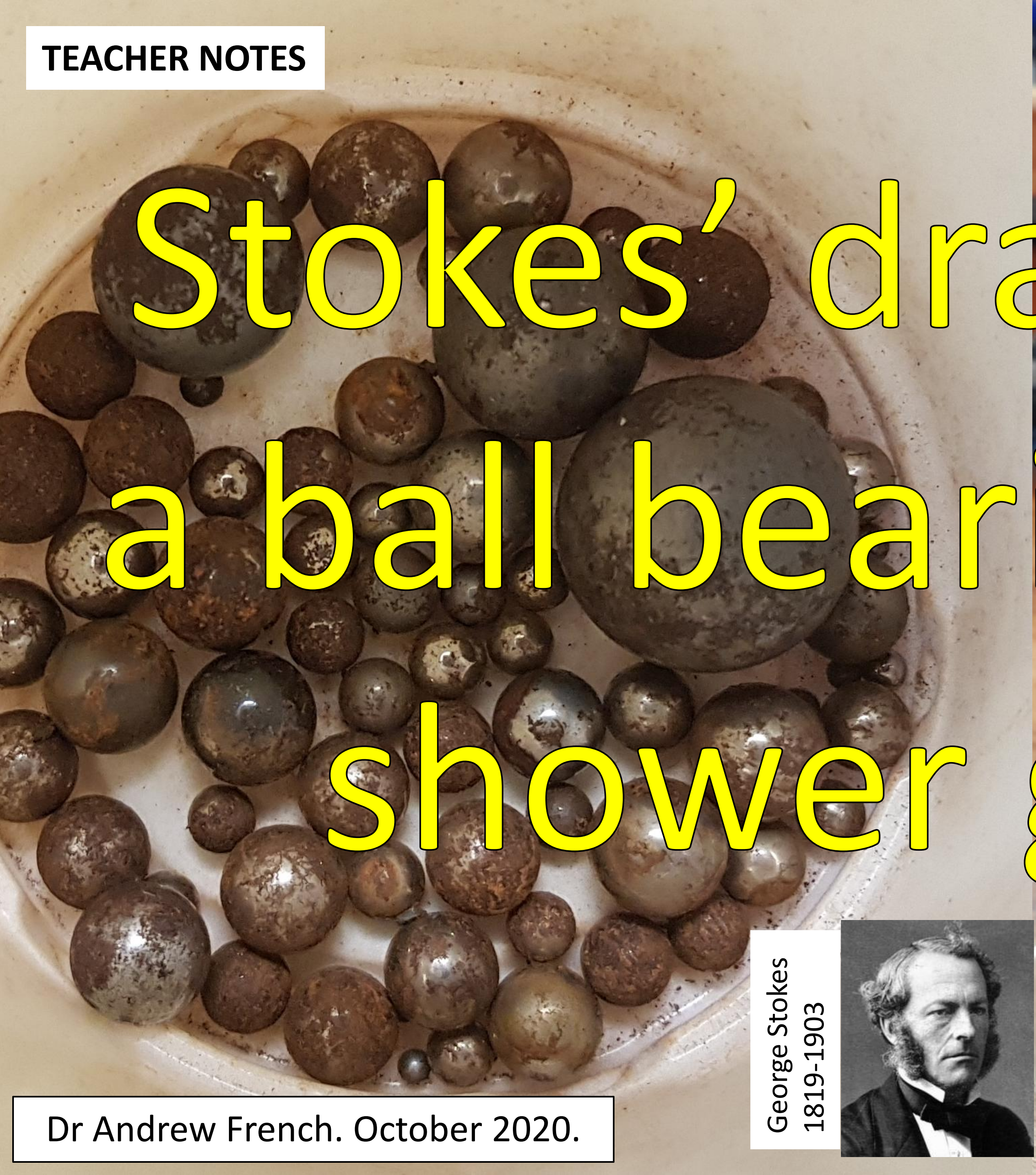
TEACHER NOTES



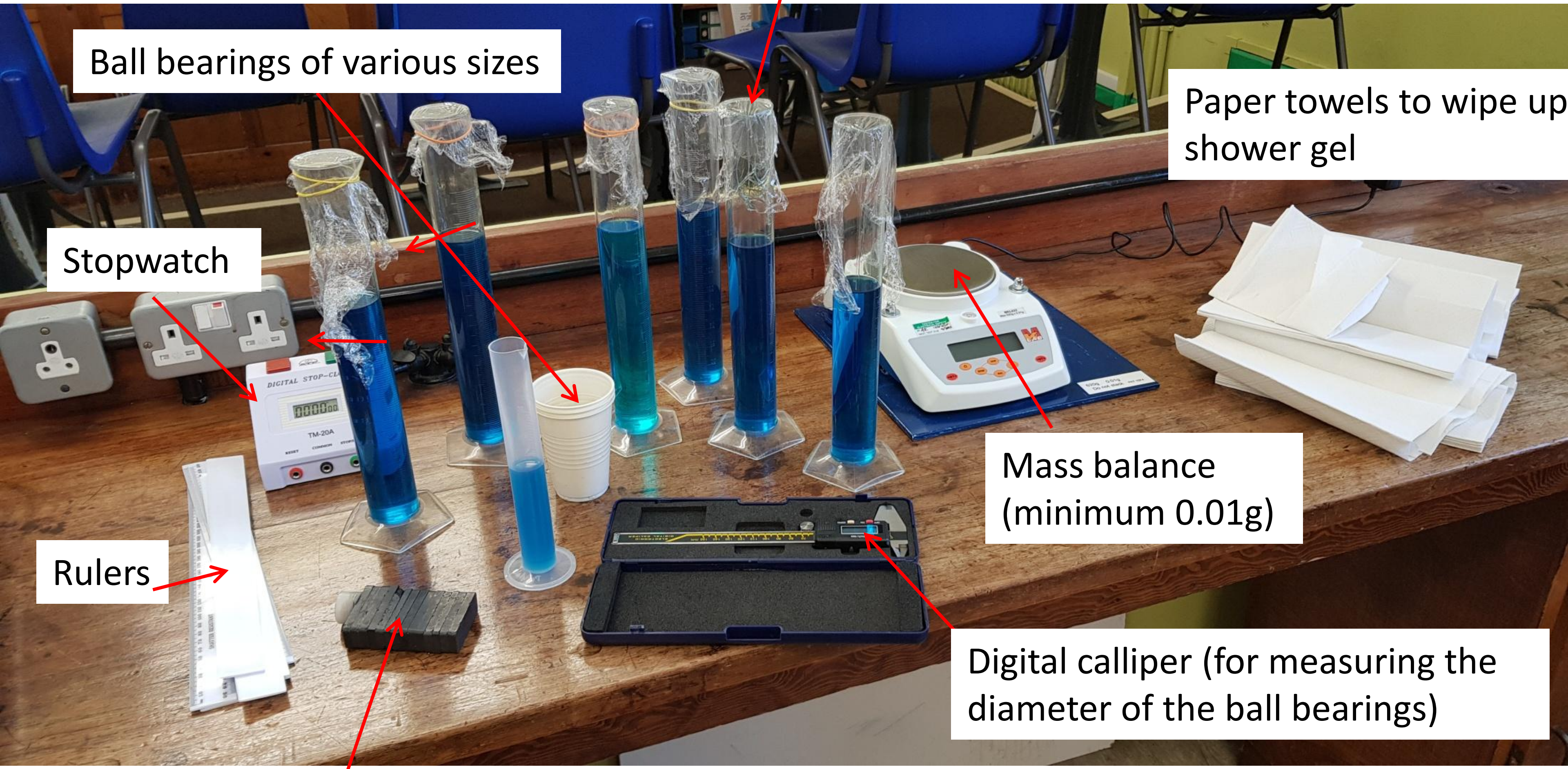
Stokes’ drag on a ball bearing in shower gel

George Stokes
1819-1903

Dr Andrew French. October 2020.

