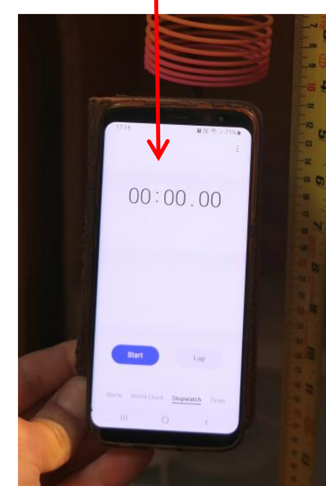
Retractable steel tape measure (or other rigid measuring system like a metre rule)



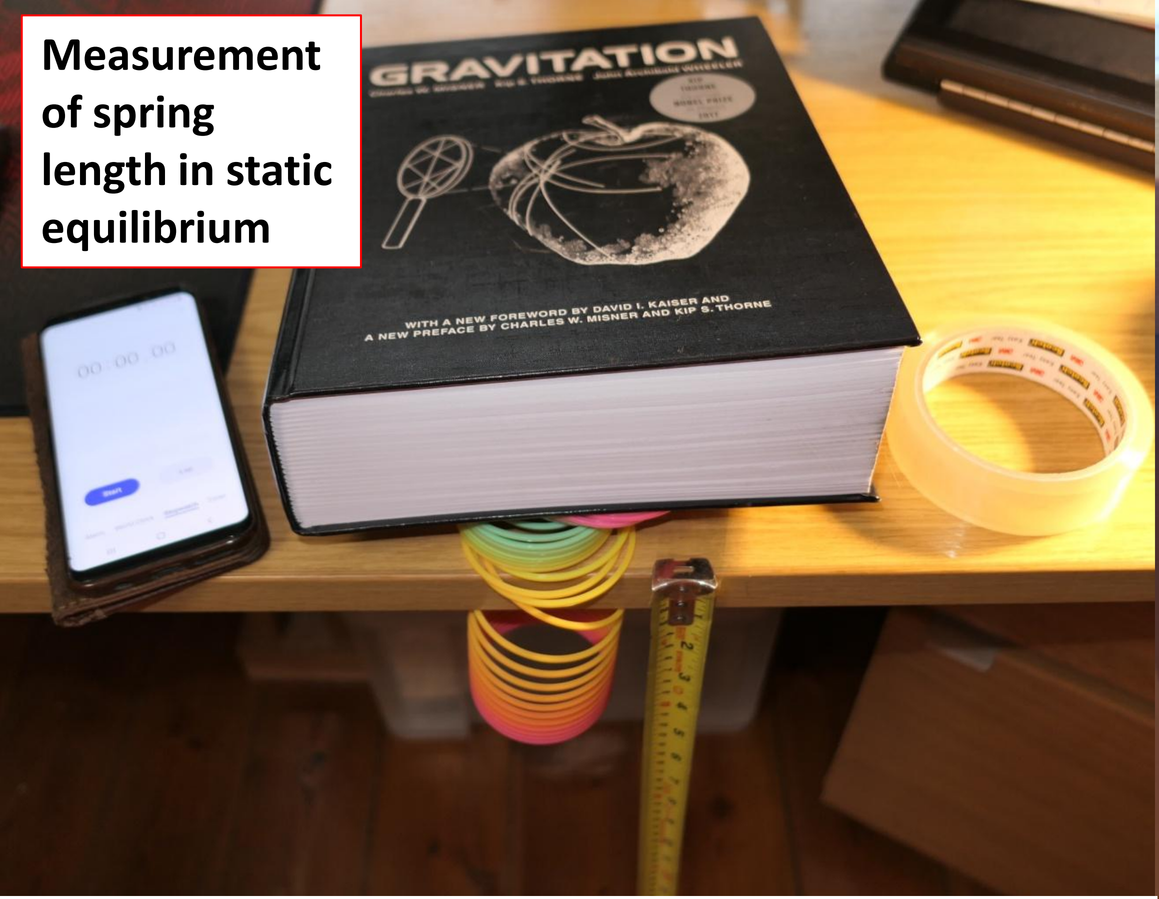
A weighty book to hold coils in place on desk

Sellotape to fix the tape measure to desk

Stopwatch (e.g. a smartphone app)



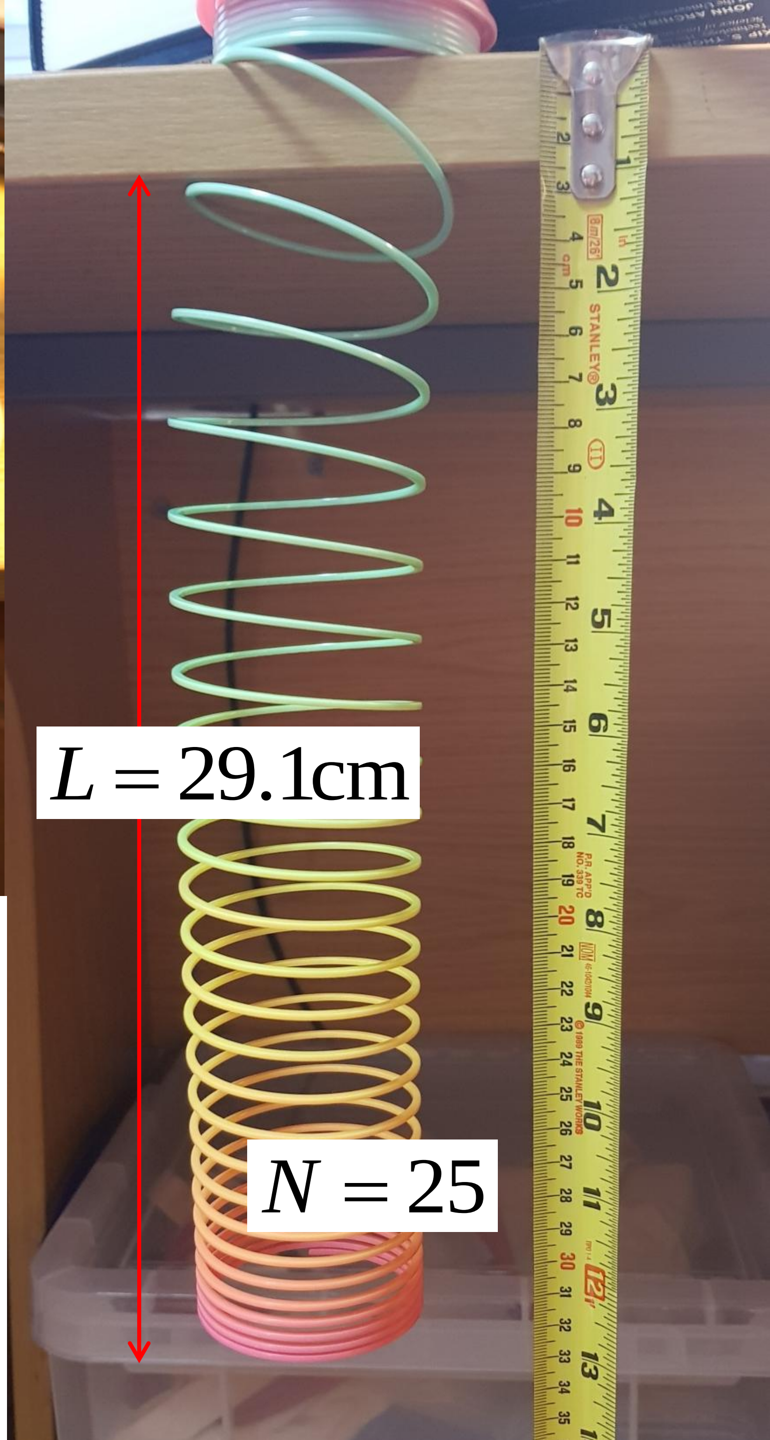
Measurement of spring length in static equilibrium

