$$
\begin{aligned}
& \text { The Art of } \\
& \text { Computer } \\
& \text { Programming }
\end{aligned}
$$

"I'd like to welcome you to this course on Computer Science. Actually, that's a terrible way to start. Computer Science is a terrible name for this business. First of all, it's not a science. It might be engineering, or it might be art, or we'll actually see that computer so-called science actually has a lot in common with magic"

Harold Abelson, MIT (1986)

# "Science is what we understand well enough to explain to a computer. Art is everything else we do." 




Donald Knuth 1938-
Stanford University

"Computer programming is an art, because it applies accumulated knowledge to the world, because it requires skill and ingenuity, and especially because it produces objects of beauty. A programmer who subconsciously views himself as an artist will enjoy what he does and will do it better."

$$
\text { - } \int_{0}^{\infty} \frac{2 x \sin x}{1+x^{2}} d x=\frac{\pi}{e}
$$

"I can't go to a restaurant and order food because I keep looking at the fonts on the menu."

## Lecture map

Fast numerical calculation + display systems

## Use of computer programming as an artistic tool

Where you can design and refine the tool
Complexity, and beauty, from simplicity
(i.e. code)

## Case studies

General thoughts on how humans best interact with information technology What You See Is What You Need!

Random rectangles
\& Mondrian


## Ciphers

Uif Dpnfez pg Fsspst
cz Xjmmjbn Tiblftqfbsf

The Sierpinski Triangle


## Case studies


for $n=1: N$
$r=r a n d ; \%$ Generate a random number
if (r $\quad$ = $1 / 3$ )
\%Move half way towards red star
$x=0.5^{*}(x R+x)$;
$y=0.5 *(y R+y) ;$
\%Plot a red dot
plot ( $\mathrm{X}, \mathrm{y}, \quad$ r.' ) ;
elseif $(r>1 / 3) \& \&(r<=2 / 3)$
\%Move ... blue star
$\mathrm{x}=0.5^{*}(\mathrm{xB}+\mathrm{x})$;
$\mathrm{y}=0.5^{*}(\mathrm{yB}+\mathrm{y})$;
\%Plot a blue dot plot ( $\mathrm{x}, \mathrm{Y}, \quad$ 'b.' ) ;
else
\%Move ... green star $x=0.5^{*}(x G+x)$;
$y=0.5^{*}(y G+y) ;$
\%Plot a green dot plot ( $\mathrm{x}, \mathrm{y}, \quad$ ' $\mathrm{g} .{ }^{\prime}$ ) ;
end

end







```
fid = fopen( filename, 'r' ); %Open file filename (read only)
```

\%Store filename text in a row vector A of characters, then close file
A = fscanf(fid, '\%c'); fclose(fid);

```
e.g A = 'The Comedy of Errors .....'
```

\%Open file for writing
fid $=$ fopen ( strrep ( filename,'.txt', ['-', cipher_mode,'.txt'] ), 'w' );
\%Step through cipher_key, replacing instances of the
\%characters with their plaintext or enciphered equivalents
B = A; dim = size(cipher_key);
if strcmp(cipher_mode,'encrypt')==1
\%Encrypt file contents
for $n=1: \operatorname{dim}(1)$
indices $=$ strfind ( A, cipher_key\{n,1\} );
B(indices) = cipher_key\{n,2\};
end
e.g.
plaintext.txt would become plaintextencrypt.txt
else
\%Decrypt file contents
for $\mathrm{n}=1: \operatorname{dim}(1)$
indices = strfind ( A, cipher_key\{n,2\} );
$B(i n d i c e s)=c i p h e r \_k e y\{n, 1\} ;$

## MATLAB code for cipher.m

end
end
\%Write encrypted character array $B$ to a appended, then close file fwrite(fid, B ); fclose(fid);


Composition with Yellow, Blue, and Red $\qquad$ 1937-42, Piet Mondrian. Oil on canvas; $72.5 \times 69 \mathrm{~cm}$. London, Tate Gallery.