Store gel cylinders with cling film top to prevent gel evaporating and reacting with the air

Measuring cylinders containing viscous shower gel



Magnets to help remove ball bearings from shower gel

Equipment setup

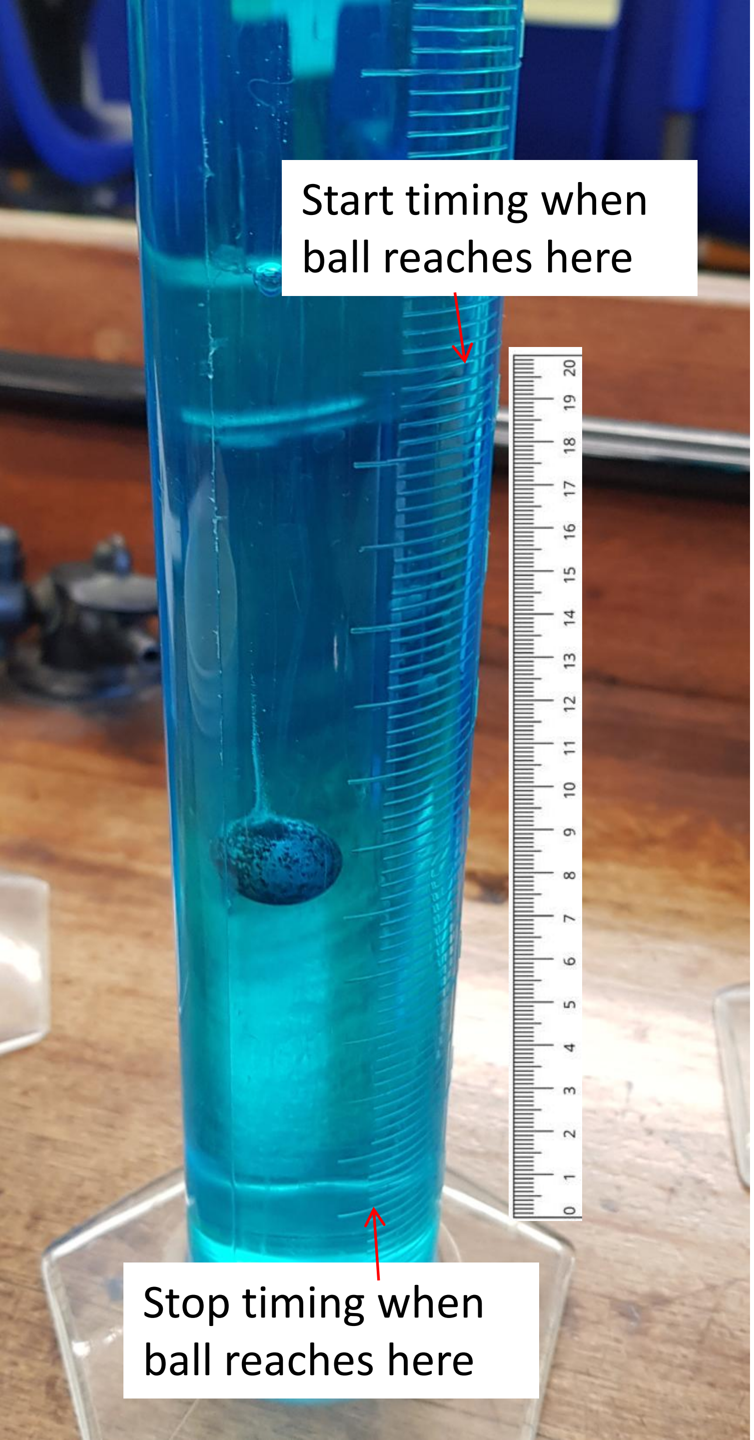
This practical is best done as a class, and then pooling results together. Students should record the terminal speed for a variety of different ball bearings, having firstly measured the radius and mass of each ball bearing (and hence calculate the density).

Measure ball bearing diameter using a digital calliper.

Ball bearings will tend to roll off a mass balance, so make a paper towel 'nest' first!

