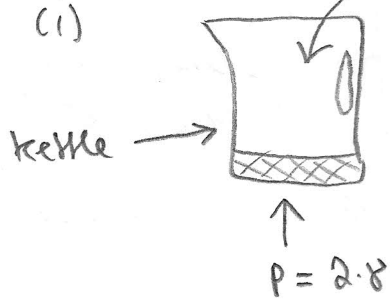
SPECIFIC HEAT CAPACITY, LATENT HEAT & NEWTON'S LAW OF COOLING

Q1 (i)

$m = 1.23\,kg$ of water at $T_0 = 18°C$



$$\boxed{\begin{aligned}\Delta E &= cm\Delta T \\ \Delta E &= Pt\end{aligned}}$$

$\Delta T = 100°C - 18°C$
$= \boxed{82°C} = 82K$

$c = 4.187\ kJ/kg/K$

$$\therefore \quad t = \frac{cm\Delta T}{P} = \frac{4.187\,kJ/kg/K \times 1.23\,kg \times 82K}{2.8\,kJ/s}$$

$$= \boxed{151\ s}$$ (2 minutes, 31 seconds) or $\approx 2\frac{1}{2}$ minutes.

(ii) let $(\overset{\Delta m}{1.23 - 0.20})$ kg of water be vaporized.

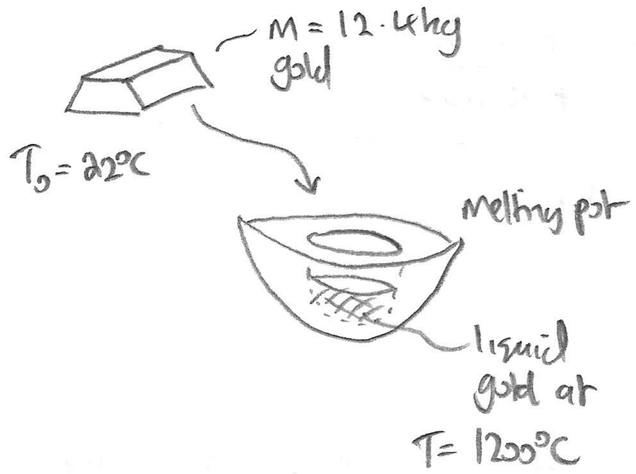
$\therefore \boxed{Pt = \Delta m L_{vap}}$ $\quad \therefore \quad t = \Delta m L_{vap}/P$

$$t = \frac{1.03\,kg \times 2260\,kJ/kg}{2.8\,kJ/s}$$

$t = 831\ s$

$= \boxed{13\ \text{minutes},\ 51\ \text{seconds}}$

(iii)

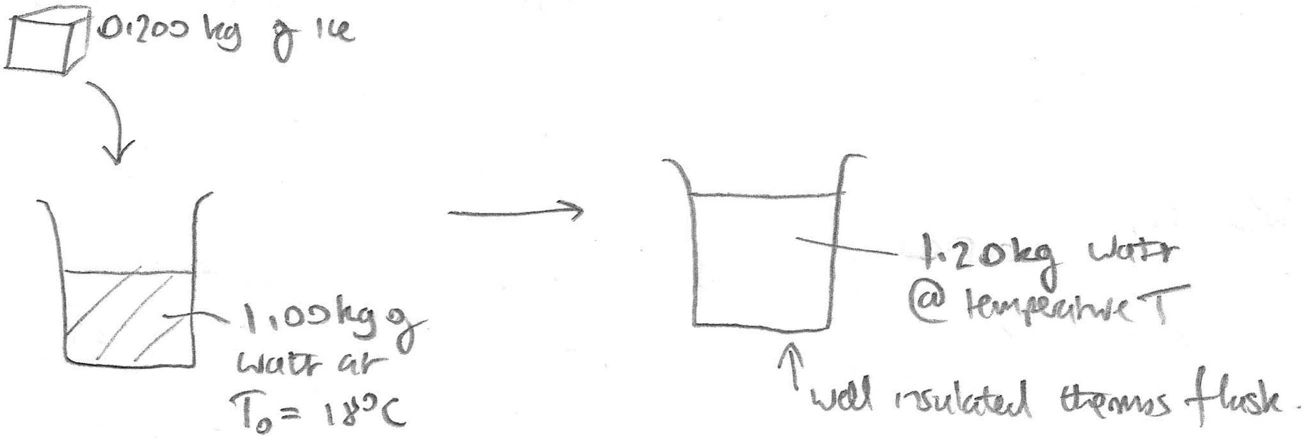


$$\boxed{\Delta E = C_S m (T_{melt} - T_0) + mL_{fus} + C_L m (T - T_{melt})}$$

$= 0.129 \times 12.4 \times (1063 - 22)$
$+ 12.4 \times 62.8$
$+ 0.15 \times 12.4 \times (1200 - 1063)$ (kJ)