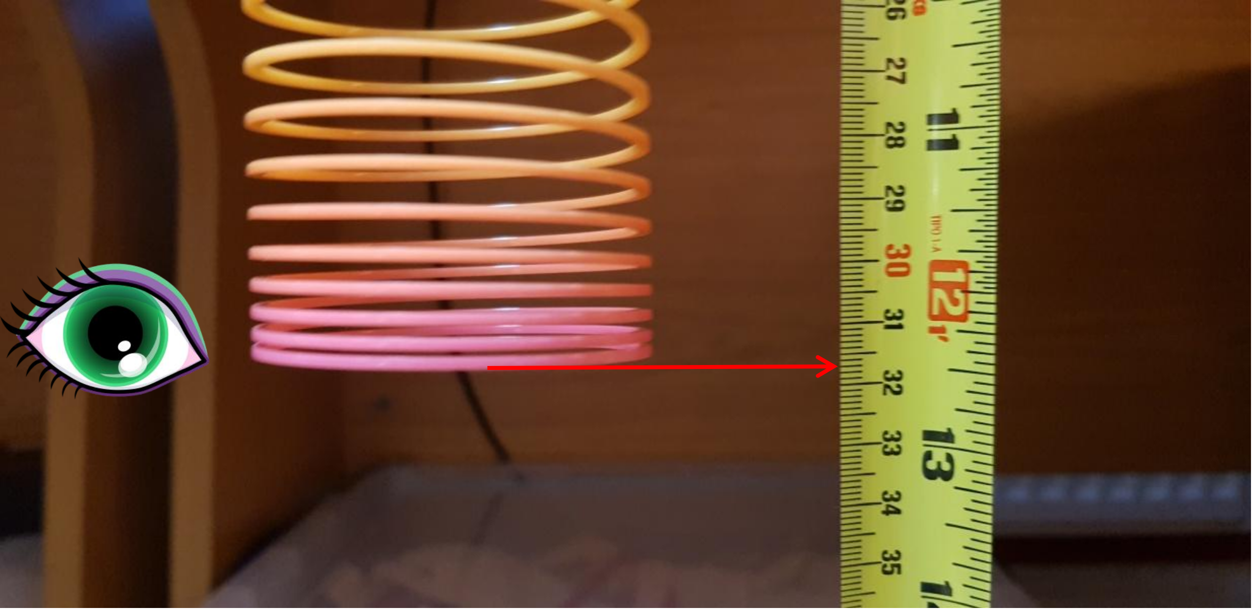


Use the heavy book to trap coils at the top of the desk.

The idea is to **measure the length L of the hung slinky vs the number of coils N** that are freely hanging from the desk.

Start with just one a coil under the book (i.e. a fully extended slinky) and then reduce by two coils until 11 are hanging freely.

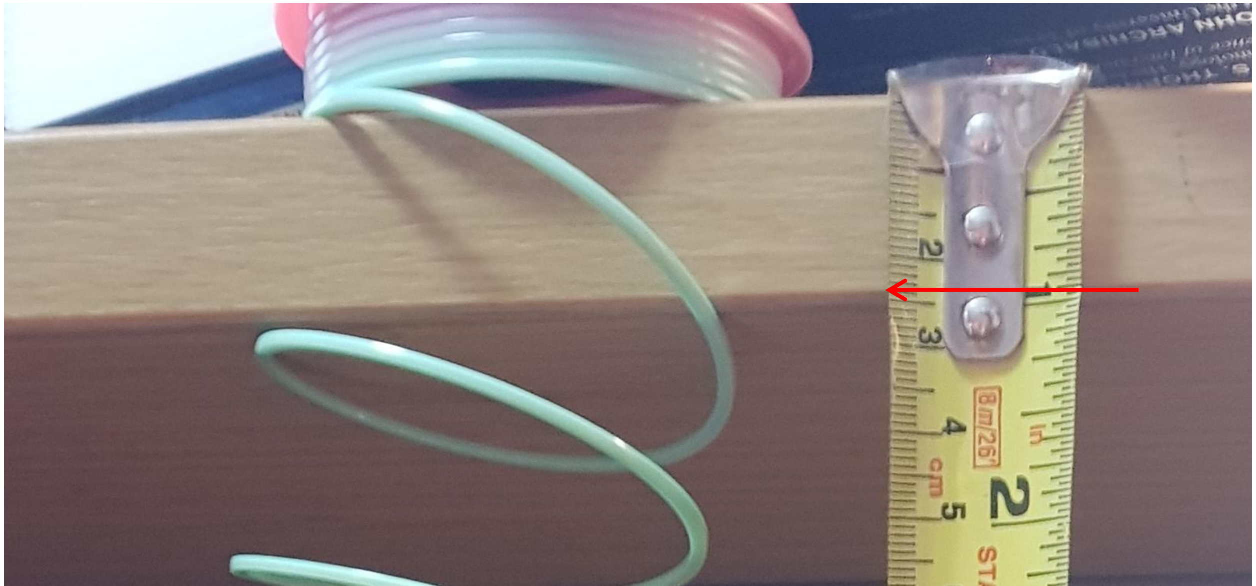




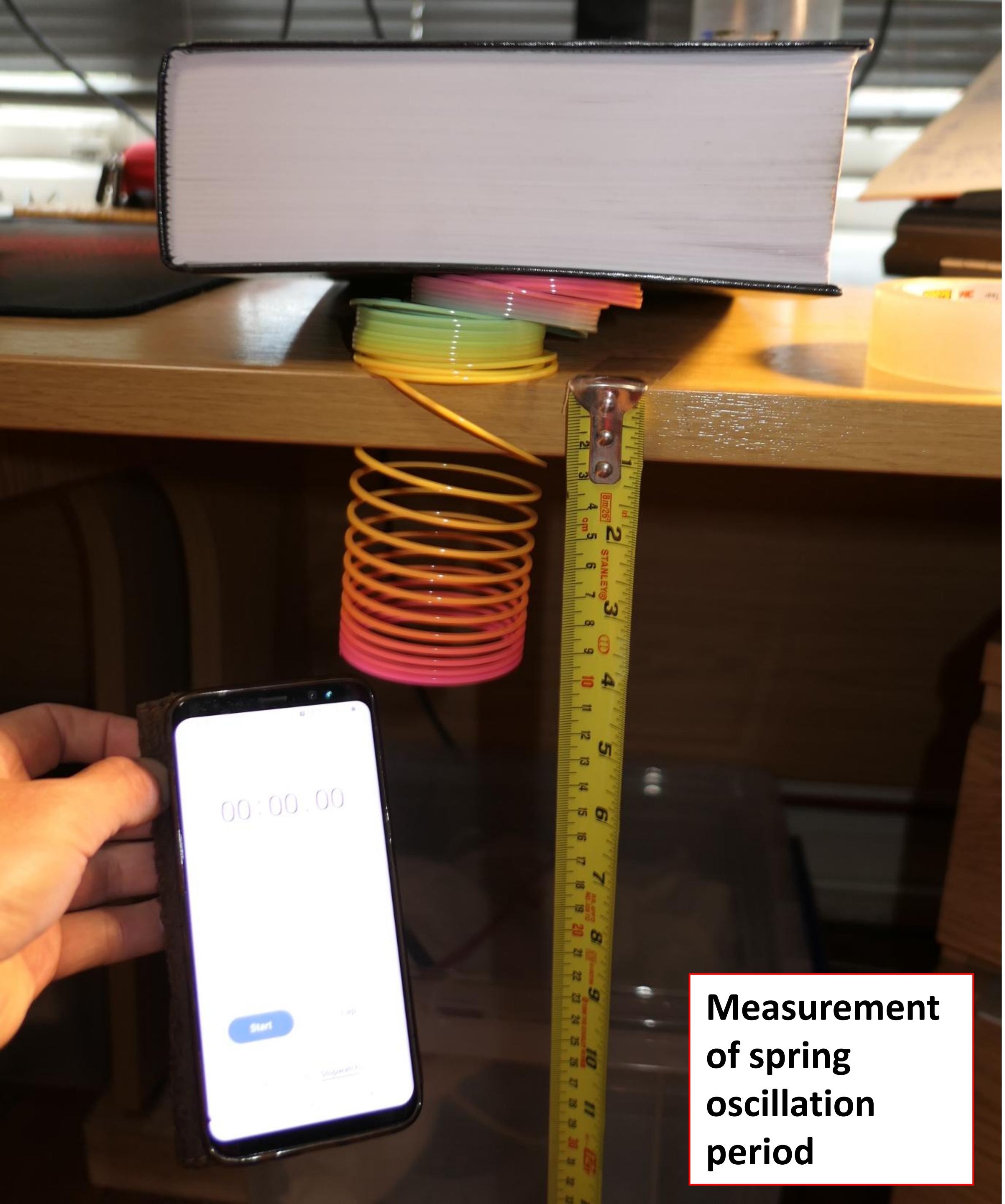
Crouch down such that your eye is level with the bottom coil of the slinky when making a measurement.

