YouTube

Verifying Stefan's Law using a filament bulb

$$IV = \pi d l \sigma T^4$$

$$\log_{10}\left(IV\right) = \log_{10}\left(\pi d l \sigma\right) + 4\log_{10} T$$

Tony Ayres & Andrew French. June 2021

Stefan's law of radiation power

$$I = \varepsilon \sigma T^4$$

Radiation power per square metre

Emmisivity

Stefan-Boltzmann constant

Absolute temperature /K

$$\sigma = 5.67 \times 10^{-8}\,\mathrm{Wm^{-2}K^{-1}}$$

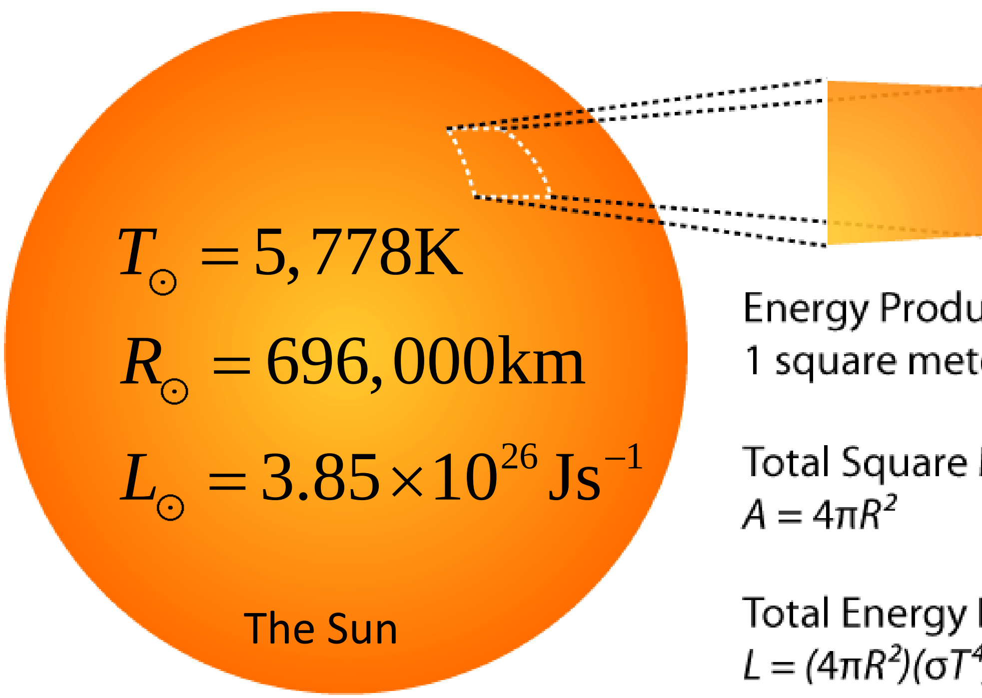
$$\varepsilon = 1$$

A '**Black Body**'. i.e. all incident radiation is absorbed and then re-radiated

For a 'Black Body' at 20°C = 293K

$$\boxed{I = 418\,\mathrm{Wm^{-2}}}$$

It is interesting to compare this to the maximum solar energy incident upon the Earth, which is on average about 1,361 $\mathrm{Wm^{-2}}$



Energy Produced by 1 square meter = σT^4

Total Square Meters $A = 4\pi R^2$

Total Energy Produced $L = (4\pi R^2)(\sigma T^4)$

Model: Assume tungsten filament of radiating area A and temperature T transforms all input electrical power IV into radiation. i.e. we can ignore other forms of energy transfer. Perhaps likely to be a better approximation at the filament gets hotter (and brighter).

$$\sigma = 5.67\times10^{-8}\,\mathrm{Wm^{-2}K^{-1}}$$

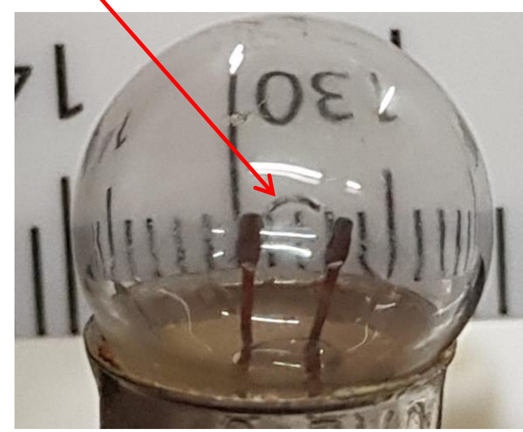
$$IV = A\sigma T^4$$

So we need to see how T might vary with V and I, *independent* of this relationship....

