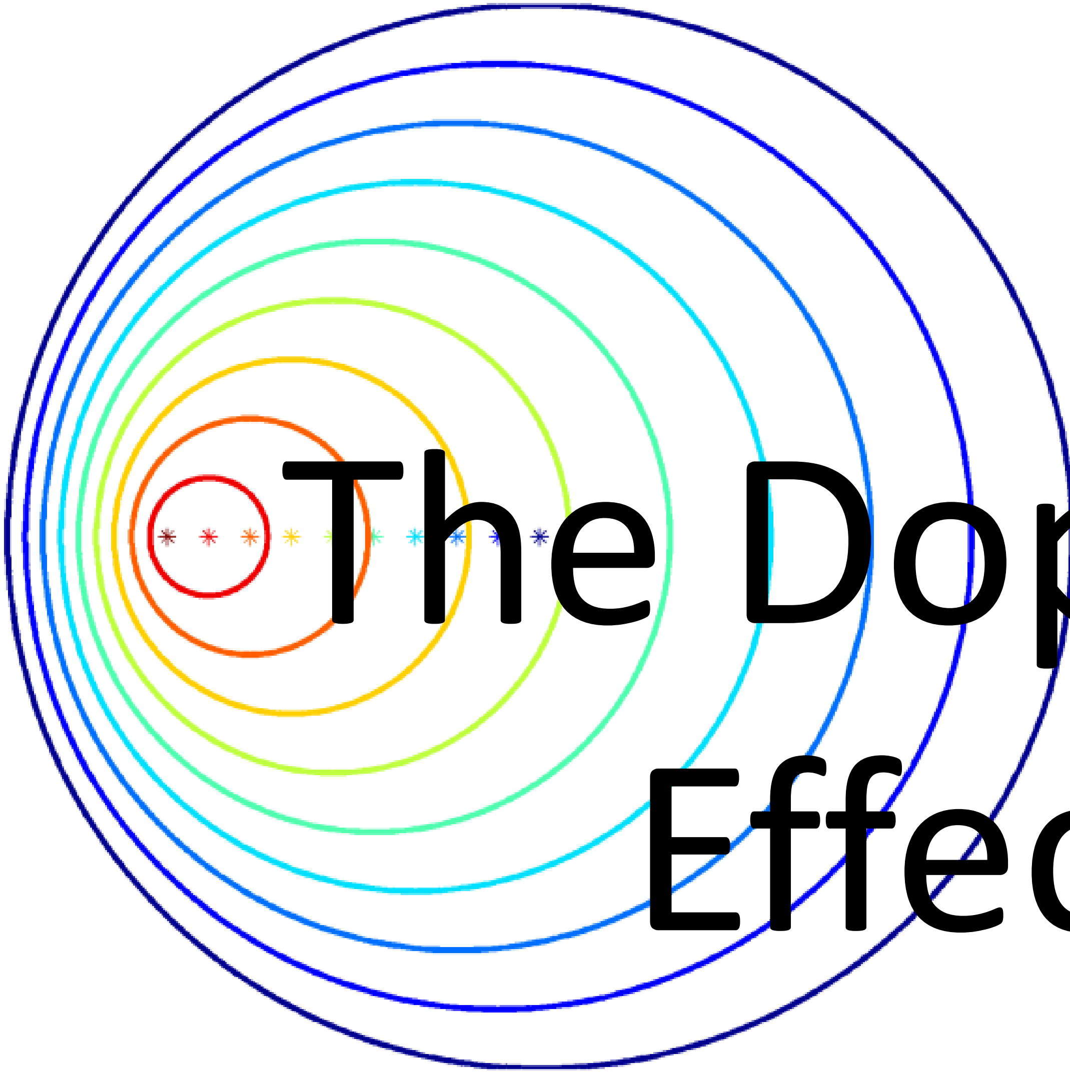
$$\Delta f \approx -\frac{u}{w} f$$

$$\frac{\lambda_o}{\lambda_e} = 1 + \frac{u\cos\theta}{w}$$

$$1 + z = \frac{\lambda_o}{\lambda_e} \Rightarrow z = \frac{u\cos\theta}{w}$$



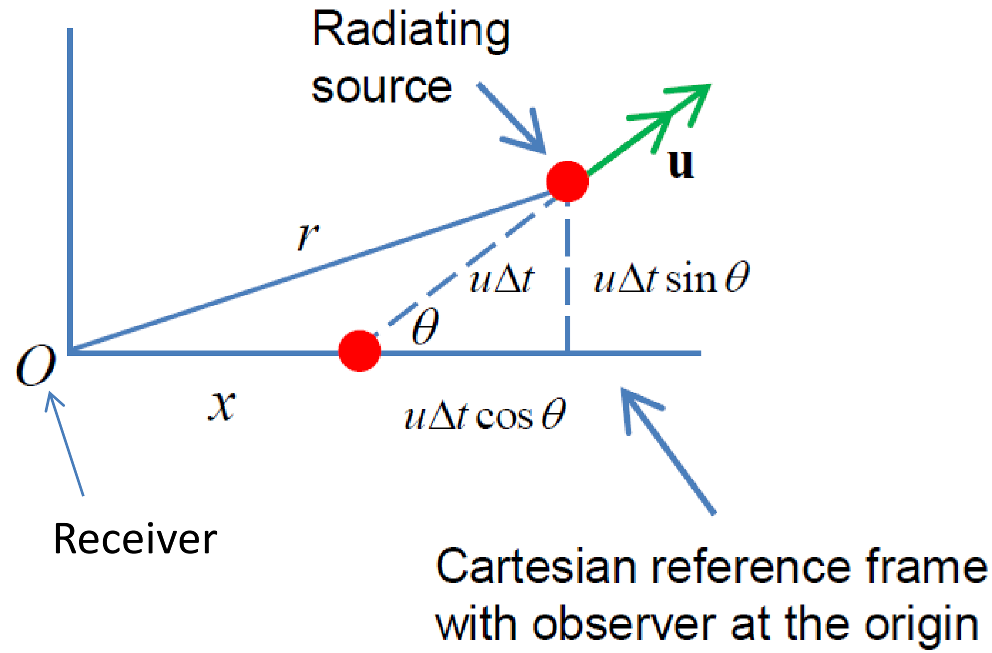
The Doppler Effect

Dr Andrew French

Christian Doppler
1803-1853

Consider a **receding** wave source of frequency f. It crosses the x axis of a Cartesian reference frame at angle θ with speed u. The receiver of the waves is stationary at the origin of the Cartesian frame.

The speed of waves, relative to the observer, is w.



Depending on the velocity of the wave source relative to the observer, the observer will experience a *frequency shift* from f. If the source *recedes*, the frequency *diminishes* and the *wavelength increases* ('redshift'). If the source is *approaching*, the observed *frequency will increase* and the *wavelength will decrease* ('blueshift').