This will improve the **precision** and **accuracy** of your length measurement and avoid *parallax error*.

Don’t forget to subtract the distance to the bottom edge of your desk from your measuring tape values of the bottom of the spring. (In my case this was 2.4cm).



Measurement of spring length in static equilibrium



Measurement of spring oscillation period

For a given number of coils hung N, pull down the slinky by about 5cm and record **ten oscillation periods using a stopwatch.**

To improve accuracy, first practice hitting the stopwatch start and stop button using your thumb *without taking your eyes of the spring*.

If you can start and stop while observing the spring all the time, you will probably reduce your timing errors to less than a tenth of a second over ten periods. (Which means we might be able to *ignore* random timing errors, and you can get away with a single measurement for each N).

(There will still be a small *systematic* error! – See later)

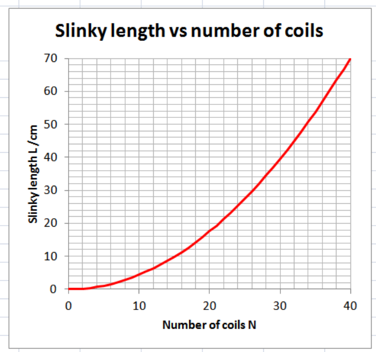
Summary of instructions

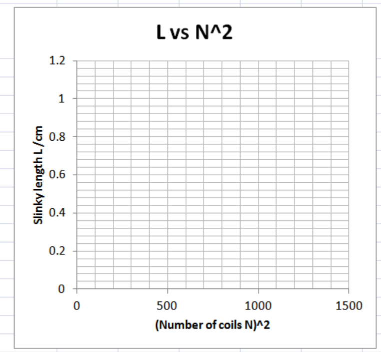
1. **Set up slinky experiment**, and check you can extend the slinky, two coils at a time. Check your book is heavy enough. Check that the fully extended slinky doesn't hit the floor.
2. Before you take any oscillation period measurements, **practice that you can start and stop the stopwatch *without taking your eyes off the slinky*!**
3. **Set up a spreadsheet** (or use my template). Have a computer to hand so you can record measurements directly as you go, and plot graphs as you go also. ***This is a really good experimental habit*** as (i) you will conduct the experiment efficiently (ii) you are well aware of what you are measuring and why (iii) you can spot any anomalies (iv) you can take more data at interesting phases (e.g. when a curve reaches a point of curvature or maximum)

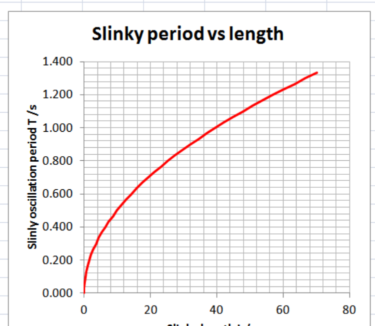
SLINKY PHYSICS
NAME. DATE

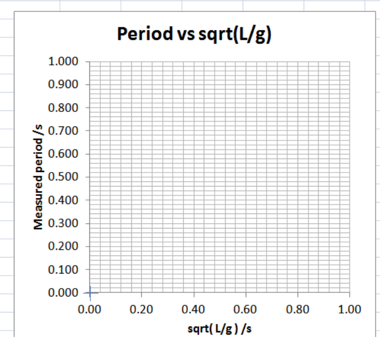
Number of coils N	Slinky length L /cm	10 periods run 1	10 periods run 2	10 periods run 3	Period T /s	Period error /s
11					0.000	
13					0.000	
15					0.000	
17					0.000	
19					0.000	
21					0.000	
23					0.000	
25					0.000	
27					0.000	
29					0.000	
31					0.000	
33					0.000	
35					0.000	
37					0.000	

N^2	sqrt(L/g) /s
121	0.00
169	0.00
225	0.00
289	0.00
361	0.00
441	0.00
529	0.00
625	0.00
729	0.00
841	0.00
961	0.00
1089	0.00
1225	0.00
1369	0.00









Enter length and ten-period measurements here.

Repeats are probably not necessary unless you have plenty of lab time!

$$L = \beta N^2$$
$$\beta = 0.0439\text{cm}$$
$$T = \alpha\sqrt{\frac{L}{g}}$$
$$\alpha = 4.98$$

You might find slightly different constants!

Graphs should *auto-plot* as you take measurements

MODEL

N	L/cm	T /s
0	0.000	0.000
1	0.044	0.033
2	0.176	0.067
3	0.395	0.100
4	0.702	0.133
5	1.098	0.167
6	1.580	0.200
7	2.151	0.233
8	2.810	0.267
9	3.556	0.300
10	4.390	0.333
11	5.312	0.366
12	6.322	0.400
13	7.419	0.433
14	8.604	0.466
15	9.878	0.500
16	11.238	0.533
17	12.687	0.566
18	14.224	0.600
19	15.848	0.633
20	17.560	0.666
21	19.360	0.700
22	21.248	0.733
23	23.223	0.766
24	25.286	0.800
25	27.438	0.833
26	29.676	0.866
27	32.003	0.899
28	34.418	0.933
29	36.920	0.966
30	39.510	0.999
31	42.188	1.033
32	44.954	1.066
33	47.807	1.099
34	50.748	1.133
35	53.778	1.166
36	56.894	1.199
37	60.099	1.233
38	63.392	1.266
39	66.772	1.299
40	70.240	1.333

Analysis of slinky measurements

Average of three runs is probably overkill – only do this if you have plenty of time. Notice how small the *errors* are (I calculated these from a standard deviation over three repeats).

Add a $y = mx$ *trendline* to a graph of L vs N^2 to work out the constant of proportionality β of model: $L = \beta N^2$

SLINKY PHYSICS

Dr Andrew French. 19/06/2022

Number of coils N	Slinky length L /cm	10 periods run 1	10 periods run 2	10 periods run 3	Period T /s	Period error /s
11	7.5	3.92	3.94	3.88	0.391	0.003
13	9.6	4.56	4.63	4.58	0.459	0.004
15	11.9	4.95	5.17	5.11	0.508	0.011
17	14.4	5.74	5.69	5.65	0.569	0.005
19	16.9	6.22	6.15	6.21	0.619	0.004
21	20.4	6.81	6.78	6.94	0.684	0.009
23	24.5	7.73	7.73	7.77	0.774	0.002
25	29.1	8.77	8.71	8.73	0.874	0.003
27	34	9.42	9.36	9.31	0.936	0.006
29	38.4	9.87	9.99	9.86	0.991	0.007
31	42.5	10.47	10.39	10.49	1.045	0.005
33	46.9	11.03	11.03	10.97	1.101	0.003
35	51.7	11.91	11.92	11.7	1.184	0.012
37	57.3	12.35	12.42	12.38	1.238	0.004

N^2	sqrt(L/g) /s
121	0.09
169	0.10
225	0.11
289	0.12
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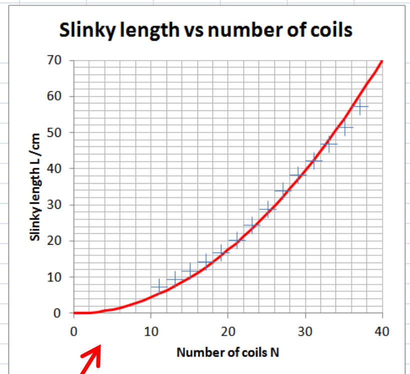
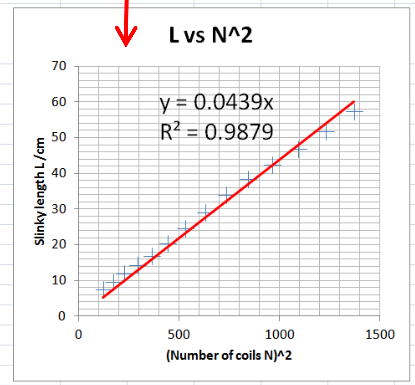
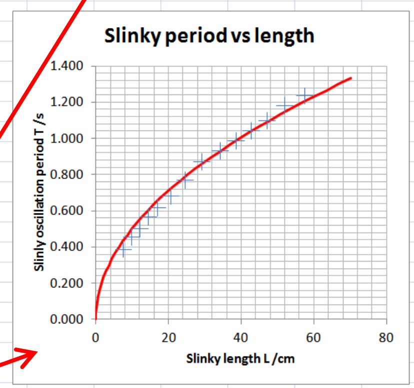
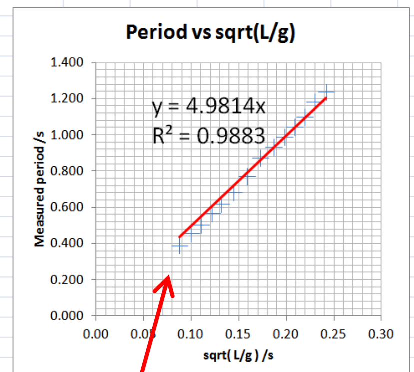
Calculate N^2 and $\sqrt{L/g}$

$$L = \beta N^2$$
$$\beta = 0.0439\text{cm}$$
$$T = \alpha \sqrt{\frac{L}{g}}$$
$$\alpha = 4.98$$









