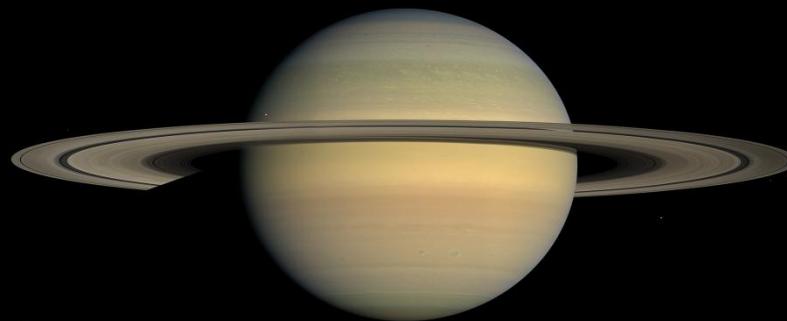
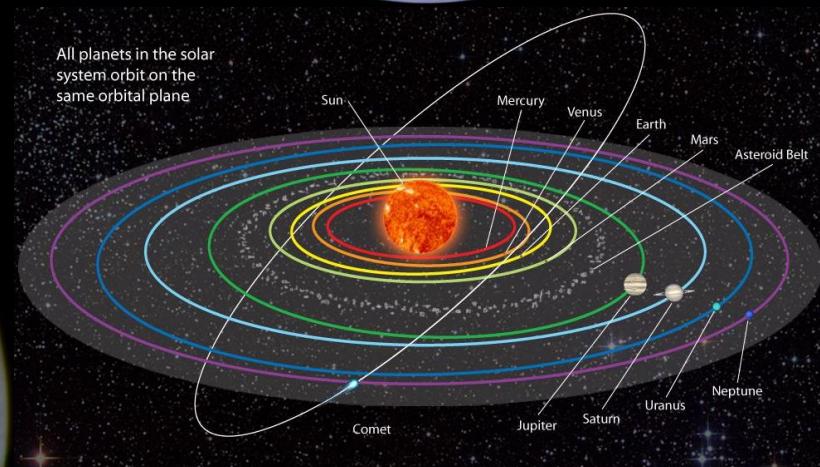
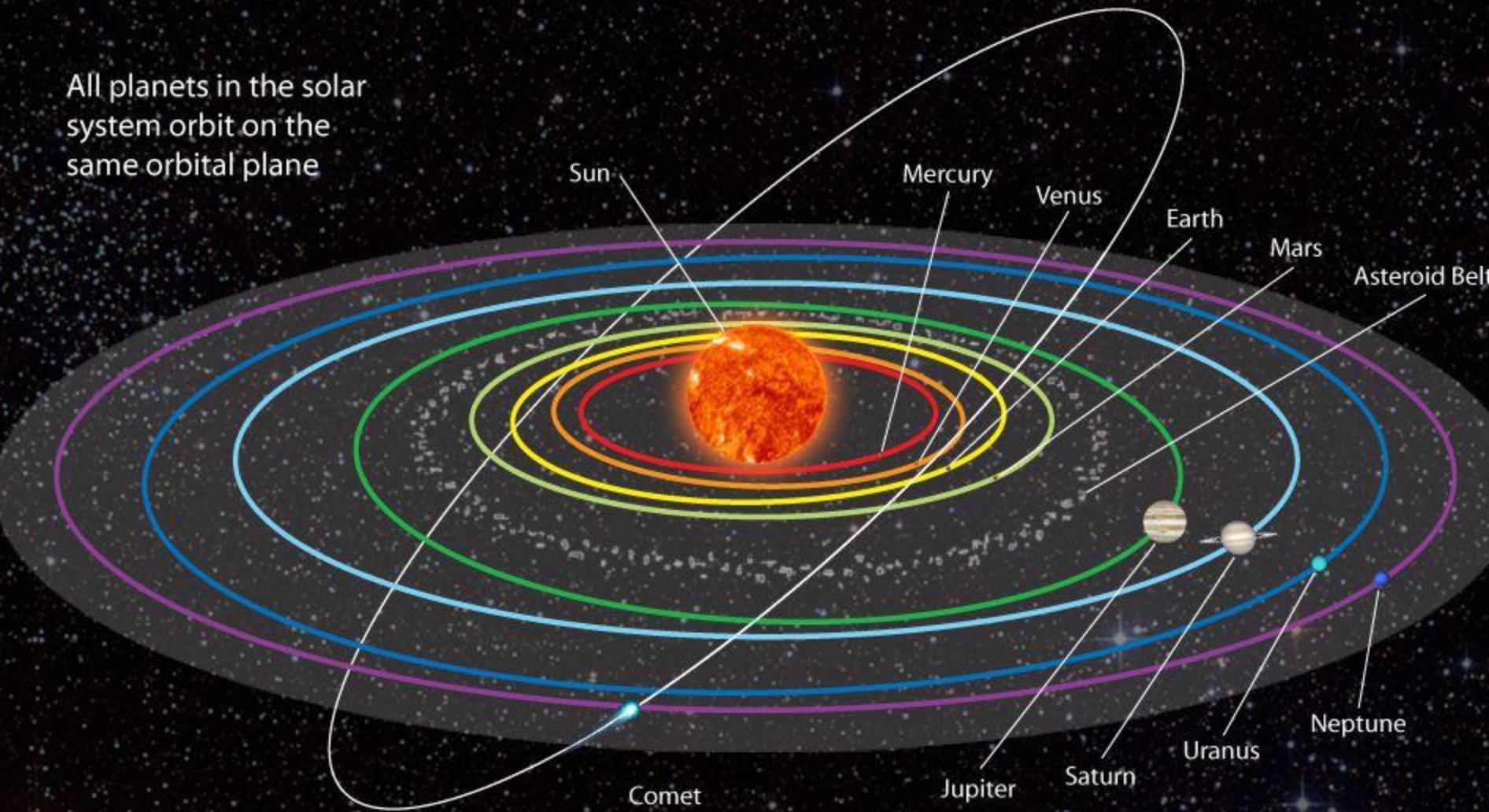