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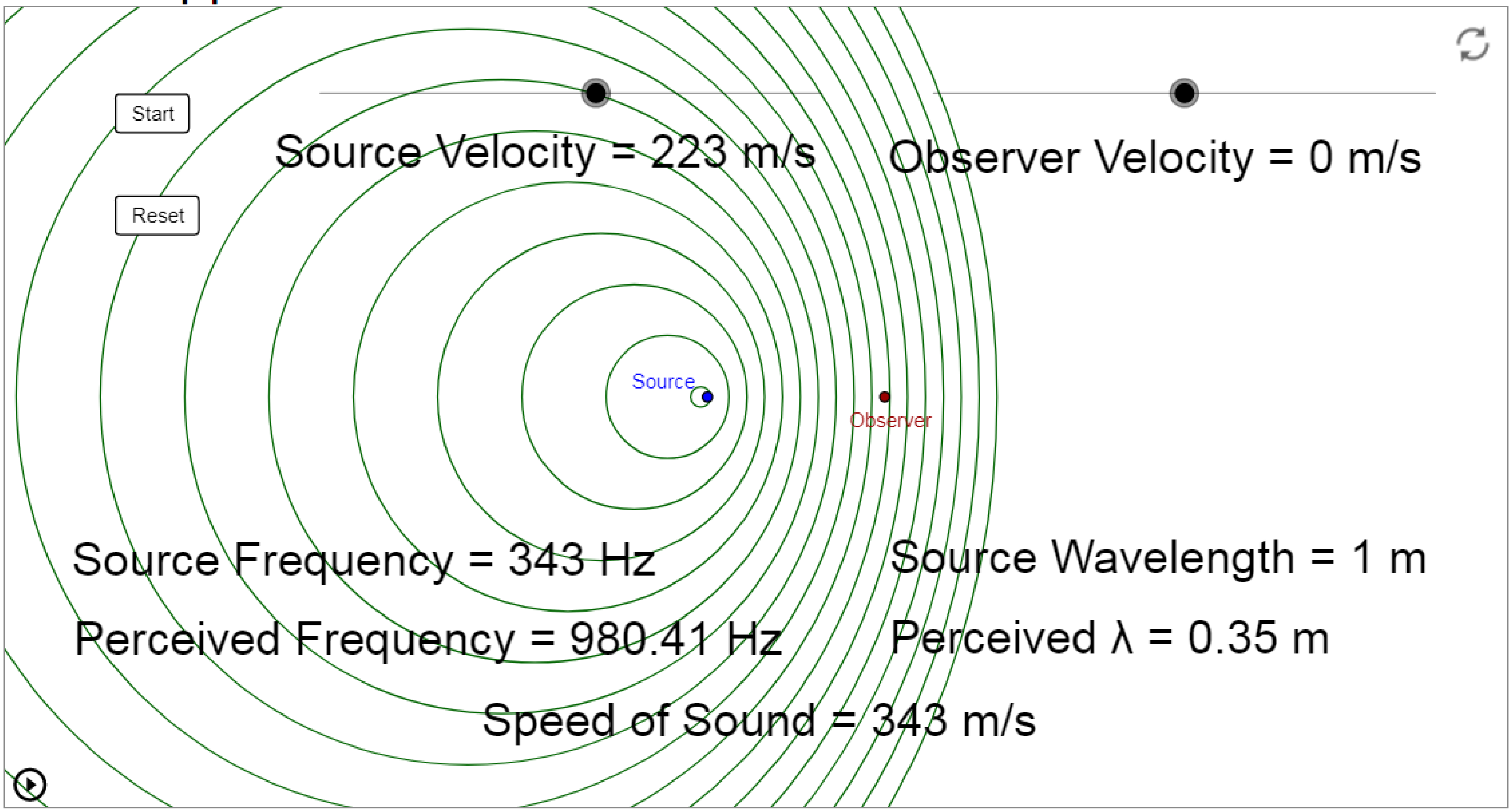
The Doppler Effect & Sonic Boom



https://ophysics.com/w11.html

Excel model of Doppler Shift

- Plot the circular wavefront(s) when the source has got to position 8.
- i.e. you see a wavefront radius which *increases* as the *initial position decreases*, and also a *shift of wavefront centre.* The combination of these two effects causes the bunching up of wavefronts