$= 1665.2 + 778.7 + 254.8$ (kJ)
$= \boxed{2699\,kJ}$

(iv)



Energy balance: $\underbrace{M_{ice} L_{fus} + C_L M_{ice} T}_{\text{energy gain by ice}} = \underbrace{C_L M_W (T_0 - T)}_{\text{energy loss from water.}}$

$$\therefore \quad T(C_L M_{ice} + C_L M_W) = C_L M_W T_0 - M_{ice} L_{fus}$$

$$\therefore \quad \boxed{T = \frac{C_L M_W T_0 - M_{ice} L_{fus}}{C_L M_{ice} + C_L M_W}}$$

$$\therefore \quad \boxed{T = \frac{T_0 - M_{ice} L_{fus} / C_L M_W}{M_{ice}/M_W + 1}} \quad (*)$$

$$\therefore \quad T = \frac{18 - \frac{0.200 \times 335}{4.187 \times 1.00}}{\frac{0.200}{0.500} + 1}$$

$$\boxed{T = 1.43°C}$$

Now in order to melt all the ice by cooling the water, it is assume the remaining water must be liquid, if $T > 0°C$

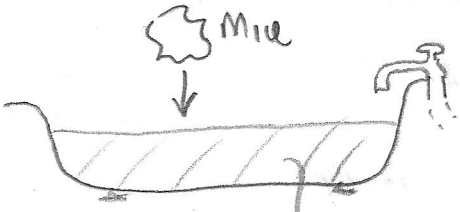
From $(*)$ $\Rightarrow$ $T_0 > M_{ice} L_{fus} / C_L M_W$ $\Rightarrow$ $\boxed{M_W > \frac{M_{ice} L_{fus}}{C_L T_0}}$

So the minimum mass of water is $\dfrac{0.200 \times 335}{4.187 \times 18}$ kg

$= \boxed{0.889 \text{ kg}}$

If less water is used, then one assumes not all the ice will be melted. There will be a mixture of ice and water (at 0°C), if maintained isolated from the surroundings.

v)



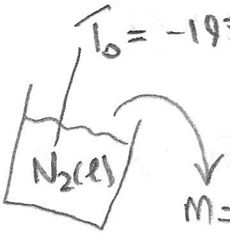
let final T be 6°C.

Energy transfer: $M_{ice} L_{fus} + C_L M_{ice} T = C_L M_W (T_0 - T)$

$$\therefore \boxed{M_{ice} = \frac{C_L M_W (T_0 - T)}{L_{fus} + C_L T}} = \frac{4.187 \times 65 \times (14-6)}{335 + 4.187 \times 6}$$

$$= \boxed{6.05 \text{ kg}}$$

(vi)



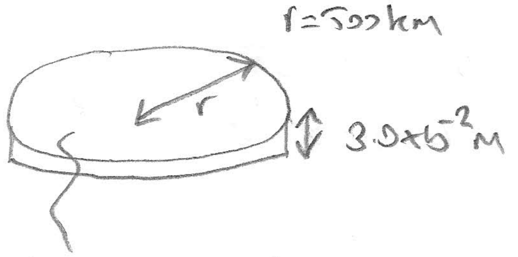
$T_a = 25°C$

$$\Delta E_{vap} = m L_{vap}$$
$$= 0.321 \times 200 \quad \text{(kJ)}$$
$$= \boxed{64.2 \text{ kJ}}$$

$$\Delta E_g = m c_g (T_a - T_0)$$
$$= 0.321 \times 1.040 \times (25 - -195.8)$$
$$= \boxed{73.7 \text{ kJ}}$$

So ΔE_g is only 15% larger than ΔE_{vap}.

(vii)



'Rain cylinder' of hurricane

Volume of rain / day is:

$V = \pi r^2 \times 3.0\times10^{-2}$ m^3

$V = \pi \times (500\times10^3)^2 \times 3.0\times10^{-2}$ m^3

$\boxed{V = 2.36\times10^{10}\ m^3}$

$\therefore$ Energy/s released from fusion of water vapour to rain is

$$P = \frac{m L_{fus}}{t_{day}} = P = \frac{\underbrace{1000\ kg/m^3 \times 2.36\times10^{10}\ m^3}_{\text{mass of rain}} \times \underbrace{335\ kJ/kg}_{L_{fus}}}{24\times3600\ s}$$

$$\Rightarrow \boxed{P = 9.14\times10^{10}\ kJ/s}$$

$$\left(\text{ie } P = \boxed{9.14\times10^{13}\ W}\right)$$

If a typical power station produces 2 → 6 GW, so if 4 GW is an average:

$$P/4GW = \frac{9.14\times10^{13}\ W}{4\times10^{9}\ W} = \boxed{22,800 \text{ power stations!}}$$