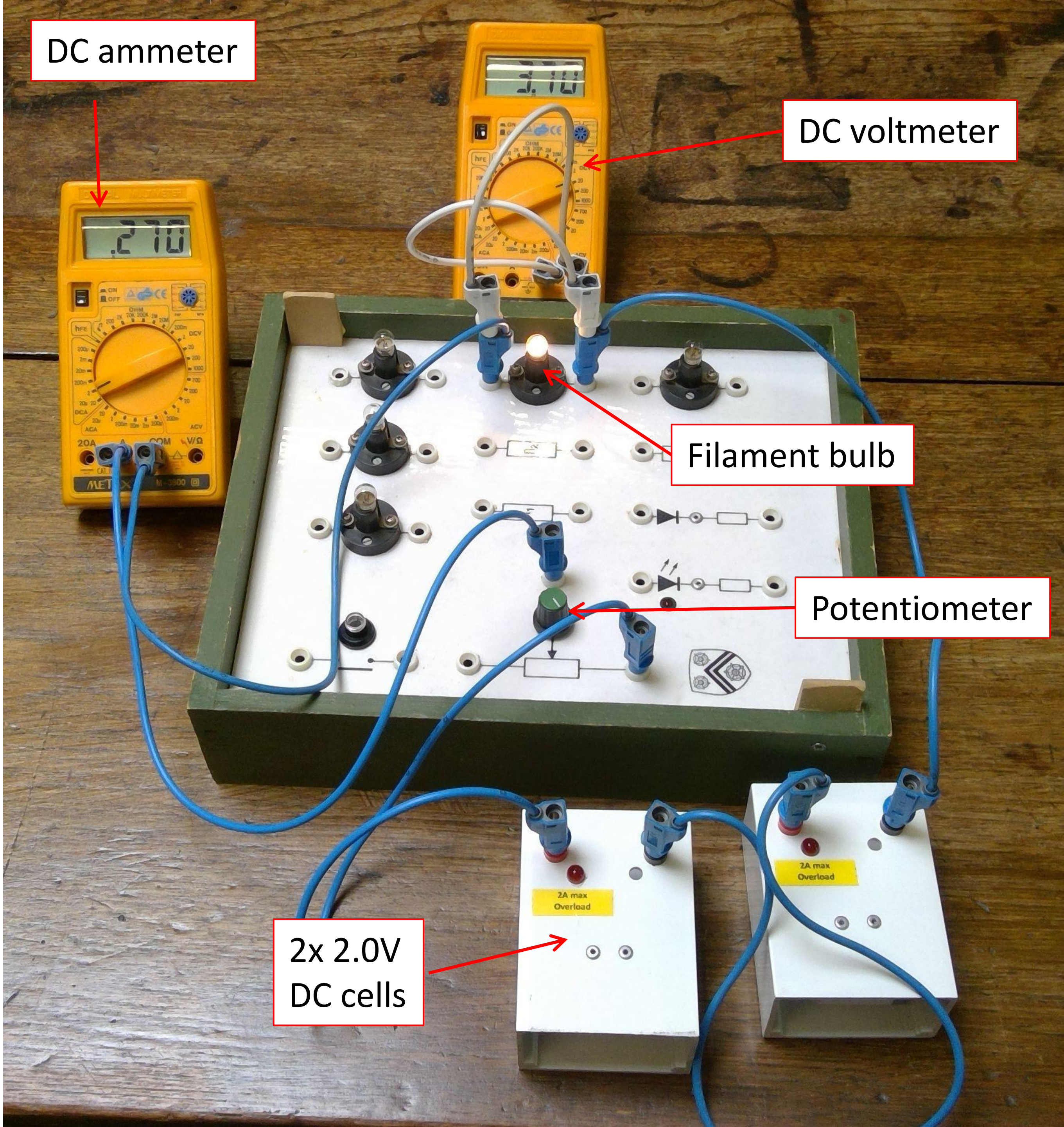
Electrical power transformed by bulb

Power radiated

Bulb filament



Equipment

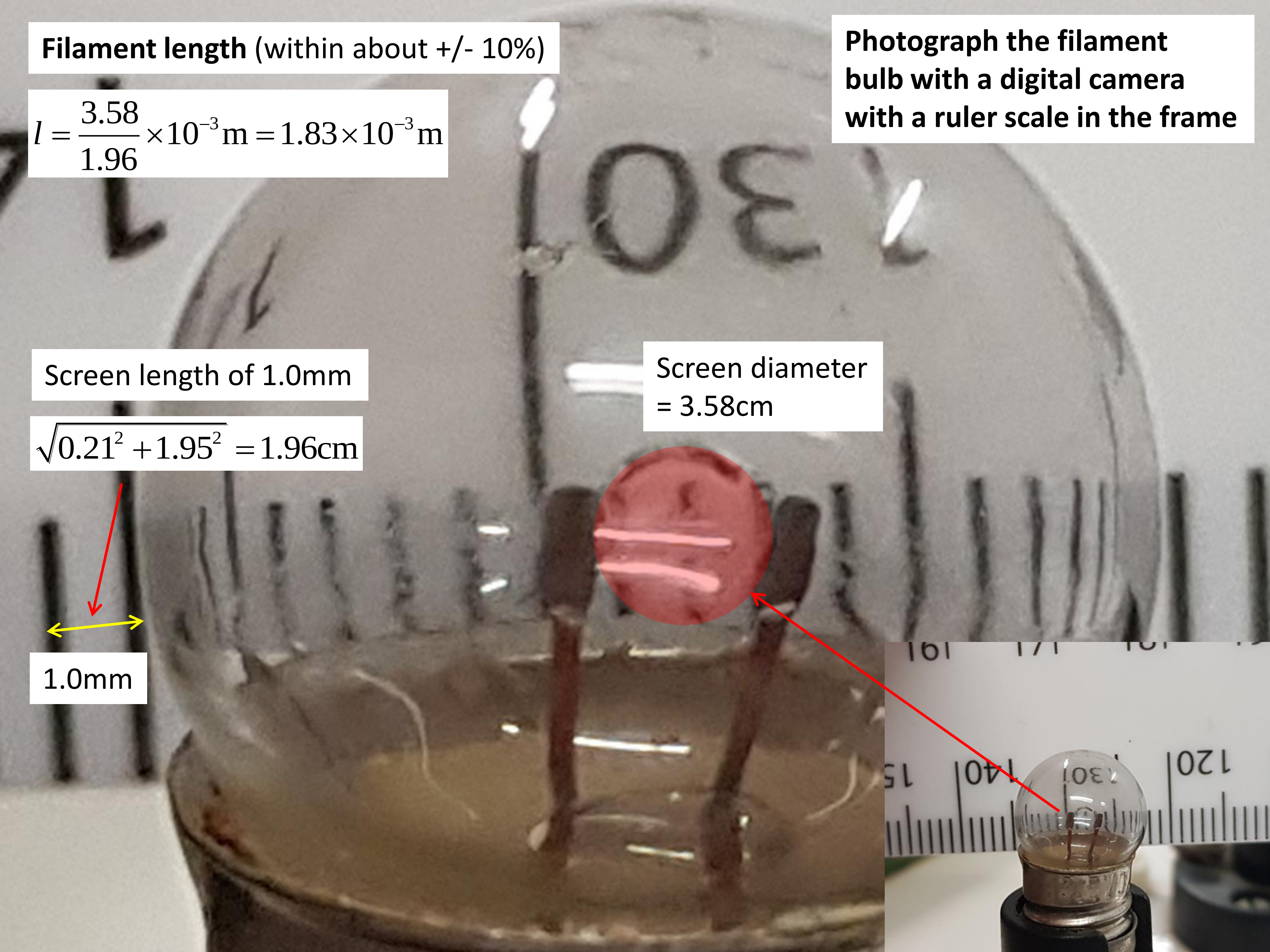


Measurements

Record current I and potential difference V for