Solar System orbits

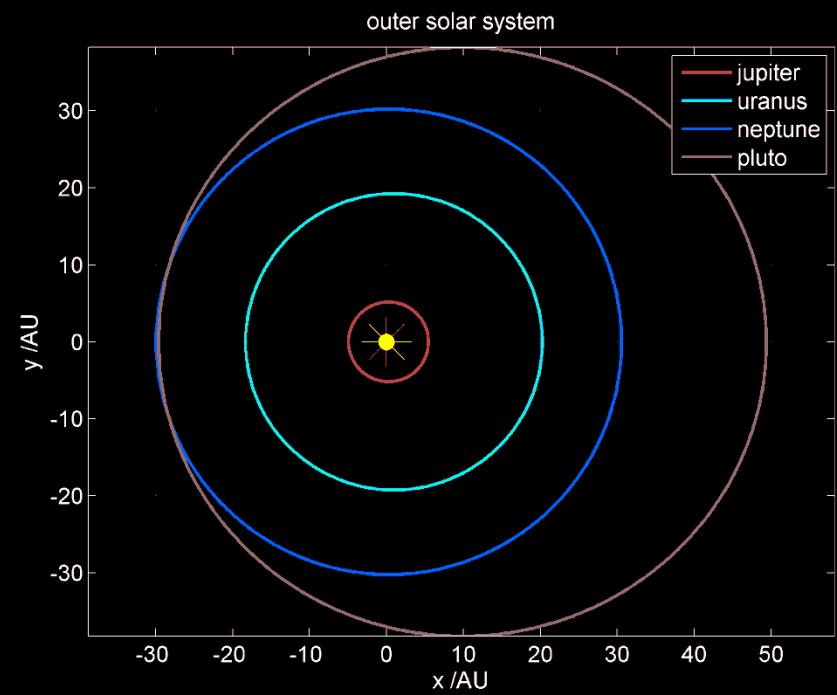
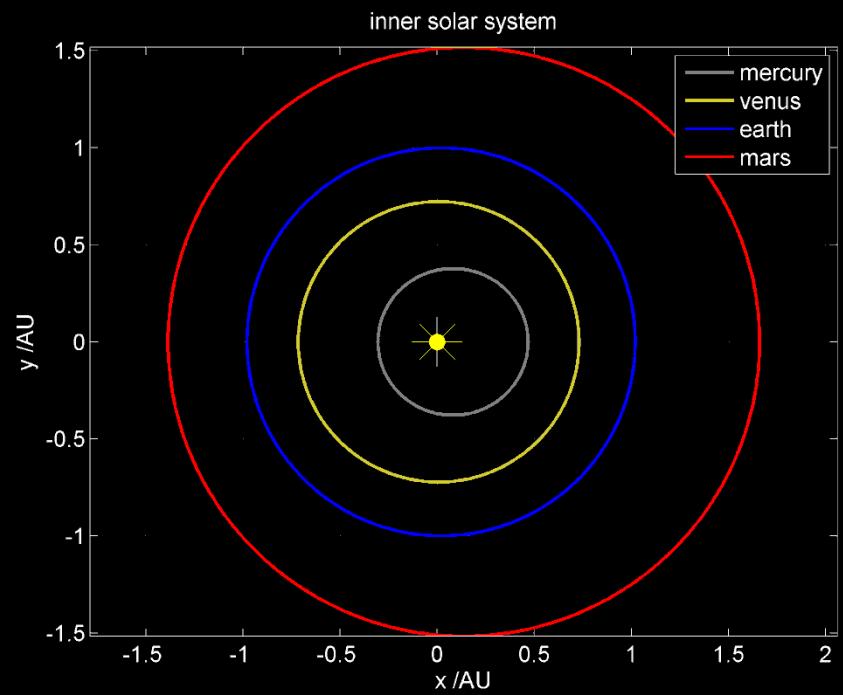
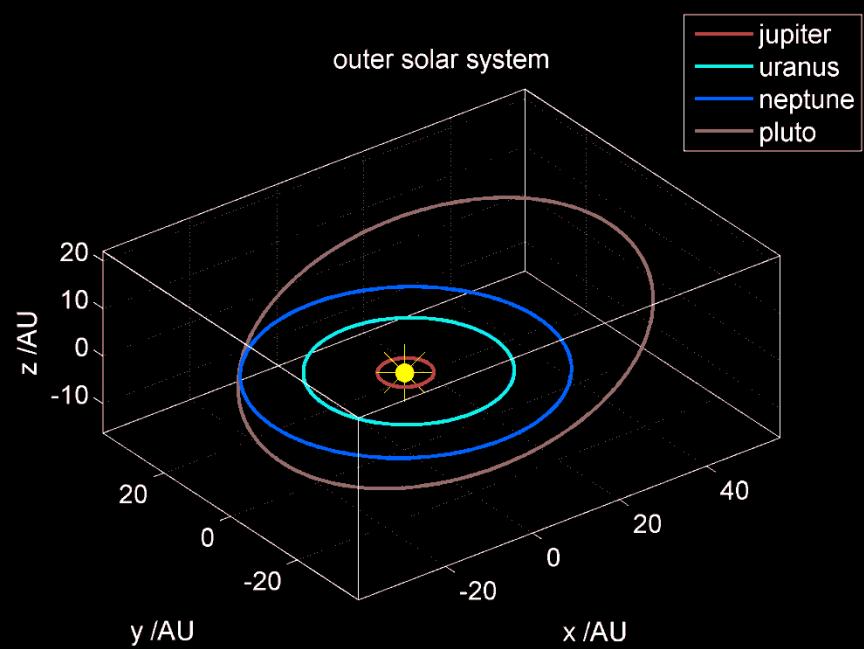
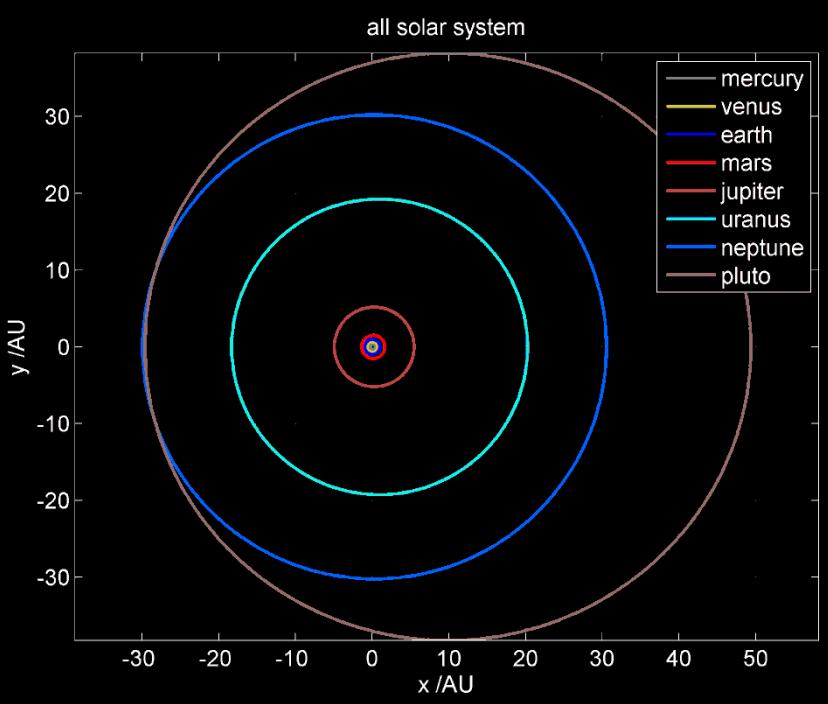


Orbital Plane

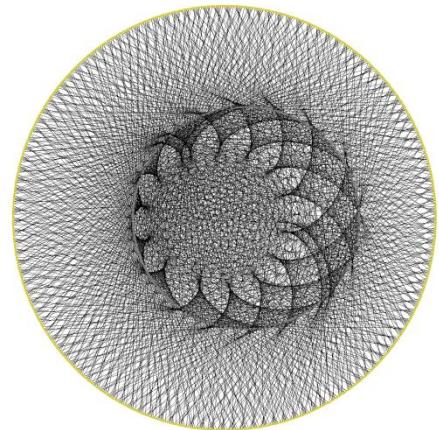
All planets in the solar system orbit on the same orbital plane



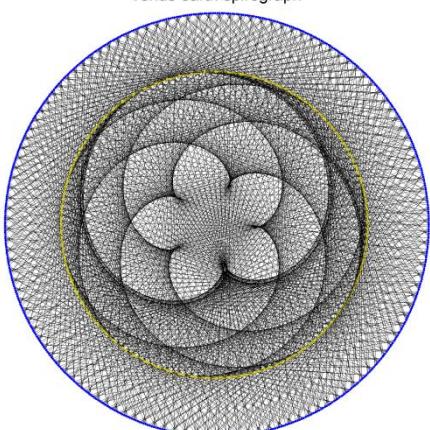
* Many comets exist outside the orbital plane



