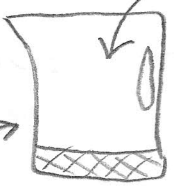
SPECIFIC HEAT CAPACITY, LATENT HEAT & NEWTON'S LAW OF COOLING

Q1 (i)



$$\boxed{\begin{aligned} \Delta E &= cM\Delta T \\ \Delta E &= Pt \end{aligned}}$$

$\Delta T = 100°C - 18°C$

$= \boxed{82°C} = 82K$

$c = 4.187$ kJ/kg/K

$$\therefore \quad t = \frac{cM\Delta T}{P} = \frac{4.187\text{ kJ/kg/K} \times 1.23\text{ kg} \times 82\text{K}}{2.8\text{ kJ/s}}$$

$= \boxed{151\text{ s}}$ (2 minutes, 31 seconds) or $\approx 2\frac{1}{2}$ minutes.

(ii) let $\underbrace{(1.23 - 0.20)}_{\Delta M}$ kg of water be vaporized.

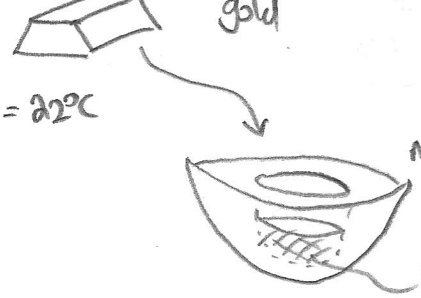
$\therefore \boxed{Pt = \Delta M L_{vap}}$ $\quad\therefore\quad$ $t = \Delta M L_{vap}/P$

$$t = \frac{1.03\text{ kg} \times 2260\text{ kJ/kg}}{2.8\text{ kJ/s}}$$

$t = 831$ s

$= \boxed{13\text{ minutes, } 51\text{ seconds}}$

(iii)



$$\boxed{\begin{aligned} \Delta E = {} & C_S M (T_{melt} - T_0) \\ & + M L_{fus} + C_L M (T - T_{melt}) \end{aligned}}$$

$= 0.129 \times 12.4 \times (1063 - 22)$

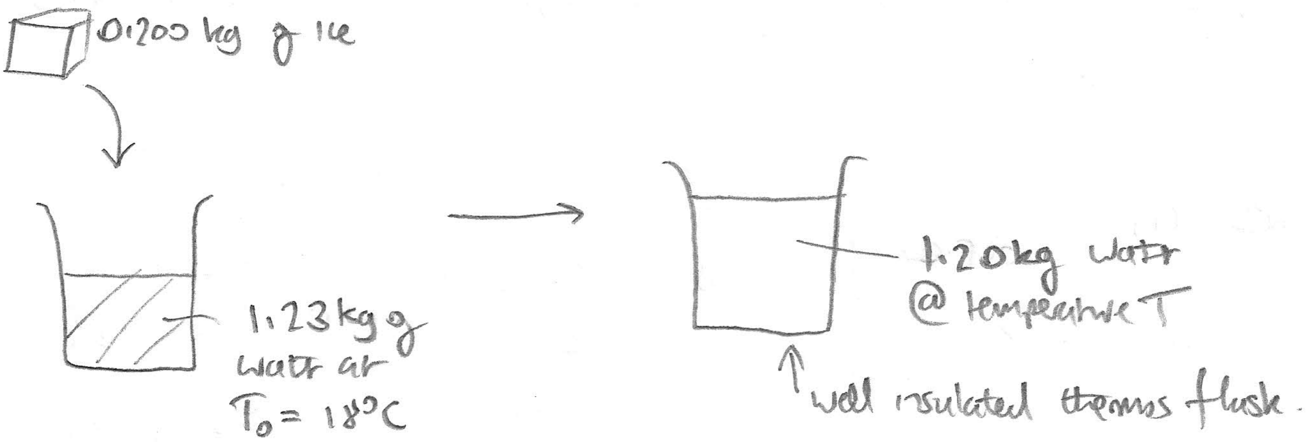
$+ 12.4 \times 62.8$

$+ 0.15 \times 12.4 \times (1200 - 1063)$ (kJ)

$= 1665.2 + 778.7 + 254.8$ (kJ)

$= \boxed{2699\text{ kJ}}$

(iv)