filament bulb using the full range of the potentiometer.

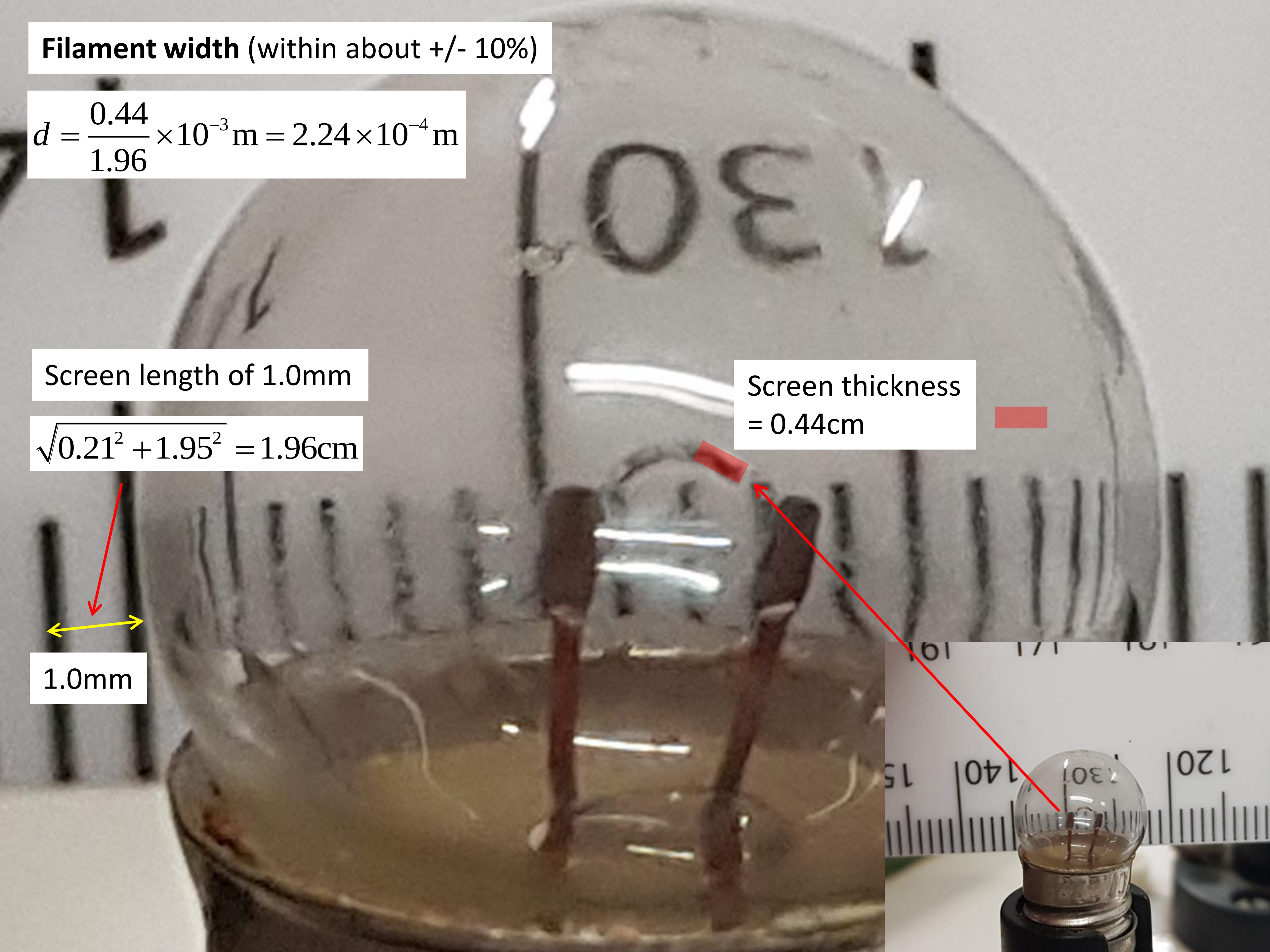
Anticipated current ranges are about 0.1A to 0.3A and voltages between 0.9V and 3.8V.

Aim for about 20 to 30 measurements.