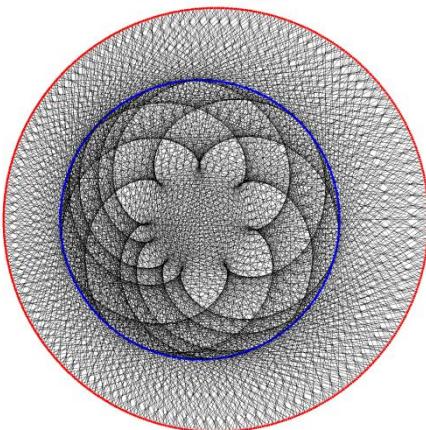
mercury venus spirograph



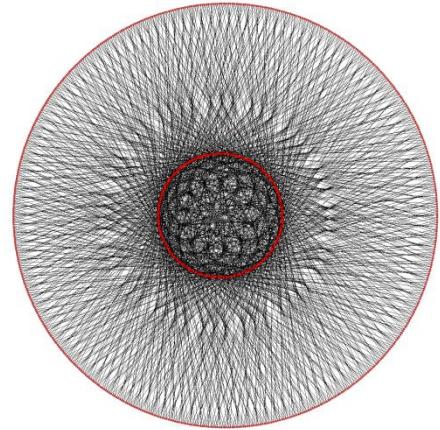
venus earth spirograph



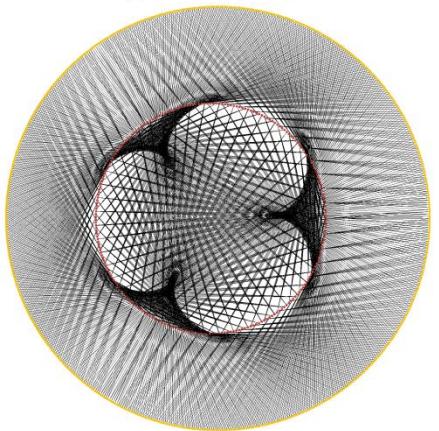
earth mars spirograph



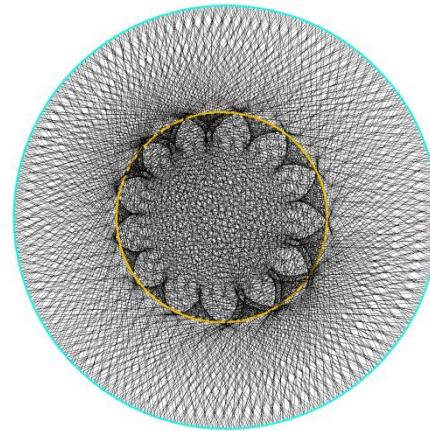
mars jupiter spirograph



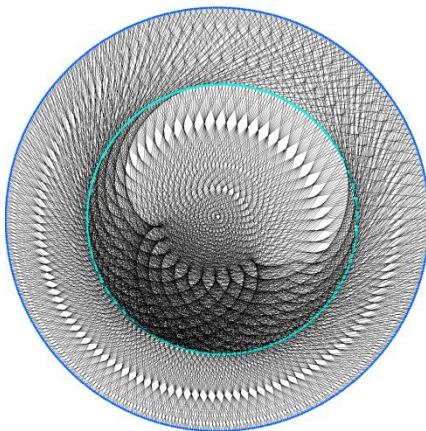
jupiter saturn spirograph



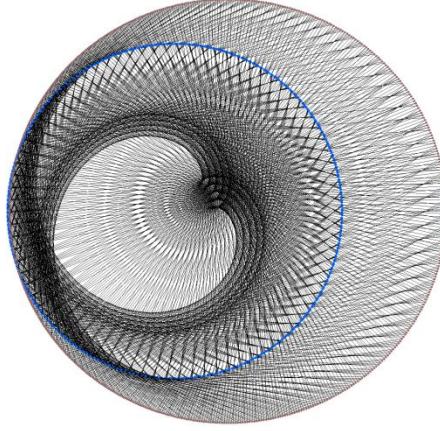
saturn uranus spirograph

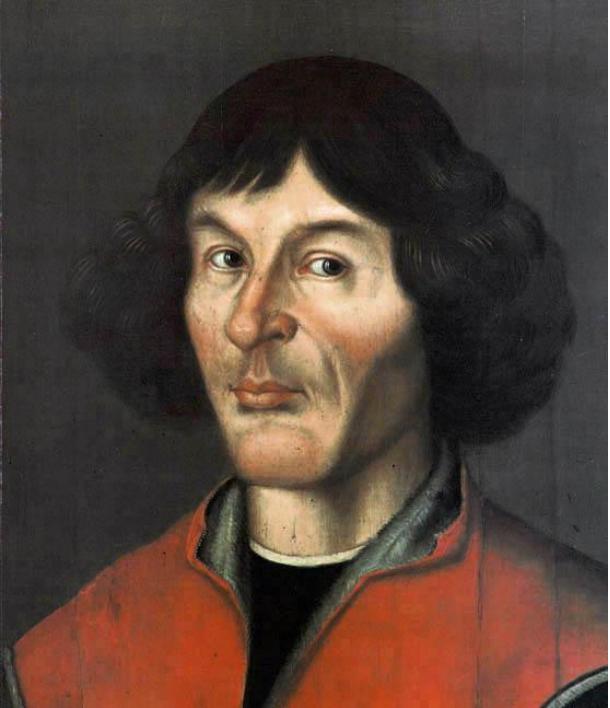


uranus neptune spirograph



neptune pluto spirograph





Nicolaus Copernicus
1473-1543



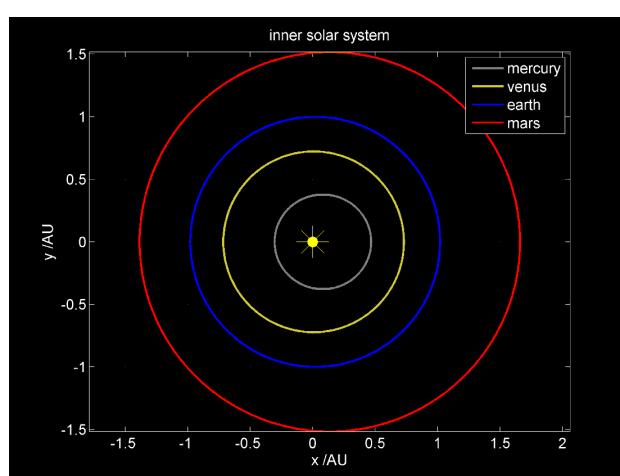
Johannes
Kepler
1571-1630



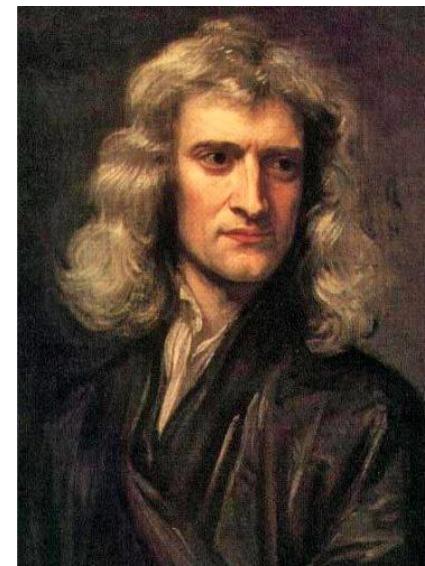
Tycho Brahe
1546-1601



Nose lost in 1566 following a sword duel with third cousin Manderup Parsberg over the legitimacy of a mathematical formula!



Isaac
Newton
1642-
1727



Kepler's three laws are:

1. *The orbit of every planet in the solar system is an ellipse with the Sun at one of the two foci.*
2. *A line joining a planet and the Sun sweeps out equal areas during equal intervals of time.*
3. *The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.*

The wording of Kepler's laws implies a specific application to the solar system. However, the laws are more generally applicable to any system of two masses whose mutual attraction is an inverse-square law.

$$r = \frac{a(1 - \varepsilon^2)}{1 - \varepsilon \cos \theta}$$

Polar
equation
of ellipse

$$\varepsilon = \sqrt{1 - \frac{b^2}{a^2}}$$

Eccentricity of
ellipse

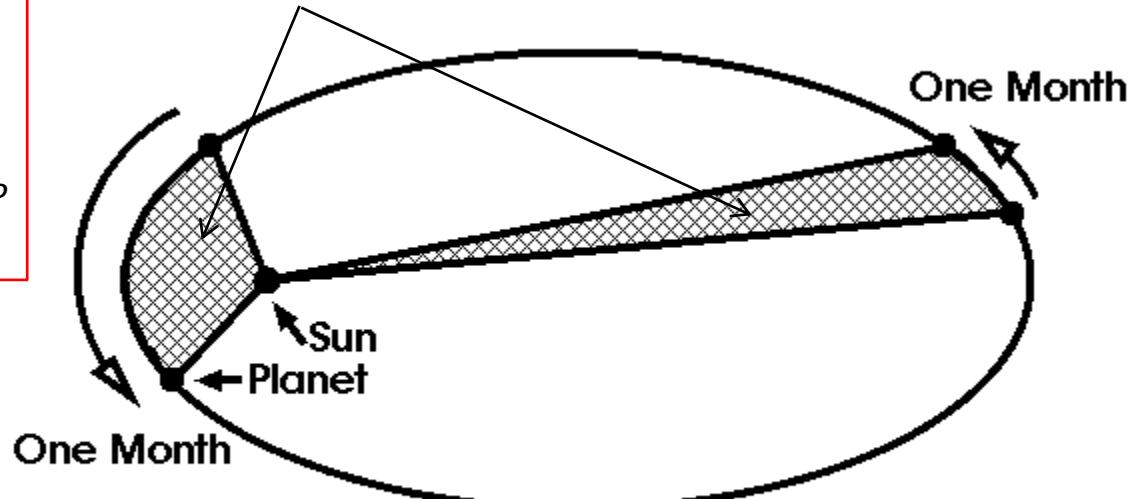
$$P^2 = \frac{4\pi^2}{G(m + M_{\odot})} a^3$$

Orbital
period P

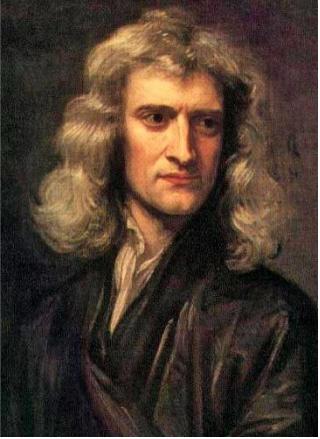
$$\frac{dA}{dt} = \frac{1}{2} \sqrt{G(m + M_{\odot})(1 - \varepsilon^2)a}$$

This is a constant

Equal areas swept out in
equal times



Johannes Kepler
1571-1630



Isaac Newton

(1642-1727) developed a mathematical model of Gravity which predicted the elliptical orbits proposed by Kepler