Randomly generated from mondrian.m


Piet Mondrian (1872-1944)
"a post or support"
De Stijl movement (Amsterdam, 1917-1931) "Neoplasticism" "Ultimate simplicity and abstraction"


Cut a rectangle randomly in horizontal and vertical directions. Randomly divide into two types


2 Shrink the 'red' type to the black lines


1. Choose an intersection at random $\star$
2. Find nearest $\star$ intersection which has the same $y$ coordinate
3. Find the nearest $\star$ intersection from this which has the same $x$ coordinate

5

4. Construct a rectangle


Find coordinates of all line segment intersections

Repeat from
4
cycling through red, blue and yellow colours


## Soundsnipper GUI




## Mandlebrot transformations of complex numbers

$$
\begin{array}{|l|}
i^{2}=-1 \\
z=x+i y \\
x=\operatorname{Re}(z) \\
y=\operatorname{Im}(z) \\
|z|=\sqrt{x^{2}+y^{2}}
\end{array}
$$

$(1+i)(1+i)$
$=1+2 i+i^{2}$
$=1+2 i-1$
$=2 i$

julia.m plot option abs diverge

## Plot a surface with

 height $h(x, y)$. This is the iteration number when |z/ exceeds a certain value e.g. 4In this case colours indicate height $h(x, y)$. It is a 'colour-map'.

julia.m plotoption plot z
Plot a surface with height $h(x, y)$

$$
\begin{aligned}
& x=\operatorname{Re}(z), \quad y=\operatorname{Im}(z) \\
& h(x, y)=e^{-\sqrt{x^{2}+y^{2}}}
\end{aligned}
$$



Benoit Mandlebrot (1924-2010)

Mandlebrot deep zoom (YouTube)

## The Mandlebrot Set has infinite complexity! <br> ... But a recursive fractal geometry




Count the green squares that contain the points





Fractal dimension=1.4549 (+/-) 0.05104






The light bulb

$$
z_{n+1}=\log \left(z_{n}^{2}+z_{0}\right)
$$




The Mandlerocket!

$$
z_{n+1}=\sin ^{-1}\left(z_{n}^{2}+z_{0}\right)
$$




The profusion of power

$$
z_{n+1}=\left(z_{n}^{2}+z_{0}\right)^{z_{n}}
$$

Remember $h(x, y)$ is a surface ....

$$
z_{n+1}=z_{n}^{2}+z_{0}
$$



$$
z_{n+1}=z_{n}^{2}+z_{0}
$$



$$
z_{n+1}=z_{n}^{2}+z_{0}
$$

Mandlebrot surface: iteration 64




Selection from Day of Julia. Mathematicon Exhibition, 2014

7 steps to enlightenment

$$
z_{n+1}=\tan ^{-1}\left(z_{n}^{2}+z_{0}\right)
$$



## The Mandlerocket

$$
z_{n+1}=\sin ^{-1}\left(z_{n}^{2}+z_{0}\right)
$$

Mandlebrot surface: iteration 25


$$
\begin{aligned}
& x=\operatorname{Re}(z), \quad y=\operatorname{Im}(z) \\
& h(x, y)=e^{-\sqrt{x^{2}+y^{2}}}
\end{aligned}
$$

$$
m_{i} \frac{d \mathbf{v}_{i}}{d t}=A m_{i} \sum_{j \neq i} \frac{m_{j}\left(\mathbf{r}_{j}-\mathbf{r}_{i}\right)}{\left|\mathbf{r}_{j}-\mathbf{r}_{i}\right|^{P+1}}-B m_{i} \sum_{j \neq i} R_{j} \frac{m_{j}\left(\mathbf{r}_{j}-\mathbf{r}_{i}\right)}{\left|\mathbf{r}_{j}-\mathbf{r}_{i}\right|^{Q+1}}
$$



| - Sun |
| :---: |
| - Mercury |
| Venus |
| Earth |
| Mars |



$t=0$




$$
t=2
$$


$t=2.5$




Movie


Movie HD




Camera roll increment

| Surface and colour | - Lighting |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Colourmap | Select light |  |  | 3D light mode |
| bespoke - | Light 2 | Light colour |  |  |
| Colour function | Lighting style | Lighting mode |  | Camlight |
| Log * | local | phong | $\checkmark$ |  |
| $\square$ Add colorbar $\square$ Add axis |  | Light range | Light azi | Light elev |
| $\square$ Transparency $\square$ Texture | $\square$ Light arrow | 1500 | 143,1567 | 56.8352 |


| Camera position |  |  |  |
| :---: | :---: | :---: | :---: |
|  |  |  | Default |
| 14.4769 | 5.042 | -8.1226 | Camera view angle /deg |
| Camera target |  | $z$ |  |
| 0.11471 | -0.092678 | 0.20539 | 4.83 |
| Camera u x | rector | $z$ |  |
| 0.43953 | 0.19331 | 0.87718 |  |