Filament length (within about +/- 10%)
$l = \frac{3.58}{1.96} \times 10^{-3}\,\mathrm{m} = 1.83 \times 10^{-3}\,\mathrm{m}$
Photograph the filament bulb with a digital camera with a ruler scale in the frame
Screen length of 1.0mm
$\sqrt{0.21^2 + 1.95^2} = 1.96\mathrm{cm}$
Screen diameter = 3.58cm
1.0mm

Filament width (within about +/- 10%)

$$d = \frac{0.44}{1.96} \times 10^{-3}\,\mathrm{m} = 2.24 \times 10^{-4}\,\mathrm{m}$$



Screen length of 1.0mm

$$\sqrt{0.21^2 + 1.95^2} = 1.96\mathrm{cm}$$

1.0mm

Screen thickness = 0.44cm

The filament is *probably* made from tungsten...

The melting point of tungsten is **3,695K**. This is why it is useful as a bulb filament. It will radiate in the visible spectrum when hot, but won't melt at these temperatures.

Rather than an unwound coil, we can imagine the filament to be a radiating cylinder of length *l* and diameter *d*. The radiating area is (slightly less than):

$$A = \pi d l$$

$$A = \pi \times 2.24 \times 10^{-4}\,\mathrm{m} \times 1.83 \times 10^{-3}\,\mathrm{m}$$

$$\boxed{A = 1.29 \times 10^{-6}\,\mathrm{m}^2}$$

$$I = \int_0^\infty B(\lambda, T)\,d\lambda = \sigma T^4$$

$$\sigma = \frac{2\pi^5 k_B^4}{15c^2h^3}$$

$$B(\lambda, T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

This formula for irradiance is the Planck law

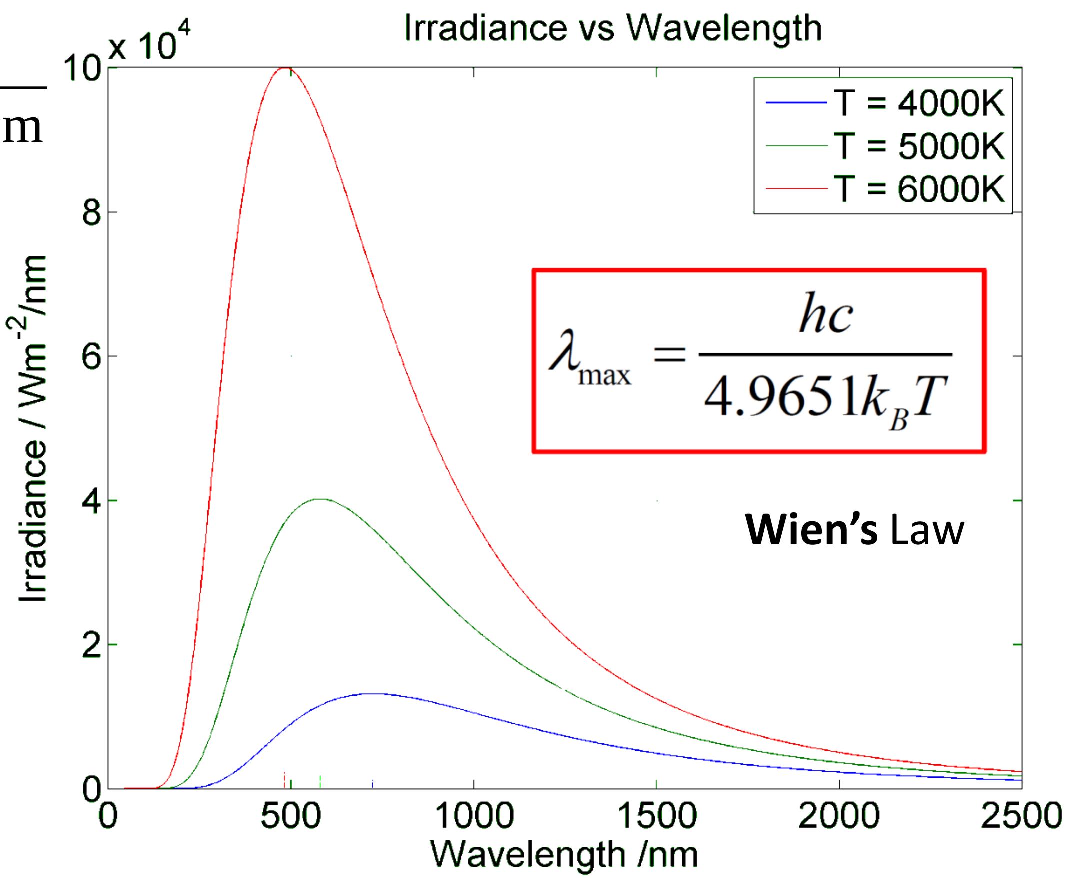
$k_B = 1.381 \times 10^{-23}\,\mathrm{m^2kgs^{-2}K^{-1}}$	Boltzmann's constant
$h = 6.626 \times 10^{-34}\,\mathrm{m^2kgs^{-1}}$	Planck's constant
$c = 2.998 \times 10^{8}\,\mathrm{ms^{-1}}$	Speed of light

$$\lambda_{\max} = 590 \times 10^{-9}\,\mathrm{m}$$

$$T = \frac{6.626 \times 10^{-34} \times 2.998 \times 10^{8}}{4.965 \times 1.381 \times 10^{-23} \times 590 \times 10^{-9}\,\mathrm{m}}$$

$$\boxed{T = 4{,}910\mathrm{K}}$$

So *peak* of radiation spectrum from a Tungsten filament must be for a larger wavelength than yellow, otherwise the filament would melt.



$$\boxed{\lambda_{\max} = \frac{hc}{4.9651 k_B T}}$$

Wien's Law

Red	620-750nm
Yellow	570-590nm
Green	495-570nm
Blue	450-495nm

The melting point of tungsten is **3,695K**.