Planet and Solar masses

Force of gravity

$$F = \frac{GmM_{\odot}}{r^2}$$

$$G = 6.67 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{s}^{-2}$$

$$r = \frac{a(1 - \varepsilon^2)}{1 - \varepsilon \cos \theta}$$

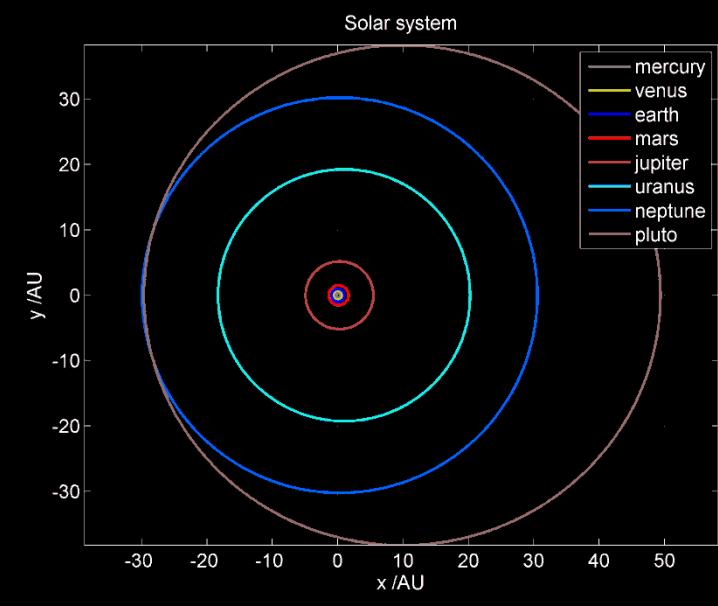
Polar equation of ellipse

$$\varepsilon = \sqrt{1 - \frac{b^2}{a^2}}$$

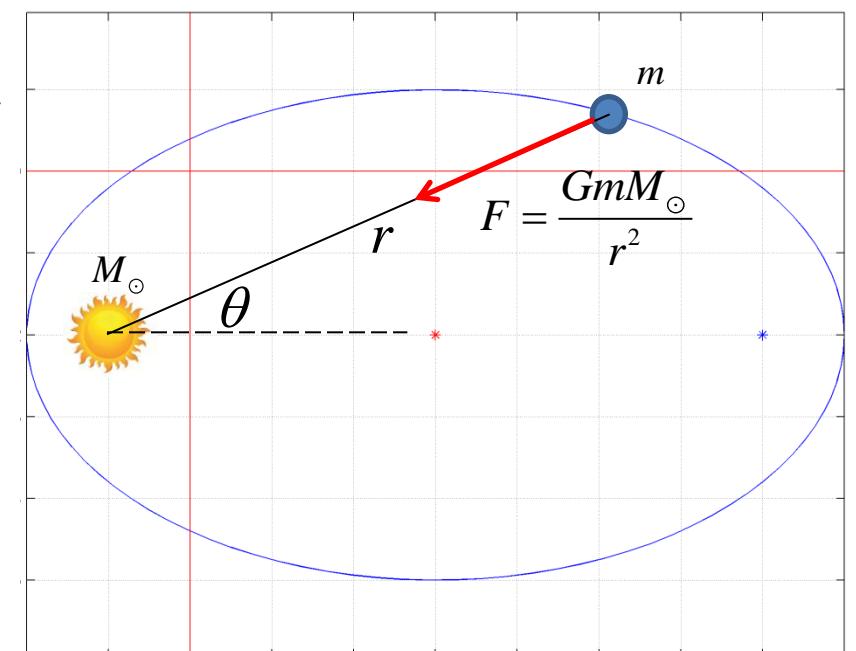
Eccentricity of ellipse

$$P^2 = \frac{4\pi^2}{G(m + M_{\odot})} a^3$$

Orbital period P



Semi-major axis $2a$



Object	Mass in Earth masses	Distance from Sun in AU	Radius in Earth radii	Rotational period /days	Orbital period /years	Gravitational field (in terms of g = 9.81 ms^-2)
Saturn	95.16	9.58	9.45	0.44	29.63	1.07
Uranus	14.50	19.29	4.01	0.72	84.75	0.90
Jupiter	317.85	5.20	11.21	0.41	11.86	2.53
Sun	332,837	-	109.12	-	-	27.95
Neptune	17.20	30.25	3.88	0.67	166.34	1.14
Pluto	0.00	39.51	0.19	6.39	248.35	0.09
Mars	0.107	1.523	0.53	1.03	1.88	0.38
Venus	0.815	0.723	0.95	243.02	0.62	0.90
Mercury	0.055	0.387	0.38	58.65	0.24	0.37
Earth	1.000	1.000	1.00	1.00	1.00	1.00

For our Solar System:

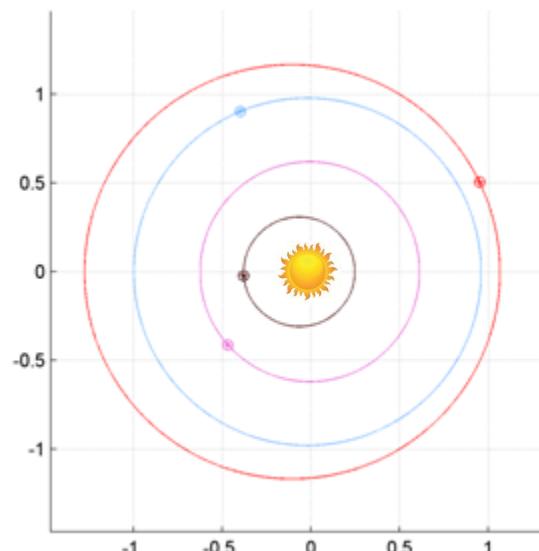
$$P^2 = \frac{4\pi^2}{G(m + M_{\odot})} a^3$$