Ball bearing selection

Removing ball bearing using a neodymium magnet



It is fairly clear to observe that **terminal velocity** is attained very rapidly for all ball bearings.

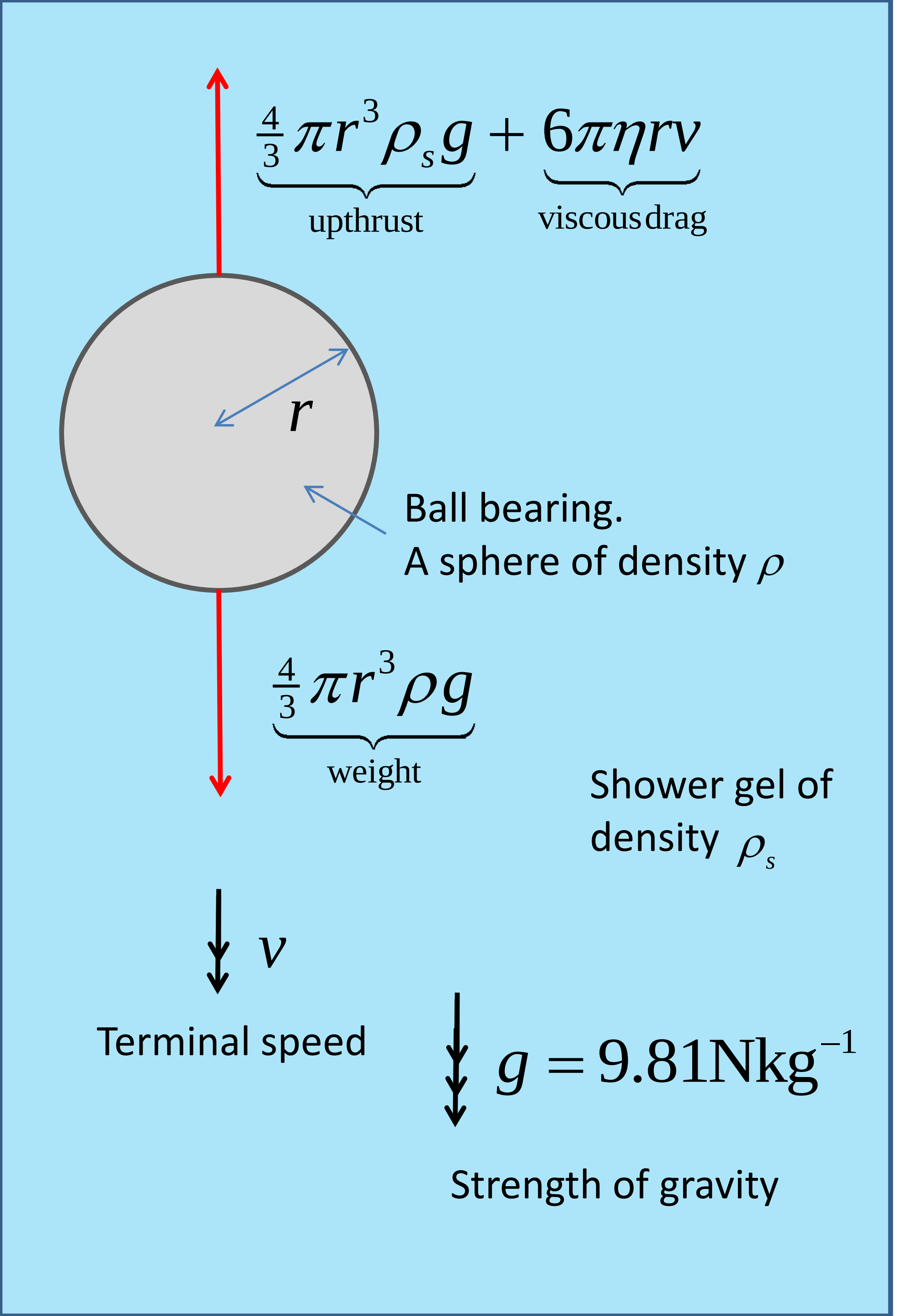
Hence terminal speed is simply a fixed distance travelled divided by the time taken.

