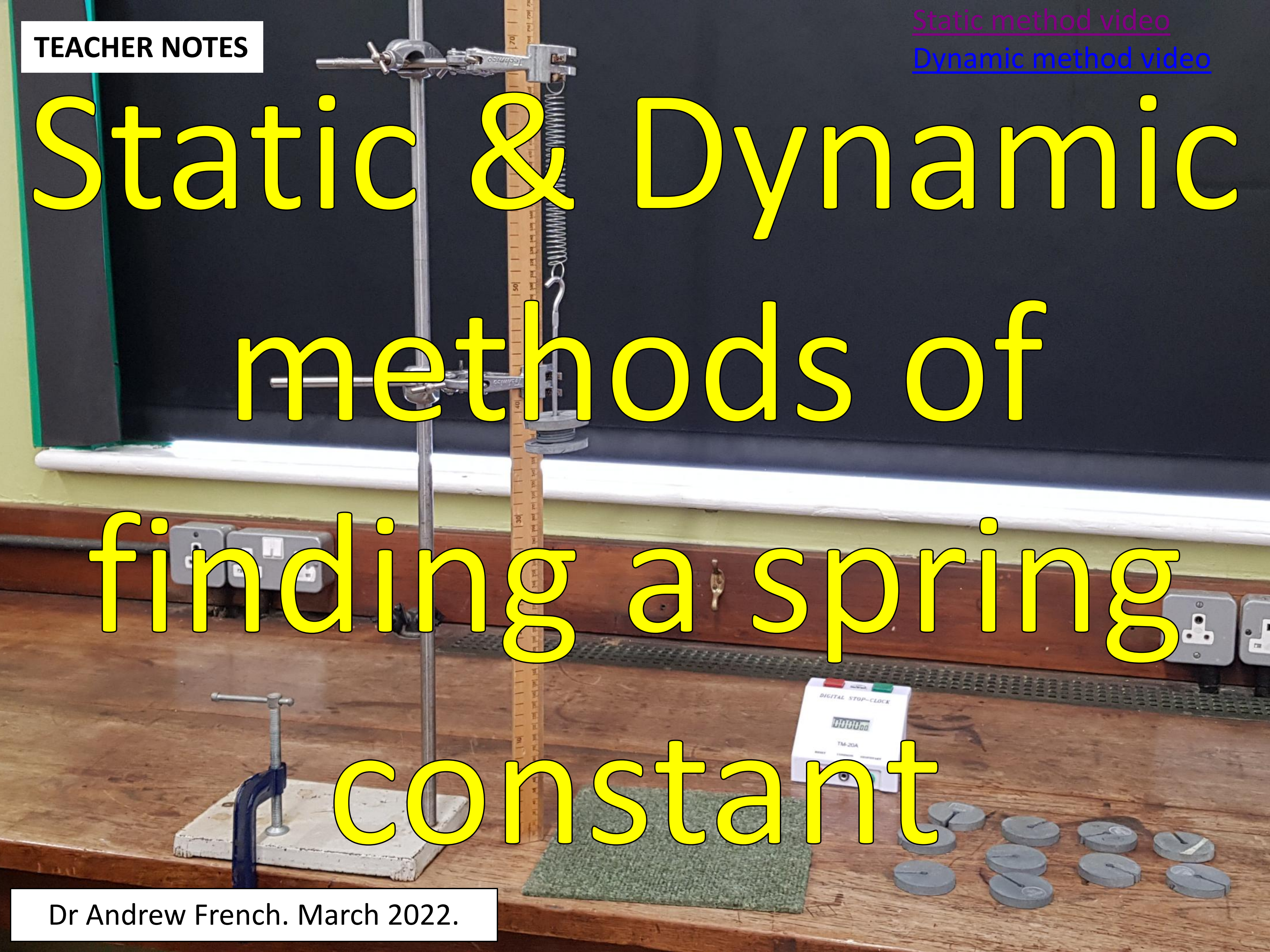