MODEL

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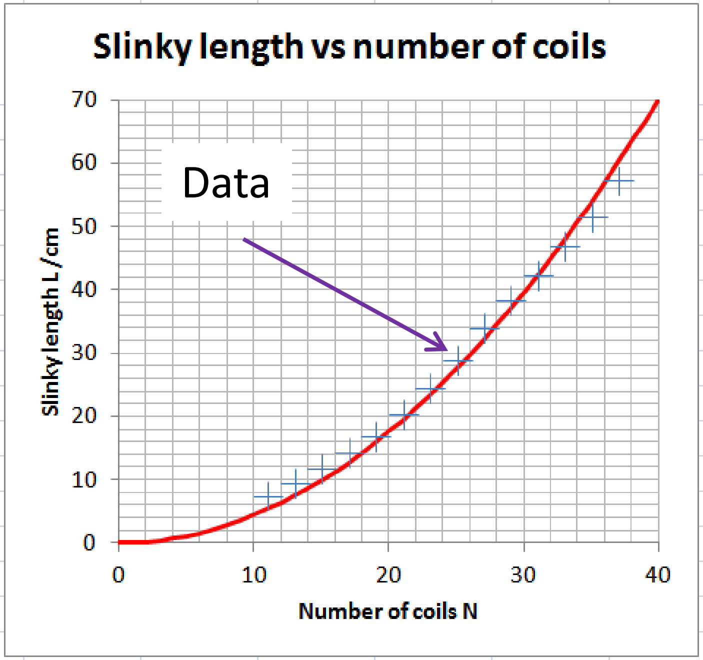
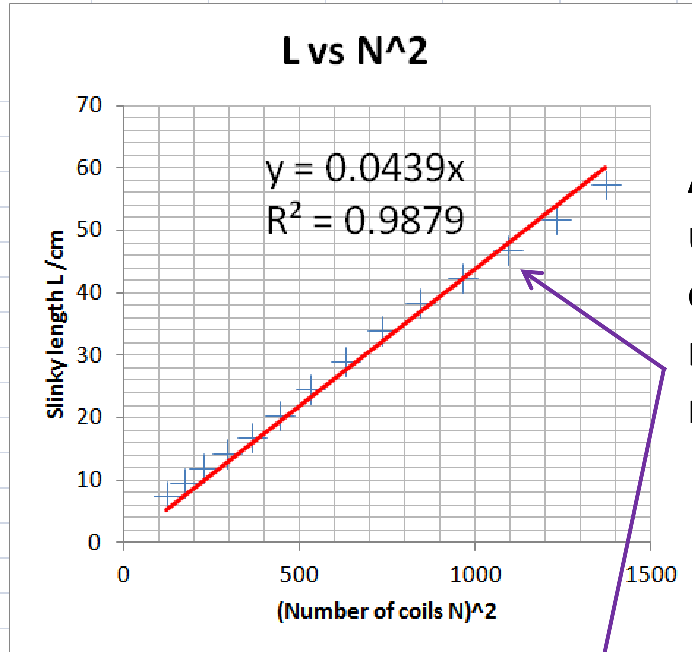
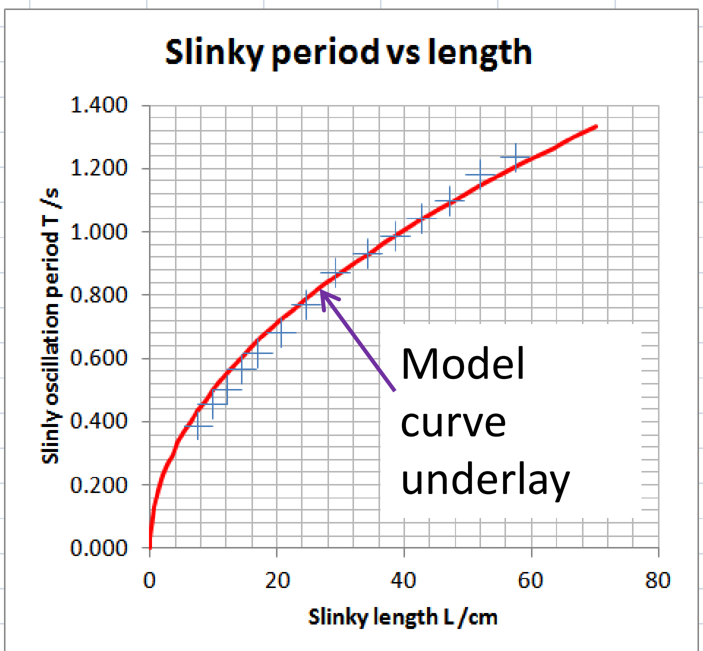
Underlay L vs N and T vs L **measurements** (crosses **+**, no joining lines) with **model curves** (no markers)

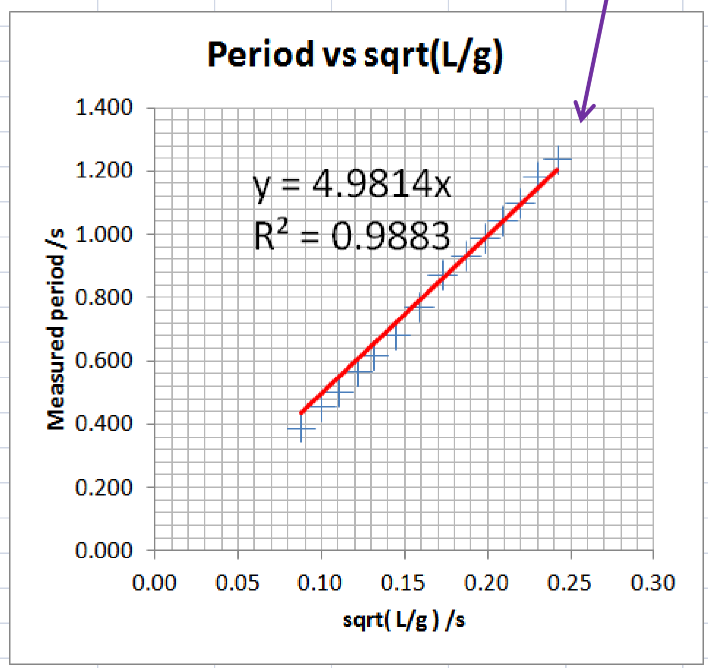
Add a $y = mx$ *trendline* to a graph of period T vs $\sqrt{L/g}$ to work out the constant of proportionality α of model: $T = \alpha\sqrt{L/g}$





An R^2 value close to unity implies a strong correlation between model and measurement.





$$L = \beta N^2$$

$$\beta = 0.0439\text{cm}$$

$$T = \alpha\sqrt{\frac{L}{g}}$$

$$\alpha = 4.98$$

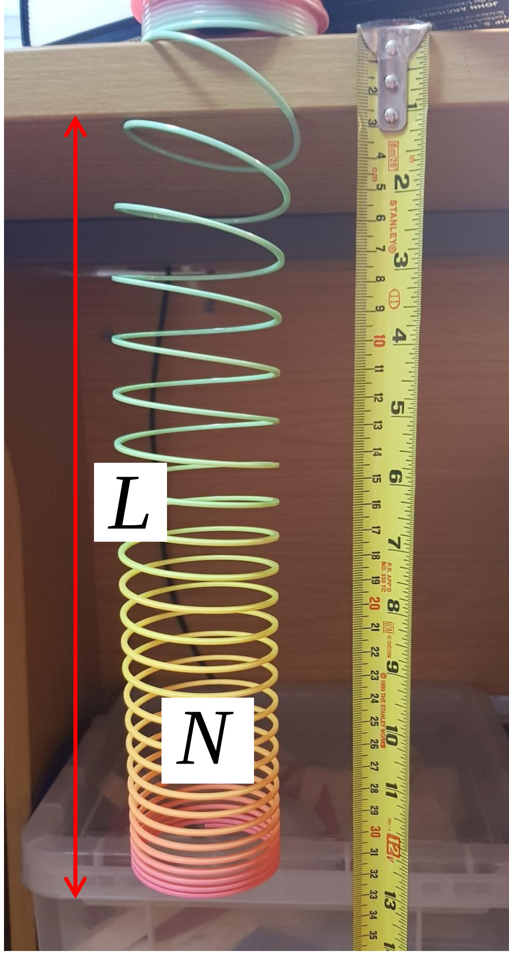
Model, with parameters α and β determined from the line of best fit gradients