Work this out for each ball bearing, using three repeats.

For the larger balls in particular, wait until the balls have dropped a few cm before timing, just to make sure terminal speed is attained.

Balls inevitably strike the gel surface at a non-zero speed, so this precaution is sensible for all balls.

You can also establish the motion before starting a stopwatch, which should reduce timing error.



If ball bearing is falling at terminal speed then **upthrust** (the weight of fluid displaced) + **drag** must balance the weight of the ball bearing.

A low *Reynolds Numbers* (i.e. low speed, 'viscous' flow), the drag force can be modelled using Stokes' law of viscous drag. η is the ***viscosity***.

$$\mathrm{Re} = \frac{\rho v L}{\eta}$$

Reynolds number. L is a characteristic dimension of the flow. In this case the ball bearing radius, but also the radius of the measuring cylinder.

$$[6\pi\eta r v] = \mathrm{kgms}^{-2}$$

$$\therefore [\eta] = \frac{\mathrm{kgms}^{-2}}{\mathrm{m}^2\mathrm{s}^{-1}} = \mathrm{kgm}^{-1}\mathrm{s}^{-1}$$

$$[6\pi\eta r v] = \mathrm{kgms^{-2}}$$

$$\therefore [\eta] = \frac{\mathrm{kgms^{-2}}}{\mathrm{m^2 s^{-1}}} = \mathrm{kgm^{-1}s^{-1}}$$

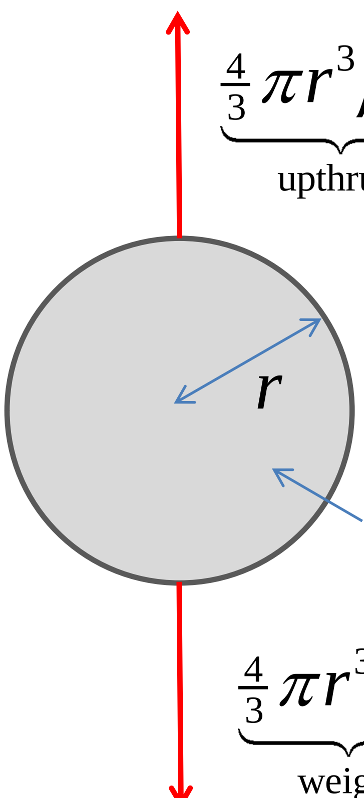
$$\left[\frac{\rho v L}{\eta}\right] = \frac{\mathrm{kgm^{-3}ms^{-1}m}}{\mathrm{kgm^{-1}s^{-1}}} = \frac{\mathrm{kgm^{-1}s^{-1}}}{\mathrm{kgm^{-1}s^{-1}}} = 1$$

$$\boxed{\mathrm{Re} = \frac{\rho v L}{\eta}}$$

See that the Reynolds number is a *dimensionless quantity.*

For viscous drag,
Re is typically << 1

$$\underbrace{\tfrac{4}{3}\pi r^3 \rho_s g}_{\text{upthrust}} + \underbrace{6\pi\eta r v}_{\text{viscous drag}}$$



Ball bearing.
A sphere of density ρ

$$\underbrace{\tfrac{4}{3}\pi r^3 \rho g}_{\text{weight}}$$

Equating upthrust + drag with weight:

$$\underbrace{\tfrac{4}{3}\pi r^3 \rho_s g}_{\text{upthrust}} + \underbrace{6\pi\eta r v}_{\text{viscous drag}} = \underbrace{\tfrac{4}{3}\pi r^3 \rho g}_{\text{weight}}$$

$$v = \frac{\tfrac{4}{3}\pi r^3 g\left(\rho - \rho_s\right)}{6\pi\eta r}$$

$$\therefore \boxed{v = \frac{\tfrac{2}{9} r^2 g\left(\rho - \rho_s\right)}{\eta}}$$

$$\therefore \eta v = \tfrac{2}{9} r^2 g \left(\rho - \rho_s \right)$$

So plot terminal speed v vs $\tfrac{2}{9} r^2 g \left(\rho - \rho_s \right)$ and one should obtain a **straight line through the origin** of *gradient* equal to the viscosity η.