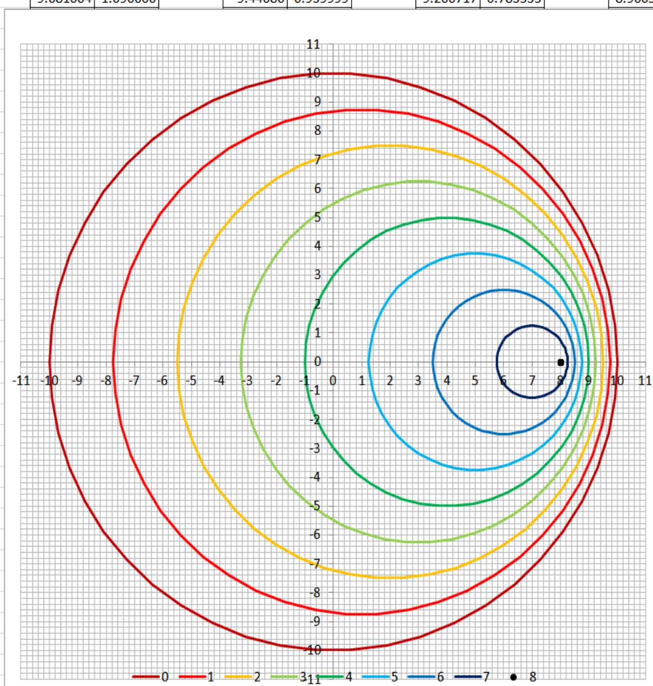
SIMULATION OF DOPPLER EFFECT OR MACH CONE

