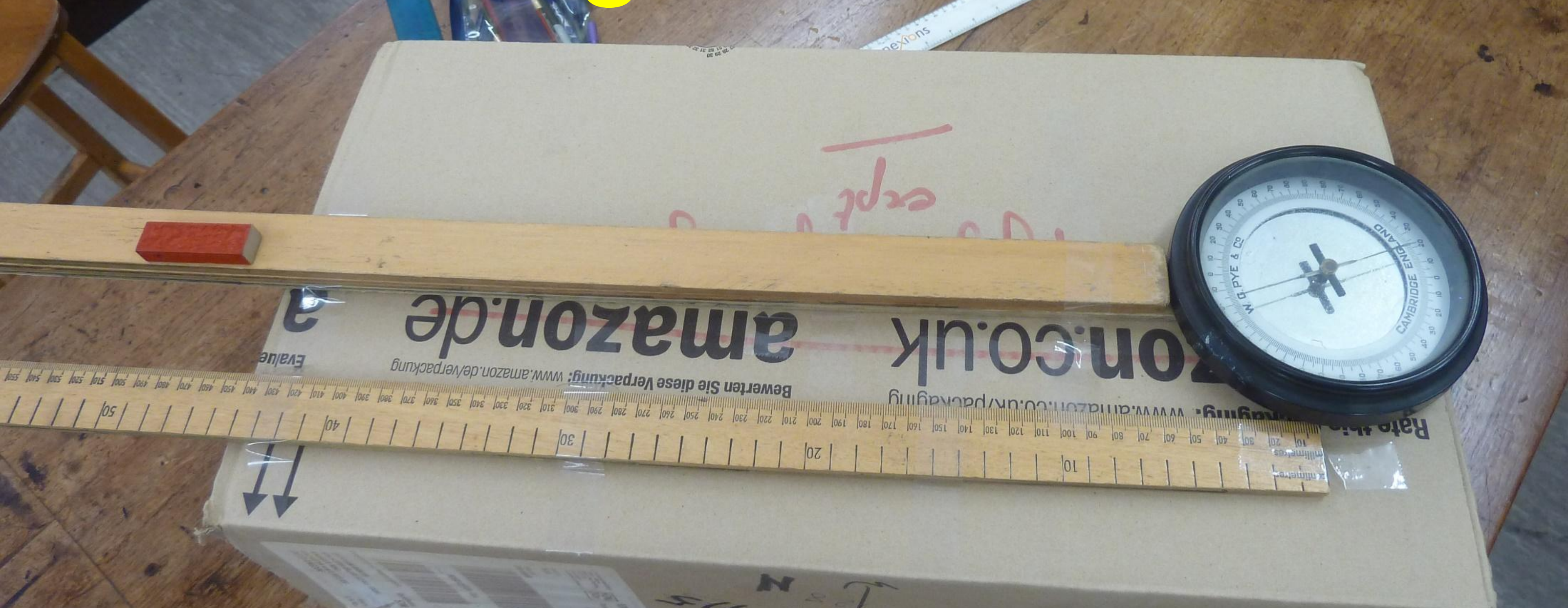
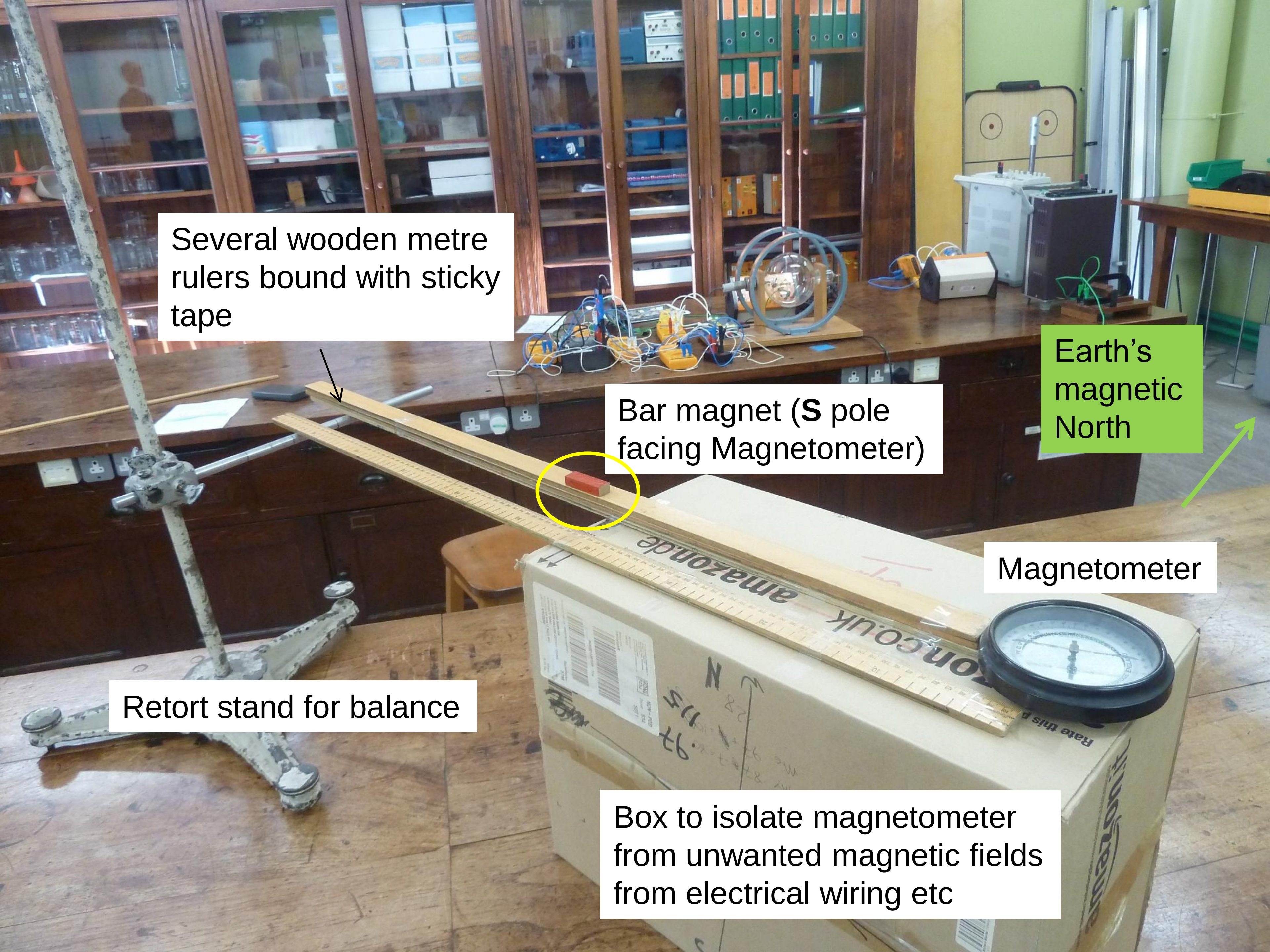
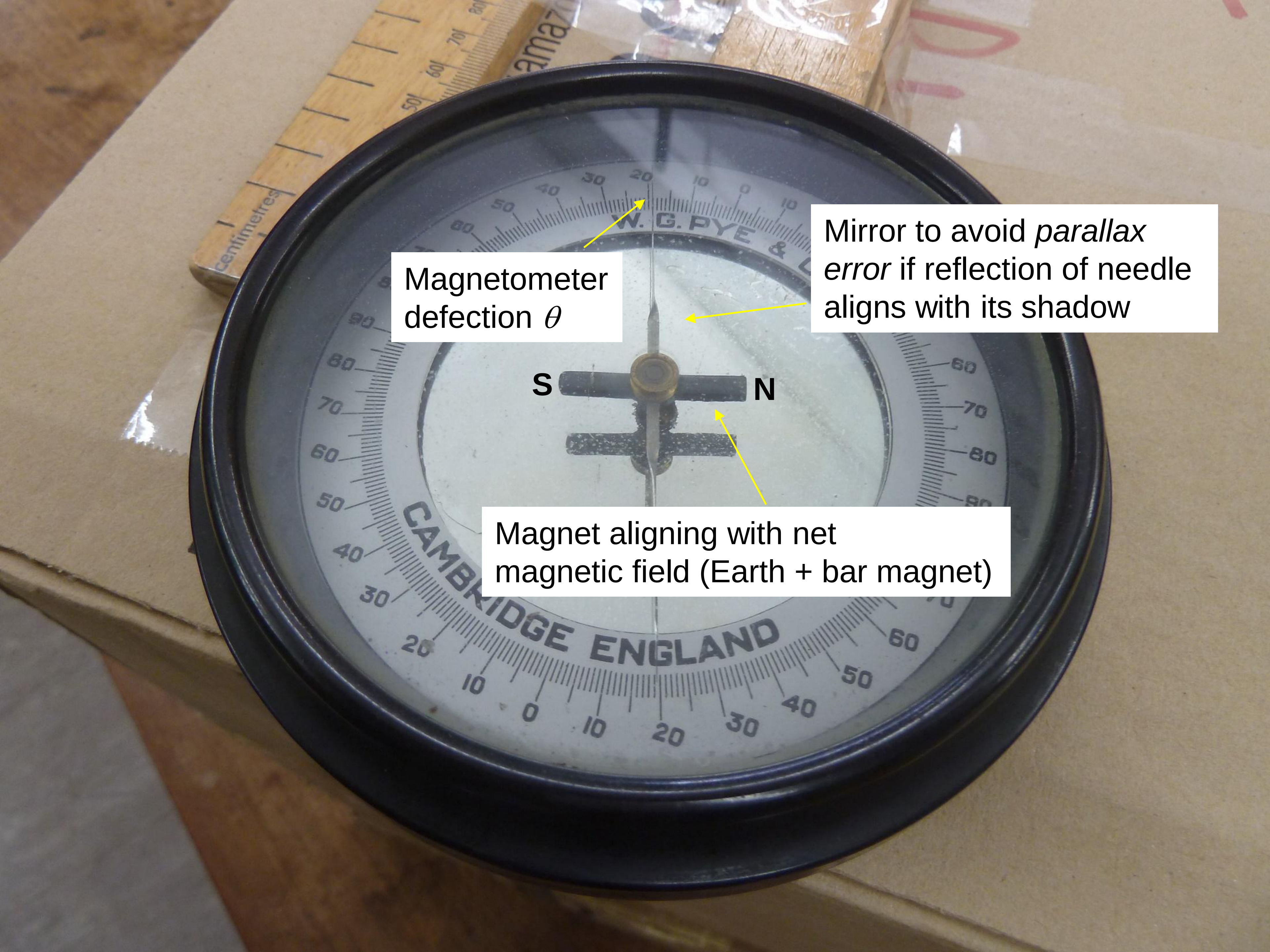
