Special relativity questions

Adapted from "Solved Problems in Special Relativity" by Charles Asman, Adam Monahan and Malcolm McMillan. University of British Columbia, Canada. 2011.

1. At what speed must a clock move if it runs at a rate which is one-half the rate of a clock at rest?

2. At what speed does a metre stick move if its length is observed to shrink by 10cm?

3. The average lifetime of a π meson in its own frame of reference is 26 ns.

   (This is its proper lifetime.)

   (i) If the π meson moves with speed 0.95*c* with respect to the Earth, what is its lifetime as measured by an observer at rest on Earth?

   (ii) What is the average distance it travels before decaying as measured by an observer at rest on Earth?

4. (i) Show that the interval of time Δt elapsed in a rest frame *S* is approximately

$$\Delta t = \Delta t' + \delta t$$
$$\delta t \approx \frac{\Delta t' v^2}{2c^2}$$

   compared to the time interval measured by a clock at rest with respect to a frame *S′* moving at velocity v relative to S.

   (ii) An atomic clock is placed in a jet airplane. The clock measures a time interval of 3600s when the jet moves with speed 400 m/s. How much larger a time interval does an identical clock held by an observer at rest on the ground measure?

5. The muon is an unstable particle that spontaneously decays into an electron and two neutrinos.

   The number of muons at t = 0 is N_0 , the number N at time t is $N = N_0 e^{-\frac{t}{\tau}}$ where τ = 2.20μs is the mean lifetime of the muon. Suppose the muons move at speed 0.95*c*.

   (i) What is the observed lifetime of the muons?

   (ii) How many muons remain after travelling a distance of 3.0 km?

6. A rod of length L_0 moves with speed v along the horizontal direction. The rod makes an angle θ' with respect to the x' axis of the rest frame of the rod.

(i) Determine the angle θ the rod makes with the x axis of a stationary frame of reference *S*.

(ii) Use the identity $\sec^2\theta = 1 + \tan^2\theta$ or otherwise to show that $L = L_0\sqrt{1 - \frac{v^2}{c^2}\cos^2\theta'}$ and hence explain why the rod both *rotates* and *contracts* when viewed in the *S* frame.

7. A relativistic taxi has the same coloured light front and back, as viewed in its rest frame. To a stationary observer, the headlights appears green if the taxi is coming towards you, and the rear lights appear red if the taxi is receding.

Green light has wavelength 500nm and Red light is 700nm. Use this information to work out the speed of the taxi (as a fraction of the speed of light) and the wavelength of the lights in the rest frame of the taxi. You may assume the taxi always travels at the same speed once it has picked up its passengers.

8. Two spaceships approach each other, each moving with the same speed as measured by a stationary observer on the Earth. In the rest frame of Spaceship 1, Spaceship 2 approaches with speed 0.7*c.*

Determine the velocities of each spaceship as measured by the stationary observer on Earth.

9. A stationary observer on Earth observes spaceships A and B moving in the same direction toward the Earth. Spaceship A has speed 0.5c and spaceship B has speed 0.8*c*.

Determine the velocity of spaceship A as measured by an observer at rest in spaceship B.

10. The motion of a medium such as water influences the speed of light. This effect was first observed by Fizeau in 1851.

Consider a light beam passing through a horizontal column of water moving with velocity v . Let u be the speed of light measured in the laboratory frame, when the beam travels in the same direction as the flow of the water, and let u' be the velocity of light in a frame co-moving with the water. If the refractive index of *stationary* water is $n = \frac{c}{u'}$ show that, when $v \ll c$

$$u \approx \frac{c}{n} + v\left(1 - \frac{1}{n^2}\right)$$

11. As seen from Earth, two spaceships A and B are approaching along perpendicular directions.

If A is observed by a stationary Earth observer to have velocity u_y = -0.9*c* and B to have velocity u_x = +0.9*c*, determine the speed of ship A as measured by the pilot of ship B.

12. Consider the relativistic form of *Newton's Second Law*

Show that when a force $\mathbf{F}$ is parallel to velocity $\mathbf{u}$

$$F = m\left(1-\frac{u^2}{c^2}\right)^{-\frac{3}{2}}\frac{du}{dt}$$

where m is the mass of the object.

If force F is constant, show that the velocity u at time t, if the particle starts from rest, is

$$\frac{u}{c} = \left(1+\left(\frac{mc}{Ft}\right)^2\right)^{-\frac{1}{2}}$$

Show that this is consistent with the Classical expression when $u \ll c$

13. Electrons in projection television sets are accelerated through a potential difference of 50 kV.

(i) Calculate the speed of the electrons using the relativistic form of kinetic energy, assuming the electrons start from rest. Express the answer in terms of speed of light *c*.

(ii) Calculate the speed of the electrons, as a fraction of *c*, using the classical form of kinetic energy.

(iii) Is the difference in speed significant in the design of this set?

[Electron mass is $m_e = 9.109\times10^{-31}\,\mathrm{kg}$, charge of the electron is $e = 1.602\times10^{-19}\,\mathrm{C}$]

14. An electron e^- with kinetic energy 1.000 MeV makes a head-on collision with a positron e^+ at rest. (A positron is an antimatter particle that has the same mass as the electron but opposite charge.)

In the collision the two particles annihilate each other and are replaced by two photons of equal energy, each travelling at angles θ with the electron's direction of motion. (A photon is a massless particle of electromagnetic radiation having energy $E_\gamma = p_\gamma c$.) The reaction is:

$$e^- + e^+ \rightarrow 2\gamma$$

Determine the energy E_γ (in MeV) , momentum p_γ (in MeV/c) and angle of emission θ of each photon.

15. The meson decays into two charged pions according to

$$K^0 \rightarrow \pi^+ + \pi^-$$

The pions have equal and opposite charges as indicated and the same rest mass

$$m_\pi = 140\,\text{MeV}/c^2$$

Suppose that a K^0 at rest decays into two pions in a bubble chamber in which a magnetic field of *B* = 2T is present. If the radius of curvature r of the pions is 34.4cm, determine the momenta and speeds of the pions, and the rest mass of the K^0 meson.

[The charge of a pion is the same as an electron, so the magnitude of the force on the pion resulting from the magnetic field is $F = evB$, which acts radially inwards i.e. perpendicular to the motion).

16. A spaceship moves away from Earth with speed v and fires a shuttle craft in the forward direction at a speed v relative to the spaceship. The pilot of the shuttle craft launches a probe in the forward direction at speed v relative to the shuttle craft.

(i) Determine the speed of the shuttle craft relative to the Earth.

(ii) Determine the speed of the probe relative to the Earth.

17. Two powerless rockets are heading towards each other on a collision course. As measured by Liz, a stationary Earth observer, Rocket 1 has speed 0.8*c*, Rocket 2 has speed 0.6*c*, both rockets are 50.0m in length, and they are initially 2.52 x 10^{12} m apart.

(i) What are their respective proper lengths?

(ii) What is the length of each rocket as observed by a stationary observer in the other rocket?

(iii) According to Liz, how long before the rockets collide?

(iv) According to Rocket 1, how long before they collide?

(v) According to Rocket 2, how long before they collide?

(vi) If the crews are able to evacuate their rockets safely within 50 min (their own time), will they be able to do so before the collision?