Energy balance: $\underbrace{M_{ice} L_{fus} + C_L M_{ice} T}_{\text{energy gain by ice}} = \underbrace{C_L M_w (T_0 - T)}_{\text{energy loss from water}}$

$$\therefore \quad T (C_L M_{ice} + C_L M_w) = C_L M_w T_0 - M_{ice} L_{fus}$$

$$\therefore \quad \boxed{T = \frac{C_L M_w T_0 - M_{ice} L_{fus}}{C_L M_{ice} + C_L M_w}}$$

$$\therefore \quad \boxed{T = \frac{T_0 - M_{ice} L_{fus} / C_L M_w}{M_{ice}/M_w + 1}} \quad (*)$$

$$\therefore \quad T = \frac{18 - \frac{0.200 \times 335}{4.187 \times 1.23}}{\frac{0.200}{1.23} + 1}$$

$$\boxed{T = 4.29°C}$$

Now in order to melt all the ice by cooling the water, it is assume the remaining water must be liquid, ie $T > 0°C$

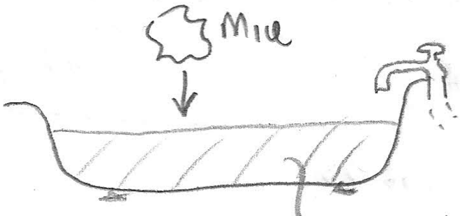
From $(*)$ $\Rightarrow$ $T_0 > M_{ice} L_{fus} / C_L M_w$ $\Rightarrow$ $\boxed{M_w > \frac{M_{ice} L_{fus}}{C_L T_0}}$

So the minimum mass of water is $\dfrac{0.200 \times 335}{4.187 \times 18}$ kg

$= \boxed{0.889 \text{ kg}}$

If less water is used, then one assumes not all the ice will be melted. There will be a mixture of ice and water (at 0°C), if maintained isolated from the surroundings.

v)



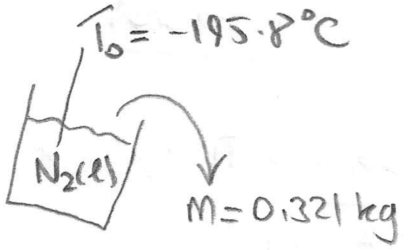
let final T be 6°C.

Energy transfer: $M_{ice} L_{fus} + C_L M_{ice} T = C_L M_W (T_0 - T)$

$\therefore \boxed{M_{ice} = \dfrac{C_L M_W (T_0 - T)}{L_{fus} + C_L T}} = \dfrac{4.187 \times 65 \times (14-6)}{335 + 4.187 \times 6}$

$= \boxed{6.05 \text{ kg}}$

(vi)



$T_a = 25°C$

$\Delta E_{vap} = m L_{vap}$

$= 0.321 \times 200$ (kJ)

$= \boxed{64.2 \text{ kJ}}$

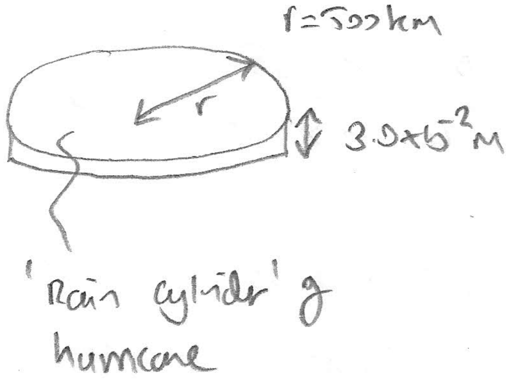
$\Delta E_g = m c_g (T_a - T_0)$

$= 0.321 \times 1.040 \times (25 - -195.8)$

$= \boxed{73.7 \text{ kJ}}$

So ΔE_g is only 15% larger than ΔE_{vap}.

(vii)



Volume of rain / day is:

$V = \pi r^2 \times 3.0\times10^{-2}$ m^3

$V = \pi \times (500\times10^3)^2 \times 3.0\times10^{-2}$ m^3

$\boxed{V = 2.36\times10^{10}\ m^3}$

∴ Energy/s released from condensing of water vapour to rain is

$$P = \frac{m L_{vap}}{t_{day}} = \quad P = \frac{\underbrace{1000\ kg/m^3 \times 2.36\times10^{10}\ m^3}_{\text{Mass of rain}} \times \underbrace{2260\ kJ/kg}_{L_{vap}}}{24\times3600\ s}$$

$$\Rightarrow \boxed{P = 6.17\times10^{11}\ kJ/s}$$

(ie $P = \boxed{6.17\times10^{14}\ W}$

If a typical power station produces 2 → 6 GW, so if 4 GW is an average:

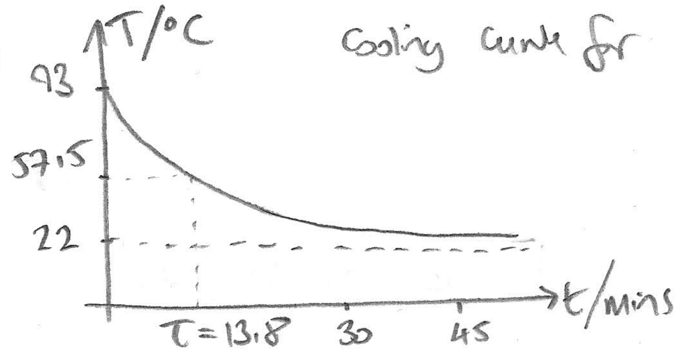
$$P/4GW = \frac{6.17\times10^{14}\ W}{4\times10^9\ W} = \boxed{154{,}330 \text{ power stations!}}$$

If the global generating capacity is 30,000 TWh/year

$$= \frac{30{,}000\times3.6\times10^{15}\ J}{365\times24\times3600\ s} = \boxed{3.42\times10^{12}\ W}$$

So $\frac{P}{3.42\times10^{12}\ W} = \boxed{181}$ So a typical hurricane may be ≈ 181 times as powerful as all the power stations on Earth! If only we could harness it!

(viii) Cooling curve for coffee.



$T_a = 22°C \quad T_0 = 93°C$

Assume Newton's law of cooling: $\boxed{T = T_a + (T_0 - T_a)e^{-\ln 2\, t/\tau}}$

so: $\ln\left(\frac{T - T_a}{T_0 - T_a}\right) = -\ln 2\, t/\tau$

$\therefore \ln\left(\frac{T_0 - T_a}{T - T_a}\right) = \ln 2\, t/\tau$

$\therefore \boxed{\tau = \frac{\ln 2\, t}{\ln\left(\frac{T_0 - T_a}{T - T_a}\right)}}$

so if $T = 65°C$ when $t = 10$ minutes

$\Rightarrow \tau = \frac{\ln 2 \times 10}{\ln\left(\frac{93 - 22}{65 - 22}\right)} = \boxed{13.8 \text{ Minutes}}$

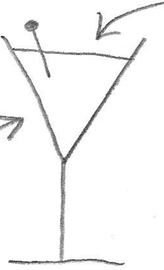
To reach 30°C from start is: $t = \frac{\tau \ln\left(\frac{T_0 - T_a}{T - T_a}\right)}{\ln 2}$

$= \frac{13.8 \text{ mins} \times \ln\left(\frac{93 - 22}{30 - 22}\right)}{\ln 2}$

$= \boxed{43.5 \text{ Minutes}}$ So if 65°C after 10 minutes,

Dr F will have to wait $\boxed{33.5 \text{ Minutes}}$ for the coffee to reach 30°C, after starting timing from when the coffee is 65°C.

(ix)



Huge x litre glass!

0.2 kg of alcohol (M_a)
+ 0.8 kg of water (M_w)
$T_0 = 25°C$

For both the alcohol and water to be frozen
$T \leq -114°C$

To cool both alcohol and water to $T = -114°C$, and freeze both, you will need to extract:

WATER: $\Delta E_w = C_L M_w T_0 + M_w L_{fus} + C_S M_w (-T)$

$= 4.187 \times 0.8 \times 25 + 0.8 \times 335 + 2.108 \times 0.8 \times 114$

$= \boxed{544\ kJ}$

ALCOHOL: $\Delta E_a = C_L M_a (T_0 - T) + M_a L_{fus}$

$= 2.440 \times 0.2 \times (25 + 114) + 0.2 \times 108$

$= \boxed{89.4\ kJ}$

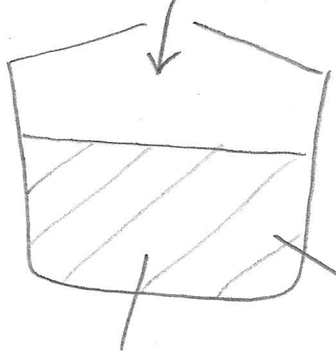
So minimum energy is $\Delta E_w + \Delta E_a = \boxed{633.4\ kJ}$

↑

If you could cool both further

(x)

$m_L = 2.0$ kg of liquid lead
@ $T_0 = 500°C$



$T_W = 20°C$

M

(mass of water)