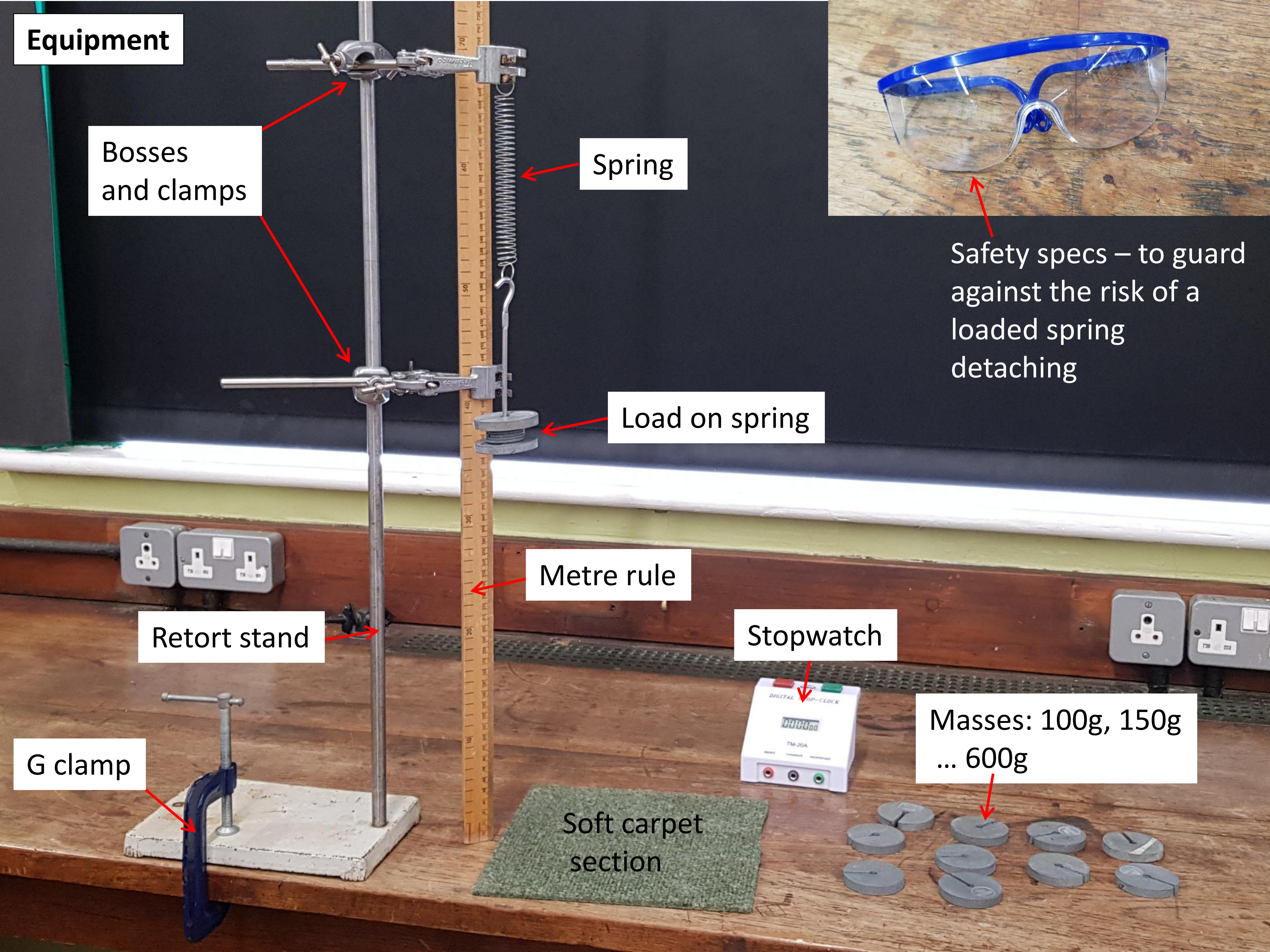
TEACHER NOTES