Empirical model of tungsten wire resistivity vs temperature

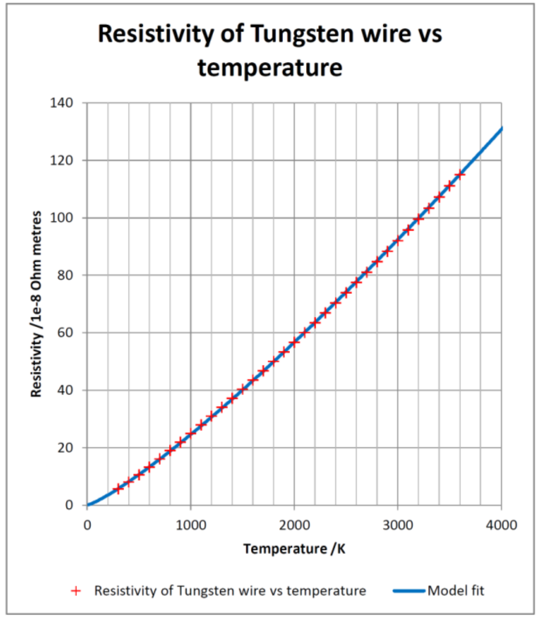
https://hypertextbook.com/facts/2004/DeannaStewart.shtml

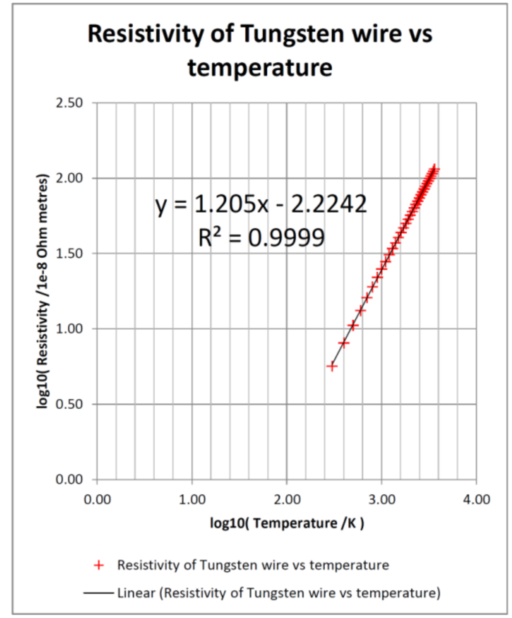
Resistivity of Tungsten wire vs temperature

https://hypertextbook.com/facts/2004/DeannaStewart.shtml

T /K	T/ degC	Resistivity /microOhms*cm (i.e. units of 10^-8 ohm metres)	log10 (T/K)	log10(resistivity)
300	27	5.65	2.48	0.75
400	127	8.06	2.60	0.91
500	227	10.56	2.70	1.02
600	327	13.23	2.78	1.12
700	427	16.09	2.85	1.21
800	527	19	2.90	1.28
900	627	21.94	2.95	1.34
1000	727	24.93	3.00	1.40
1100	827	27.94	3.04	1.45
1200	927	30.98	3.08	1.49
1300	1027	34.08	3.11	1.53
1400	1127	37.19	3.15	1.57
1500	1227	40.36	3.18	1.61
1600	1327	43.55	3.20	1.64
1700	1427	46.78	3.23	1.67
1800	1527	50.05	3.26	1.70
1900	1627	53.35	3.28	1.73
2000	1727	56.67	3.30	1.75
2100	1827	60.06	3.32	1.78
2200	1927	63.48	3.34	1.80
2300	2027	66.91	3.36	1.83
2400	2127	70.39	3.38	1.85
2500	2227	73.91	3.40	1.87
2600	2327	77.49	3.41	1.89
2700	2427	81.04	3.43	1.91
2800	2527	84.7	3.45	1.93
2900	2627	88.33	3.46	1.95
3000	2727	92.04	3.48	1.96
3100	2827	95.76	3.49	1.98
3200	2927	99.54	3.51	2.00
3300	3027	103.3	3.52	2.01
3400	3127	107.2	3.53	2.03
3500	3227	111.1	3.54	2.05
3600	3327	115	3.56	2.06

Andy French. June 2021.





$$\log_{10}\left(\frac{\rho}{10^{-8}\,\Omega\mathrm{m}}\right) = 1.205\log_{10}\left(T/\mathrm{K}\right) - 2.2242$$

$$\rho = 10^{-2.2242}\left(T/\mathrm{K}\right)^{1.205} \times 10^{-8}\,\Omega\mathrm{m}$$

Empirical model fit to resistivity vs temperature data for Tungsten wire

$$\rho = 5.968\times10^{-11}\times\left(T/\mathrm{K}\right)^{1.205}\ \Omega\mathrm{m}$$

MODEL

T /K	Resistivity /microOhms*cm (i.e. units of 10^-8 ohm metres)
10	0.10
20	0.22
30	0.36
40	0.51
50	0.67
60	0.83
70	1.00
80	1.17
90	1.35
100	1.53
110	1.72
120	1.91
130	2.10
140	2.30
150	2.50
160	2.70
170	2.91
180	3.11
190	3.32
200	3.54
210	3.75
220	3.97
230	4.18
240	4.41
250	4.63
260	4.85
270	5.08
280	5.30
290	5.53
300	5.76
310	6.00
320	6.23
330	6.47
340	6.70
350	6.94
360	7.18
370	7.42
380	7.66
390	7.91
400	8.15