Theoretical model of vertically hung slinky

$$L = \beta N^2$$

oscillation period →

$$T = \alpha \sqrt{\frac{L}{g}}$$



References:

1. BPhO Challenge experiments by Keith Gibbs (static stretching of a plastic helical spring)
2. Jörg Pretz (2021) Oscillations of a suspended slinky. *Eur. J. Phys*. **42** 045008
3. A. P. French (1994) The suspended Slinky—A problem in static equilibrium. *The Physics Teacher* 32, 244 https://doi.org/10.1119/1.2343983
4. P. Gluck (2010) A project on soft springs and the slinky. *Phys. Educ.* **45** 178
5. Richard A. Young (1993) Longitudinal standing waves on a vertically suspended slinky. *American Journal of Physics* **61**, 353; https:// doi.org/10.1119/1.17270

A coil at displacement x from the bottom of the desk is separated by a from the coil above it.

If the spring is Hookean, the tension F at x is:

$$F = \kappa a$$

where κ is the inter-coil spring stiffness (assumed to be a constant property of a given slinky).

In *equilibrium* the **tension** F is balanced by the **weight** of the slinky below the coil at x

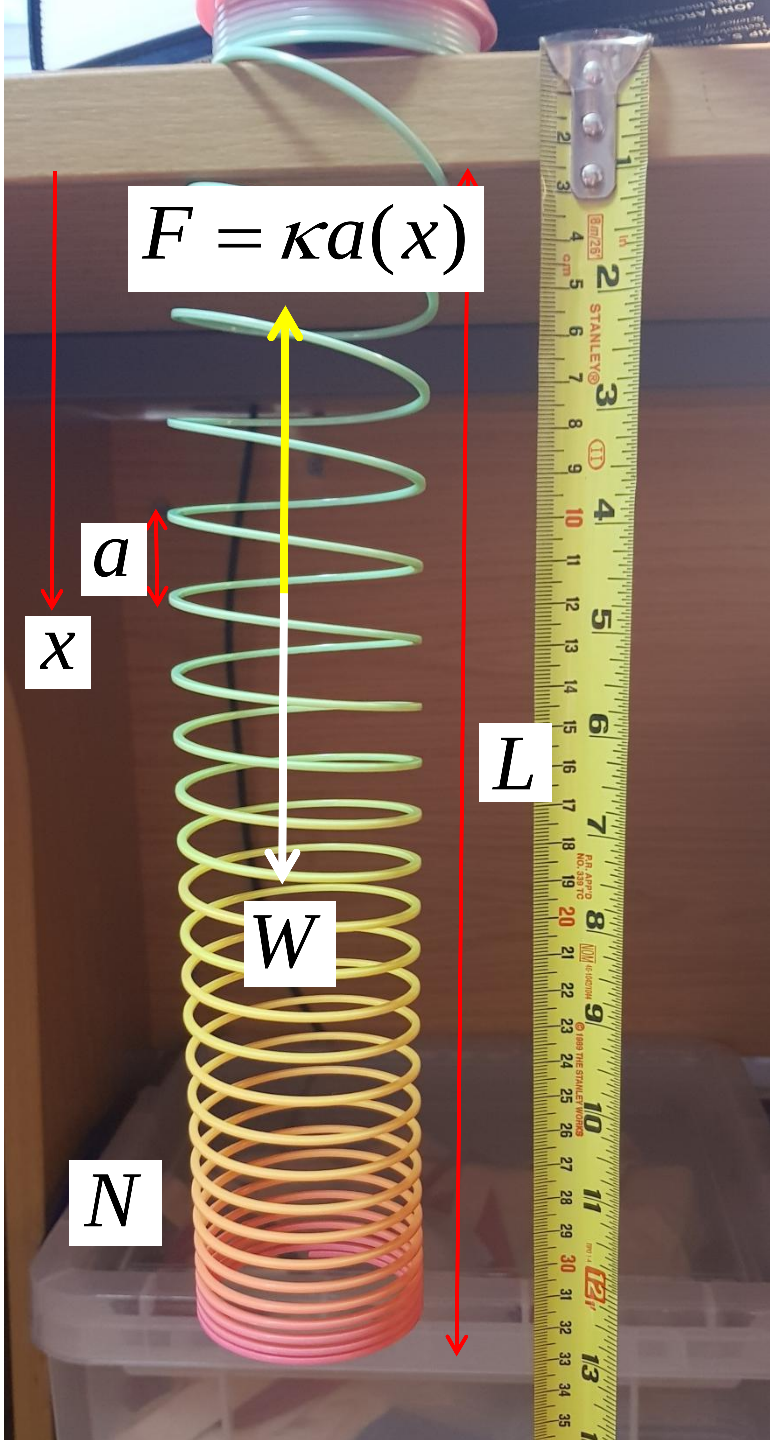
$$F = W$$

$$W = n(x)mg$$

where m is the mass of a coil (i.e. total spring mass divided by total number of coils) and g is the strength of gravity, which we shall take as g = 9.81Nkg^{-1}.

The number of coils hanging below x is:

$$n(x) = \int_x^L \frac{dx'}{a(x')} = -\int_L^x \frac{dx'}{a(x')}$$



Hence in equilibrium where $F = W$:

$$\kappa a(x) = -mg\int_L^x \frac{dx'}{a(x')}$$

Differentiating:

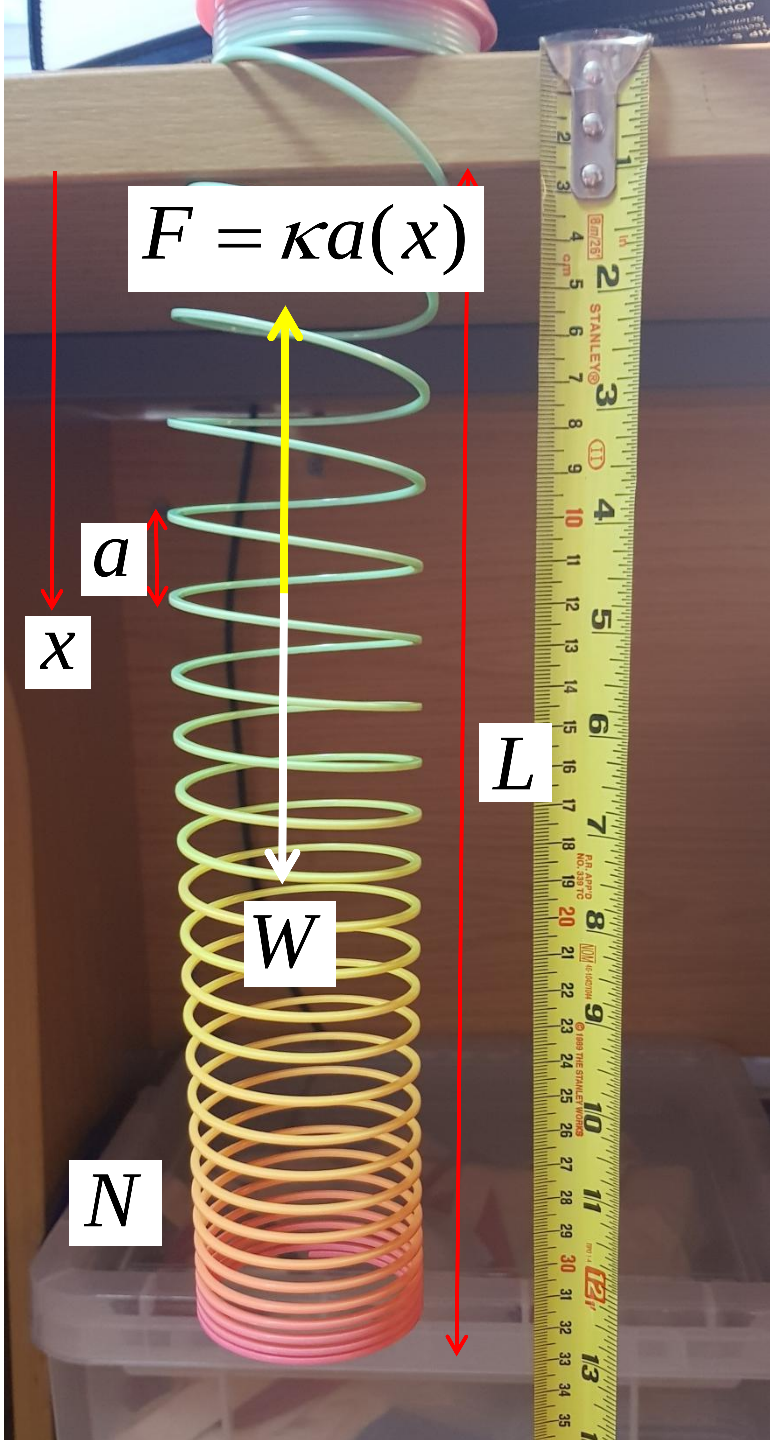
$$\kappa \frac{da}{dx} = -\frac{mg}{a}$$