[Static method video]
[Dynamic method video]

Static & Dynamic methods of finding a spring constant

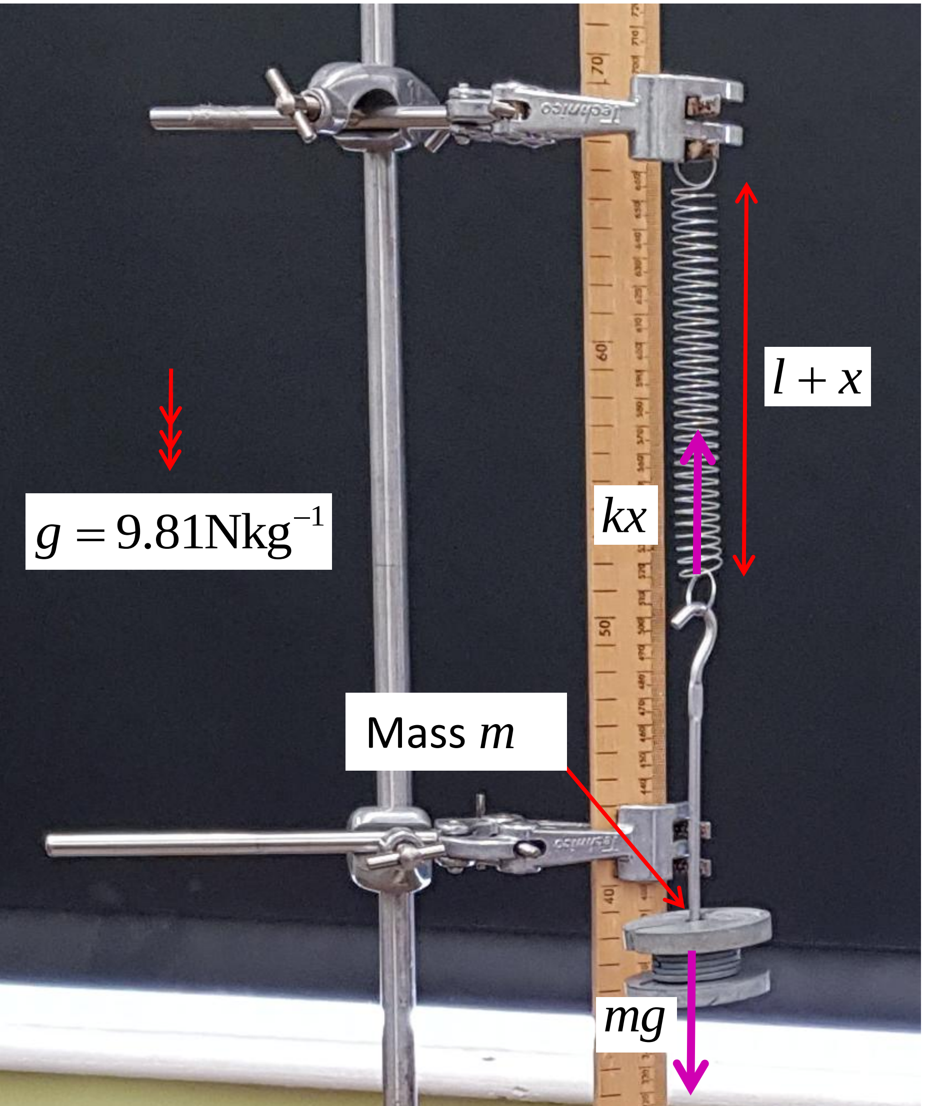


Dr Andrew French. March 2022.

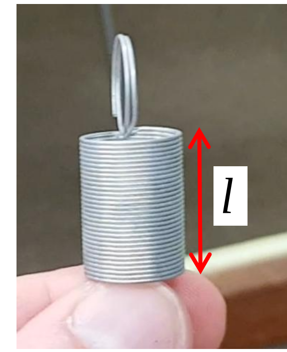

Equipment
Bosses and clamps
Spring
Safety specs – to guard against the risk of a loaded spring detaching
Load on spring
Metre rule
Retort stand
Stopwatch
Masses: 100g, 150g … 600g
G clamp
Soft carpet section

PART 1: FIND A SPRING CONSTANT USING A STATIC METHOD

Suspend masses from a spring and measure equilibrium extension x vs weight of load. If the spring is *Hookean*, then load weight equates to the spring constant k multiplied by extension.



Weight mg and spring tension kx must balance in equilibrium.