Guess this

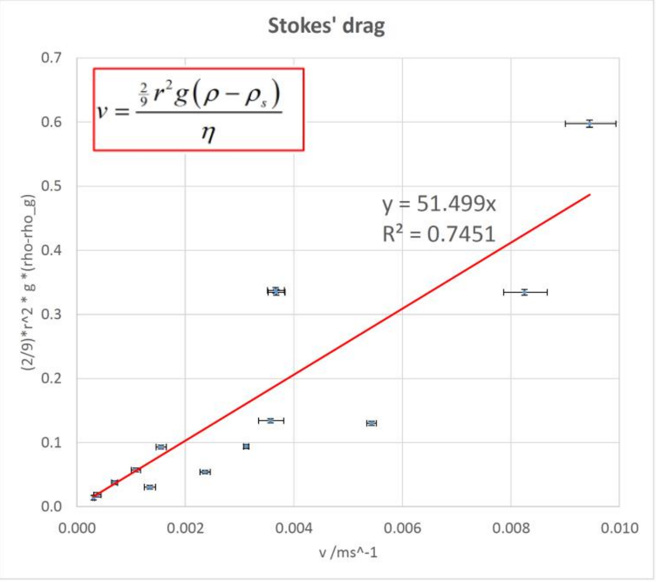
Shower gel density /kgm^-3	1000

Strength of gravity /Nkg^-1	9.81

BALL BEARING FALLING AT TERMINAL SPEED IN SHOWER GEL
10/10/2020

+-0.01mm ball bearing radius /mm	+-0.005g ball bearing mass /g	mean ball bearing density /kg/m^3	lower bound ball bearing density /kg/m^4	upper bound ball bearing density /kg/m^5	t1/s	t2/s	t3/s	mean t /s	t error /s	+-0.1 cm d/cm	mean drop speed /ms^-1	lower bound error /ms^-1 drop speed error /ms^-1	upper bound error /ms^-1 drop speed error/ms ^-1
1.99	0.24	7270	7013	7535	15.93	16.36	16.07	16.12	0.22	3.80	2.36E-03	9.28E-05	9.54E-05
1.48	0.10	7439	6926	7972	13.68	13.46	13.15	13.43	0.27	1.80	1.34E-03	9.91E-05	1.03E-04
0.98	0.03	7609	6151	9155	66.35	65.30	65.29	65.65	0.61	2.00	3.05E-04	1.79E-05	1.82E-05
2.99	0.86	7681	7560	7803	27.95	28.07	28.62	28.21	0.36	11.70	4.15E-03	8.69E-05	8.91E-05
1.98	0.25	7689	7422	7963	18.17	17.27	17.47	17.64	0.47	1.91	1.08E-03	8.35E-05	8.81E-05
4.77	3.52	7743	7683	7803	13.53	14.54	14.49	14.19	0.57	11.70	8.25E-03	3.86E-04	4.18E-04
4.77	3.52	7743	7683	7803	27.28	27.61	25.99	26.96	0.86	9.90	3.67E-03	1.49E-04	1.59E-04
6.36	8.38	7776	7735	7818	14.10	14.20	15.20	14.50	0.61	13.70	9.45E-03	4.47E-04	4.86E-04
3.01	0.89	7791	7671	7914	15.40	16.90	16.50	16.27	0.78	5.80	3.57E-03	2.21E-04	2.43E-04
2.49	0.51	7886	7716	8061	25.47	23.87	25.20	24.85	0.86	3.85	1.55E-03	9.05E-05	9.70E-05
4.74	3.52	7891	7830	7952	22.03	20.85	21.87	21.58	0.64	7.90	3.66E-03	1.50E-04	1.60E-04
2.48	0.51	8031	7856	8209	37.45	37.83	37.33	37.54	0.26	11.70	3.12E-03	4.80E-05	4.87E-05
1.48	0.12	8837	8299	9394	29.40	28.00	29.60	29.00	0.87	2.00	6.90E-04	5.36E-05	5.69E-05
1.01	0.04	9268	7874	10743	24.30	22.50	25.80	24.20	1.65	0.90	3.72E-04	6.25E-05	7.16E-05

