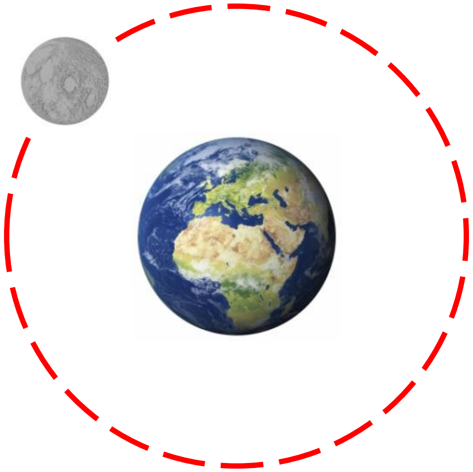
What is Chaos?



Dynamics, the *physics of motion*, provides us with *equations* which can be used to **predict the future position of objects** if we know (i) their present **position** and **velocity** and (ii) the **forces** which act on each object.

This works *very well* for planetary motion, tides etc.
Not so well for weather or indeed the position of pool balls....

This is because ***most* systems cannot be solved exactly**.
An *approximate numerical method* is required to work out what happens next.
Many systems, even really simple ones, are ***highly sensitive to initial conditions.***

This means future behaviour becomes increasingly difficult to predict

Nonlinearity is often the problem!

Strange attractors and even more chaos!

Edward Lorenz was using a **Royal McBee LGP-30 computer** in 1961 to model weather patterns. He accidentally fed in 3 digit precision numbers into the model from a printout rather than the 6 digits used by the computer. These tiny errors created a hugely different weather forecast....

Lorenz's weather model was *very* sensitive to initial conditions.

His equations looked a bit like these:

$$\frac{dx}{dt} = s\left(y - x\right)$$

$$\frac{dy}{dt} = x\left(r - z\right) - y$$

$$\frac{dz}{dt} = xy - bz$$

$$s = 10$$

$$r = 28$$

$$b = \tfrac{8}{3}$$

Edward Lorenz
1917-2008

Although x,y,z trajectories are **chaotic**, they tend to *gravitate towards a particular region.*

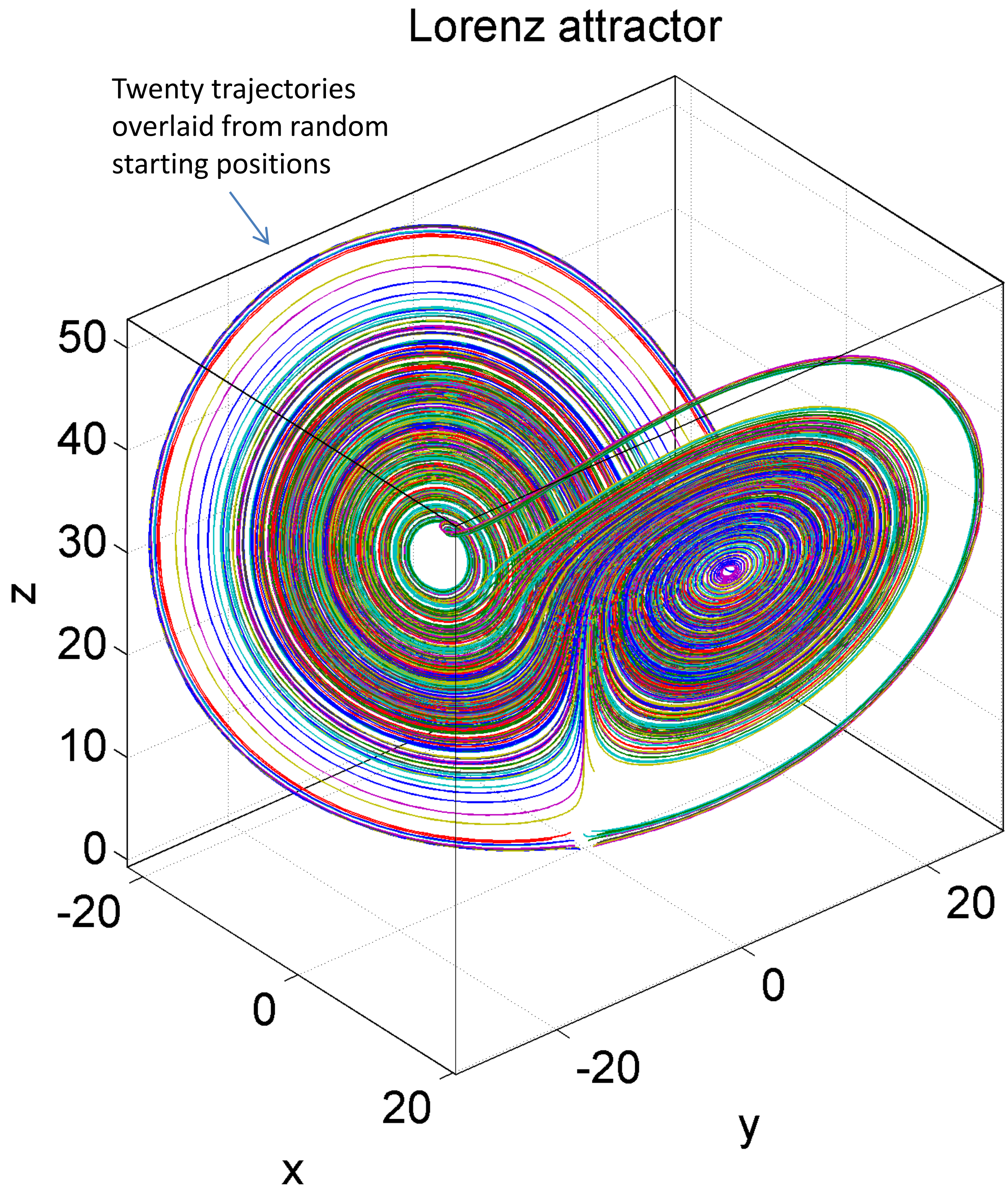
This region is called a **Strange Attractor**