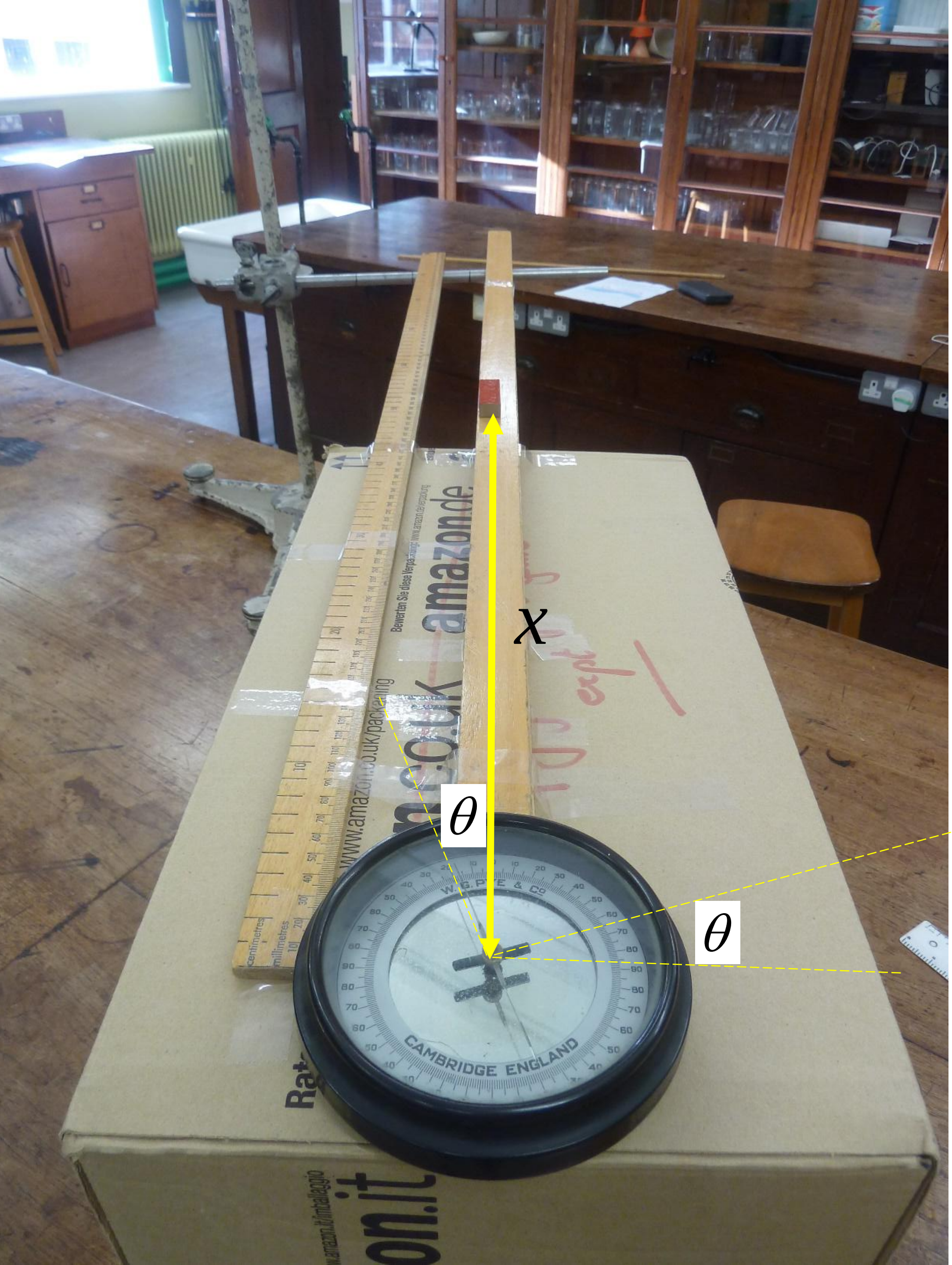
Magnetic field of a bar magnet using a tangent magnetometer



