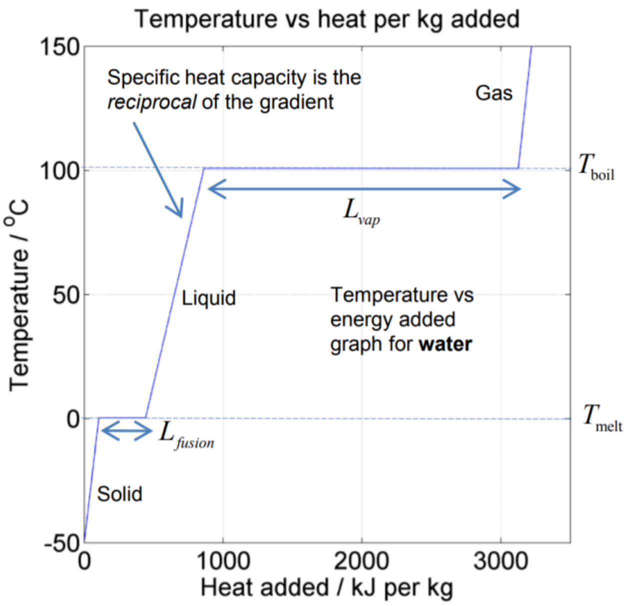
During distinct *solid*, *liquid* and *gaseous* phases, if losses can be ignored, *'temperature rise is proportional to the amount of thermal energy added,'* is a reasonable approximate model.

This tallies with the definition of (absolute) temperature (in Kelvin, K), that **temperature is proportional to the mean kinetic energy of (random) molecular motion.**

The **specific heat capacity** c is the *energy required to raise 1kg of a substance by 1 degree of temperature* (in Celsius or Kelvin scales). So if the energy change is ΔE, thermal mass is m and ΔT is the temperature rise: $\boxed{\Delta E = cm\Delta T}$. The specific heat capacity c is assumed to be *constant* within each phase. In reality c varies with temperature, but this is only really significant at very cold temperatures, and indeed very hot temperatures when additional modes of molecular movement are activated.



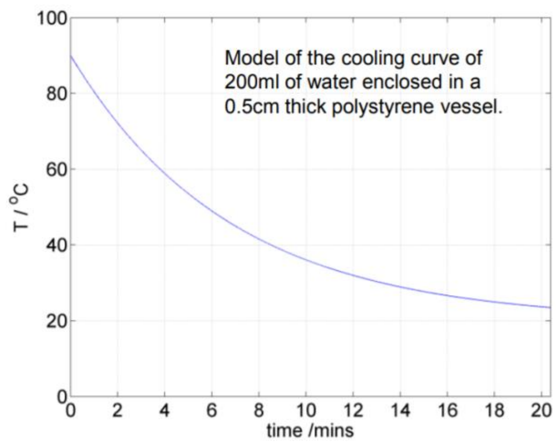
At a bulk *phase transition temperature* (e.g. freezing or boiling point)[1], the energy required to change state (per kg of substance) is the **specific latent heat** L_{fus} or L_{vap}. $\boxed{\Delta E_{SL} = mL_{fus}}$ is the **latent heat of fusion** required for a solid to liquid transition. This is also the energy released when a liquid freezes and solidifies. $\boxed{\Delta E_{LG} = mL_{vap}}$ is the **latent heat of vaporization** required for a liquid to gas transition. This is also the energy released when a gas condenses and becomes a liquid. The hydrogen bonding in water molecules means L_{vap} is *particularly large*, and helps to explain the energy source of hurricanes. (Water vapor spirals around the eye wall of a hurricane from a hot and humid sea surface, rises, cools and falls as rain. The release of energy during this gas to liquid phase transition powers the rotation of the hurricane vortex).

Data from: https://www.engineeringtoolbox.com/specific-heat-metals-d_152.html. All values correspond to standard atmospheric pressure 101,325Pa.

Substance	T_{melt} /°C	T_{boil} /°C	L_{fus} / kJkg^{-1}	L_{vap} / kJkg^{-1}	c_S / kJkg^{-1}K^{-1}	c_L / kJkg^{-1}K^{-1}	c_G / kJkg^{-1}K^{-1}
Water	0	100	335	2,260	2.108	4.187	1.930
Ethanol	-114	78.3	108	855	0.970	2.440	1.900
Nitrogen	-210	-195.8	25.7	200	0.890	2.042	1.040
Oxygen	-218.8	-183	13.9	213	0.779	1.669	0.919
Copper	1,083	2,566	207	4,730	0.385	0.386	0.380
Gold	1,063	2,808	62.8	1,720	0.129	0.15	-
Lead	327.3	1,750	23.2	859	0.136	0.118	-

Newton's law of cooling is a simplification of *Fourier's law* of heat transport via *conduction*. If we ignore *convection* (bulk transport of heated molecules) and *radiation* (transfer of energy via emission or absorption of electromagnetic waves), the rate of transfer of heat from a hot body to an adjacent cooler body is proportional to the *temperature gradient* between them.