clock anticlock


Andy French v3. 02014 Spheria
NG image saved in 136.2825 s . Welcome to Spherium. Dragging the mouse in the main axes will result in a 3D rotation. Use the + and - buttons to zoom and out, and the >> < etc to translate the figure. Hydrogenic orbital spheria take the form H XN e.g. H P1. Note the blue square must be pressed to update Spherium following 3D rotation
ammonite

| Ammonite options |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| - Plot spiral? | V Add ridges | Ridge frequency | Cross section ratio | \# spiral turns | \# surface points per turn |
| V Add bumps | $\checkmark$ Add ridges to colour | 5 | 0.9 | 5 | 200 |
|  | (1) Add bumps to colour | Ridge ampilude | Spiral bump amplitud |  |  |
| Spiral type |  | 0.3 | 0.2 |  |  |
| Logarithmic |  | Helicity | Spiral bump frequency |  |  |
|  |  | 0 | 14 |  |  |

Atomic dragon spiral Mathematicon, 2014

$$
\text { PNG image saved in } 136.2825 \mathrm{~s} \text {. Welcome to Spherium. Dragging the mouse in the main axes will result in a 3D rotation. Use the }+ \text { and - buttons to zoom }
$$ in and out, and the $>,<$ etc to translate the figure. Hydrogenic orbital spheria take the form HXN e.g. $\mathrm{HP1}$. Note the blue square must be pressed to update Spherium following 3D rotation.



| Camera pos | ion | z | Default |
| :---: | :---: | :---: | :---: |
| 11.0504 | 9.6644 | -8.7085 | Camera view angle /deg |
| Camera target |  | $z$ |  |
| -0.04044: | -0.20037 | 0.32887 | 4.0273 |
| Camera up <br> $\times$ | ${ }^{\text {ector }}$ | $z$ |  |
| 0.4116 | 0.31926 | 0.85361 |  |

Camera roll increment
clock anticlock
 v3. 02014

|  | Student Version> : klein | $\square$ |
| :--- | :--- | :--- |

- Kiein bottle

| Pipe granularity | Rotational granularity | Radius of top bend | : small pipe | Radius of base : small pipe | Base height : small pipe |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1000 | 1000 | 3 |  | 7 | 8 |



## Klein bottle with cloudy holes transparency map

$\square$ 回 $x$
translate the figure. Hydrogenic orbital spheria take the form H XN e.g. H P1. Note the blue square must be pressed to update Spherium following 3 D rotation. $\qquad$
<Student Version> : polyspike

| Polyspike- |  |  |
| :---: | :---: | :---: |
| \# azi spikes | \# elev spikes | spikiness |
| 20 | 10 | 1 |



## *harmonograph

- The Harmonograph was a Victorian curiosity attributed to Professor Blackburn in 1844
- Use two or three pendulums to create strange and beautiful patterns


Example of a lateral harmonograph


Photo from The Science Museum


Create Harmonographs from .wav files

## Rotary freq-damp



$+$

Harmonograph

## Harmonograph types

 \# loopsRotary freq-damp
50

Create Harmonographs from .wav files

## Rotary freq-damp

$\mathrm{N}=50, \mathrm{~A}=0.5, \mathrm{~F}=7.04$, phi $=121.7936^{\circ}, \mathrm{D}=2.15$



Harmonograph

| Play tones <br> Save .PNG <br> Written by Andy French <br> v1 2012$\quad$DPI |
| :--- |

Harmonograph types
Rotary freq-damp
$-$
\# loops
50

Create Harmonographs from .wav files

## Rotary freq-damp

$N=50, A=0.5, F=5.92$, phi $=121.7936^{\circ}, D=2.15$



Default Load settings Save settings

Harmonograph types
\# loops
Rotary freq-damp


Create Harmonographs from .wav files

## Rotary freq-damp

$\mathrm{N}=50, \mathrm{~A}=0.5, \mathrm{~F}=3.04$, phi $=121.7936^{\circ}, \mathrm{D}=2.15$



## Create Harmonographs from .wav files

## Rotary freq-damp

$\mathrm{N}=50, \mathrm{~A}=0.5, \mathrm{~F}=0.96, \mathrm{phi}=121.7936^{\circ}, \mathrm{D}=2.15$

