



S Lorentz transform relates distance x and time t between 'static' frame S and moving frame S' , which moves $\parallel x$ at velocity V

$$t = \gamma \left(t' + \frac{Vx'}{c^2} \right)$$

$$\gamma = \left(1 - \frac{v^2}{c^2} \right)^{-\frac{1}{2}}$$

Now if clock is stationary in S' , $x' = 0$

$$\therefore t = \gamma t'$$

let $t' = \frac{1}{2}t \quad \therefore t = \gamma \frac{1}{2}t \Rightarrow \boxed{\gamma = 2}$

i.e. "moving clock runs at half the rate of an equivalent clock in S "

$$\Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{4} \Rightarrow \frac{v^2}{c^2} = \frac{3}{4}$$

$$\Rightarrow \boxed{v = \frac{\sqrt{3}}{2}c} \quad \approx \boxed{0.866c}$$

2,

A metre stick travels at speed V relative to frame S \parallel to the x direction. In a frame S' where the metre stick is stationary

$$\boxed{x' = \gamma(x - vt)}$$
 from the Lorentz Transform

Now $x' = 1$ metre and let stick length be measured in S when $t = 0$.

Here $1 = \gamma x \quad \therefore x = \frac{1}{\gamma}$ i.e. 'lengths contract'

Now if $x = 0.9$ m

$$\gamma = \frac{10}{9}$$

$$\left(1 - \frac{v^2}{c^2} \right)^{-\frac{1}{2}} = \frac{10}{9}$$

$$1 - \frac{v^2}{c^2} = \frac{81}{100}$$

$$\frac{v^2}{c^2} = 1 - \frac{81}{100}$$

$$\frac{v^2}{c^2} = \frac{19}{100}$$

$$\boxed{v = \frac{\sqrt{19}}{10}c}$$

$$\approx \boxed{0.436c}$$

①

3/ let $t' = 26 \text{ ns}$ be lifetime of the π meson in its rest frame.
 Hence taking the Earth to be a frame S and the meson to be at rest in a frame S' moving at speed $v = 0.95c$ relative to S in the x direction

Lorentz transform $\Rightarrow t = \gamma(t' + vx'/c^2)$ $x' = 0$

so $t = (1 - 0.95^2)^{-\frac{1}{2}} \times 26 \text{ ns}$

$t = 3.20 \times 26 \text{ ns}$

$t = 83.3 \text{ ns}$

so meson will travel $\approx 0.95c \times 83.3 \text{ ns} \approx 23.7 \text{ m}$

before it decays.

4/ Atomic clock measures time $t' = 3600 \text{ s}$ on-board a jet moving at 400 ms^{-1}

on the ground, the equivalent time interval is $t = \gamma t'$

(Note $x' = 0$ as it is assumed the clock is at rest relative to the S' frame co-moving with the aircraft)

$t = \left(1 - \left(\frac{400}{2.9979 \times 10^8}\right)^2\right)^{-\frac{1}{2}} \times 3600$

Now calculator precision problems working this out directly

So let $t = 3600 + \delta t$, and since $\frac{v^2}{c^2} \ll 1$

use a single term Binomial expansion

$t \approx t' \left(1 + (-\frac{1}{2}) \left(-\frac{v^2}{c^2}\right) + \frac{(-\frac{1}{2})(-\frac{3}{2}) \left(-\frac{v^2}{c^2}\right)^2}{2} + \dots\right)$

$\approx t' \left(1 + \frac{v^2}{2c^2}\right)$

$\delta t \approx \frac{t' v^2}{2c^2}$

$\delta t = \frac{3600 \times 400^2}{2 \times (2.9979 \times 10^8)^2} \approx 3.2 \text{ ns}$

②

