Measurement in Quantum Mechanics (QM)

The Copenhagen Interpretation of measurement in QM
(Niels Bohr, Werner Heisenberg 1925-1927). For the discussion below, 'state' might mean position, velocity, spin, polarization etc).

• The state of all physical systems can be represented mathematically by a ***wavefunction***. This is a solution to the **Schrödinger Equation** (the time dependent version if one predicts a system will change with time). This is essentially a combination of the **wave equation** with the **conservation of energy.**

• The **modulus squared** of the **wavefunction** is related to the **probability** of a particular state existing. (Born interpretation).

• Prior to measurement, a wavefunction can always be written as a superposition of **eigenstates** (i.e. possible outputs) *of the measurement device.* When a measurement is made the wavefunction '**collapses**' to one of the possible eigenstates associated with the measurement device.

• This *could* mean that *realism is rejected*. This means *until it is measured* the system *does not exist in a particular state*. It is genuinely *a superposition of all possible states*. This explains the 'state lottery' (!) when a system is measured. This is also what the famous **Schrödinger's Cat** thought experiment is all about. A cat is placed in a box containing a poisonous gas which is released as a result of a (random) radioactive decay. Until the box is opened (i.e. the alive or dead state of the cat is 'measured') the cat is said to be a *superposition of both alive and dead states*. ***Alternatively***, the original state (e.g the polarization of a photon) *is fixed*, but *unknown* prior to measurement by a measuring device. It is the measuring process itself which, probabilistically, selects one of its eigenstates (e.g. a polarization direction at an angle to the prior-to-measurement state). This state is *now what exists* following measurement. The latter interpretation shall be used later on in an example to explore the EPR paradox described below. In other words, we shall *assume realism is true*. e.g. a photon did indeed have a particular state prior to measurement.

The EPR paradox
Einstein, Boris Podolsky and Nathan Rosen wrote a paper in 1935 which stated that Quantum Mechanics might be *non-local*, and indeed violates one of the axioms of Relativity. It is possible to produce a system consisting of two **entangled** particles. For example a pair or particles with mutually opposite spin. Imagine the particles are produced at the centre of an enormous laboratory with detectors placed, possibly astronomical distances away. However, if one detector measures an 'up' spin then we *immediately* know that the other particle must be 'down.' This is obvious logic if (i) we assume the entanglement of the particles and (ii) assume the particle was indeed in one state or another *before it was measured*. In other words, *realism is true.*

However, QM states that the particle heading to either detector is indeed in *both spin eigenstates* ***of the detector*** *until it is measured*. Once it *has* been measured, how is it possible for the other particle's wavefunction to collapse to the opposite 'eigenstate' without faster-than-light instantaneous 'spooky action at a distance' communication? This appears to be a violation of the **Principle of Locality**, which means an interaction between two particles is mediated via an interaction which must travel though the space between the particles. The Principle of Relativity means this *cannot* be performed faster than the speed of light.

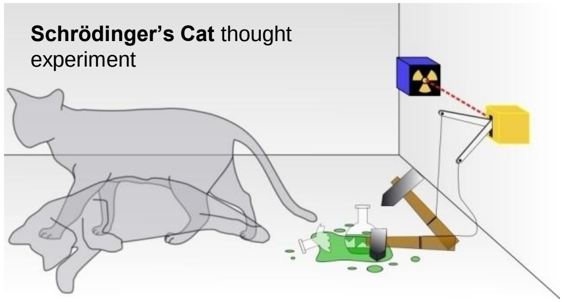
Experiments by Alain Aspect (1972) and many more recently have shown that entangled particles have the expected properties that are consistent with the QM hypothesis. This of course means if QM is true then QM *violates locality.*

Why is *realism* a problem for Quantum Mechanics? At no point in standard probability theory do we assert a superposition of states until measurement. The inherent uncertainty of measurement (e.g. the roll of a dice, choice of a card etc) arises from the distribution of possible outcomes, and our *incomplete knowledge* of which of these outcomes has occurred until we measure it. When a deck of cards is properly shuffled, there is a 1/52 chance that the top card is the Queen of Diamonds. If the Queen of Diamonds *was indeed picked*, it was clearly there *before* we chose to reveal it. Surely probability is a measure of the information known to an observer about a system *before* a measurement of the system state is made? It is nothing intrinsic to the actual state of the system. However .....