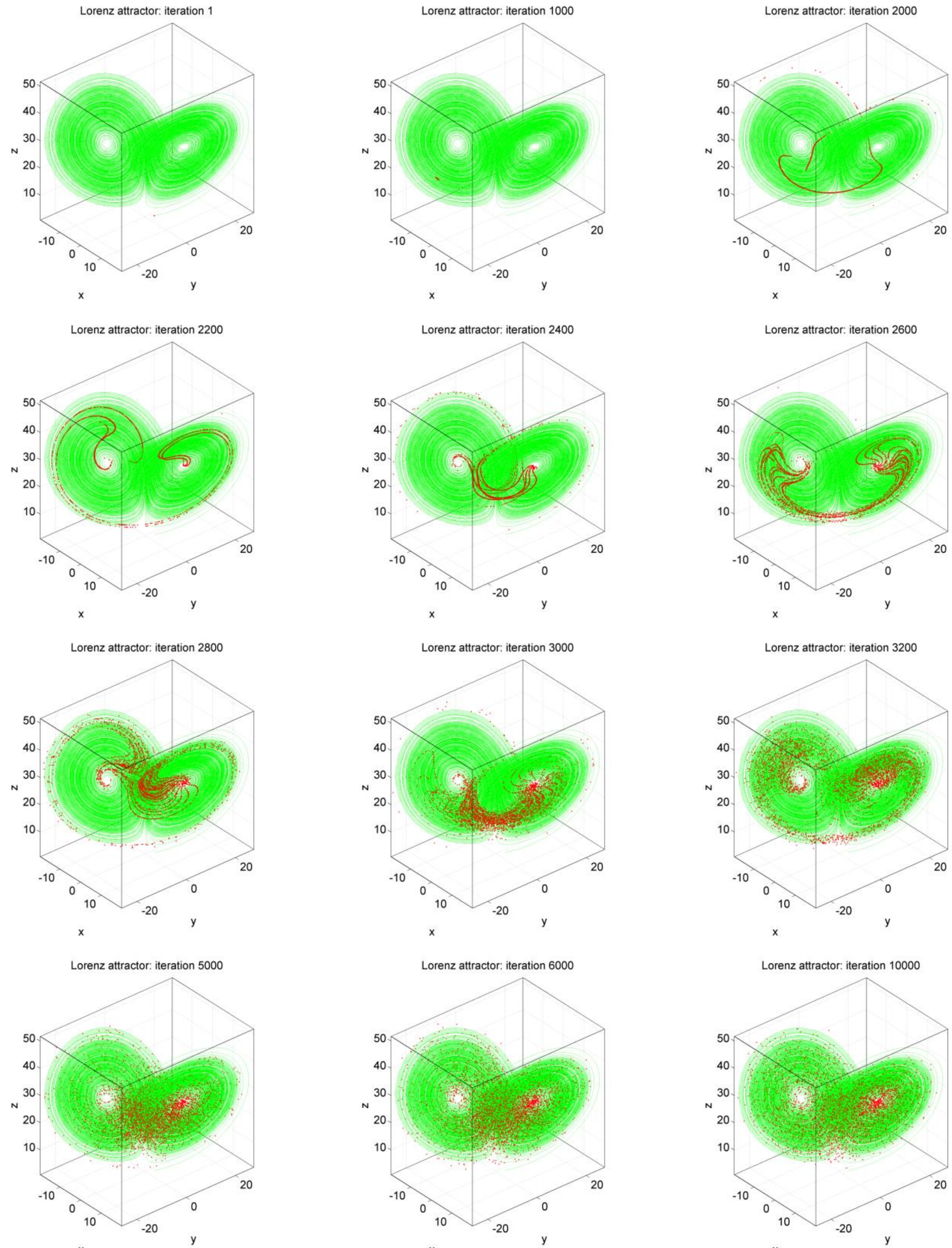
$$\frac{dx}{dt} = s(y - x)$$

$$\frac{dy}{dt} = x(r - z) - y$$

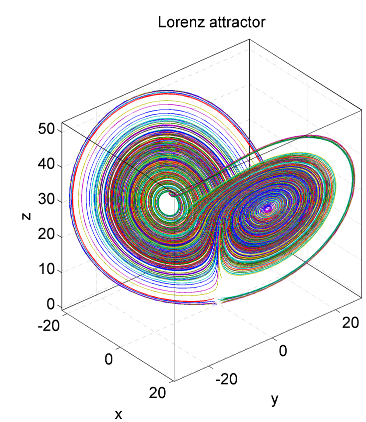
$$\frac{dz}{dt} = xy - bz$$

$s = 10 \quad r = 28 \quad b = \frac{8}{3}$