Dr Andrew French & 5P1. Winchester College. November 2017.


Several wooden metre rulers bound with sticky tape
Bar magnet (**S** pole facing Magnetometer)
Earth's magnetic North
Magnetometer
Retort stand for balance
Box to isolate magnetometer from unwanted magnetic fields from electrical wiring etc

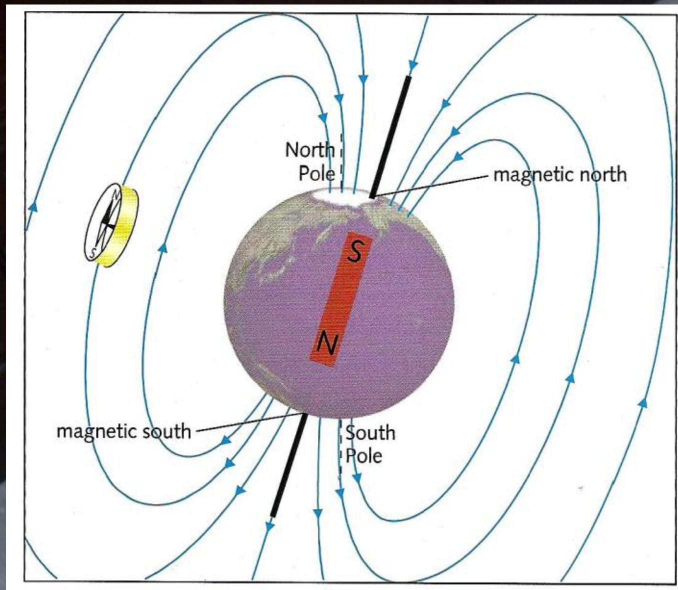

Magnetometer defection θ
Mirror to avoid *parallax error* if reflection of needle aligns with its shadow
S
N
Magnet aligning with net magnetic field (Earth + bar magnet)
CAMBRIDGE ENGLAND
centimetres



Distance x /m of magnet
from centre of magnetometer

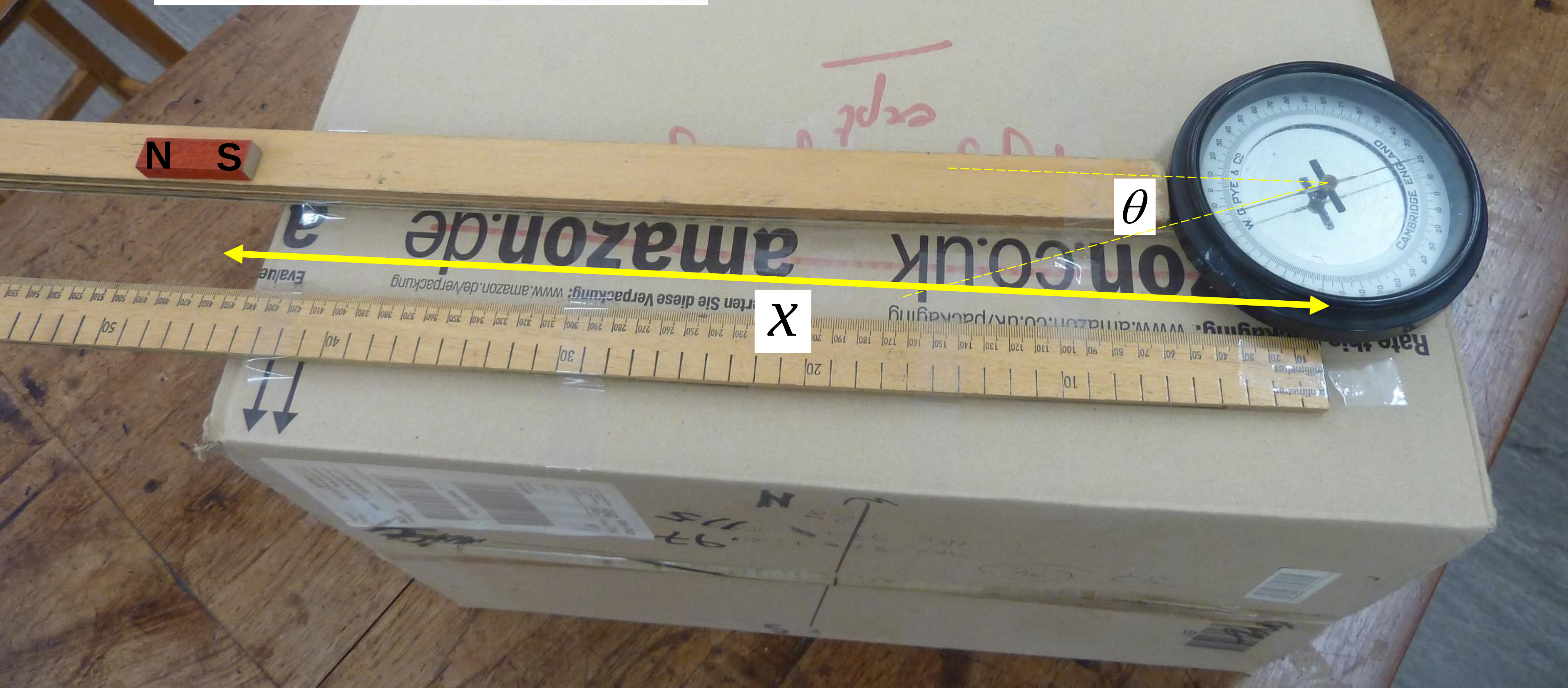
Deflection θ of magnetometer
needle from position
corresponding to alignment
with Earth's magnetic field.