....The problem with this is how to explain the **Double Slit experiment** with a beam of particles. If you work out which slit a particle is heading towards you will *not* get the diffraction-style pattern that you get if you simply measure detections of a screen down-stream of the slit. When you *don't* measure the trajectory of the particle pre-slit it *retains its wave-like character* and appears to travel through *both* slits, resulting in a diffraction-style pattern of detections over time.

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+V\psi=i\hbar\frac{\partial\psi}{\partial t}$$

Schrödinger Equation

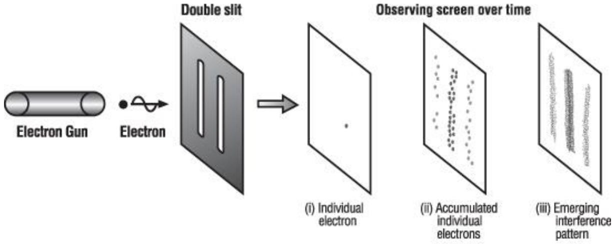



Boris Podolsky 1896-1966


Nathan Rosen 1909-1995

Source of entangled particles

If you measure a down spin at the left detector, you will know that the right detector will have up spin.
This is logical *if indeed the particle had that spin* before you measured it!



Double-slit apparatus showing the pattern of electron hits on the observing screen building up over time.

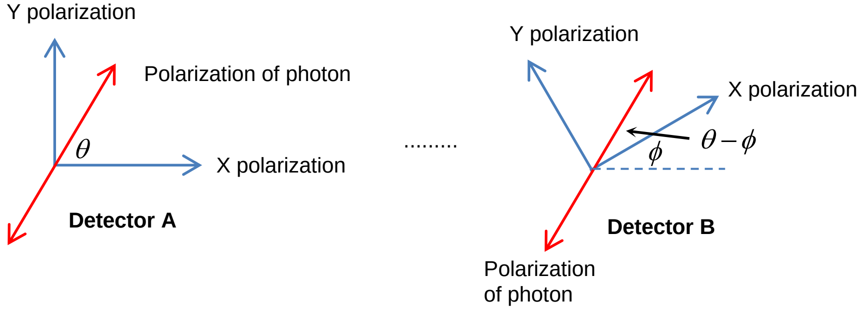
Worked example relating to the EPR paradox* - A Classical approach

Imagine a pair of photons are produced at the centre of a large laboratory. Each photon has the same linear polarization, i.e. it consists of an electromagnetic wave of a particular frequency, whose electric field vector is orientated at a fixed angle. (But whose magnitude varies sinusoidally over time and space).

It is assumed the process which creates the photons results in random linear polarizations.

Two detectors are placed at opposite ends of the laboratory, equidistant from the photon source. Each detector has binary detection logic based on the power transmitted through a polarizer. The relative orientation ϕ of the polarizers between detectors A and B is *known*, but the orientation of the photon polarization at source, relative to the polarizers, is not *a priori.*

Let us use **Malus' Law** to define the detection logic. The light received by each detector is split two ways and passed through polarizers with polarization ninety degrees apart.



Detection logic
A detects X if power in X direction is larger than or equal to power in Y direction.
B detects X if power in X direction is larger than or equal to power in Y direction.

Hence by Malus' Law:

$\sin^2\theta > 0.5$	A detects Y
$\sin^2\theta \le 0.5$	A detects X

$\sin^2(\theta-\phi) > 0.5$	B detects Y
$\sin^2(\theta-\phi) \le 0.5$	B detects X

Of course I have made an **assumption of realism** here!

i.e. the photons actually do emerge from the source with a fixed (but unknown) polarization.

Let's proceed with this argument and compare it to what QM would predict.

Polarization angle θ is assumed to be a random variable, uniformly distributed throughout 360°.