Eventually
$T = 25°C$

Energy balance:

$$C_L M (T - T_W) = C_{LL} M_L (T_0 - T_M) + M_L L_{fus} + C_S M_L (T_M - T)$$

$$\therefore M = \frac{C_{LL} M_L (T_0 - T_M) + M_L L_{fus} + C_S M_L (T_M - T)}{C_L (T - T_W)}$$

[C_{LL} is specific heat capacity of liquid lead]

$\therefore\ M = \dfrac{0.118 \times 2.0 \times (500 - 327.3) + 2.0 \times 23.2 + 0.136 \times 2.0 \times (327.3 - 25)}{4.187 \times (25 - 20)}$

$\boxed{M = 8.09\ kg}$ of water

Q2/ (i) $M = 10{,}000$ kg of frozen Oxygen at $T_0 = -250°C$

To convert the oxygen to gas at $25°C = T$

$$\Delta E = C_s M (T_M - T_0) + M L_{fus} + C_L M (T_{boil} - T_M) + M L_{vap} + C_g M (T - T_{boil})$$

$$= [0.779 \times (-218.8 - -250) + 13.9 + 1.669 \times (-183 - -218.8) + 213 + 0.919 \times (25 - -183)] \times 10{,}000 \quad (kJ)$$

$$= \boxed{5.02 \times 10^6\ kJ}$$

(ii) one breath is $0.21 \times 500\ cm^3 = 0.21 \times 500\ (10^{-2} m)^3$ of O_2.

so mass of O_2 per breath is:

$M_b = 0.21 \times 500 \times 10^{-6} \times 1.43$ kg

$= \boxed{1.50 \times 10^{-4}\ kg}$

Density of O_2 (g) is $1.43\ kg/m^3$

So 10,000 kg should equate to $\dfrac{10{,}000}{1.50 \times 10^{-4}}$ breaths

$\approx 6.66 \times 10^7$. If 20 breaths / minute

$\Rightarrow$ 10,000 kg of O_2 should provide a human with the O_2 air content for $\dfrac{6.66 \times 10^7}{20}$ minutes

$= 3.33 \times 10^6$ minutes $\approx$ $\boxed{6.34\ years}$

(iii) If the O_2 is converted at just the rate sufficient to supply one human with oxygen

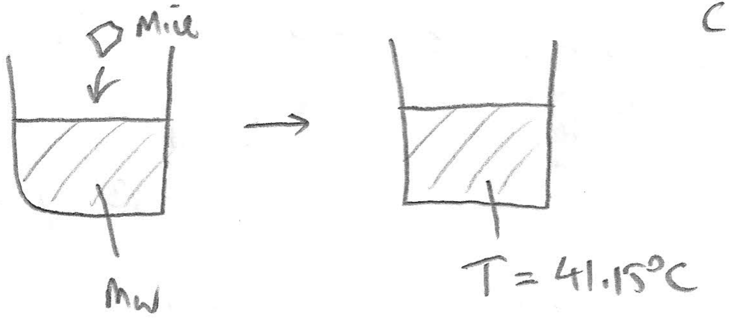
$$\text{Power} = \frac{5.02 \times 10^6 \text{ kJ}}{3.33 \times 10^6 \times 60 \text{ s}}$$

$$= \boxed{0.025 \text{ kW}}$$

So not much! But expect many more humans than one to need the O_2. Also assume losses. And also assume the humans will occasionally need more than 1.50×10^{-4} kg per breath, and indeed breath at a greater rate than 20/min. So perhaps about 1 kW is needed! (i.e. factor of 100 larger).

↳ However this does reduce the time to use up the O_2 by $\times$ 100 also, so rather than 6.34 years you might have more like 23 days.

Q3/ $M_{ice} = 26.98$ g @ 0°C
$M_W = 85.40$ g @ 79.37°C $= T_0$



$$\underbrace{C_L M_{ice} T + M_{ice} L_{fus}}_{\text{Energy gain of ice}} = \underbrace{C_L M_W (T_0 - T)}_{\text{energy loss from water}}$$

$$\therefore \boxed{L_{fus} = \frac{C_L M_W (T_0 - T) - C_L M_{ice} T}{M_{ice}}}$$

$$\therefore L_{fus} = \frac{4.187 \times 85.40 \times 10^{-3} \times (79.37 - 41.15) - 4.187 \times \frac{26.98}{1000} \times 41.15}{26.98 \times 10^{-3}}$$

$$= \boxed{334 \text{ kJ/kg}}$$