Newton's law simplifies this, and incorporates the spatial aspects of the temperature gradient (i.e. resulting from container wall thickness etc) into a single constant k. Newton's law states the rate of change of temperature T of a body is proportional to *the temperature difference* between T and ambient temperature T_a.



[1] A more complete definition of specific latent heat is the 'energy required, per kg, to change phase, *at constant temperature*'.

$$\frac{dT}{dt} = -k(T - T_a) \Rightarrow \int_{T_0}^{T} \frac{dT'}{T' - T_a} = -kt \Rightarrow \ln\left(\frac{T - T_a}{T_0 - T_a}\right) = -kt$$

$$\Rightarrow \boxed{T = T_a + (T_0 - T_a)e^{-kt}}$$

i.e. an *exponential decay* of temperature with time.

Much like radioactive decay, and the discharge of a capacitor, k is perhaps more usefully defined in terms of a *time constant* τ . Let the time constant correspond to the time taken for the substance to reach the average temperature: $\bar{T} = \frac{1}{2}(T_0 + T_a)$.

$$\ln\left(\frac{\bar{T} - T_a}{T_0 - T_a}\right) = -k\tau \Rightarrow \ln\left(\frac{\frac{1}{2}(T_0 + T_a) - T_a}{T_0 - T_a}\right) = -k\tau \Rightarrow k = -\frac{1}{\tau}\ln\left(\frac{\frac{1}{2}(T_0 - T_a)}{T_0 - T_a}\right)$$

$$\Rightarrow \boxed{k = \frac{\ln 2}{\tau}}$$

Question 1

(i) 1.23kg of water at 18°C is poured from a tap into a well insulated kettle. Taking in account losses such as heating the kettle itself, the kettle supplies 2.8kW of power to the water in the form of heating. How long does it take (in minutes) to boil the water? Assume a negligible amount of water evaporates during the heating process.

(ii) The same kettle is (i) is allowed to boil until only 0.20kg of liquid water remains. The lid is removed to facilitate the escape of water vapour. How long (in minutes) from the onset of boiling does this take?

(iii) A standard gold bar has a mass of 12.4kg. It is placed in a melting pot at 22°C and electrically heated with minimal losses. Calculate the amount of energy required to produce molten gold at 1200°C.

(iv) 0.200kg of ice at 0°C is added to 1.23kg of water, initially at 18°C. The water & ice are in a thermos flask, so energy transfers beyond the water + ice system can be ignored. Calculate the temperature of the water once all the ice has melted. What is the minimum mass of water required to melt all the ice?

(v) An athlete pours herself a cold water bath at 14°C. She uses 65kg of cold water. Calculate the mass (in kg) of ice required to reduce the bath temperature to a very chilly 6°. Assume at this point all the ice has melted, and there is no energy transfer to from the bath to the surroundings.

(vi) 0.321kg of boiling liquid Nitrogen is poured onto a desk during a science demonstration. Calculate the amount of energy to vaporize all the liquid nitrogen. Contrast this energy to the amount required to raise the 'just boiled' nitrogen gas, to a laboratory ambient temperature of 25°C.

(vii) If a hurricane of radius 500km precipitates an average of 3.0cm of rain per day, calculate the energy/s released from the fusion of water vapour to rain. Compare this to a typical power station (4GW) and the world's generating capacity (about 30,000TWh per year[2]). Assume the density of rain water is 1000kg/m^3.

(viii) Dr French makes a cup of coffee at the optimum brewing temperature of 93°C. Unfortunately he forgets about it, and ten minutes later it is 65°C. If the ambient temperature is 22°C, find the characteristic time constant τ for the cooling coffee. Hence calculate how much longer it will take for the coffee to reach 30°C. Also sketch the cooling curve of the coffee.

(ix) A cocktail mixture containing 0.2kg of alcohol and 0.8kg of flavoured water is mixed at 25°C. Calculate the minimum energy that must be extracted in order for the *entire* mixture to be frozen.

(x) In a lead smelting workshop, 2.0kg of liquid lead at 500°C is dropped into a vat of water which is initially at 20°C. The vat is insulated, and sealed to prevent any significant amount of steam escaping. After a short time, the solid lead is in thermal equilibrium with the water, which is now 25°C. Calculate the mass of water (in kg).

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[2] $1\text{TWh} = 10^{12}\text{Js}^{-1} \times 3600\text{s} = 3.6 \times 10^{15}\text{J}$

Question 2 An ordinary breath for a human adult is about 500ml. Air is about 21% oxygen by volume, and the density of oxygen at sea level is about 1.43kg/m^3. On a future space mission, a deposit of 10,000kg of solid oxygen is discovered on a remote planet, at a temperature of -250^{o}C (i.e. 23K).

(i) Calculate the amount of energy required to convert all the frozen oxygen to gas at 25^{o}C.

(ii) If an average breathing rate is 20 breaths per minute, show the oxygen could sustain a single human for 6.34 years. Assume there is a plentiful supply of nitrogen and other air molecules, apart from oxygen.

(iii) Ignoring the effect of losses, calculate the average power of an electrical heater required to convert the frozen oxygen to breathable gas (for one 'average' human). Suggest what a more practical power should be for a mission.