Hence, by separating ‘the variables’

$$\int a\,da = -\frac{mg}{\kappa}\int dx$$

$$\tfrac{1}{2}a^2 = -\frac{mgx}{\kappa} + c$$

When $x = L, a = 0$ $\quad \therefore c = \dfrac{mgL}{\kappa}$



Hence:

$$a(x)=\sqrt{\frac{2mg}{\kappa}}(L-x)^{\frac{1}{2}}$$

The number of coils below x is therefore:

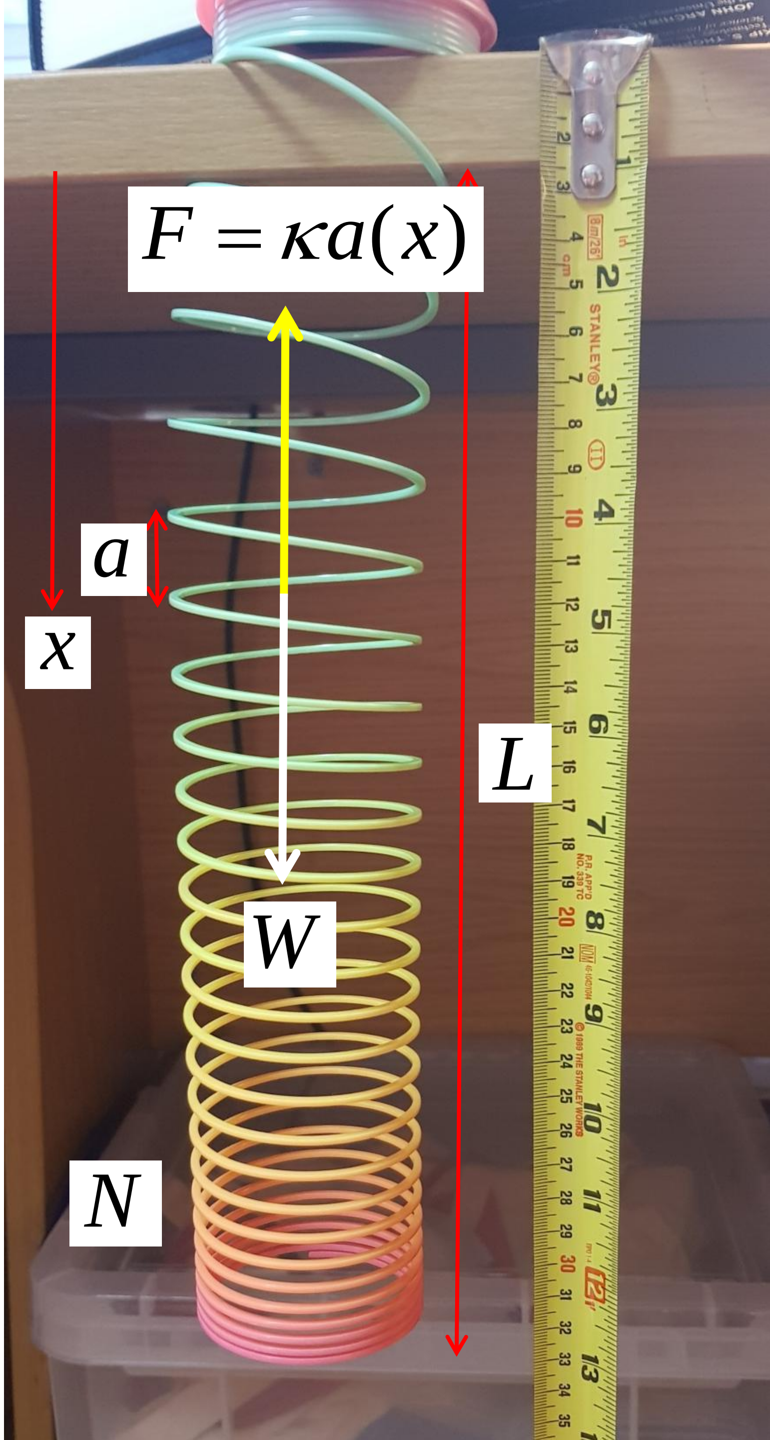
$$n(x)=\int_x^L \frac{dx'}{a(x')}=\int_x^L \frac{dx'}{\sqrt{\frac{2mg}{\kappa}}(L-x')^{\frac{1}{2}}}$$

$$n(x)=\sqrt{\frac{\kappa}{2mg}}\int_x^L (L-x')^{-\frac{1}{2}}\,dx'$$

$$=\sqrt{\frac{\kappa}{2mg}}\left[-2(L-x')^{\frac{1}{2}}\right]_x^L$$

$$=\sqrt{\frac{\kappa}{2mg}}\left(0-(-2)(L-x)^{\frac{1}{2}}\right)$$

$$\therefore n(x)=\sqrt{\frac{2\kappa(L-x)}{mg}}$$



In summary:

Coil separation at displacement x from bottom of table:

$$a(x) = \sqrt{\frac{2mg}{\kappa}}\left(L - x\right)^{\frac{1}{2}}$$

Number of coils $n(x)$ below x is:

$$n(x) = \sqrt{\frac{2\kappa(L - x)}{mg}}$$

Total number of coils N (i.e. below x = 0)