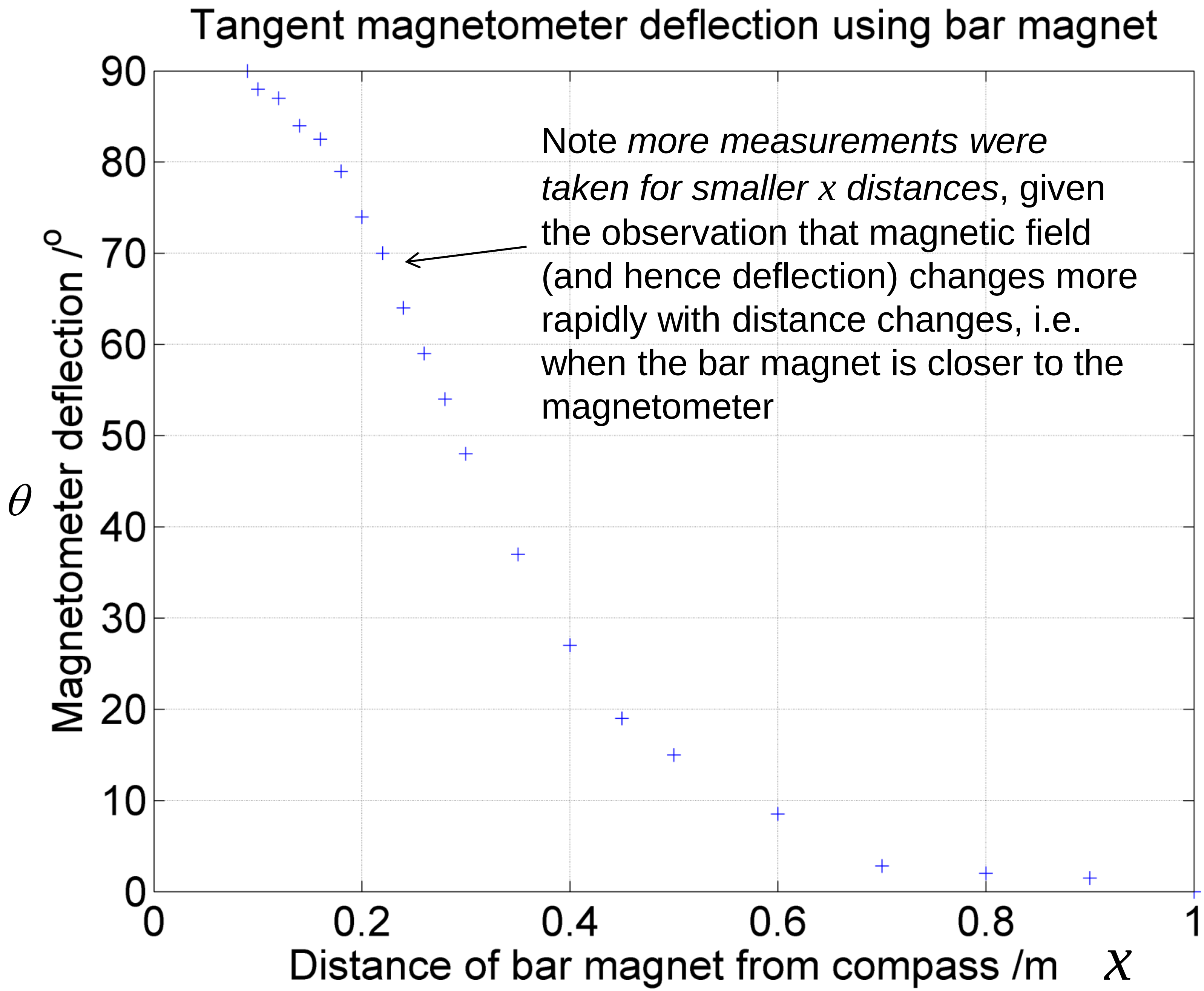
For very sensible reasons, we
firstly aligned to magnetic
north it *sans magnet*, i.e. so
we were able to start from a
zero deflection.



Note by convention magnetic field lines point **towards the south pole** and **emerge from the north pole**.

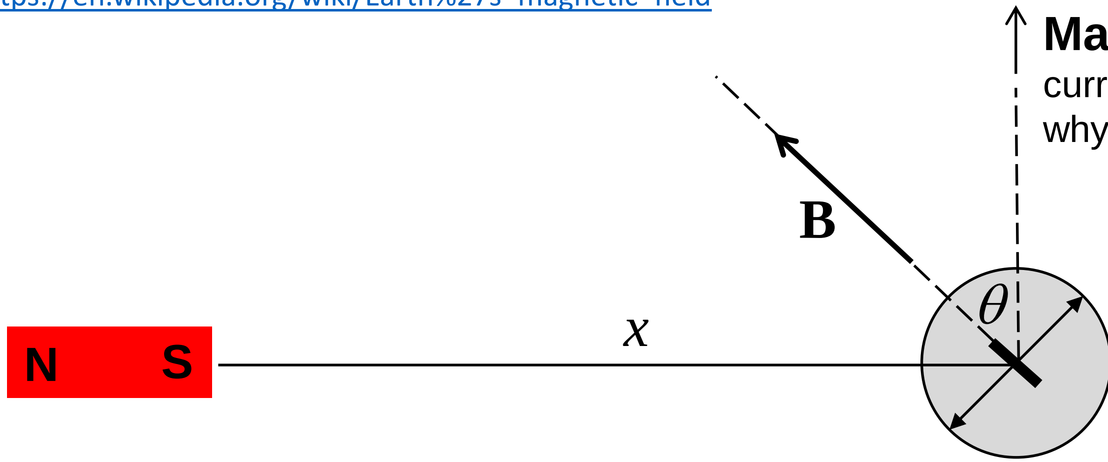
Note also that, as of 11 Nov 2017, geomagnetic north is actually a south pole! (i.e. *field lines point north*, not south).

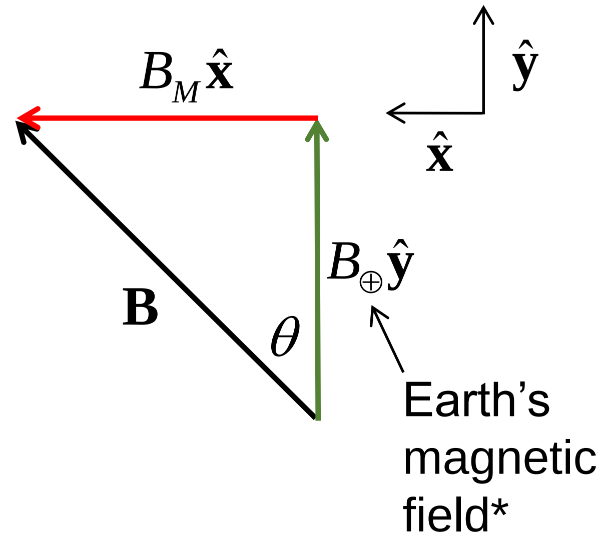



Tangent magnetometer deflection using bar magnet
90
80
70
60
50
40
30
20
10
0
0
0.2
0.4
0.6
0.8
1
Magnetometer deflection /°
θ
Distance of bar magnet from compass /m
x
Note more measurements were taken for smaller x distances, given the observation that magnetic field (and hence deflection) changes more rapidly with distance changes, i.e. when the bar magnet is closer to the magnetometer

* https://en.wikipedia.org/wiki/Earth%27s_magnetic_field

Magnetic North (note this is currently a magnetic *south pole*, which is why the magnetic field points towards it)