What is the **probability** of detectors A and B both measuring the same polarization?

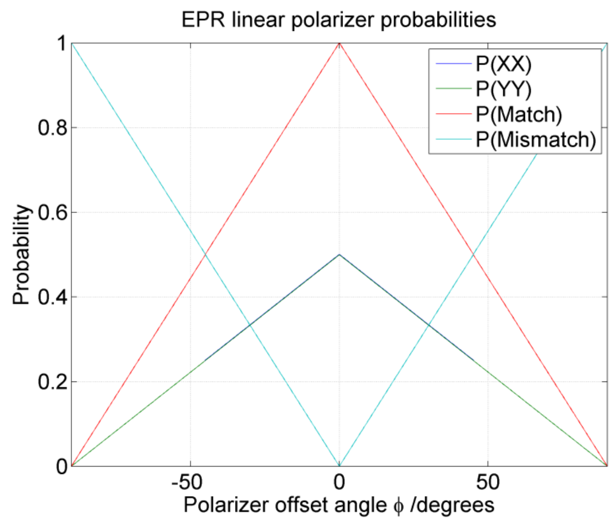
$$P(X_A \,\&\, X_B) = \frac{\text{range of } \theta \text{ such that } \left(\sin^2\theta \le 0.5\right) \,\&\, \left(\sin^2(\theta-\phi) \le 0.5\right)}{360^\circ}$$

$$P(Y_A \,\&\, Y_B) = \frac{\text{range of } \theta \text{ such that } \left(\sin^2\theta > 0.5\right) \,\&\, \left(\sin^2(\theta-\phi) > 0.5\right)}{360^\circ}$$

The probability of a matching output is therefore $P(\text{match}) = P(X_A \,\&\, X_B) + P(Y_A \,\&\, Y_B)$

The probability of a mismatch is therefore 1 – this value.

This predicts a *linear* model for matching probabilities. When the polarizers are aligned there is a 100% chance of matching whereas at 90° angular separation a match is impossible.

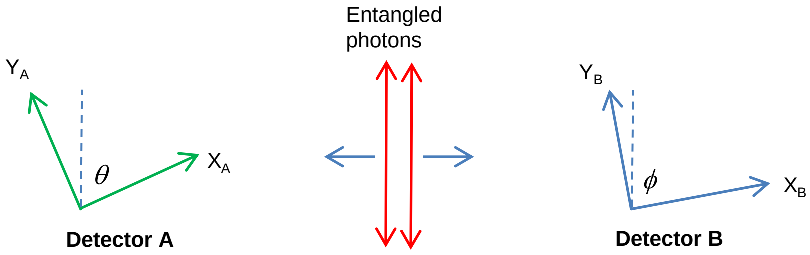


*Based upon discussions with John Cullerne & Jeremy Douglas, Winchester College

Worked example relating to the EPR paradox – Quantum Mechanical model

In this scenario assume a pair of entangled photons with the *same vertical polarization* are produced in the centre of a large laboratory. Detectors based upon linear polarizers are placed a significant distance from the source, such that communication at light-speed between the detectors will have a measurable time delay.

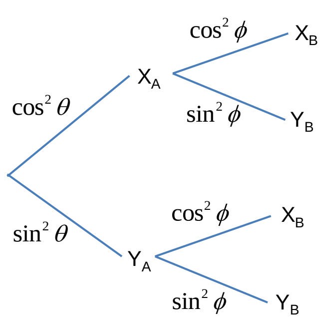
The detectors are placed at angles θ and ϕ from the vertical. A single photon will either be detected or not. Detection means the detected photon has an X polarization, not detected implies the photon has a Y polarization. X and Y directions are of course in general *different* for each detector.