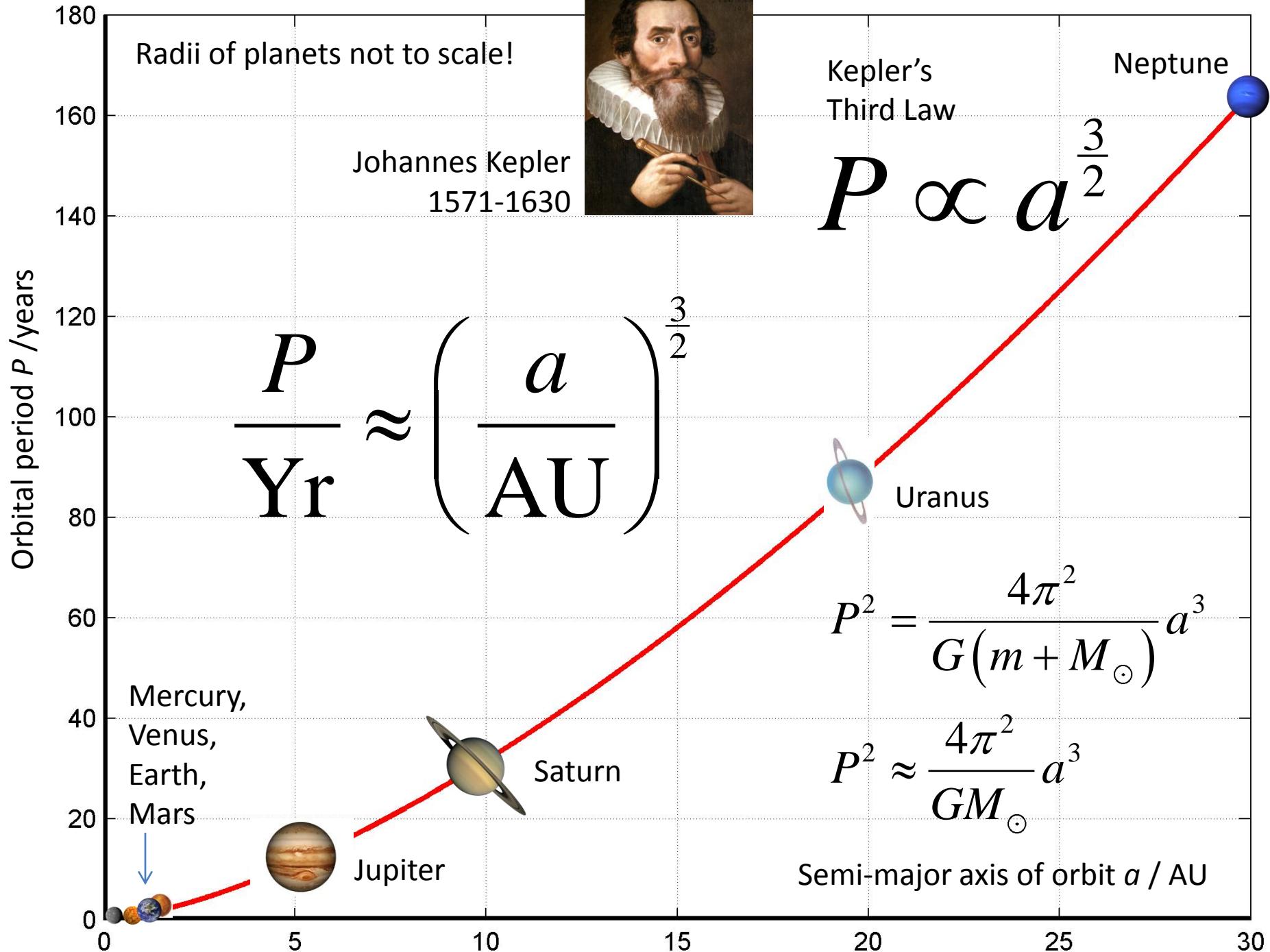
$$m \ll M_{\odot}$$

$$P^2 \approx \frac{4\pi^2}{GM_{\odot}} a^3$$

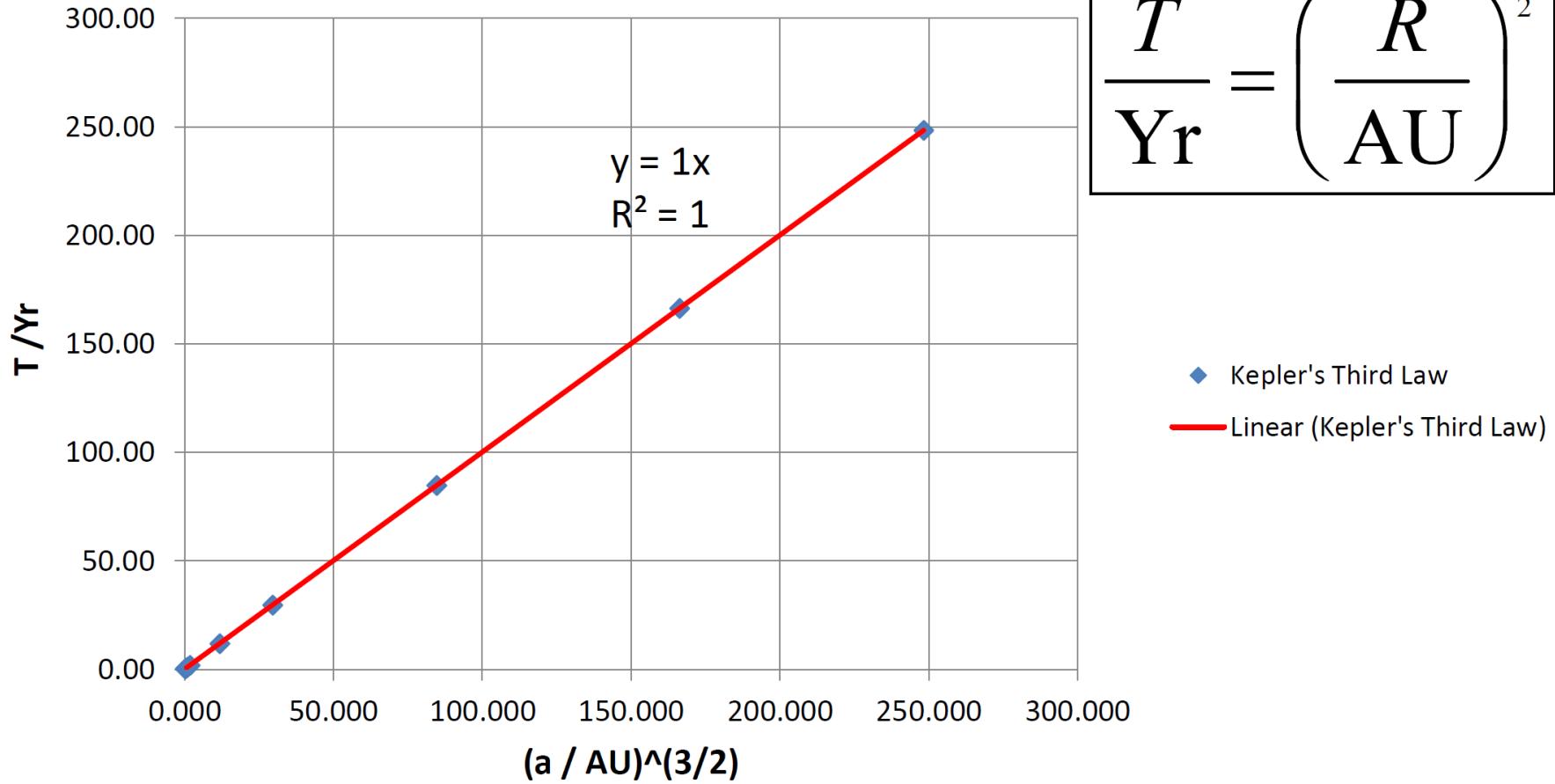
$$Yr^2 = \frac{4\pi^2}{GM_{\odot}} AU^3$$

$$\therefore \frac{P}{Yr} \approx \left(\frac{a}{AU} \right)^{\frac{3}{2}}$$





Kepler's Third Law



A *very strong* correlation of Kepler III to orbital data for planets in our solar system!

Challenge #1: Replicate this Kepler III correlation in Excel or Python or MATLAB

Object	M/M_\oplus	a / AU	ε	θ_0	β
Sun	332,837	-	-	-	-
Mercury	0.055	0.387	0.21	*	7.00
Venus [†]	0.815	0.723	0.01	*	3.39
Earth	1.000	1.000	0.02	*	0.00
Mars	0.107	1.523	0.09	*	1.85
Jupiter	317.85	5.202	0.05	*	1.31
Saturn	95.159	9.576	0.06	*	2.49
Uranus [†]	14.500	19.293	0.05	*	0.77
Neptune	17.204	30.246	0.01	*	1.77
Pluto [†]	0.003	39.509	0.25	*	17.5

R/R_\oplus	T_{rot} / days	P / Yr
109.123	-	-
0.383	58.646	0.241
0.949	243.018	0.615
1.000	0.997	1.000
0.533	1.026	1.881
11.209	0.413	11.861
9.449	0.444	29.628
4.007	0.718	84.747
3.883	0.671	166.344
0.187	6.387	248.348

β is the orbital inclination /degrees. In all cases the semi-major axis pointing direction is

$$\mathbf{d} = d_x \hat{\mathbf{x}} + d_y \hat{\mathbf{y}} + d_z \hat{\mathbf{z}} = \cos \beta \hat{\mathbf{x}} + \sin \beta \hat{\mathbf{z}}$$

$$r = \frac{a(1 - \varepsilon^2)}{1 - \varepsilon \cos \theta}$$

$$\varepsilon = \sqrt{1 - \frac{b^2}{a^2}}$$

$$P^2 = \frac{4\pi^2}{G(m + M_\odot)} a^3$$

$$M_\odot = 1.9891 \times 10^{30} \text{ kg}$$

$$R_\odot = 6.960 \times 10^8 \text{ m}$$

$$M_\oplus = 5.9742 \times 10^{24} \text{ kg}$$

$$R_\oplus = 6.37814 \times 10^6 \text{ m}$$

$$1 \text{AU} = 1.495979 \times 10^{11} \text{ m}$$

* For the current orbital polar angle θ_0 (and indeed more accurate values for solar system parameters) see the website of the Jet Propulsion Laboratory (JPL) <http://ssd.jpl.nasa.gov/>

†These planets rotate clockwise about their own internal polar axis. ("Retrograde"). All the other planets rotate anti-clockwise about their own internal axis. All the planets orbit the sun in an anticlockwise direction.

Challenge #2: Plot elliptical orbits of the planets using Excel, Python, MATLAB etc....

Assume all orbits are **ellipses** with the Sun at the (left) focus. Let this sun position be the origin of a Cartesian coordinate system, and assume the sun is stationary.

$$r = \frac{a(1 - \varepsilon^2)}{1 - \varepsilon \cos \theta}$$

$$\varepsilon = \sqrt{1 - \frac{b^2}{a^2}} \quad \therefore b = a(1 - \varepsilon^2)$$

$$P^2 = \frac{4\pi^2}{G(m + M_{\odot})} a^3$$

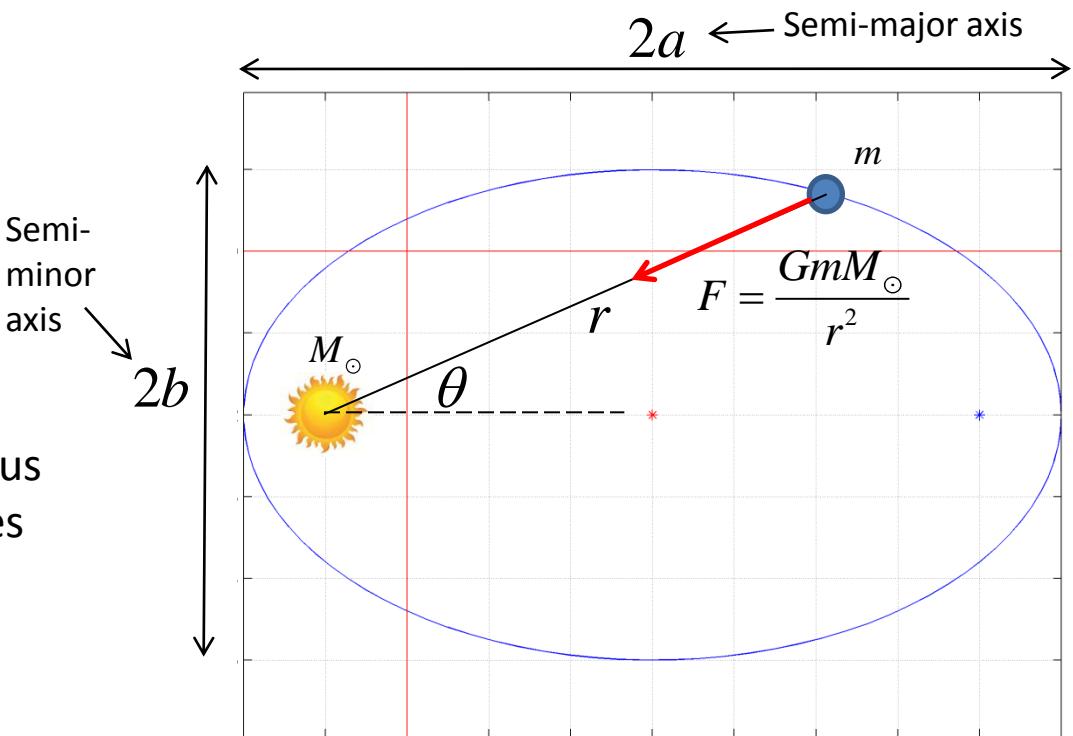
$$x = r \cos \theta, \quad y = r \sin \theta \\ \theta = 0 \dots 2\pi$$