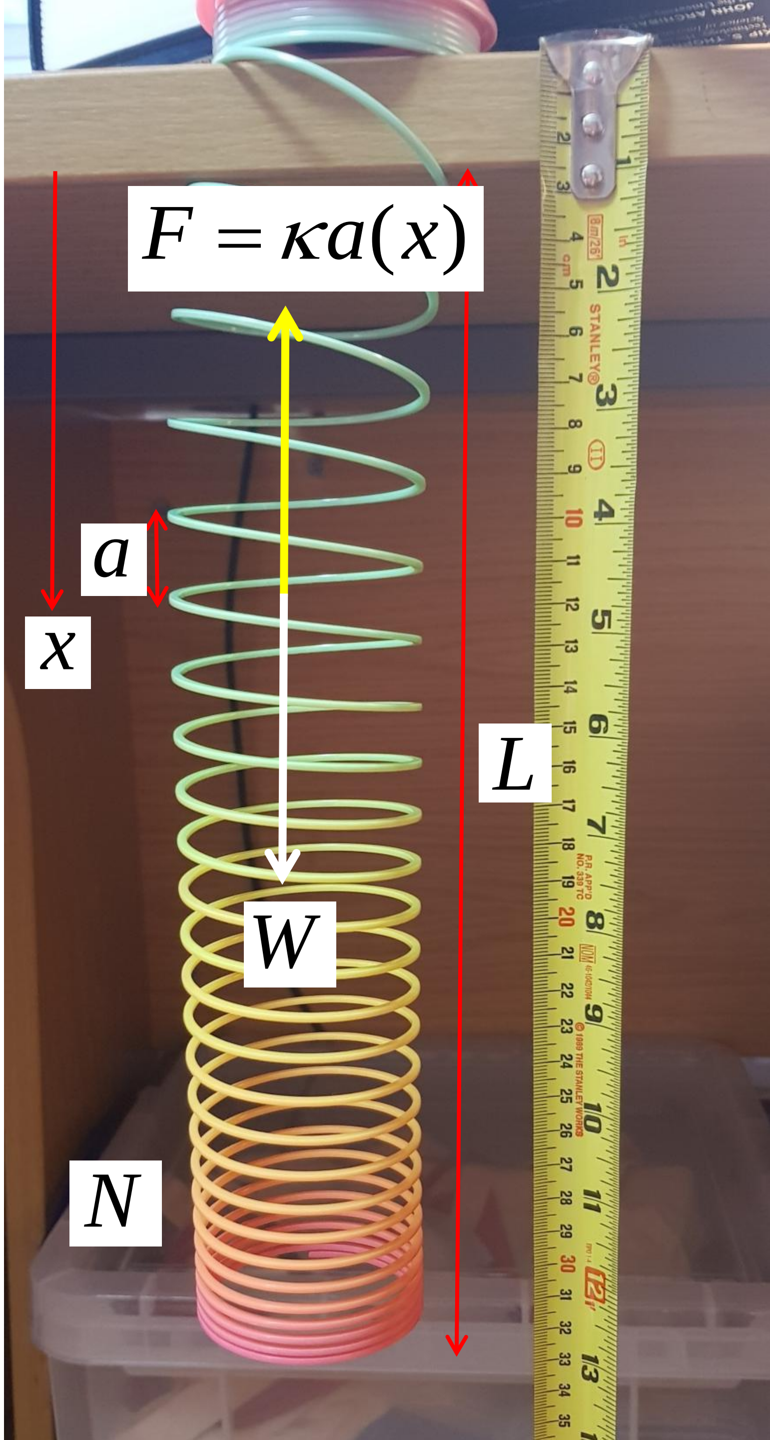
$$N = \sqrt{\frac{2\kappa L}{mg}}$$

Hence length L of vertical hung slinky is:

$$L = \tfrac{1}{2}\frac{mg}{\kappa}N^2$$

so constant

$$\beta = \tfrac{1}{2}\frac{mg}{\kappa}$$



Elastic waves on a tensioned string travel at speed

$$v = \sqrt{\frac{F}{\mu}}$$

where F is the string tension and m is the mass per unit length. For our slinky model:

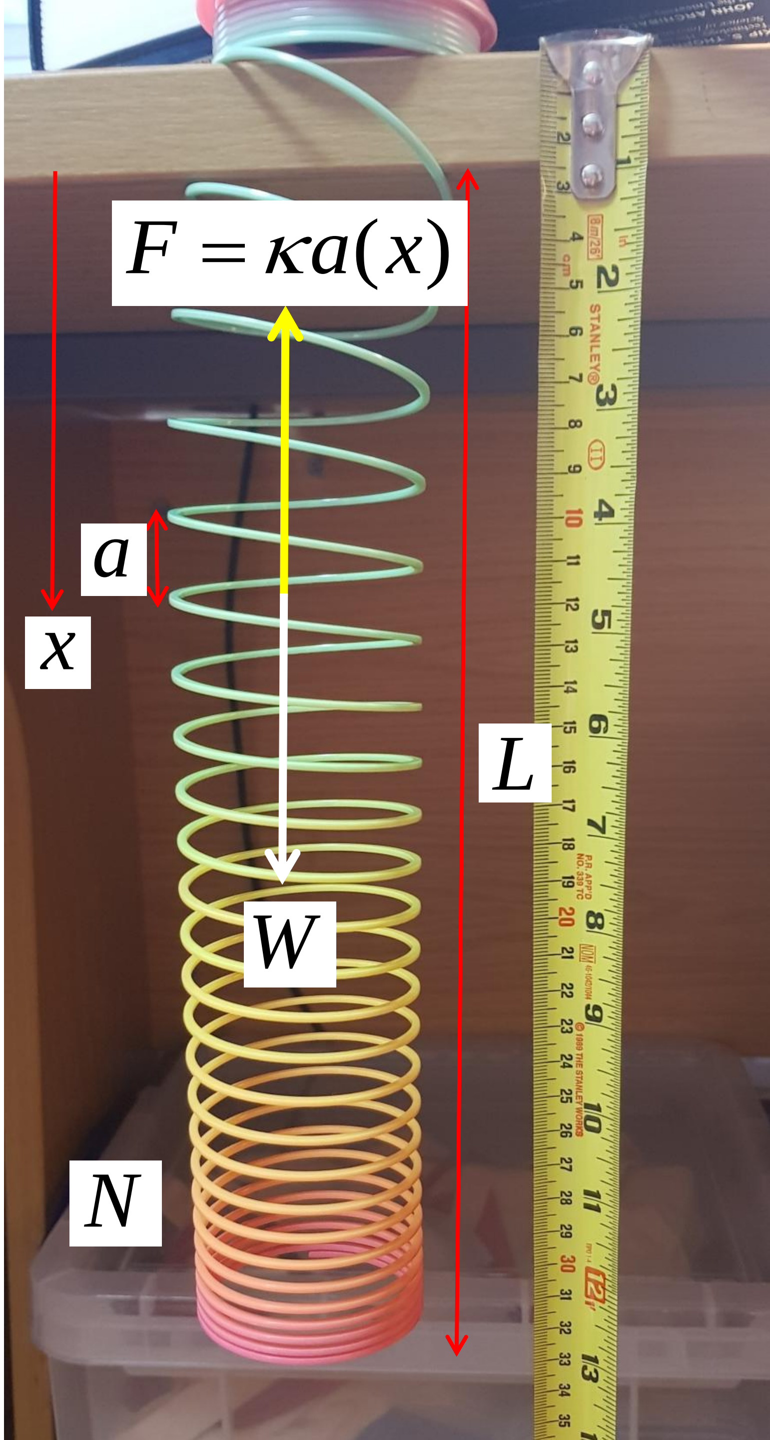
$$F(x) = \kappa a(x), \quad \mu(x) = \frac{m}{a(x)}$$

$$\therefore v = \sqrt{\frac{\kappa a}{m/a}} = \sqrt{\frac{\kappa}{m}}a$$

$$\boxed{a(x) = \sqrt{\frac{2mg}{\kappa}}(L-x)^{\frac{1}{2}}}$$ is the inter-coil spacing

Hence: $$v = \sqrt{\frac{\kappa}{m}}\sqrt{\frac{2mg}{\kappa}}(L-x)^{\frac{1}{2}}$$

$$\boxed{\therefore v = \sqrt{2g}(L-x)^{\frac{1}{2}}}$$ i.e. wave speed *varies* with x



The variation of wave speed v vs x allows us to calculate the time for longitudinal waves to traverse the length of the slinky:

$$t = \int_0^L \frac{dx}{v(x)}, \qquad v = \sqrt{2g}\left(L - x\right)^{\frac{1}{2}}$$

$$\therefore t = \frac{1}{\sqrt{2g}} \int_0^L \left(L - x\right)^{-\frac{1}{2}} dx$$

$$\therefore t = \frac{1}{\sqrt{2g}} \left[-2\left(L - x\right)^{\frac{1}{2}}\right]_0^L = \frac{1}{\sqrt{2g}} 2\sqrt{L}$$

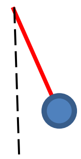
Therefore a ‘round trip’ of waves from the bottom to the top of the slinky and back should take:

$$\tau = \frac{4}{\sqrt{2}}\sqrt{\frac{L}{g}}$$

This expression is *similar* to our empirical model result for oscillation period

$$T = \alpha\sqrt{\frac{L}{g}}$$

This is also like the period of a *pendulum* of length L



$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\frac{4}{\sqrt{2}} \approx 2.83$$

Is this the same as the longitudinal wave period T? No.

...... It is a ***factor of two*** out!