Find temperature from V and I

$$R = \frac{V}{I}$$

Using empirical resistivity formula for tungsten

$$R = \frac{\rho L}{\pi r^2} = \frac{La}{\pi r^2} T^{1.205}$$

$$a = 5.968 \times 10^{-11}$$

$$\boxed{\therefore T = \left(\frac{V}{I}\right)^{\frac{1}{1.205}} \left(\frac{\pi r^2}{La}\right)^{\frac{1}{1.205}}}$$

If we use our filament measurements (i.e. ignore the coiling)

$$2r = d = 2.24 \times 10^{-4}\,\mathrm{m}$$

$$L = l = 1.83 \times 10^{-3}\,\mathrm{m}$$

$$\left(\frac{\pi r^2}{La}\right)^{\frac{1}{1.205}} \approx 40{,}914$$

This will result in excessively high temperatures i.e. above tungsten melting point

Filament is actually a *coil* of thin wire, so the lengths l and diameter d measured are both likely to be significantly inaccurate. (But perhaps OK for the radiation calculation since the outer coils are likely to be the major contributors).

Sensible guesses

https://physics.info/electric-resistance/practice.shtml

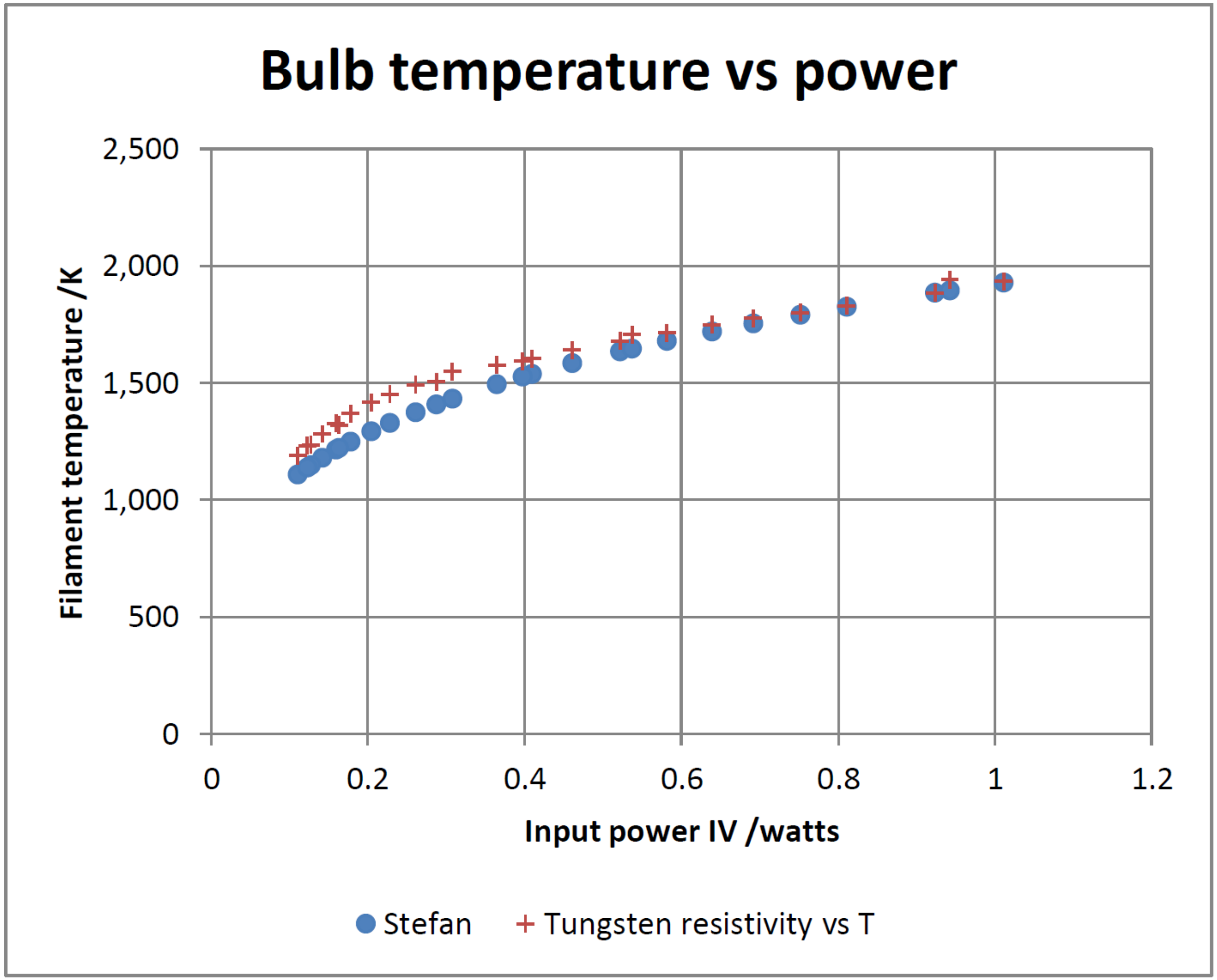
Filament diameter

$$2r = 4.6 \times 10^{-5}\,\mathrm{m}$$

Filament length

$$L = 4.2\,\mathrm{cm}$$

$$\left(\frac{\pi r^2}{La}\right)^{\frac{1}{1.205}} \approx 220$$



From Tungsten resistivity vs T

$$\left(\frac{\pi r^2}{La}\right)^{\frac{1}{1.205}} \approx 220$$

$$T = \left(\frac{V}{I}\right)^{\frac{1}{1.205}} \left(\frac{\pi r^2}{La}\right)^{\frac{1}{1.205}}$$

$$T = \left(\frac{IV}{A\sigma}\right)^{\frac{1}{4}}$$

From Stefan's law

Predict temperature well below tungsten melting point of 3,695K.