Play tones

Harmonograph
\# loops
Harmonograph types \# loops


Rotary freq-damp
$\mathrm{N}=50, \mathrm{~A}=0.5, \mathrm{~F}=-1.5204$, phi $=121.7936^{\circ}, \mathrm{D}=2.15$








Play tones


Written by Andy French
v1 2012

Default

Harmonograph types
Rotary freq-damp
Load settings Save settings \# loops

1000 \# points per loop


$$
\begin{aligned}
& x=A_{1} e^{-\frac{t}{T_{1}}} \sin \left(t W_{1}+P_{1}\right)+A_{2} e^{-\frac{t}{T_{2}}} \sin \left(t W_{2}+P_{2}\right) \\
& y=A_{3} e^{-\frac{t}{T_{3}}} \sin \left(t W_{3}+P_{3}\right)+A_{4} e^{-\frac{t}{T_{4}}} \sin \left(t W_{4}+P_{4}\right)
\end{aligned}
$$

## Rotary harmonograph with frequency damping



$$
\begin{aligned}
T & =\frac{2 \pi}{\omega \log \left(\frac{100}{100-D}\right)}\left[1, \frac{1}{F}, 1, \frac{1}{F}\right] \\
A & =[1, a, 1, a] \\
W & =[\omega,-F \omega, \omega,-F \omega] \\
P & =\left[0, \phi, \frac{\pi}{2}, \frac{\pi}{2}+\phi\right]
\end{aligned}
$$

Parameters
$t$ is time /seconds
$\omega$ is $2 \pi$ times the first pendulum swing frequency $/ \mathrm{Hz}$
$a$ is the amplitude ratio
$F$ is the frequency ratio
$D$ is the damping factor (typically between 0 and 5 )
$\phi$ is the phase difference /radians between the pendula

## Musical harmony

- The mathematics of music has been known since the time of Pythagoras, 2500 years ago
- Frequency intervals of simple fractions e.g. 3:2 (a fifth) yield 'harmonious' music
- An octave means a frequency ratio of 2. An octave above concert A ( 440 Hz ) is therefore 880 Hz . An octave below is 220 Hz .
- The modern ‘equal-tempered scale’ divides an octave (the frequency ratio 2 ) into twelve parts such that

$$
F_{n}=2^{n / 12}=\sqrt[\frac{n}{12}]{2}
$$

## Musical harmony

| Name | Exact value in 12 -TET | Decimal value in 12-TET | Cents | Just intonation interval |
| :---: | :---: | :---: | :---: | :---: |
| Unison (C) | $2^{0 / 12}=1$ | 1.000000 | 0 | $\frac{1}{1}=1.000000$ |
| Minor second (Cझ/Db) | $2^{1 / 12}=\sqrt[12]{2}$ | 1.059463 | 100 | $\frac{16}{15}=1.066667$ |
| Major second (D) | $2^{2 / 12}=\sqrt[6]{2}$ | 1.122462 | 200 | $\frac{9}{8}=1.125000$ |
| Minor third (D\#/Eb) | $2^{3 / 12}=\sqrt[4]{2}$ | 1.189207 | 300 | $\frac{6}{5}=1.200000$ |
| Major third (E) | $2^{4 / 12}=\sqrt[3]{2}$ | 1.259921 | 400 | $\frac{5}{4}=1.250000$ |
| Perfect fourth (F) | $2^{5 / 12}=\sqrt[12]{32}$ | 1.334840 | 500 | $\frac{4}{3}=1.333333$ |
| Augmented fourth (F\#/Gb) | $2^{6 / 12}=\sqrt{2}$ | 1.414214 | 600 | $\frac{7}{5}=1.400000$ |
| Perfect fifth (G) | $2^{7 / 12}=\sqrt[12]{128}$ | 1.498307 | 700 | $\frac{3}{2}=1.500000$ |
| Minor sixth (G\#/Ab) | $2^{8 / 12}=\sqrt[3]{4}$ | 1.587401 | 800 | $\frac{8}{5}=1.600000$ |
| Major sixth (A) | $2^{9 / 12}=\sqrt[4]{8}$ | 1.681793 | 900 | $\frac{5}{3}=1.666667$ |
| Minor seventh (A\#/Bb) | $2^{10 / 12}=\sqrt[6]{32}$ | 1.781797 | 1000 | $\frac{7}{4}=1.750000$ |
| Major seventh (B) | $2^{11 / 12}=\sqrt[12]{2048}$ | 1.887749 | 1100 | $\frac{15}{8}=1.875000$ |
| Octave (C) | $2^{12 / 12}=2$ | 2.000000 | 1200 | $\frac{2}{1}=2.000000$ |

## Represent musical harmonies visually with the harmonograph!


Rotary
$\mathrm{F}=2.01, \mathrm{D}=0.7$,
$\mathrm{A}=1, \mathrm{phi}=0$

Note the difference a small change in F makes....