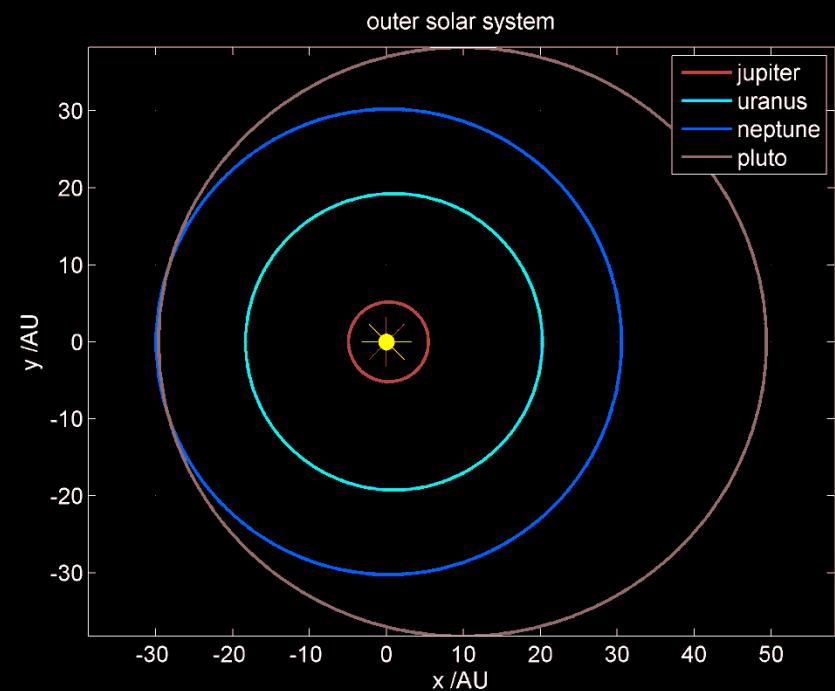
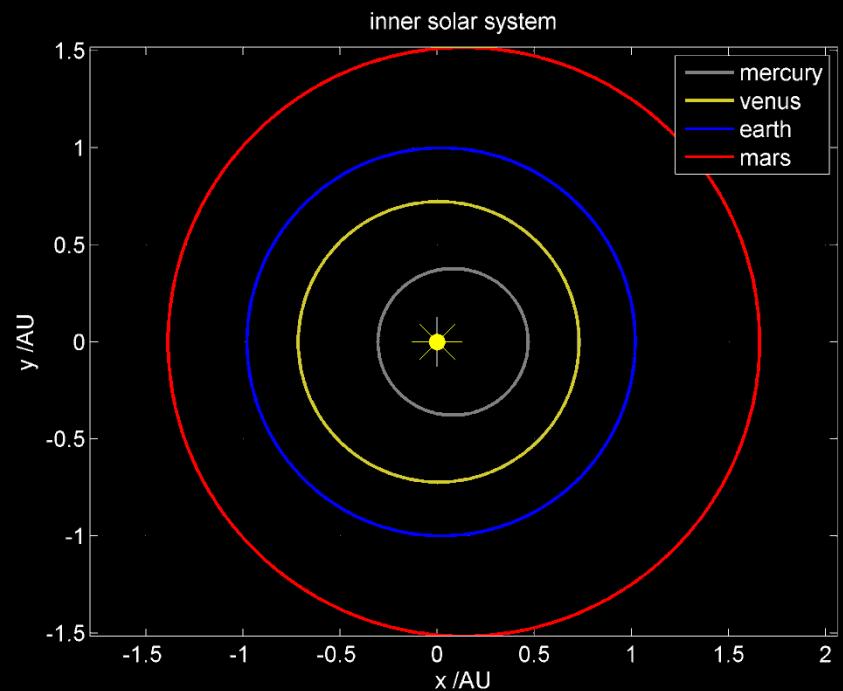
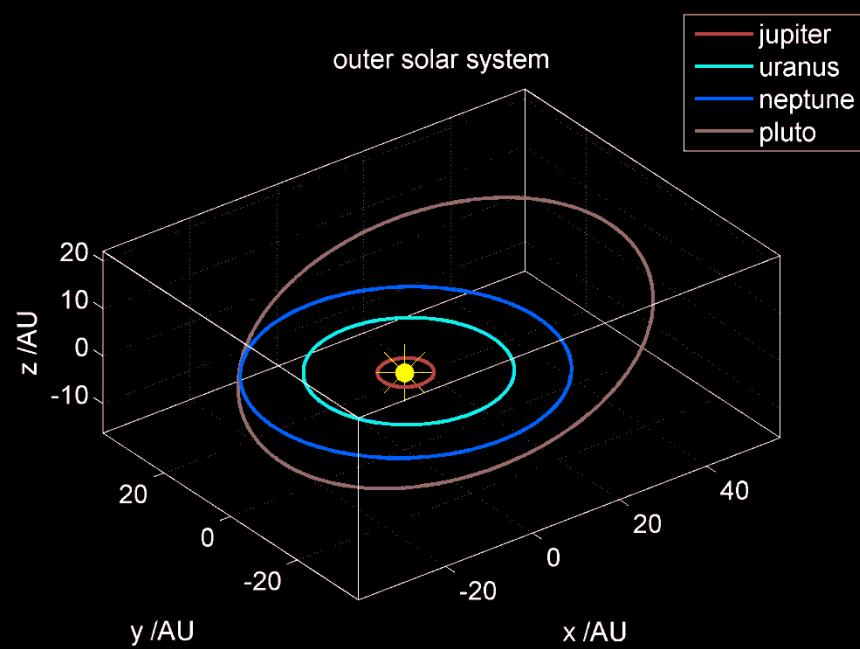
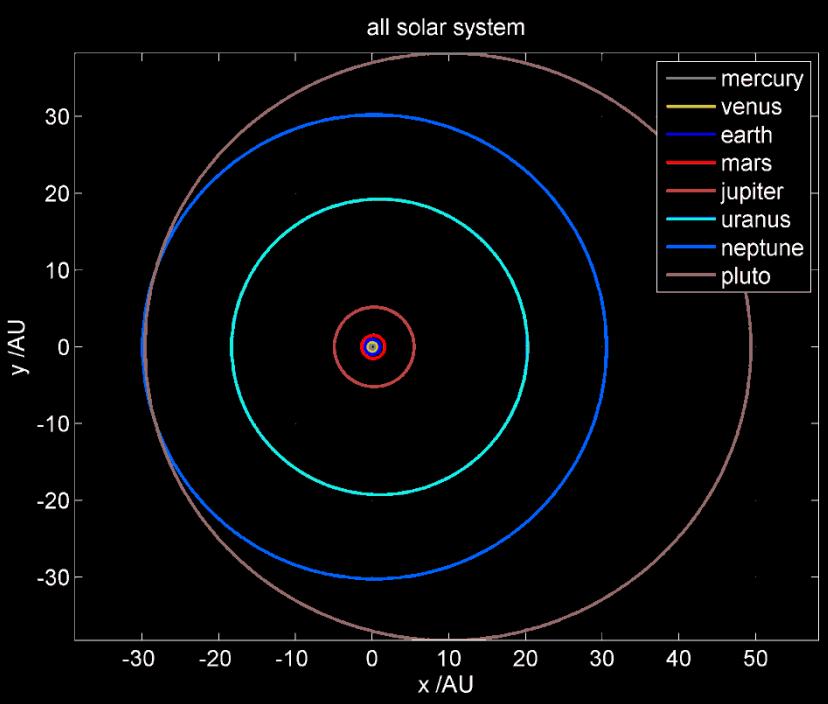
Use the data in the table on the previous slide. Use a 1,000 linearly spaced angles θ for each orbit.

Use an axis scale of AU

Plot the inner five planets on a separate scale to the outer planets

Ignore the inclination angle β (i.e. set it to be zero for now)





Challenge #3: Create a 2D animation of the solar system orbits

Use an axis scale of AU

Plot the inner five planets on a separate scale to the outer planets

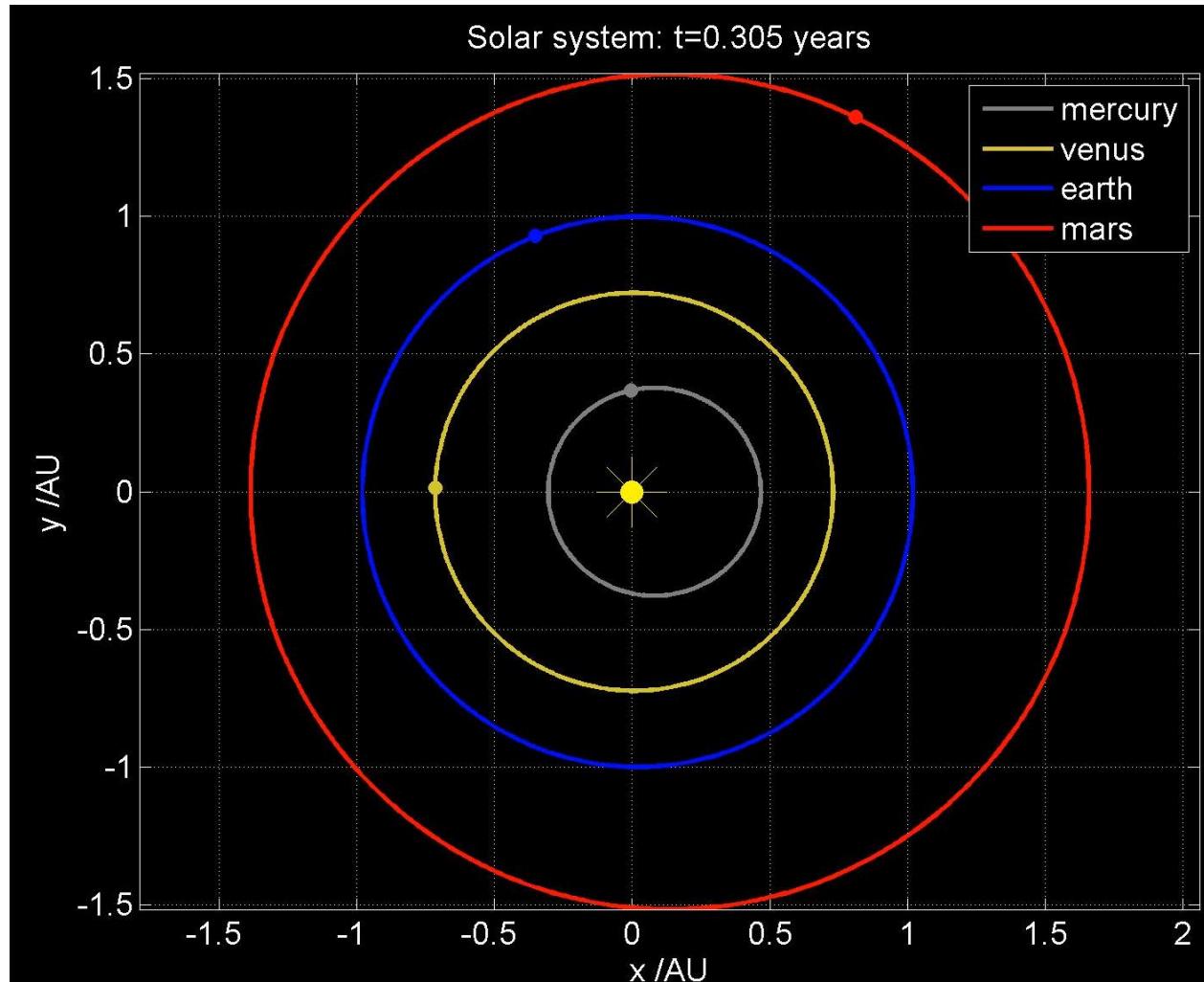
For the *inner* planets, set a frame rate such that one orbit of the Earth takes a second i.e. **one year is one second**. For the *outer* planets, **set the orbit of Jupiter to take one second**.