$$\boxed{\sigma = 5.67\times10^{-8}\,\mathrm{Wm^{-2}K^{-1}}}$$

$$A = \pi d l$$

i.e. we ignore coiling for the radiating area

$$A = 1.29\times10^{-6}\,\mathrm{m}^2$$

$$R=\frac{V}{I} \qquad R=\frac{\rho L}{\pi r^2}=\frac{La}{\pi r^2}T^{1.205} \qquad a=5.968\times10^{-11}$$

Radiating area of filament

$$A=\pi dl$$

$$A=1.29\times10^{-6}\,\mathrm{m}^2$$

$$\therefore T=\left(\frac{V}{I}\right)^{\frac{1}{1.205}}\left(\frac{\pi r^2}{La}\right)^{\frac{1}{1.205}}$$

Assume electrical power equates to radiated power

$$\boxed{IV=A\sigma T^4}$$

Verifying Stefan's law

(assuming $R(T)$ correlation)

$$\log_{10}(IV)=\log_{10}(A\sigma)+4\log_{10}T$$

$$T=\left(\frac{V}{I}\right)^{\frac{1}{1.205}}\left(\frac{\pi r^2}{La}\right)^{\frac{1}{1.205}}$$

$$\left(\frac{\pi r^2}{La}\right)^{\frac{1}{1.205}}\approx 220$$

$$\therefore \underbrace{\log_{10}(IV)}_{y}=\underbrace{\log_{10}(A\sigma)+\frac{4}{1.205}\log_{10}\left(\frac{\pi r^2}{La}\right)}_{c}+\frac{4}{1.205}\underbrace{\log_{10}\left(\frac{V}{I}\right)}_{x}$$

$$y=c+\frac{4}{1.205}x$$

Let's see if this relationship holds for a filament bulb ...

Verifying Stefan's Law using a filament bulb. Andy French June 2021.

Potential difference V across bulb /V	Current I through bulb /A	Resistance R /ohms	Power P /watts	log10(P)	log10(R)	T /K from V/I	T /K from Stefan's Law
3.73	0.271	13.76383764	1.01083	0.004678	1.13874	1,934	1,929
3.51	0.263	13.3460076	0.92313	-0.03474	1.125351	1,885	1,886
3.23	0.251	12.8685259	0.81073	-0.09112	1.109529	1,829	1,825
3.08	0.244	12.62295082	0.75152	-0.12406	1.101161	1,800	1,791
2.93	0.236	12.41525424	0.69148	-0.16022	1.093956	1,776	1,754
2.79	0.229	12.18340611	0.63891	-0.19456	1.085769	1,748	1,720
2.63	0.221	11.90045249	0.58123	-0.23565	1.075563	1,714	1,680
2.46	0.212	11.60377358	0.52152	-0.28273	1.064599	1,679	1,635
2.28	0.202	11.28712871	0.46056	-0.33671	1.052583	1,641	1,585
2.12	0.193	10.98445596	0.40916	-0.38811	1.040779	1,604	1,539
1.98	0.184	10.76086957	0.36432	-0.43852	1.031847	1,577	1,495
1.8	0.171	10.52631579	0.3078	-0.51173	1.022276	1,548	1,433
1.62	0.161	10.0621118	0.26082	-0.58366	1.002689	1,492	1,375
1.49	0.153	9.738562092	0.22797	-0.64212	0.988495	1,452	1,329
1.39	0.147	9.455782313	0.20433	-0.68967	0.975697	1,417	1,293
1.27	0.14	9.071428571	0.1778	-0.75007	0.957676	1,369	1,249
1.18	0.135	8.740740741	0.1593	-0.79778	0.941548	1,327	1,215
1.09	0.13	8.384615385	0.1417	-0.84863	0.923483	1,282	1,180
1.01	0.126	8.015873016	0.12726	-0.89531	0.903951	1,235	1,149
0.92	0.12	7.666666667	0.1104	-0.95703	0.884607	1,190	1,109
0.99	0.124	7.983870968	0.12276	-0.91094	0.902214	1,231	1,139
1.19	0.137	8.686131387	0.16303	-0.78773	0.938826	1,320	1,222
1.71	0.168	10.17857143	0.28728	-0.54169	1.007687	1,506	1,408
2.08	0.191	10.89005236	0.39728	-0.4009	1.03703	1,593	1,527
2.52	0.213	11.83098592	0.53676	-0.27022	1.073021	1,706	1,647
3.61	0.261	13.83141762	0.94221	-0.02585	1.140867	1,942	1,895

Filament radius /m 2.30E-05 Guess - since coil

Filament length /m 4.20E-02

$$\left(\frac{\pi r^2}{La}\right)^{\frac{1}{1.205}}$$ 219.54

$$T = \left(\frac{V}{I}\right)^{\frac{1}{1.205}}\left(\frac{\pi r^2}{La}\right)^{\frac{1}{1.205}}$$

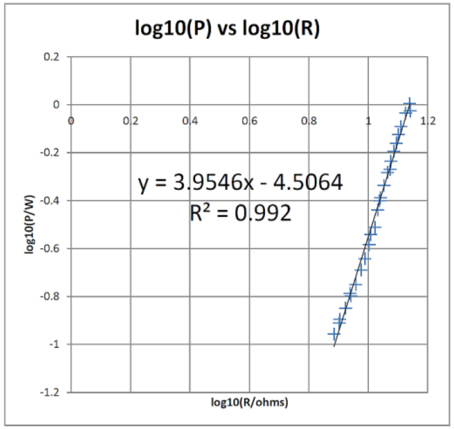
$$T = \left(\frac{IV}{A\sigma}\right)^{\frac{1}{4}}$$

Stefan Boltzmann constant calculator

1.042E-08

$$\sigma = \frac{10^c}{A}\left\{\left(\frac{\pi r^2}{La}\right)^{\frac{1}{1.205}}\right\}^{-4}$$

$$\sigma = 5.67\times10^{-8}\,\mathrm{Wm^{-2}K^{-1}}$$



4/1.205 = 3.32

Perhaps not pure tungsten?