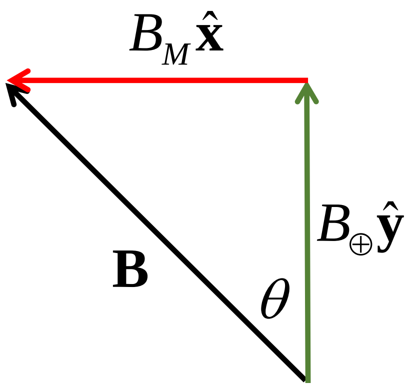




Net magnetic field acting on magnetometer magnet (which it aligns with) is:

$$\mathbf{B} = B_M\hat{\mathbf{x}} + B_\oplus\hat{\mathbf{y}}$$

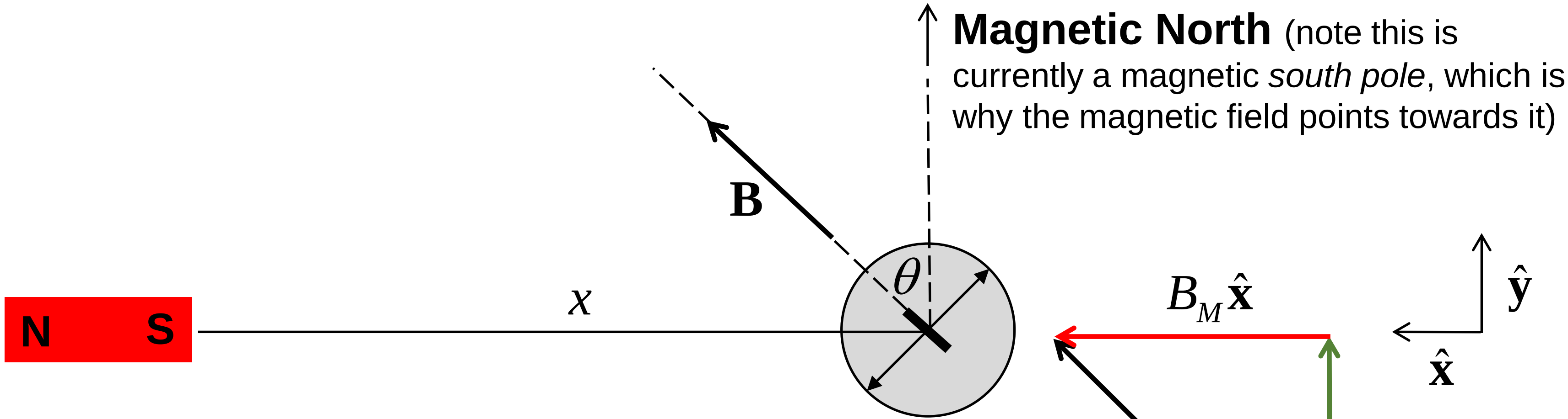
$$25\mu\mathrm{T} < B_\oplus < 65\mu\mathrm{T}$$

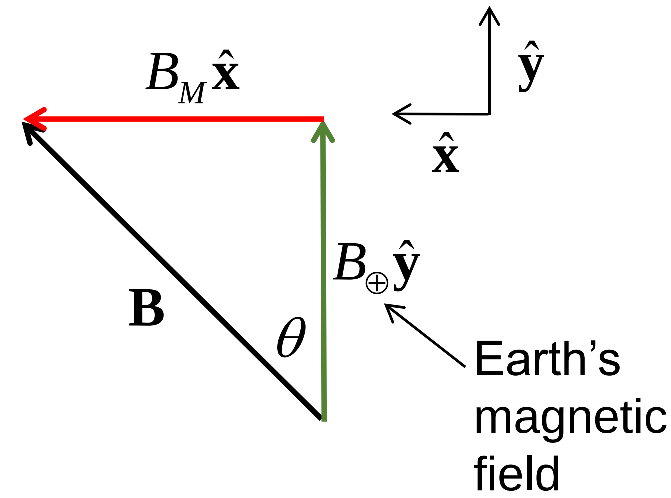


Hence:

$$\boxed{\frac{B_M}{B_\oplus} = \tan\theta}$$

$B_M\hat{\mathbf{x}}$ is the magnetic field due to the bar magnet





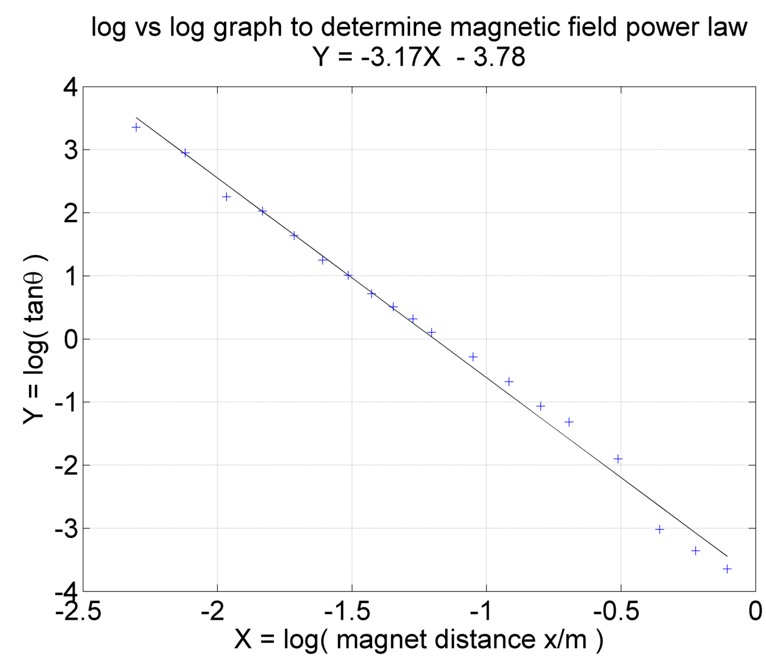
$$\frac{B_M}{B_\oplus} = \frac{k}{x^n}$$