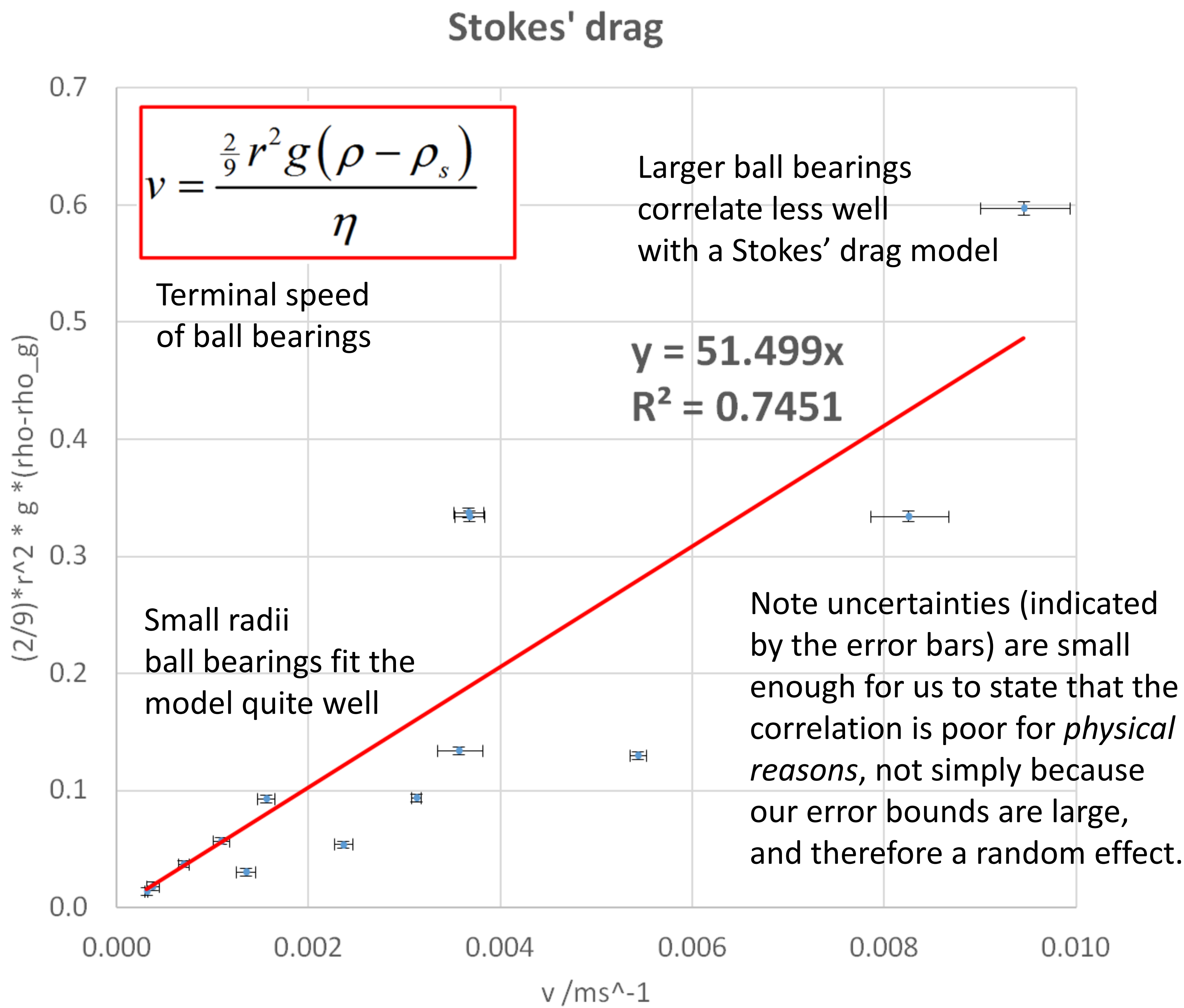
(2/9)*(r^2)*g*(rho-rho_g)	lower bound error	upper bound error
5.41E-02	2.75E-03	2.85E-03
3.05E-02	2.82E-03	2.98E-03
1.38E-02	3.27E-03	3.59E-03
1.30E-01	3.21E-03	3.28E-03
5.72E-02	2.83E-03	2.94E-03
3.34E-01	4.34E-03	4.39E-03
3.34E-01	4.34E-03	4.39E-03
5.98E-01	5.50E-03	5.55E-03
1.34E-01	3.25E-03	3.33E-03
9.31E-02	3.03E-03	3.12E-03
3.38E-01	4.39E-03	4.45E-03
9.39E-02	3.07E-03	3.15E-03
3.74E-02	3.04E-03	3.20E-03
1.84E-02	3.40E-03	3.71E-03