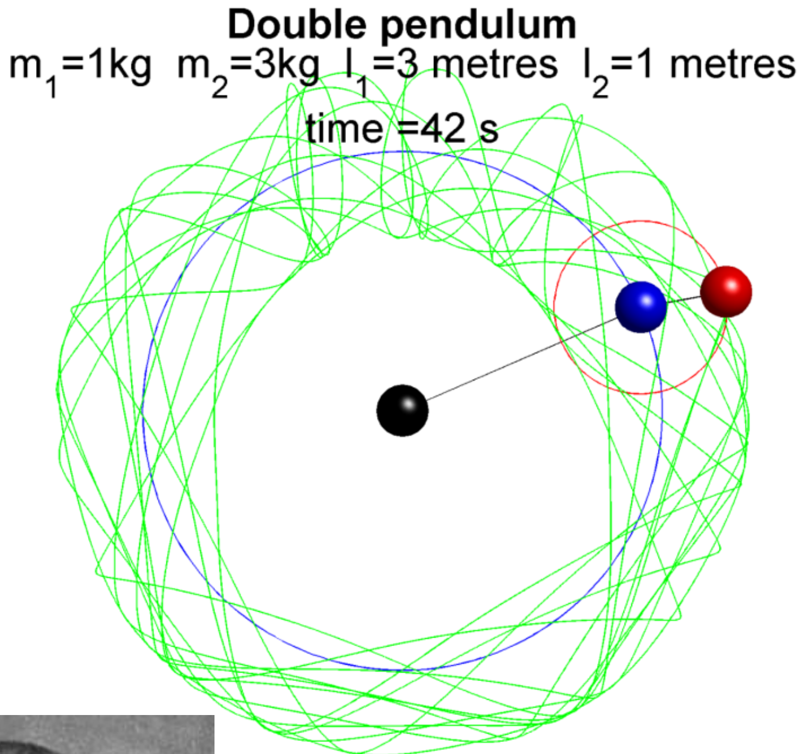


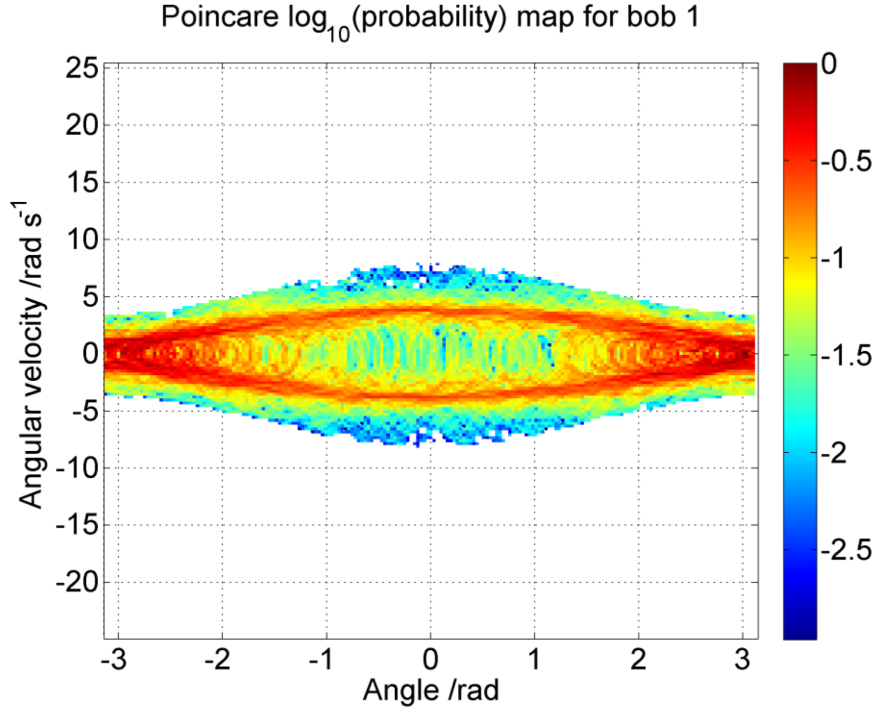


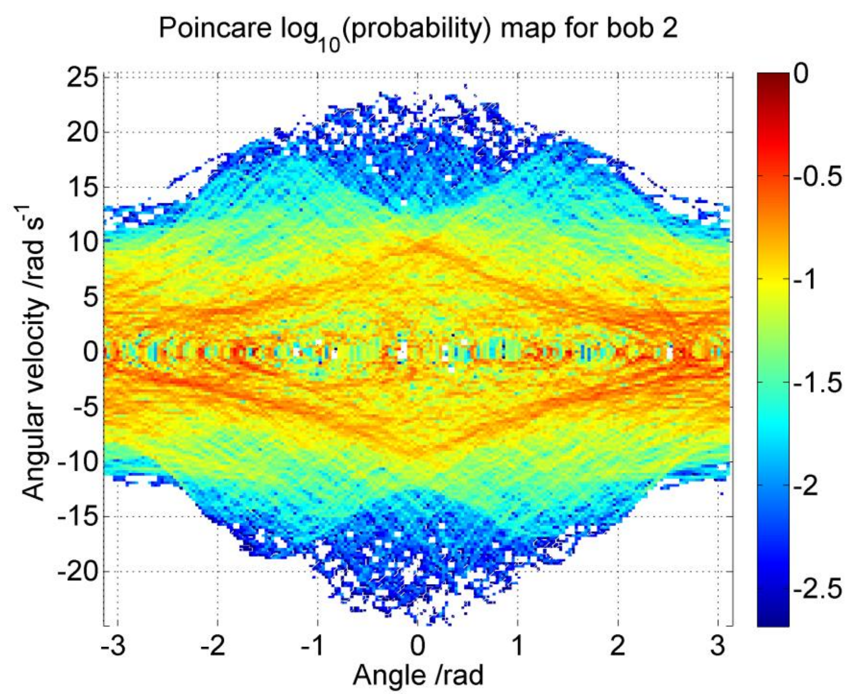
Applying the Lorenz equations, a cluster of initial x,y,z values separated by a *tiny* random deviation will eventually **spread out evenly throughout the strange attractor.**



Based upon Shaw *et al*; "Chaos", Scientific American 54:12 (1986) 46-57







[not chaotic version]()

Henri Poincaré (1854 – 1912)