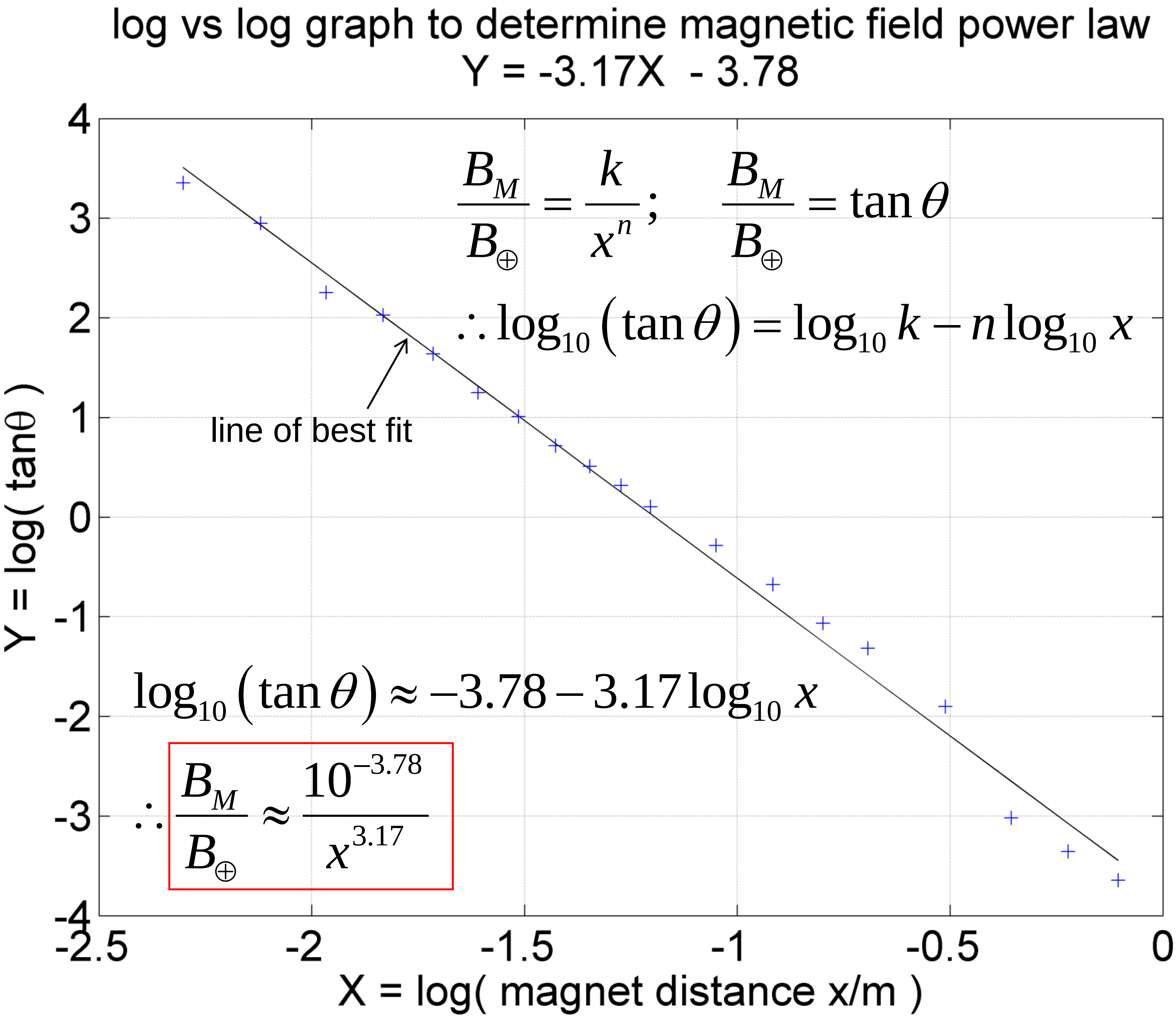
Postulate a **power-law** variation of bar magnet field strength with distance. k is a constant.

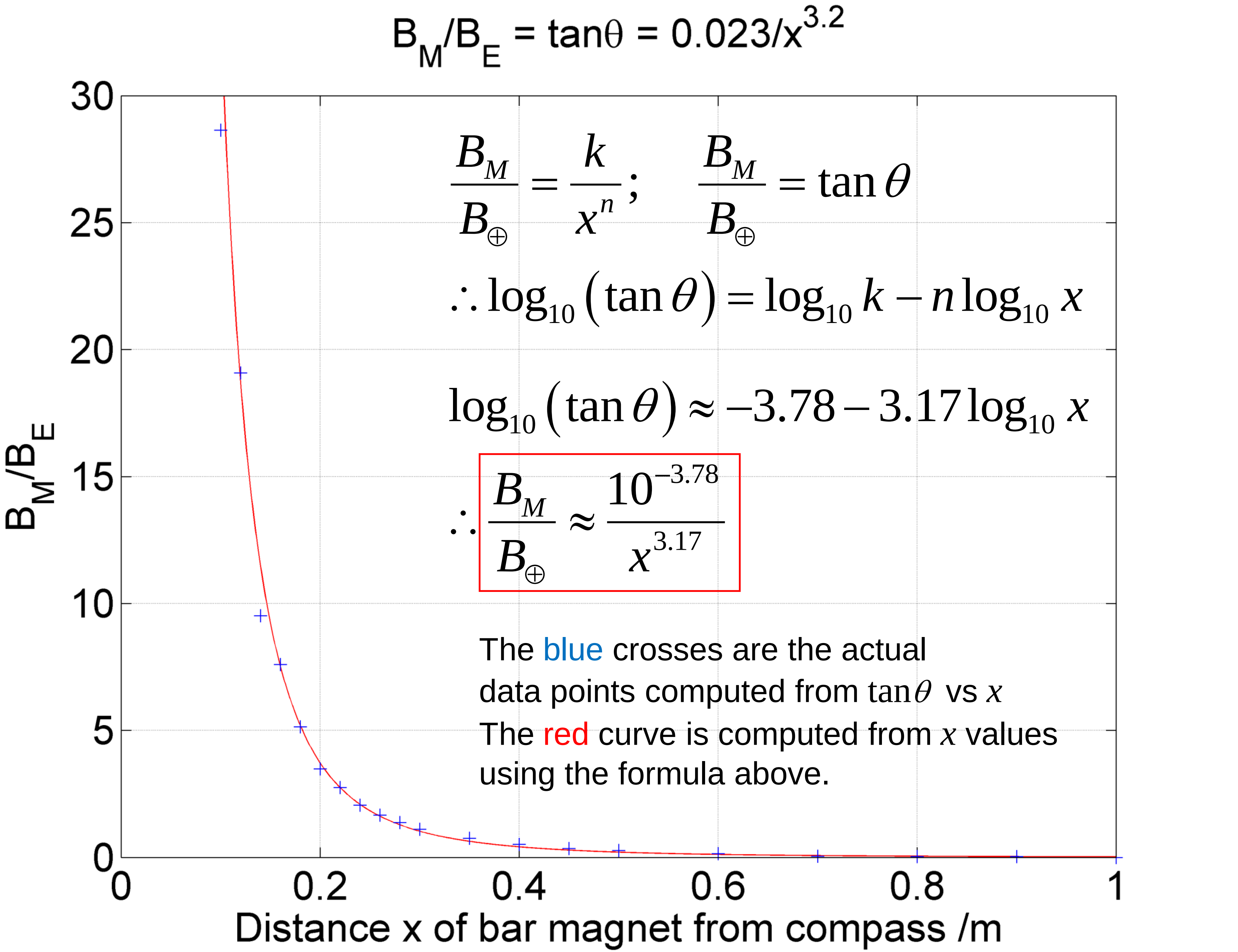
$$\frac{B_M}{B_\oplus} = \tan\theta$$

$$\therefore \boxed{\log_{10}(\tan\theta) = \log_{10} k - n\log_{10} x}$$

So plotting $Y = \log_{10}(\tan\theta)$ vs $X = \log_{10} x$ should yield a **straight line** with gradient -n