Question 3 85.40g of water at 79.37^{o}C is added to a styrofoam beaker. A thermocouple is placed in the water and continuous measurements of temperature are recorded by a datalogger.

26.98g of ice at 0^{o}C is added to the water. After about 30s the ice has all melted, and the water temperature is now 41.15^{o}C. Assume negligible heat exchange with the beaker and surroundings during this time.

Use the information above, plus the specific heat capacities of liquid and solid water in the table on page 1, to calculate the specific latent heat of fusion L_{fus} of water.

Question 4 A thermocouple is placed in a kettle containing 0.9542kg of water. The RMS input voltage is 235V and a constant current of 8.7A is drawn by the heating element. Use the following experimental data to determine the specific heat capacity of water, and comment on your result, relative to the value presented in the table on page 1. Time t is in seconds, and temperature T is in ^{o}C.

$$t = [0,10,20,30,40,50,60,70,80,90,100,110,120,130,140,150,160,170]$$
$$T = [19.01,23.13,28.7,34.06,39.13,44.54,49.56,54.49,58.72,63.49,67.84,72.55,76.90,81.42,85.69,90.7,95.56,98.87]$$

Question 5 The same kettle in Q4 is placed upon a mass-balance, which is zeroed once boiling is fully established. The lid of the kettle is left open so vapour can escape, and the kettle continues to boil while the (same) electrical power is supplied. The following measurements of cumulative mass ΔM /g lost are recorded vs time t /s.

$$t = [0,30,60,90,120,150,180,210,240,270,300]$$
$$\Delta M = [0,26.2,49.5,80.2,106.7,129.8,157.1,186.1,210.9,235.5,264.8]$$

Use the data to determine the specific latent heat of vaporization L_{vap} of water.

Question 6 Temperature vs time measurements for a cooling metal block are used to investigate Newton's model of cooling: $T(t) = T_a + (T_0 - T_a)e^{-\frac{t}{\tau}\ln 2}$

Each temperature measurement was taken every 60s.

$$T = [95.0,86.3,78.6,71.8,65.9,60.7,56.1,52.1,48.5,45.4,42.7,40.3,38.2,36.4,34.7,33.3,32.1,31.0,30.0,29.1,28.4]$$

The ambient temperature was $T_a = 23^o\text{C}$. Show that $y = t\ln 2$ vs $x = \ln\left(\frac{T_0 - T_a}{T - T_a}\right)$ is an appropriate straight line graph if the model is correct. Hence, from a line of best fit through the origin, determine the characteristic time constant τ (in s).

Question 7

The Fahrenheit temperature scale T_F relates to the Celsius scale T_C via the equation: $T_F = \frac{9}{5}T_C + 32$

(i) At what temperature in K does $T_F = T_C$?

(ii) United States temperature extremes are (based on a Google search Oct 2019)

Coldest:	-80 °F at Prospect Creek Camp, Alaska on January 23, 1971
Hottest :	134 °F at Greenland Ranch in Death Valley, California on July 10, 1913

Convert these temperatures in to Celsius and Kelvin.

(iii) An ice sculpture of Daniel Fahrenheit of mass 4.2kg is constructed at Prospect Creek Camp, Alaska on January 23, 1971. It is taken in a temperature controlled refrigerated box (at -80^{o}F) to Death Valley and then the box is opened. The ambient temperature at Death Valley is 125°F.
Calculate the total amount of heat energy (in kJ) that the sculpture must absorb once it has been entirely converted to liquid water, which is in thermal equilibrium with the surroundings.

(iv) What will happen to the liquid water over time?

Question 8

Dr French accidentally pours himself a drink of water from the hot tap. The water temperature is 42^{o}C.

The drink is 568.3cm^3 (1 pint) in volume, and he adds 100g of ice (at 0 ^{o}C). The drink and all the ice are poured into a thermos flask and the top is screwed on tight. After a few minutes, Dr French opens the flask and notices all the ice has melted. If liquid water has a density of 0.997g/cm^3, calculate the new temperature of the drink.

You may assume the thermos flask prevents any heat exchange with the ambient environment.

How much ice would you need to reduce the drink to 7 ^{o}C ?

Question 9

James Bond asks for a Martini, shaken not stirred. A dry martini contains gin and vermouth in various proportions, is shaken with ice cubes, and is usually topped with a olive. For simplicity assume the following:

- The cocktail liquid is 250cm^3
- It is served at $T = 5^{o}\mathrm{C}$
- The average density is 0.9g/cm^3 (pure ethanol is 0.79 g/cm^3, water is about 1 g/cm^3)
- The specific heat capacity is about 1200J/kg/K (pure ethanol is 970 J/kg/K, but the cocktail includes some melted ice)

James wears a 10g metal ring (specific heat capacity 370J/kg/K) from *Q* department which unfortunately malfunctions and immediately heats up to a temperature of 500^{o}C. James smoothly removes the ring and drops it into the Martini.

(i) Assuming no evaporation occurs, calculate the final temperature of the Martini.

(ii) In order to lower the entire mixture to 007^{o}C, calculate the mass of ice that must be added, assuming all the ice melts and no heat is transferred to the surroundings. The ring is removed first.