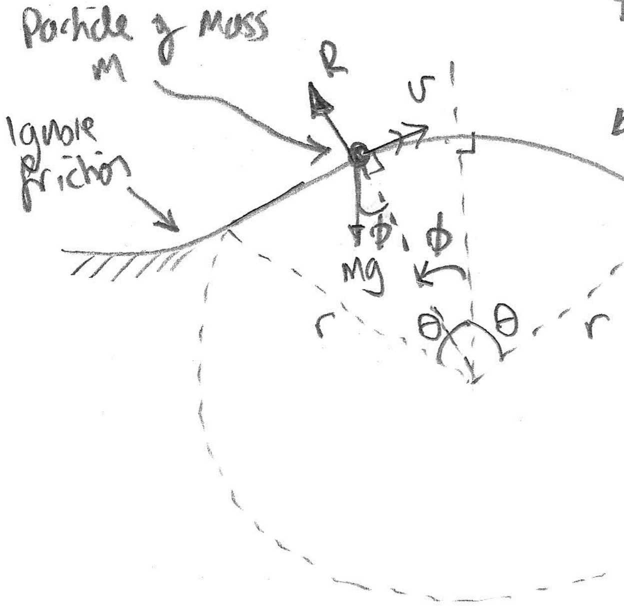
SPEED BUMP

$g = 9.81$ N/kg

[speed bump arc $-\theta < \phi < \theta$]



speed bump, an arc of a circle of radius r

CASE 1 $v =$ constant.

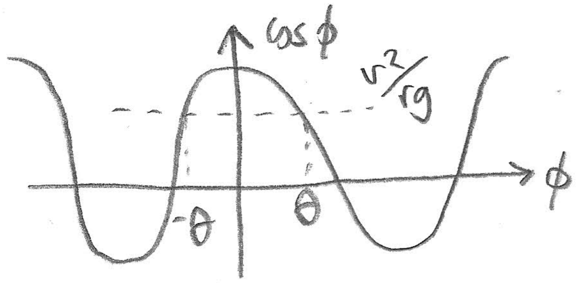
(ie driven car, tangential force from engine maintains speed)

Newton II radially inwards: $\frac{mv^2}{r} = mg\cos\phi - R$

$\therefore$ contact force $\boxed{R = mg\cos\phi - \frac{mv^2}{r}}$

To maintain contact with speed bump $\boxed{R \geq 0}$

$\therefore \; mg\cos\phi - \frac{mv^2}{r} \geq 0 \qquad \therefore \; \cos\phi \geq \frac{v^2}{rg}$



Minimum values for $\cos\phi$ are $\cos\theta$.

$\therefore$ if $\cos\theta \geq \frac{v^2}{rg}$ ~~then~~

$\cos\phi \geq \frac{v^2}{rg}$

[This means if $R \geq 0$ when particle hits speed bump ie when $\phi = \theta$, then it will stay on the speed bump]

so if $\boxed{\cos\theta \geq \frac{v^2}{rg}}$ this means $\boxed{\theta \leq \cos^{-1}\left(\frac{v^2}{rg}\right)}$

So if $v = 30$ mph $= \frac{30}{2.237}$ ms^{-1}

$g = 9.81$ N/kg, $\theta = 30°$, $\quad r \geq \frac{v^2}{g\cos\theta} = r_{min}$

$\Rightarrow r_{min} = \left(\frac{30}{2.237}\right)^2 \Big/ 9.81 \times \cos 30° = \boxed{21.2\text{m}}$