$$x = r \cos \theta, \quad y = r \sin \theta$$

$$\theta = \frac{2\pi t}{P}$$

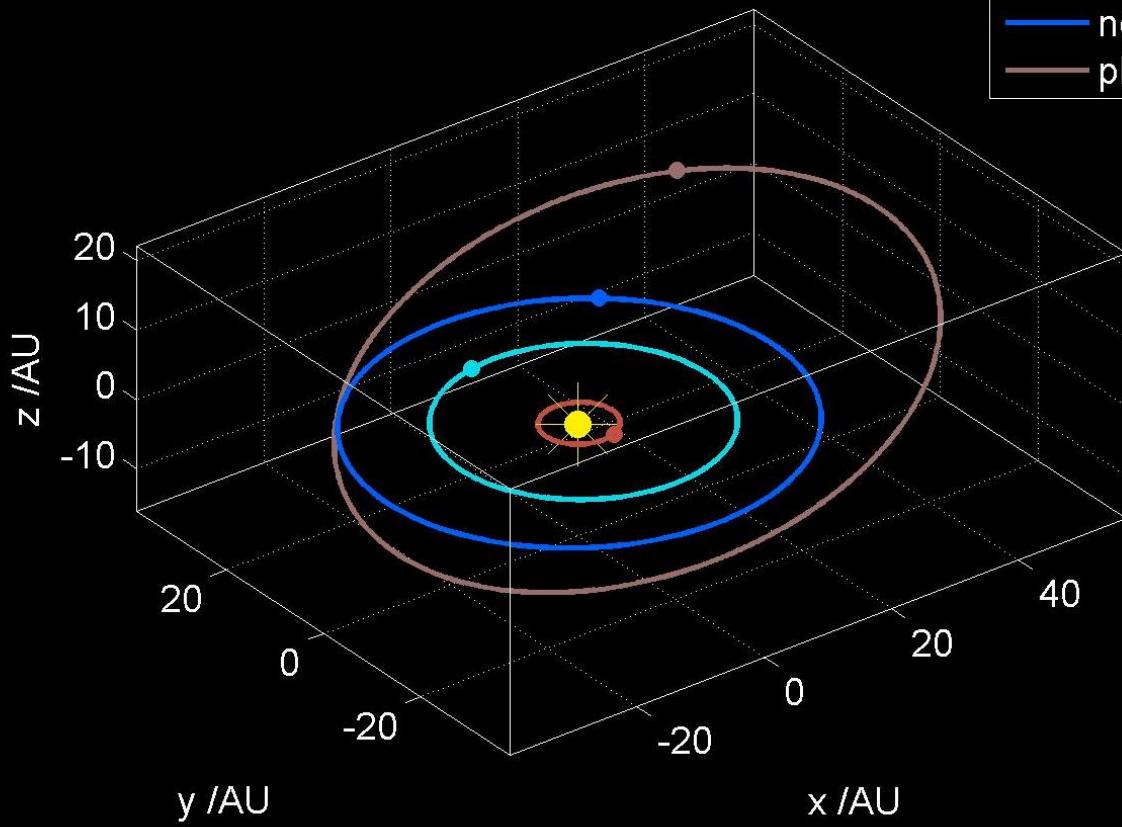
Run the simulation till the outermost planet completes at least one orbit.

[YouTube example video](#)



Solar system: t=21.9 years

- jupiter
- uranus
- neptune
- pluto



Challenge #4: Create a 3D animation of the solar system orbits

[YouTube example video](#)

Use the elliptical inclination angle β . Most orbits won't change much, but Pluto is the exception! The coordinate change is:

$$x' = x \cos \beta \quad z' = x \sin \beta \quad y' = y$$

β is the orbital inclination /degrees. In all cases the semi-major axis pointing direction is

$$\mathbf{d} = d_x \hat{\mathbf{x}} + d_y \hat{\mathbf{y}} + d_z \hat{\mathbf{z}} = \cos \beta \hat{\mathbf{x}} + \sin \beta \hat{\mathbf{z}}$$

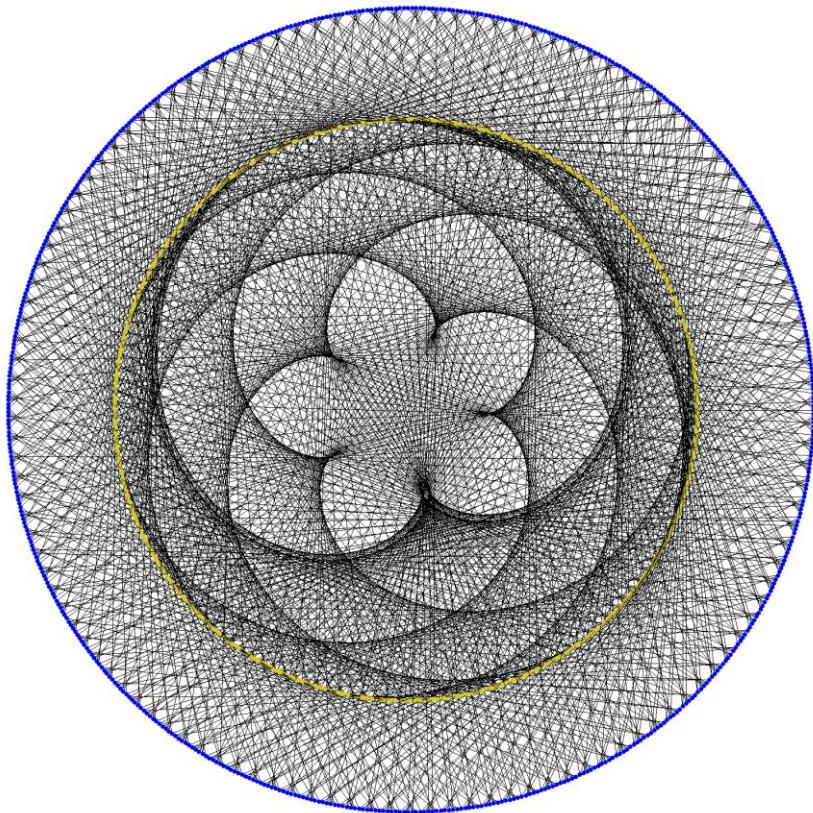
Challenge #5: Solar system spirograph!

inspired by: <https://engaging-data.com/planetary-spirograph>

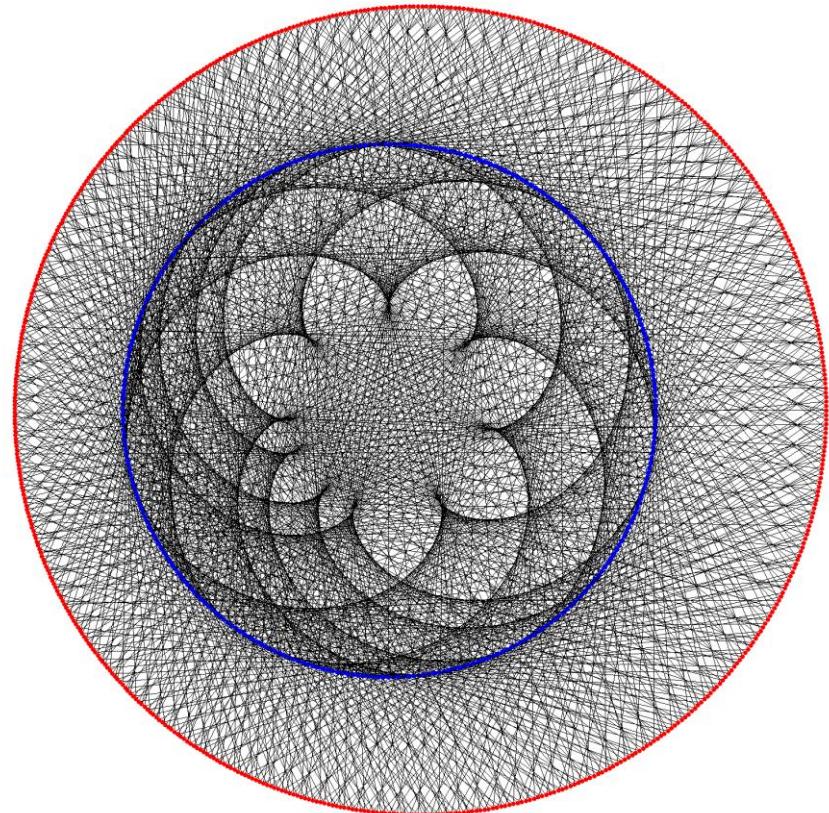
Choose a pair of planets and determine their orbits vs time. At time intervals of Δt , draw a line between the planets and plot this line. Keep going for N orbits of the outermost planet.