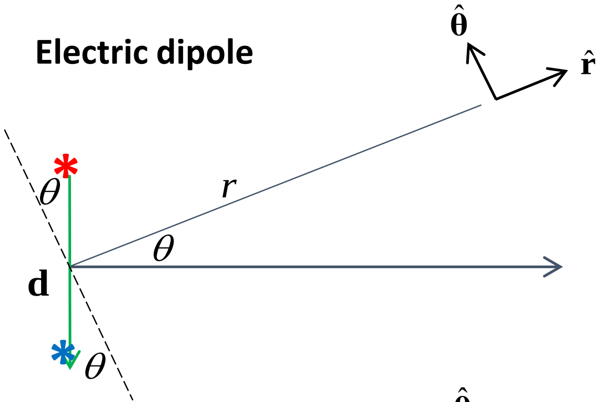



log vs log graph to determine magnetic field power law
Y = -3.17X - 3.78
$\frac{B_M}{B_\oplus} = \frac{k}{x^n}; \quad \frac{B_M}{B_\oplus} = \tan\theta$
$\therefore \log_{10}(\tan\theta) = \log_{10} k - n \log_{10} x$
line of best fit
$\log_{10}(\tan\theta) \approx -3.78 - 3.17 \log_{10} x$
$\therefore \boxed{\frac{B_M}{B_\oplus} \approx \frac{10^{-3.78}}{x^{3.17}}}$
Y = log(tanθ)
X = log(magnet distance x/m)
4
3
2
1
0
-1
-2
-3
-4
-2.5
-2
-1.5
-1
-0.5
0


$B_M/B_E = \tan\theta = 0.023/x^{3.2}$
$\frac{B_M}{B_\oplus} = \frac{k}{x^n}; \quad \frac{B_M}{B_\oplus} = \tan\theta$
$\therefore \log_{10}(\tan\theta) = \log_{10} k - n\log_{10} x$
$\log_{10}(\tan\theta) \approx -3.78 - 3.17\log_{10} x$
$\therefore \boxed{\frac{B_M}{B_\oplus} \approx \frac{10^{-3.78}}{x^{3.17}}}$
The blue crosses are the actual data points computed from $\tan\theta$ vs x
The red curve is computed from x values using the formula above.
B_M/B_E
Distance x of bar magnet from compass /m
0
5
10
15
20
25
30
0
0.2
0.4
0.6
0.8
1

The field of a **Magnetic dipole** is mathematically very similar to that of an electric dipole (see Electric dipole notes).

Electric dipole



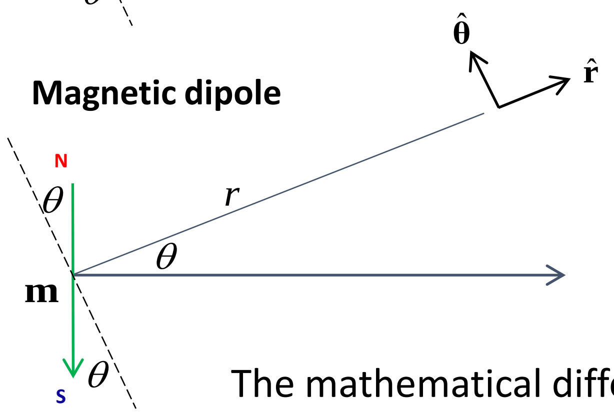
$$\boxed{\mathbf{E} = \frac{qd}{4\pi\varepsilon_0 r^3}\left(2\hat{\mathbf{r}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta\right)}$$

In both cases assume r is much greater than the dimensions associated with the dipole

This explains why the variation of magnetic field strength vs distance is $B_M \propto r^{-3}$

We measured -3.2 as the power.

Magnetic dipole



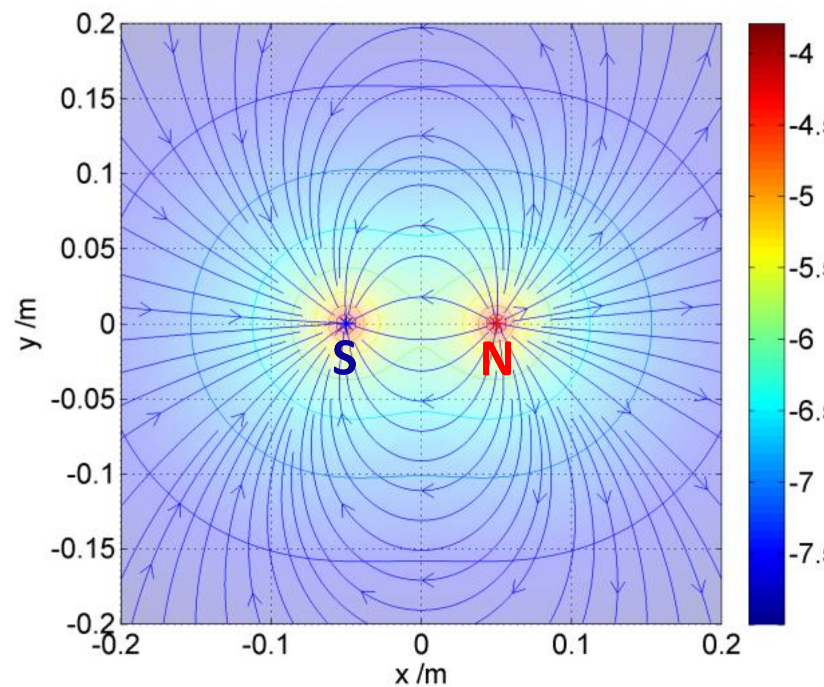
$$\boxed{\mathbf{B} = \frac{\mu_0 m}{4\pi r^3}\left(2\hat{\mathbf{r}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta\right)}$$

Permittivity of free space $\varepsilon_0 = 8.854\times10^{-12}\,\mathrm{Fm}^{-1}$

Permeability of free space $\mu_0 = 4\pi\times10^{-7}\,\mathrm{Hm}^{-1}$

The mathematical difference between the electric and magnetic dipoles is the quantity $\frac{qd}{\varepsilon_0} \rightarrow \mu_0 m$

m is the **magnetic dipole moment**



For a magnetic dipole formed from a small current loop (or indeed solenoid) of radius a $\boxed{m = I\pi a^2}$