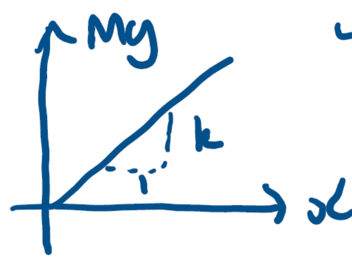


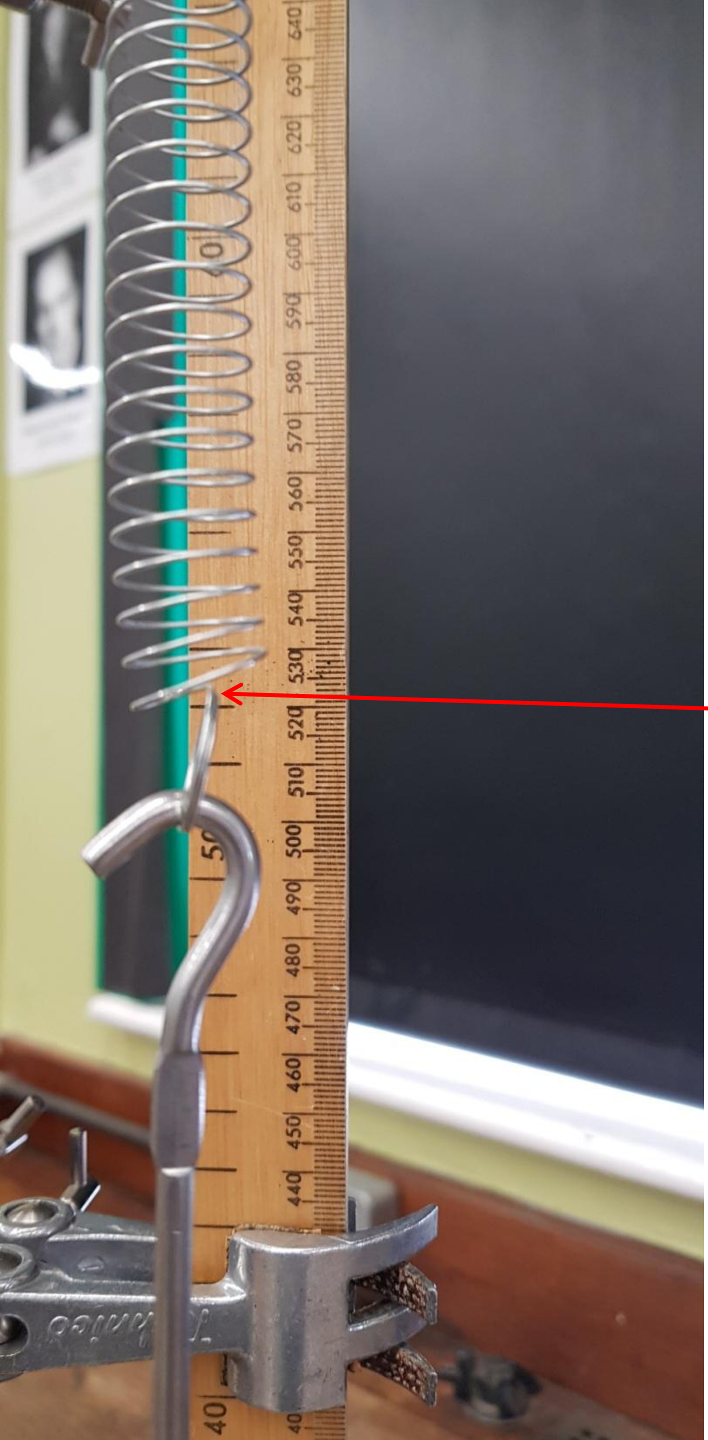
Therefore a graph of **weight vs extension** should yield a straight line graph through the origin with gradient k.

Hookean Spring:

$mg = kx$ ← extension

mg: weight; k: spring constant





Don’t forget eye protection!

Record the position of the top (or bottom) of the ring. Assume this doesn’t deform under the load. i.e. the load causes only the spring to uncoil.

Get you eye level to match the spring-ring to avoid *parallax error* in the measurement via the vertically clamped ruler.

Use 50g steps between 100g and 650g

Take a sample of '100g' to see how accurate these actually are...

CALCULATING A SPRING CONSTANT

STATIC METHOD

Strength of gravity g /Nkg^-1: 9.81

Mass /g	ring top /mm	Extension x /mm	Extension x /m	Weight /N measured	Recalibrated weight /N	Weight upper error /N	Weight lower error /N
0	625	0	0.000	0.00	0.00	0.00	0.00
100	585	40	0.040	0.98	0.96	0.01	0.01
150	563	62	0.062	1.47	1.45	0.01	0.01
200	540	85	0.085	1.96	1.93	0.02	0.02
250	518	107	0.107	2.45	2.41	0.02	0.02
300	497	128	0.128	2.94	2.89	0.03	0.03
350	475	150	0.150	3.43	3.37	0.03	0.03
400	450	175	0.175	3.92	3.85	0.04	0.04
450	429	196	0.196	4.41	4.34	0.04	0.04
500	407	218	0.218	4.91	4.82	0.04	0.04
550	385	240	0.240	5.40	5.30	0.05	0.05
600	360	265	0.265	5.89	5.78	0.05	0.05
650	340	285	0.285	6.38	6.26	0.06	0.06