We shall assign probabilities for each detector's eigenstate to be based upon the statistics of the **classical limit** i.e. billions and billions of photons! In this case we expect Malus' Law to hold i.e. the square of the projection of the polarization yields transmitted power.

$$P(X_A)=\cos^2\theta,\quad P(Y_A)=\sin^2\theta$$
$$P(X_B)=\cos^2\phi,\quad P(Y_B)=\sin^2\phi$$

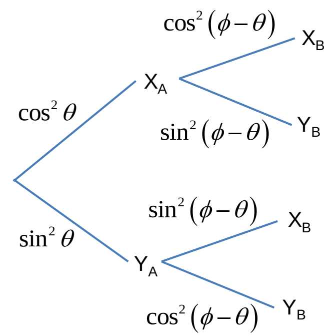
If there is *no waveform collapse upon measurement* (we shall call this the **Classical scenario**) then the measurements by each detector are *independent*. We can therefore construct the following tree diagram to work out the probability of match and hence mismatch in the outputs (XYXXYYX...) of each detector.



Classical scenario

$$P(\text{match})=P(X_A,X_B)+P(Y_A,Y_B)$$
$$P(\text{match})=\cos^2\theta\cos^2\phi+\sin^2\theta\sin^2\phi$$
$$P(\text{mismatch})=1-\cos^2\theta\cos^2\phi-\sin^2\theta\sin^2\phi$$

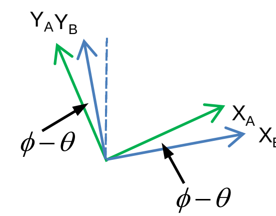
Alternatively, if we measure using detector A first, then QM says that the polarization will now be the measured eigenstate of detector A. This will *change the statistics of measurement of B*. This if course requires 'spooky action at a distance' between A and B....



Quantum scenario

$$P(\text{match})=P(X_A,X_B)+P(Y_A,Y_B)$$
$$P(\text{match})=\cos^2\theta\cos^2(\phi-\theta)+\sin^2\theta\cos^2(\phi-\theta)$$
$$P(\text{match})=(\cos^2\theta+\sin^2\theta)\cos^2(\phi-\theta)$$
$$P(\text{match})=\cos^2(\phi-\theta)$$
$$P(\text{mismatch})=1-\cos^2(\phi-\theta)$$
$$P(\text{mismatch})=\sin^2(\phi-\theta)$$

Note we get the same match and mismatch probabilities if we measure B first. However, what happens if A and B detections are simultaneous?



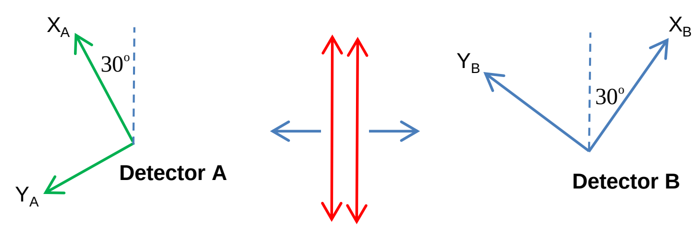
Example:
Classical

$$\theta=-30^\circ,\ \phi=30^\circ$$
$$P(\text{mismatch})=1-\cos^2\theta\cos^2\phi-\sin^2\theta\sin^2\phi$$
$$P(\text{mismatch})=1-\left(\tfrac{\sqrt{3}}{2}\right)^2\left(\tfrac{\sqrt{3}}{2}\right)^2-\left(-\tfrac{1}{2}\right)^2\left(\tfrac{1}{2}\right)^2$$
$$P(\text{mismatch})=1-\tfrac{9}{16}-\tfrac{1}{16}=\frac{16-10}{16}=\boxed{\tfrac{3}{8}}$$

Example:
QM

$$\theta=-30^\circ,\ \phi=30^\circ$$
$$P(\text{mismatch})=\sin^2(\phi-\theta)$$
$$P(\text{mismatch})=\sin^2(60^\circ)=\tfrac{3}{4}=\boxed{\tfrac{6}{8}}$$

The difference between the probabilities is significant, and therefore readily measurable. For this scenario the QM prediction is that the fraction of mismatches between the detector strings XXXYYXXXYYYX... is *double* the classical prediction.



We can summarize the comparison between the models by plotting coloured surfaces of match and mismatch probabilities against detector angles. Note: $\phi_A = \theta,\ \phi_B = \phi$