Rotary
$\mathrm{F}=1.51, \mathrm{D}=0.7$,
$\mathrm{A}=1$, phi $=0$

## What You See Is What You Need

 lengths $R$ and $D$ and will also vary with angles. However, as a first approximation, let us assume the variation of these quantities is small and or secondary importance to the overall dynamics. i.e. all $k$ parameters shall be assumed to be fixed inputs and independent of $\theta$ and $\phi$.
## Application of Newton's second law to determine

 derived quantitiesLet us apply Newton's second law in and directions to both the passenger and the parachute. If dynamic equilibrium is assumed there is no acceleration, so the sum of all the forces must equate to zero.

Passenger:

| $x:$ | $0=-T_{1} \cos \theta+F_{1}+T_{2} \cos (\phi+\theta)$ |
| :--- | :--- |
| $y:$ | $0=-T_{1} \sin \theta-M g+T_{2} \sin (\phi+\theta)$ |



Parachute:

$$
\begin{array}{|ll|}
\hline x: & 0=-T_{2} \cos (\phi+\theta)+F_{2} \\
\hline y: & 0=-T_{2} \sin (\phi+\theta)-m g+F_{L} \\
\hline
\end{array}
$$

Substituting for the $v^{2}$ models of drag and lift:
Passenger:

$$
\begin{array}{|ll|}
\hline x: & T_{1} \cos \theta=k_{1} v^{2}+T_{2} \cos (\phi+\theta) \\
\hline y: & T_{1} \sin \theta=-M g+T_{2} \sin (\phi+\theta) \\
\hline
\end{array}
$$

Parachute:

$$
\begin{array}{|ll|}
\hline x: & T_{2} \cos (\phi+\theta)=k_{2} v^{2} \\
\hline y: & T_{2} \sin (\phi+\theta)=-m g+k_{L} v^{2} \\
\hline
\end{array}
$$

\# (parachute newton 2 x y)

Hence by dividing the $y$ and $x$ components of (ref: passenger newton 2 xy )

$$
\tan \theta=\frac{-M g+T_{2} \sin (\phi+\theta)}{k_{1} v^{2}+T_{2} \cos (\phi+\theta)}
$$

and then substituting the results of (ref: parachute newton 2 xy ) we arrive at an equation relating $v$
$\qquad$
$\square$
$\square$
$\square$ Introduction - 1

facebook $\&^{5}$


Home favourite
wins silver med Okagbare seals
women's spint Wumen's spint Glasgow
GLASGOW 2014 Lord Coe praise
'sensational $G \varepsilon$ GLAGGOW \& WE
SCOTLAND SCOTLAND
Bolt. Games si
'nonsense' nonsense Silver justifies
Weightman beli Weightman be $\substack{\text { Guernsey's Dru } \\ \text { athletics }}$



> bly



What is it like being Usain Bolt?
Adulation, glory, money, weird
photocalls, 'selfie' 'equests and a photocalls, 'selfie' requests and a
bagman' to get his food - the life of the sprint superstar.

Meet Judd and Williams - the next generation
Jess Judd and Jodie Williams are determined to make their
at Glasgow 2014, as BBC Sport's Tom Fordyce finds out.
'I had tn invent mu nwn terhninuio to clear the bar
ñ hack from a

## 'High productivity multi-tasking,' or are you just being distracted?

What You See Is What You Need


The rise of Apps for Smartphones....Typically software designed for a very specific purpose

"The psychological profiling of a programmer is mostly the ability to shift levels of abstraction, from low level to high level. To see something in the small and to see something in the large."
"Email is a wonderful thing for people whose role in life is to be on top of things. But not for me; my role is to be on the bottom of things. What I do takes long hours of studying and uninterruptible concentration."

Fast numerical calculation + display systems

## Use of computer programming as an artistic tool

Where you can design and refine the tool
Complexity, and beauty, from simplicity (i.e. code)

## Case studies

General thoughts on how humans best interact with information technology What You See Is What You Need!

## Inspired?



All welcome, regardless of prior experience