Estimated measurement error (+/-) /m

0.002

100g mass sample and estimated % error

98.3	97.2	99.6	97.5

Mean average mass /g: 98.15

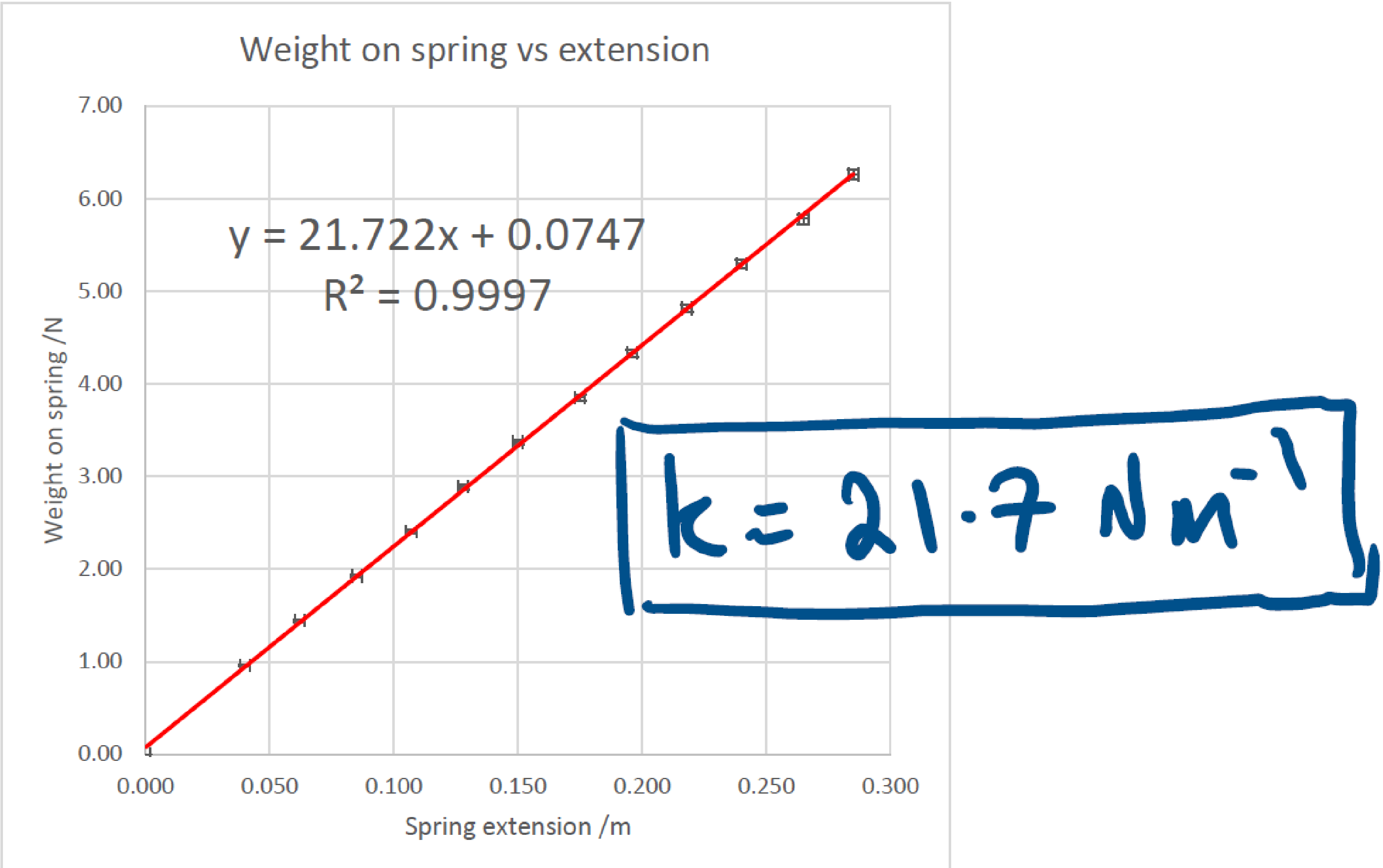
Standard deviation /g: 0.93

So expect mass to be 0.982 of quoted mass, with fractional error of 0.93/98.15 =

0.009

Weight sample – use to calibrate weights (multiply by 0.9815) and have a fractional error of about 1%

Spring constant calculation via static method



With a (symmetric) 1% error this means: $k = (21.7 \pm 0.2)\,\text{Nm}^{-1}$

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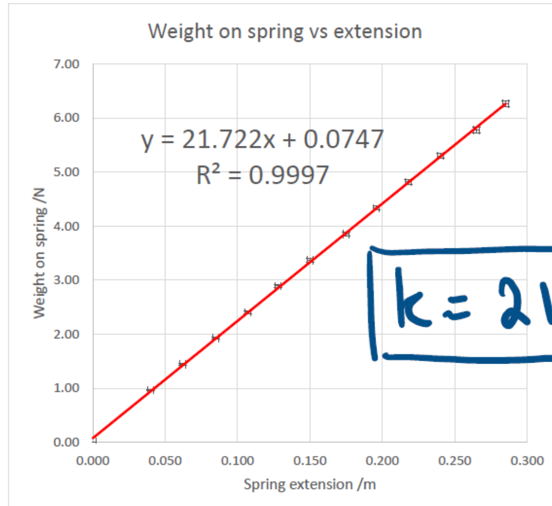


$k = 21.7\ Nm^{-1}$

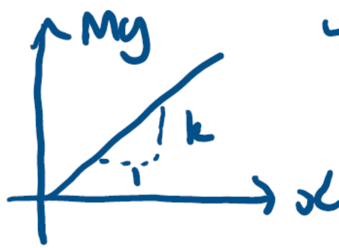
(if ignore x error, about ±1% from mass uncertainties)

Hookean Spring:

$mg = kx$ ← extension

mg: weight

k: spring constant



PART 2: FIND A SPRING CONSTANT VIA A DYNAMIC METHOD

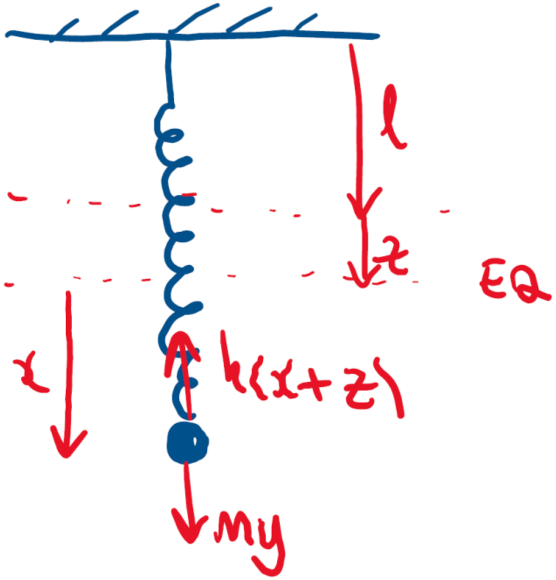
Pull down weighted spring from equilibrium position, and let it oscillate. Amplitude of a few cm!

Start stopwatch from lowest position, and carefully time ten oscillations.

Get eye at level with lowest position of oscillating mass to minimise timing error for end of ten oscillations.

Divide by ten to get the period, and perform **three repeats** for each mass

Simple Harmonic Motion (SHM)



NII:

$$m\ddot{x} = mg - k(x+z)$$

@ eq $mg = kz$

$$x = A\cos\left(\frac{2\pi t}{T}\right)$$

$$\dot{x} = -A\frac{2\pi}{T}\sin\left(\frac{2\pi t}{T}\right)$$

$$\ddot{x} = -A\frac{4\pi^2}{T^2}\cos\left(\frac{2\pi t}{T}\right)$$

$$\therefore\; m\ddot{x} = -kx$$

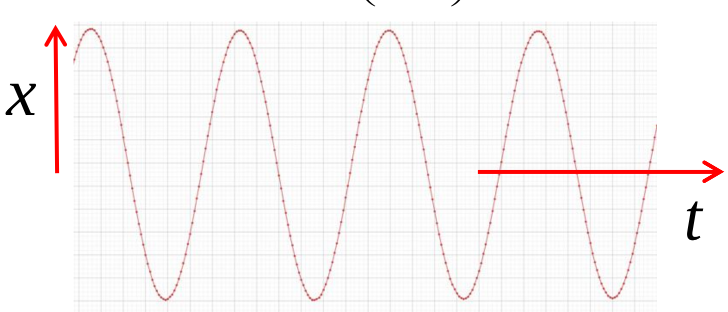
$$\ddot{x} = -\frac{k}{m}x$$

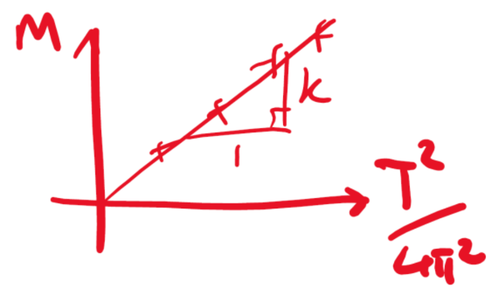
$$\ddot{x} = -\frac{4\pi^2}{T^2}x$$

$$\therefore\; \frac{4\pi^2}{T^2} = \frac{k}{m}$$

$$\therefore\; m = k\frac{T^2}{4\pi^2}$$

$$x = A\cos\left(\frac{2\pi t}{T}\right)$$





plot M vs $T^2/4\pi^2$

CALCULATING A SPRING CONSTANT

DYNAMIC METHOD

k /Nm^-1: 23.1

Use standard deviation of periods

Mass /g	10 periods /s	10 periods /s	10 periods /s	Period /s	error in T /s	Mass /kg measured	Recalibrated mass /kg	mass upper error /kg	mass lower error /kg	Period^2	Period^2 lower error	Period^2 upper error	4*pi^2*Recalibrated mass /kg	4*pi^2*mass upper error /kg	4*pi^2*mass lower error /kg
100	4.19	4.27	4.23	0.423	0.003	0.100	0.098	0.001	0.001	0.179	0.007	0.007	3.948	0.054	0.036
150	5.1	5.06	5.22	0.513	0.007	0.150	0.147	0.001	0.001	0.263	0.009	0.009	5.922	0.054	0.053
200	6.07	5.88	6	0.598	0.008	0.200	0.196	0.002	0.002	0.358	0.004	0.004	7.896	0.054	0.071
250	6.58	6.62	6.54	0.658	0.003	0.250	0.246	0.002	0.002	0.433	0.025	0.025	9.870	0.054	0.089
300	7.51	7.15	7.14	0.727	0.017	0.300	0.295	0.003	0.003	0.528	0.010	0.010	11.844	0.054	0.107
350	7.74	7.72	7.59	0.768	0.007	0.350	0.344	0.003	0.003	0.590	0.021	0.021	13.817	0.054	0.124
400	8.45	8.18	8.18	0.827	0.013	0.400	0.393	0.004	0.004	0.684	0.014	0.014	15.791	0.054	0.142
450	8.71	8.91	8.8	0.881	0.008	0.450	0.442	0.004	0.004	0.776	0.015	0.016	17.765	0.054	0.160
500	9.36	9.16	9.3	0.927	0.008	0.500	0.491	0.005	0.005	0.860	0.009	0.010	19.739	0.054	0.178
550	9.65	9.7	9.58	0.964	0.005	0.550	0.540	0.005	0.005	0.930	0.020	0.020	21.713	0.054	0.195
600	10.24	10.09	10	1.011	0.010	0.600	0.589	0.005	0.005	1.022	0.010	0.010	23.687	0.054	0.213
650	10.45	10.44	10.55	1.048	0.005	0.650	0.638	0.006	0.006	1.098	0.000	0.000	25.661	0.054	0.231

Estimated time measurement error (+/-) /s

0.02

100g mass sample and estimated % error

98.3	97.2	99.6	97.5

Mean average mass /g: 98.15

Standard deviation /g: 0.93

So expect mass to be 0.982 of quoted mass, with fractional error of 0.93/98.15 =

0.009

Hooke's law & NII

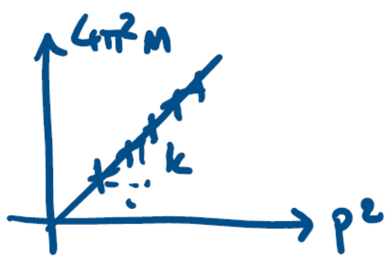
$M\ddot{x} = -kx$

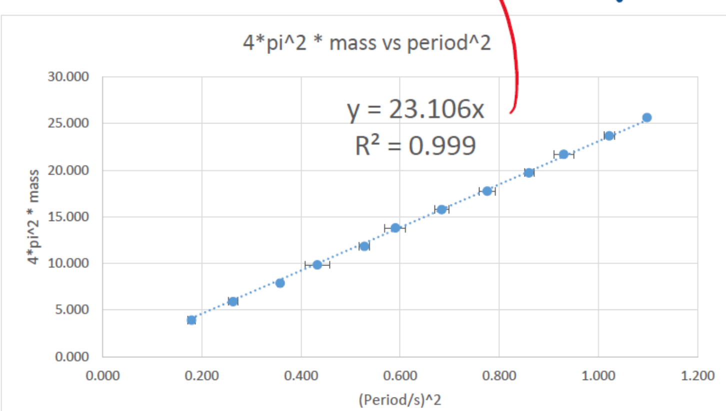
$\therefore \ddot{x} = -\frac{k}{M}x$

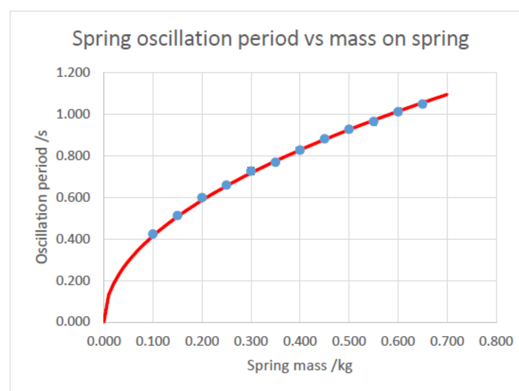
SHM: $\ddot{x} = -\left(\frac{2\pi}{P}\right)^2 x$ $\therefore P = 2\pi\sqrt{\frac{M}{k}}$

So $P^2 = \frac{4\pi^2 M}{k}$ $\therefore 4\pi^2 M = kP^2$ (y, x)