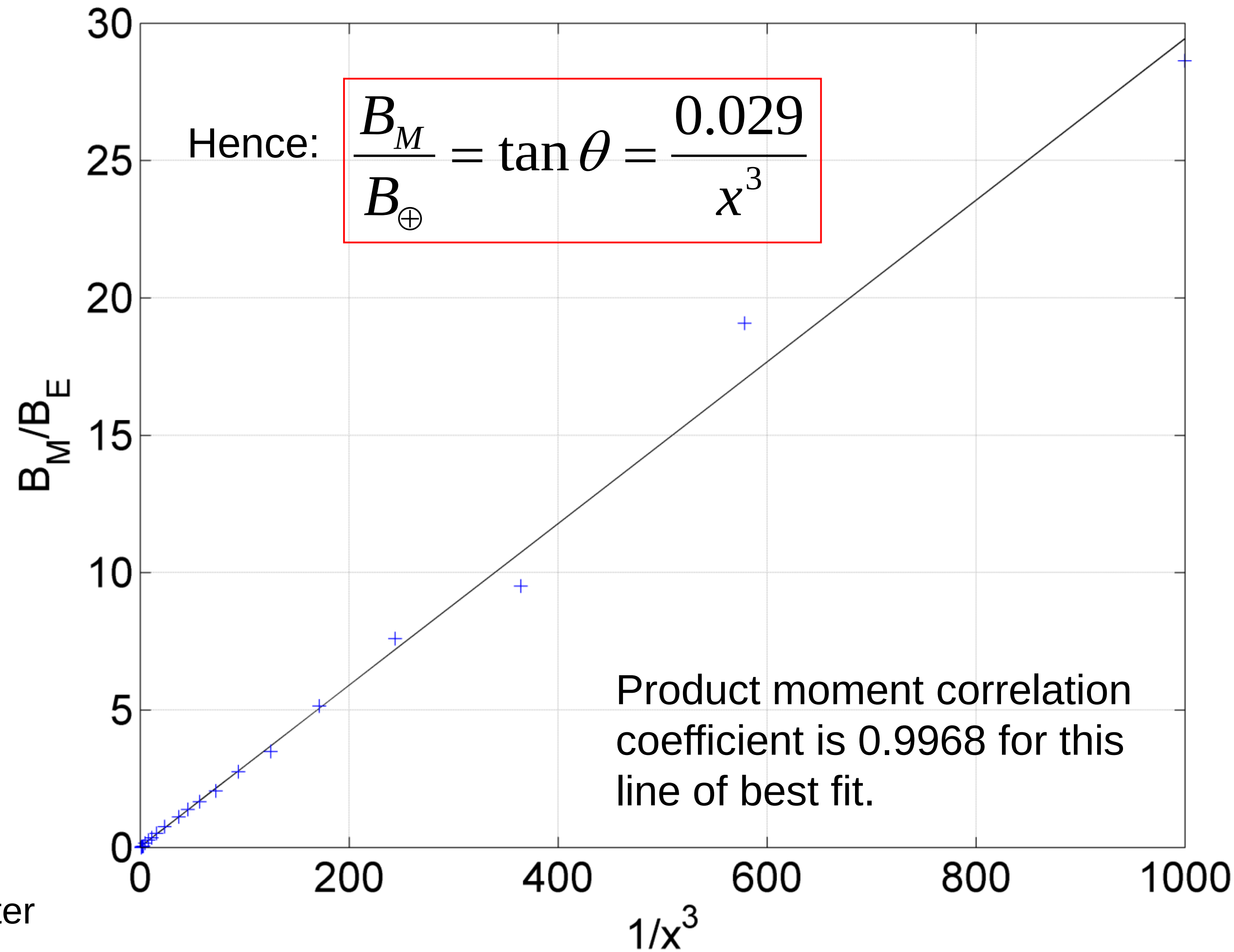
Rather than a curve fit using $B_M \propto r^{-3.17}$ we can construct an alternative linearization

$$\boxed{\frac{B_M}{B_\oplus} = \tan\theta = \frac{k}{x^3}}$$

So plotting $\tan\theta$ vs $1/x^3$ should yield a **straight line**

i.e. using our theoretical model of the magnetic dipole

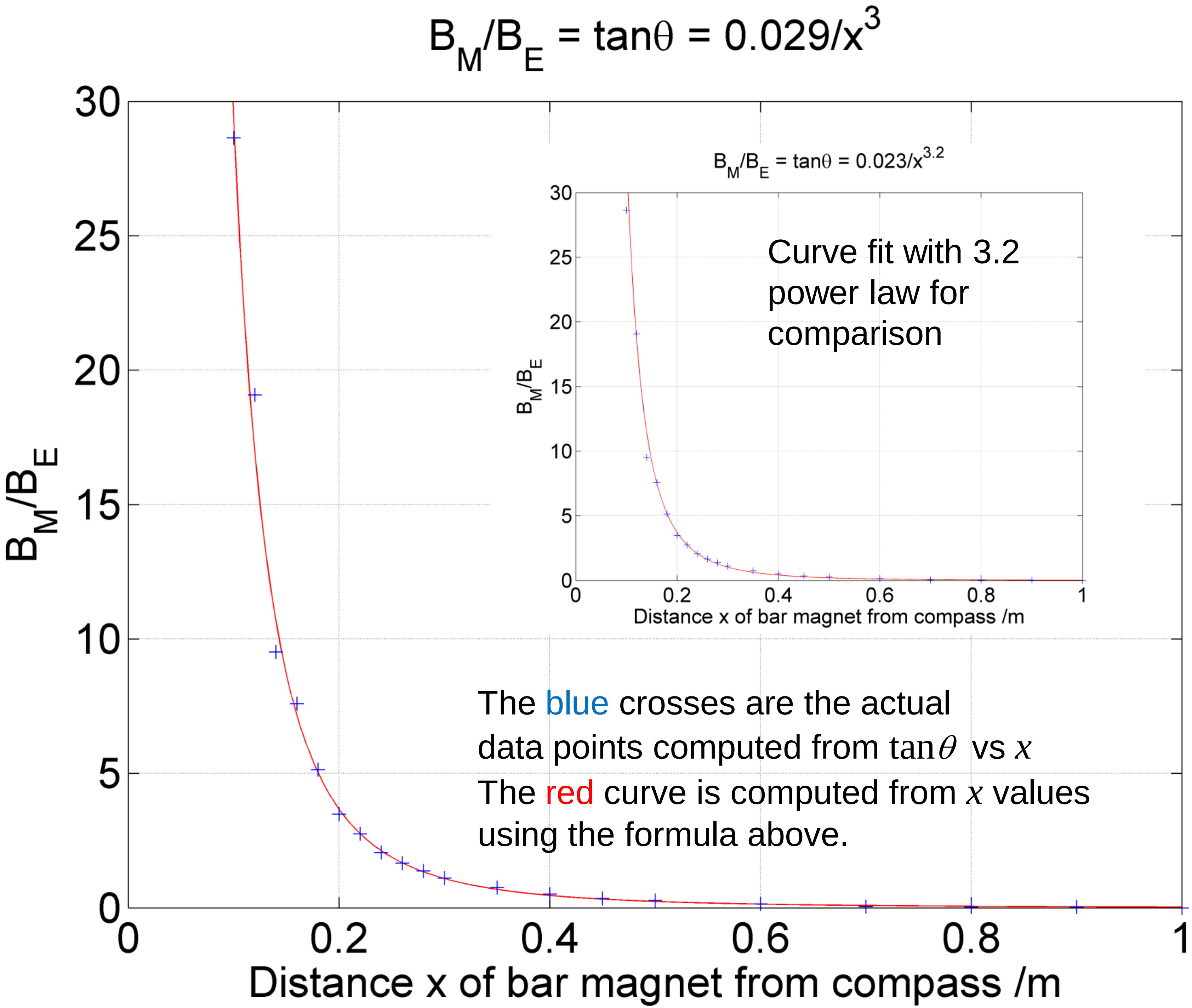
Y = 0.0294X - 0.0215



Hence: $$\boxed{\frac{B_M}{B_\oplus} = \tan\theta = \frac{0.029}{x^3}}$$

Product moment correlation coefficient is 0.9968 for this line of best fit.

Note distance x is in metres from centre of magnetometer


$B_M/B_E = \tan\theta = 0.029/x^3$
B_M/B_E
Distance x of bar magnet from compass /m
0
0.2
0.4
0.6
0.8
1
0
5
10
15
20
25
30
$B_M/B_E = \tan\theta = 0.023/x^{3.2}$
Curve fit with 3.2 power law for comparison
The blue crosses are the actual data points computed from $\tan\theta$ vs x
The red curve is computed from x values using the formula above.