Upper and lower bounds used to calculate error bars.

$$\underbrace{\tfrac{4}{3}\pi r^3 \rho_s g}_{\text{upthrust}} + \underbrace{\tfrac{1}{2} c_D \rho_s \pi r^2 v^2}_{\text{dynamic drag}} = \underbrace{\tfrac{4}{3}\pi r^3 \rho g}_{\text{weight}}$$

As an extension, you might consider *dynamic drag*. But it doesn't work very well!


Stokes' drag
$v = \frac{\frac{2}{9} r^2 g (\rho - \rho_s)}{\eta}$
Terminal speed
of ball bearings
Larger ball bearings
correlate less well
with a Stokes' drag model
y = 51.499x
R^2 = 0.7451
Small radii
ball bearings fit the
model quite well
Note uncertainties (indicated
by the error bars) are small
enough for us to state that the
correlation is poor for *physical*
reasons, not simply because
our error bounds are large,
and therefore a random effect.
(2/9)*r^2 * g *(rho-rho_g)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.000
0.002
0.004
0.006
0.008
0.010
v /ms^-1

Is viscous drag an appropriate model for the larger balls?

$$\mathrm{Re} = \frac{\rho_s v L}{\eta}$$

$$\rho_s \approx 1000\,\mathrm{kgm^{-3}}$$

$$\rho \approx 7500\,\mathrm{kgm^{-3}}$$

$$\mathrm{Re} \ll 1$$

$$L = r,\; v = \frac{\frac{2}{9} r^2 g\left(\rho - \rho_s\right)}{\eta}$$

$$\frac{\rho_s \dfrac{\frac{2}{9} r^2 g\left(\rho - \rho_s\right)}{\eta} r}{\eta} \ll 1$$

$$\boxed{\therefore r \ll \left(\frac{\eta^2}{\frac{2}{9} g \rho_s \left(\rho - \rho_s\right)}\right)^{\frac{1}{3}}}$$

Using the viscosity of shower gel as:

$$\eta = 52\ \mathrm{kgm^{-1}s^{-1}}$$

$$r \ll \left(\frac{\eta^2}{\frac{2}{9} g \rho_s \left(\rho - \rho_s\right)}\right)^{\frac{1}{3}}$$

$$r \ll \left(\frac{52^2}{\frac{2}{9} \times 9.81 \times 1000\left(7500 - 1000\right)}\right)^{\frac{1}{3}}$$

$$\boxed{r \ll 5.76 \times 10^{-3}\,\mathrm{m}}$$

So the larger balls, with a radius of 6.36mm don’t justify this approximation, of radius much less than 5.76mm.

The larger balls are also *not insignificant* compared to the radius of the cylinder, so might expect the flow to be modified in a way that we can ignore for the smaller balls.

A non viscous flow regime can perhaps be hinted at by the slightly bubbly trail left behind by the largest ball bearing.