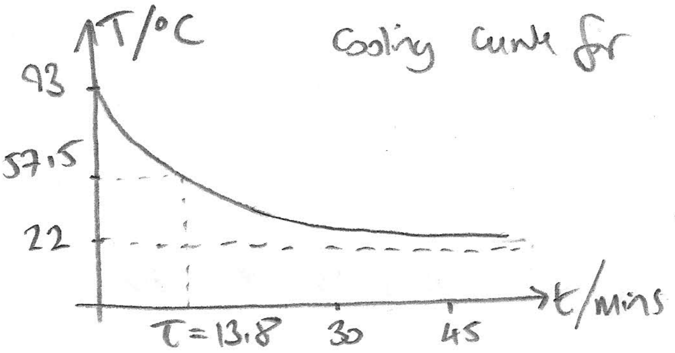
If the global generating capacity is 30,000 TWh/year

$$= \frac{30,000\times3.6\times10^{15}\ J}{365\times24\times3600\ s} = \boxed{3.42\times10^{12}\ W}$$

$$\text{So } \frac{P}{3.42\times10^{12}\ W} = \boxed{26.7}$$

So a typical hurricane may be ≈ 30 times as powerful as all the power stations on Earth! If only we could harness it!

(viii) Cooling curve for coffee.



$T_a = 22°C \qquad T_0 = 93°C$

Assume Newton's law of cooling: $\boxed{T = T_a + (T_0 - T_a)e^{-\ln 2\, t/\tau}}$

so: $\ln\left(\frac{T - T_a}{T_0 - T_a}\right) = -\ln 2\, t/\tau$

$\therefore \ln\left(\frac{T_0 - T_a}{T - T_a}\right) = \ln 2\, t/\tau$

$\therefore \boxed{\tau = \frac{\ln 2\, t}{\ln\left(\frac{T_0 - T_a}{T - T_a}\right)}}$

so if $T = 65°C$ when $t = 10$ minutes

$$\Rightarrow \tau = \frac{\ln 2 \times 10}{\ln\left(\frac{93 - 22}{65 - 22}\right)} = \boxed{13.8 \text{ minutes}}$$

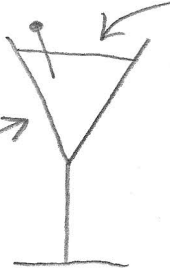
To reach 30°C from start is: $t = \frac{\tau \ln\left(\frac{T_0 - T_a}{T - T_a}\right)}{\ln 2}$

$$= \frac{13.8 \text{ mins} \times \ln\left(\frac{93 - 22}{30 - 22}\right)}{\ln 2}$$

$= \boxed{43.5 \text{ minutes}}$ So if 65°C after 10 minutes,

Dr F will have to wait $\boxed{33.5 \text{ minutes}}$ for the coffee to reach 30°C, after starting timing from when the coffee is 65°C.

(ix)



Huge x litre glass!

0.2 kg of alcohol (M_a)
+ 0.8 kg of water (M_w)
$T_0 = 25^\circ C$

For both the alcohol and water to be frozen
$T \leqslant -114^\circ C$

To cool both alcohol and water to $T = -114^\circ C$, and freeze both, you will need to extract:

WATER:
$$\Delta E_w = C_L M_w T_0 + M_w L_{fus} + C_S M_w (-T)$$
$$= 4.187 \times 0.8 \times 25 + 0.8 \times 335 + 2.108 \times 0.8 \times 114$$
$$= \boxed{544\ kJ}$$

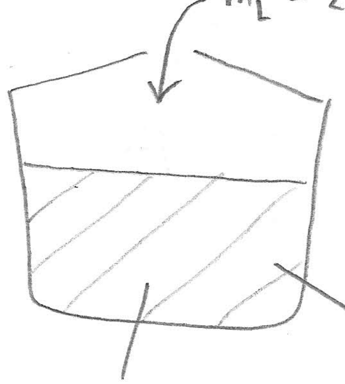
ALCOHOL:
$$\Delta E_a = C_L M_a (T_0 - T) + M_a L_{fus}$$
$$= 2.440 \times 0.2 \times (25 + 114) + 0.8 \times 108$$
$$= \boxed{154\ kJ}$$