Now for vertical longitudinal oscillations we have a node of displacement at the top of the slinky and an antinode at the base (i.e. the freely moving end). **Gluck** shows that, by **solving the *wave equation*** using a spatial variable ‘stretch distance per turn’, period T is:

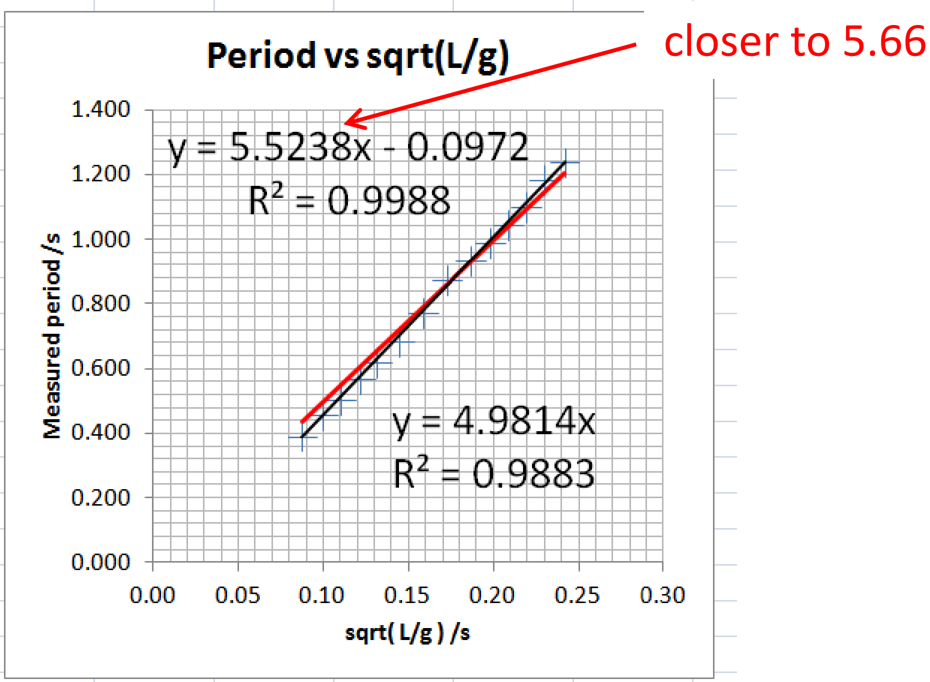
$$T = 2\tau$$

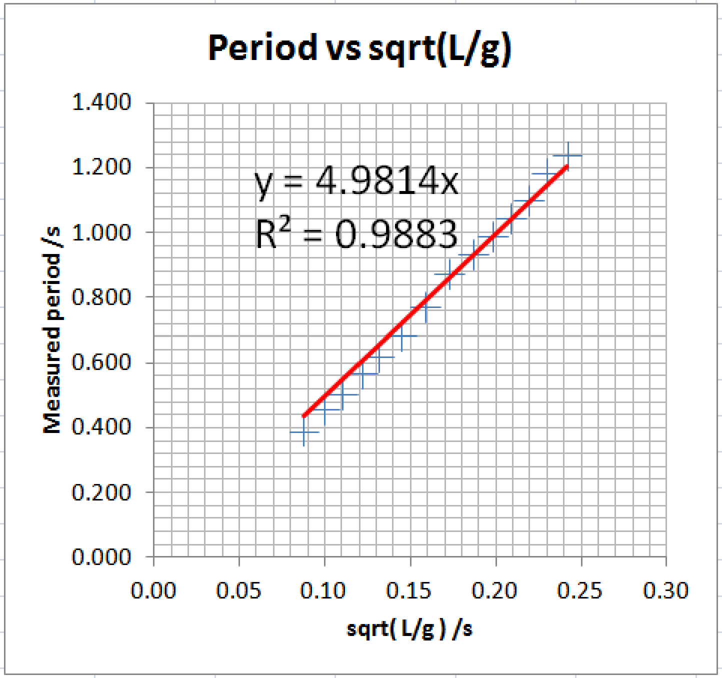
for the fundamental mode. i.e. *twice* the ‘round trip’ time. Hence:

$$T = \alpha\sqrt{\frac{L}{g}}$$

$$\alpha = 4\sqrt{2} \approx 5.66$$

If we contrast a *y = mx* with a *y = mx +c* trendline, *the latter offers closer agreement*, implying a <u>small systematic error</u> which we can attribute to delays in starting and stopping the stopwatch. It seems we are under-measuring by about 0.097s.





Analysis modification using *y* = *mx* + *c* fit of *T* vs sqrt(*L/g*)

Incorporate systematic time offset of 0.0972s in model

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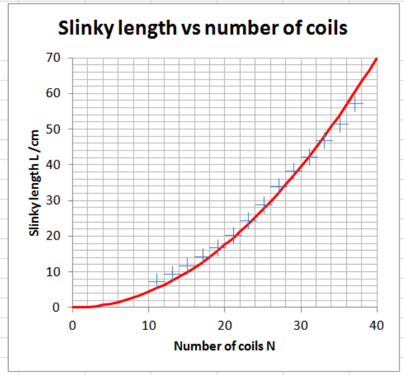
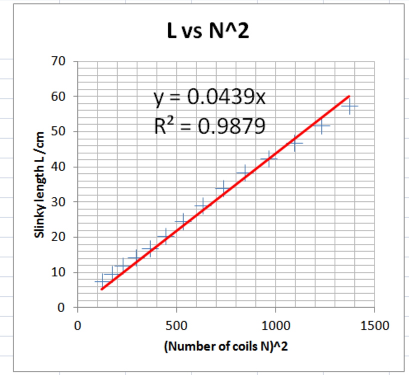
1225	0.23
1369	0.24





THEORY

$$L = \beta N^2 = \tfrac{1}{2}\frac{mg}{\kappa}N^2$$

$$T = 4\sqrt{2}\sqrt{\frac{L}{g}} \approx 5.66\sqrt{\frac{L}{g}}$$

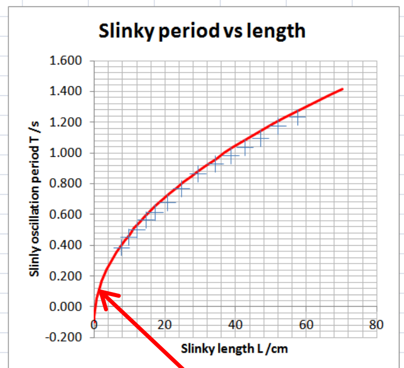
EMPIRICAL

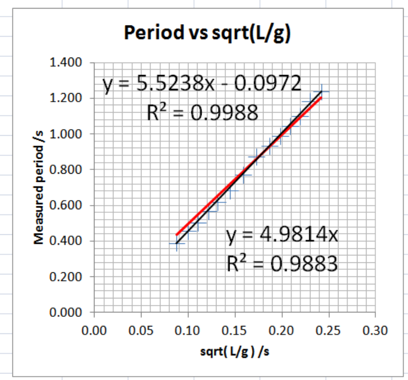
$$L = \beta N^2$$

$$\beta = 0.0439\text{cm}$$

$$T = \alpha\sqrt{\frac{L}{g}}$$

$$\alpha = 4.98$$





MODEL

N	L/cm	T /s
0	0.000	-0.097
1	0.044	-0.059
2	0.176	-0.021
3	0.395	0.016
4	0.702	0.054
5	1.098	0.092
6	1.580	0.130
7	2.151	0.168
8	2.810	0.206
9	3.556	0.244
10	4.390	0.281
11	5.312	0.319
12	6.322	0.357
13	7.419	0.395
14	8.604	0.433
15	9.878	0.471
16	11.238	0.509
17	12.687	0.546
18	14.224	0.584
19	15.848	0.622
20	17.560	0.660
21	19.360	0.698
22	21.248	0.736
23	23.223	0.774
24	25.286	0.812
25	27.438	0.849
26	29.676	0.887
27	32.003	0.925
28	34.418	0.963
29	36.920	1.001
30	39.510	1.039
31	42.188	1.077
32	44.954	1.114
33	47.807	1.152
34	50.748	1.190
35	53.778	1.228
36	56.894	1.266
37	60.099	1.304
38	63.392	1.342
39	66.772	1.379
40	70.240	1.417

$$T = 4\sqrt{2}\sqrt{\frac{L}{g}} - 0.0972$$

Predict measured oscillation period

Although perhaps a better plot would be to *add the time offset to the data, rather than the model,* which means $T = 0$ when $N = 0$.

Intuitive explanation for 'factor of two'?

Standing waves with a node at one end and an antinode at the other end require the **length of a wave-system to be an odd number of quarter wavelengths.**

Therefore for a 'round trip', *the wave must traverse half a wavelength*. So for a full wavelength to be completed, this corresponds to **two 'round trips**.'

More slinky videos!

Stair walk

Slinky falling downs stairs

World Record (2016) slinky stair walk

Bottom of slinky *doesn't move* until longitudinal wave catches up!

Slinky drop 1000 FPS

Veritasium explains slinky drop

Extension with tennis ball

Slinky drop from the *Slo-Mo-Guys*

Silly and impressive

Slinky dance double act