$k = 23.1\ Nm^{-1}$

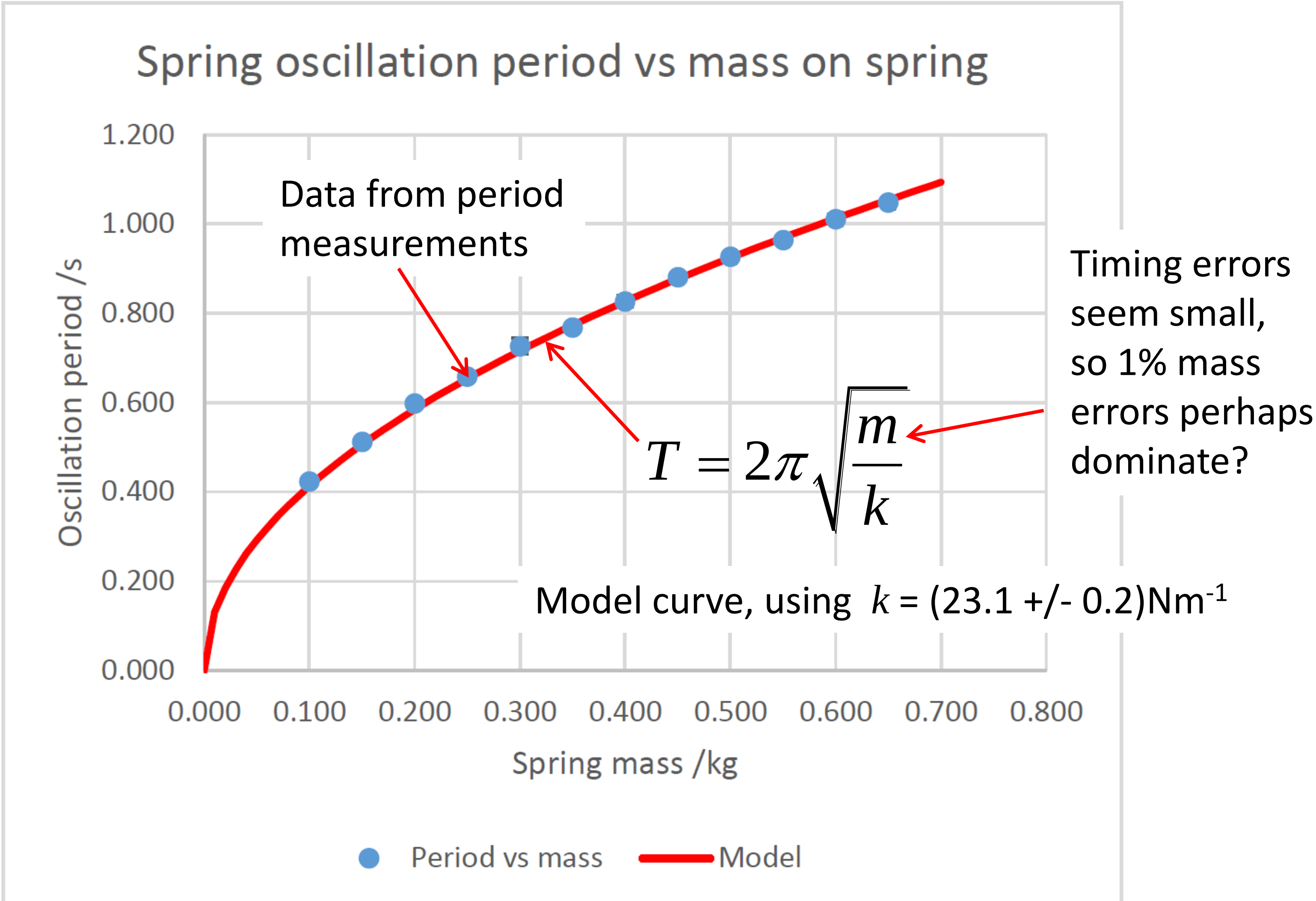






MODEL

Mass /kg	Period /s
0	0
0.01	0.13073
0.02	0.18488
0.03	0.22643
0.04	0.261459
0.05	0.29232
0.06	0.320221
0.07	0.345878
0.08	0.369759
0.09	0.392189
0.1	0.413403
0.11	0.433581
0.12	0.452861
0.13	0.471352
0.14	0.489145
0.15	0.506314
0.16	0.522918
0.17	0.539012
0.18	0.554639
0.19	0.569837
0.2	0.58464
0.21	0.599078
0.22	0.613176
0.23	[illegible]
0.24	[illegible]
0.25	[illegible]
0.26	[illegible]
0.27	0.679291
0.28	0.691756
0.29	[illegible]
[illegible]	[illegible]
0.31	0.727872
0.32	0.739518
0.33	0.750984
0.34	0.762278
0.35	0.773407
0.36	0.784378
0.37	0.795197
0.38	0.805871
0.39	0.816406
0.4	0.826806
0.41	0.837078
0.42	0.847225
0.43	0.857251
0.44	0.867162
0.45	0.876961
0.46	0.886651
0.47	0.896237
0.48	0.905721
0.49	0.915107
0.5	0.924398
0.51	0.933596



Interestingly, the spring constant (using the same spring) as calculated via the dynamic method seems to be a little higher (23.1 Nm^{-1} compared to 21.7 Nm^{-1}) than k calculated via the static method. They are not quite the same, within the (approximately 1%) error bounds. Possible permanent strain of spring, increasing k, after the static experiment?