$N = 10$, $\Delta t = N \times \text{maximum orbital period} / 1234$, might be sensible parameters.

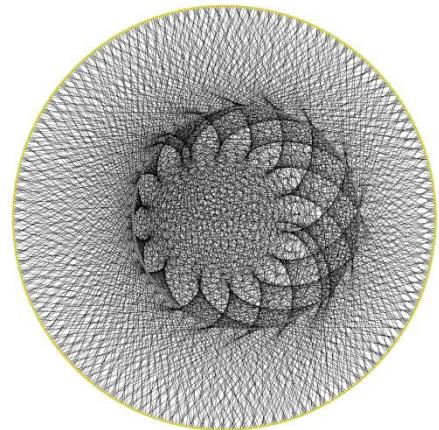
venus earth spirograph



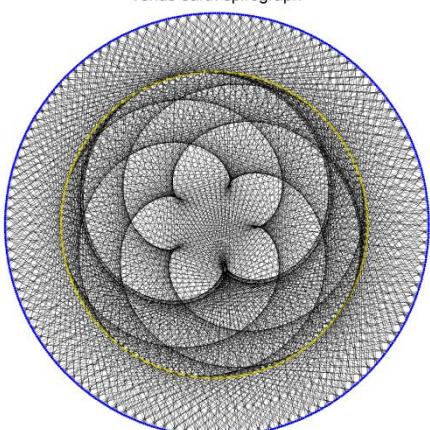
earth mars spirograph



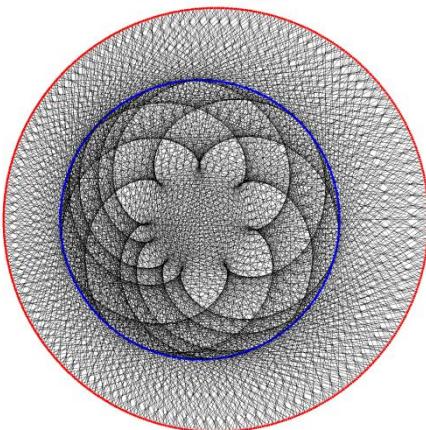
mercury venus spirograph



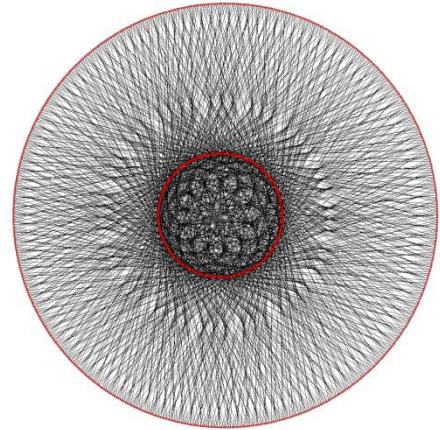
venus earth spirograph



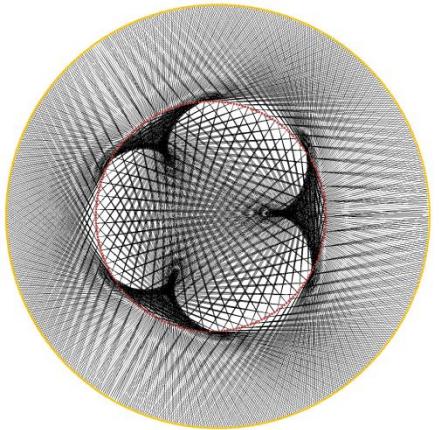
earth mars spirograph



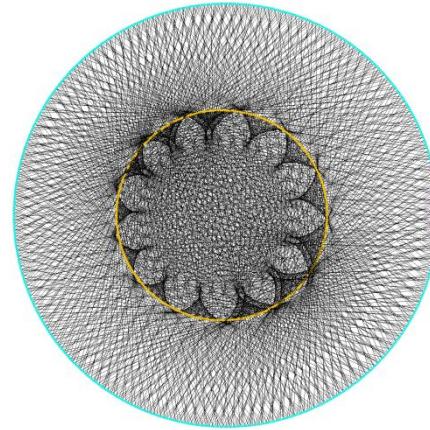
mars jupiter spirograph



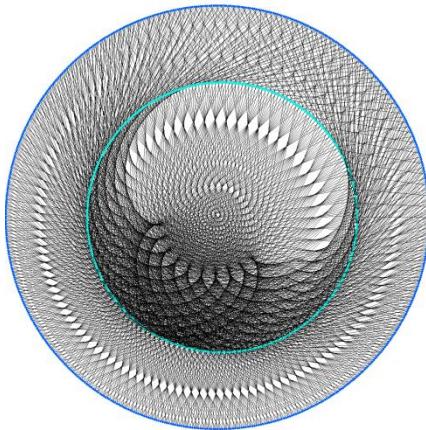
jupiter saturn spirograph



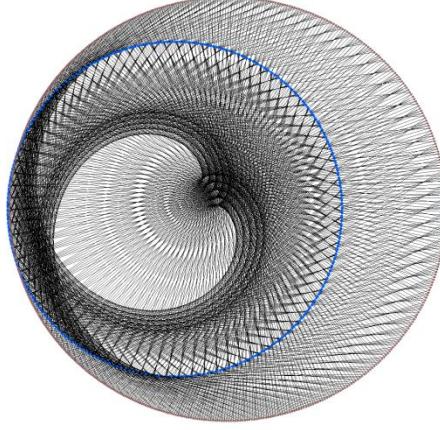
saturn uranus spirograph



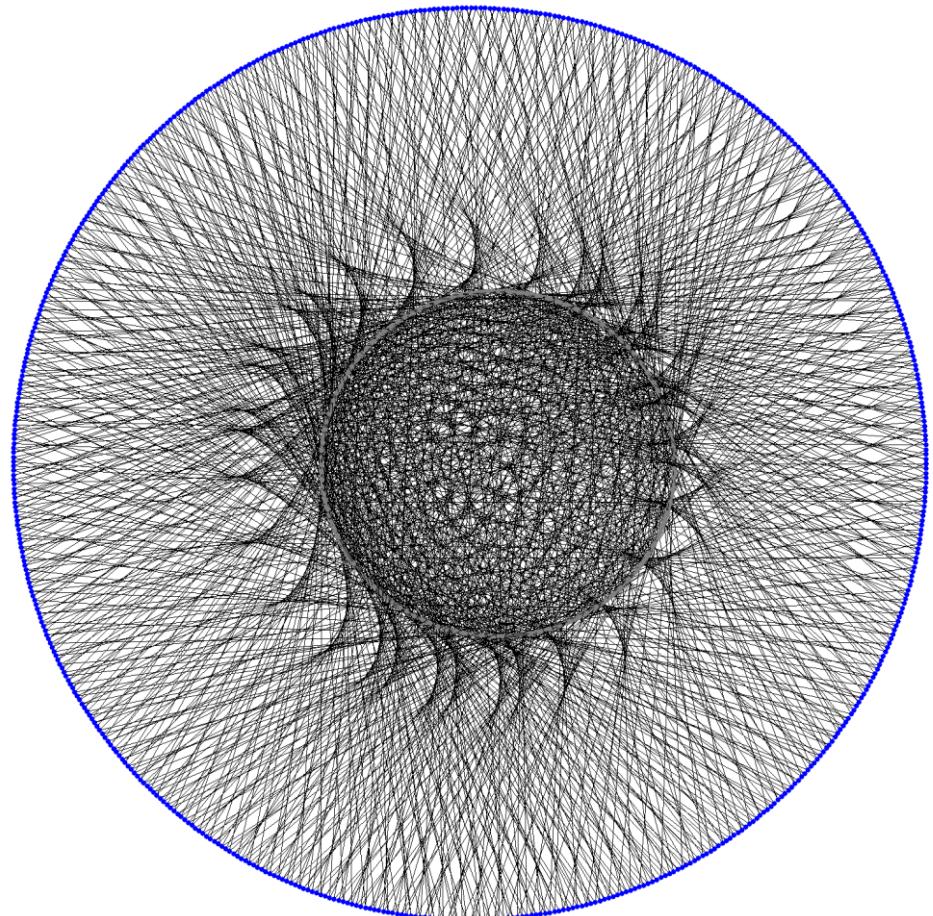
uranus neptune spirograph



neptune pluto spirograph



mercury earth spirograph



venus mars spirograph