This imples that bulb resistance is almost linear with temperature.

$$IV = A\sigma T^4$$

$$\log_{10}(IV) = \log_{10}(A\sigma) + 4\log_{10}T$$

$$T = \left(\frac{V}{I}\right)^{\frac{1}{1.205}}\left(\frac{\pi r^2}{La}\right)^{\frac{1}{1.205}}$$

$$\therefore \underbrace{\log_{10}(IV)}_{y} = \underbrace{\log_{10}(A\sigma) + \frac{4}{1.205}\log_{10}\left(\frac{\pi r^2}{La}\right)}_{c} + \frac{4}{1.205}\underbrace{\log_{10}\left(\frac{V}{I}\right)}_{x}$$

$$y = c + \frac{4}{1.205}x$$

Filament radiating area A /m^2

1.29E-06

Width /m 2.24E-04

Length /m 1.83E-03

$$\underbrace{\log_{10}(IV)}_{y} = m\underbrace{\log_{10}\left(\frac{V}{I}\right)}_{x} + c$$

$$m = 3.95$$

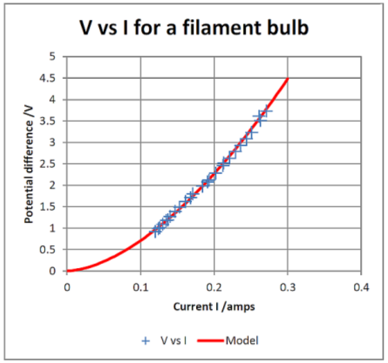
$$c = -4.51$$

$$IV = 10^c\left(\frac{V}{I}\right)^m$$

$$I^{m+1} = 10^c V^{m-1}$$

$$V = 10^{\frac{-c}{m-1}} I^{\frac{m+1}{m-1}}$$

Empirical model for bulb V vs I



Model

I /A	V/V
0	0
0.005	0.004653
0.01	0.01489
0.015	0.029401
0.02	0.047643
0.025	0.069281
0.03	0.094075
0.035	0.121845
0.04	0.152447
0.045	0.185759
0.05	0.221681
0.055	0.260127
0.06	0.301018
0.065	0.344288
0.07	0.389877
0.075	0.437728
0.08	0.487793
0.085	0.540026
0.09	0.594385
0.095	0.650831
0.1	0.709329
0.105	0.769843
0.11	0.832344
0.115	0.896802
0.12	0.963188
0.125	1.031476
0.13	1.101642
0.135	1.173662
0.14	1.247514

Interestingly, the $\log_{10}$(IV) vs $\log_{10}$(V/I) graph has a gradient much closer to 4 than 3.3. It appears that the resistance of the bulb is almost proportional to its temperature. Perhaps the assumption of all the electrical energy being radiated by the filament is flawed. There will inevitably some heat transfer into other parts of the bulb too, and indeed conduction into the low density gas within the glass bulb.

Can we calculate the Stefan Boltzmann constant?

$$\boxed{\sigma = 5.67\times10^{-8}\,\mathrm{Wm^{-2}K^{-1}}}$$

$$\underbrace{\log_{10}(IV)}_{y} = \underbrace{\log_{10}(A\sigma) + \frac{4}{1.205}\log_{10}\left(\frac{\pi r^2}{La}\right)}_{c=-4.51} + 3.95\underbrace{\log_{10}\left(\frac{V}{I}\right)}_{x}$$

$$10^c = A\sigma\left(\frac{\pi r^2}{La}\right)^{\frac{4}{1.205}} \Rightarrow \boxed{\sigma = \frac{10^c}{A}\left\{\left(\frac{\pi r^2}{La}\right)^{\frac{1}{1.205}}\right\}^{-4}}$$

$$\left(\frac{\pi r^2}{La}\right)^{\frac{1}{1.205}} \approx 40{,}914$$

With our filament measurements

T is too high

$$\sigma = 1.04\times10^{-8}\,\mathrm{Wm^{-2}K^{-1}}$$

We are out by about a factor of 5.5

$$\left(\frac{\pi r^2}{La}\right)^{\frac{1}{1.205}} \approx 220$$

Sensible guess for unwound coil

$$A = \pi d l$$

$$A = \pi\times2.24\times10^{-4}\,\mathrm{m}\times1.83\times10^{-3}\,\mathrm{m}$$

$$A = 1.29\times10^{-6}\,\mathrm{m^2}$$

Empirical I,V curve for filament bulb

$$\left(\frac{V}{\text{volts}}\right) = 33.8 \times \left(\frac{I}{\text{amps}}\right)^{1.68}$$

$$\underbrace{\log_{10}(IV)}_{y} = m \underbrace{\log_{10}\left(\frac{V}{I}\right)}_{x} + c$$

$$m = 3.95$$

$$c = -4.51$$

$$IV = 10^{c}\left(\frac{V}{I}\right)^{m}$$

$$I^{m+1} = 10^{c} V^{m-1}$$

$$V = 10^{\frac{-c}{m-1}} I^{\frac{m+1}{m-1}}$$