So minimum energy is $\Delta E_w + \Delta E_a = \boxed{698\ kJ}$

if you could cool both further

(x)

$M_L = 2.0$ kg of liquid lead
@ $T_0 = 500^\circ C$



$T_W = 20^\circ C$

M
(mass of water)

Eventually
$T = 25^\circ C$

Energy balance:
$$C_L M (T - T_W) = C_{LL} M_L (T_0 - T_M) + M_L L_{fus} + C_S M_L (T_M - T)$$
$$\therefore M = \frac{C_{LL} M_L (T_0 - T_M) + M_L L_{fus} + C_S M_L (T_M - T)}{C_L (T - T_W)}$$

[C_{LL} is specific heat capacity of liquid lead]

$$\therefore\ M = \frac{0.118\times 2.0\times(500-327.3)+2.0\times 23.2+0.136\times 2.0\times(327.3-25)}{4.187\times(25-20)}$$

$\boxed{M = 8.09\ kg}$ of water

Q2/ (i) $M = 10{,}000$ kg of frozen Oxygen at $T_0 = -250°C$

To convert the oxygen to gas at $25°C = T$

$$\Delta E = C_S M(T_M - T_0) + ML_{fus} + C_L M(T_{boil} - T_M) + C_g M(T - T_{boil})$$

$$= \left[0.779\times(-218.8--250) + 13.9 + 1.669\times(-183--218.8) + 0.919\times(25--183)\right]\times 10{,}000 \quad (kJ)$$

$$= \boxed{2.891\times 10^6\ kJ}$$

(ii) one breath is $0.21\times 500\ cm^3 = 0.21\times 500\,(10^{-2}m)^3$ of O_2.

so mass of O_2 per breath is:

$$M_b = 0.21\times 500\times 10^{-6}\times 1.43 \quad kg$$

$$= \boxed{1.50\times 10^{-4}\ kg}$$

Density of O_2 (g) is $1.43\ kg/m^3$

So 10,000 kg should equate to $\frac{10{,}000}{1.50\times 10^{-4}}$ breaths $\approx 6.66\times 10^7$. If 20 breaths / minute

$\Rightarrow$ 10,000 kg of O_2 should provide a human with the O_2 air content for $\frac{6.66\times 10^7}{20}$ minutes

$= 3.33\times 10^6$ minutes $\approx \boxed{6.34\ years}$

(iii) If the O_2 is converted at just the rate sufficient to supply one human with oxygen

$$\text{Power} = \frac{2.891 \times 10^{6}\ \text{kJ}}{3.33 \times 10^{6} \times 60\ \text{s}}$$

$$= \boxed{0.014\ \text{kW}}$$

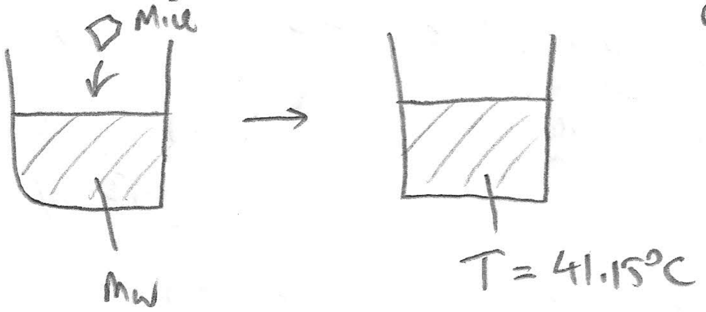
So not much! But expect many more humans than one to need the O_2. Als assume losses. And also assume the humans will occasionally need more than 1.50×10^{-4} kg per breath, and indeed breath at a greater rate than 20/min.

So perhaps about 1 kW is needed! (i.e. factor of 100 larger).

↳ However this does reduce the time to use up the O_2 by $\times$ 100 also, so rather than 6.34 years you might have more like <u>23 days</u>.

Q3/ $M_{ice} = 26.98$ g @ 0°C

$M_W = 85.40$ g @ $79.37°C = T_0$



$$\underbrace{C_L M_{ice} T + M_{ice} L_{fus}}_{\text{Energy gain of ice}} = \underbrace{C_L M_W (T_0 - T)}_{\text{energy loss from water}}$$

$$\therefore \boxed{L_{fus} = \frac{C_L M_W (T_0 - T) - C_L M_{ice} T}{M_{ice}}}$$

$$\therefore L_{fus} = \frac{4.187 \times 85.40 \times 10^{-3} \times (79.37 - 41.15) - \frac{4.187 \times 26.98 \times 41.15}{1000}}{26.98 \times 10^{-3}}$$

$$= \boxed{334\ \text{kJ/kg}}$$