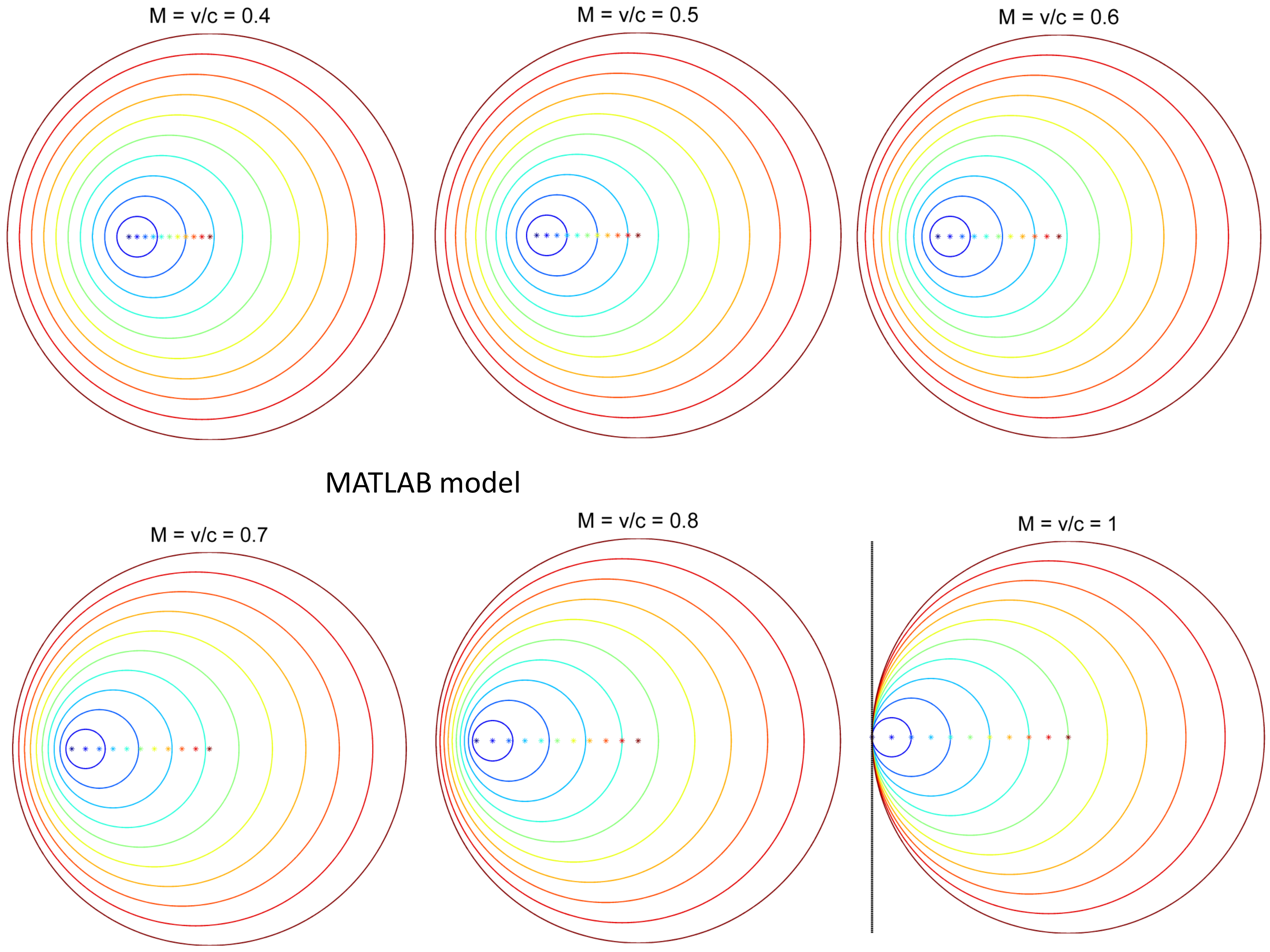
A. French Dec 2021

Mach number = speed of wave source / speed of waves M 0.8

	0	1	2	3	4	5	6	7	8
Wave source position:	0	1	2	3	4	5	6	7	8
Wavefront radius:	10.00	8.75	7.50	6.25	5.00	3.75	2.50	1.25	0.00

		Wavefront circle		Wavefront circle		Wavefront circle		Wavefront circle		Wavefront circle		Wavefront circle		Wavefront circle		Wavefront circle		Wavefront circle	
	Angle /rad	**x**	**y**	**x**	**y**	**x**	**y**	**x**	**y**	**x**	**y**	**x**	**y**	**x**	**y**	**x**	**y**	**x**	**y**
0	0	10	0	9.75	0	9.5	0	9.25	0	9	0	8.75	0	8.5	0	8.25	0	8	0
.02	0.125664	9.921147	1.253332	9.681004	1.096666	9.44086	0.939999	9.200717	0.783333	8.960574	0.626666	8.72043	0.47	8.480287	0.313333	8.240143	0.156667	8	0
.04	0.251327	9.685832	2.486899								1.243449	8.632187	0.932587	8.421458	0.621725	8.210729	0.310862	8	0
.06	0.376991	9.297765	3.681246								1.840623	8.486662	1.380467	8.324441	0.920311	8.162221	0.460156	8	0
.08	0.502655	8.763067	4.817537								2.408768	8.28615	1.806576	8.190767	1.204384	8.095383	0.602192	8	0
0.1	0.628319	8.09017	5.877853								2.938926	8.033814	2.204195	8.022542	1.469463	8.011271	0.734732	8	0
.12	0.753982	7.289686	6.845471								3.422736	7.733632	2.567052	7.822422	1.711368	7.911211	0.855684	8	0
.14	0.879646	6.37424	7.705132								3.852566	7.39034	2.889425	7.59356	1.926283	7.79678	0.963142	8	0
.16	1.00531	5.358268	8.443279								4.22164	7.00935	3.16623	7.339567	2.11082	7.669783	1.05541	8	0
.18	1.130973	4.257793	9.048271								4.524135	6.596672	3.393101	7.064448	2.262068	7.532224	1.131034	8	0
0.2	1.256637	3.09017	9.510565								4.755283	6.158814	3.566462	6.772542	2.377641	7.386271	1.188821	8	0
.22	1.382301	1.873813	9.822873								4.911436	5.70268	3.683577	6.468453	2.455718	7.234227	1.227859	8	0
.24	1.507964	0.627905	9.980267								4.990134	5.235464	3.7426	6.156976	2.495067	7.078488	1.247533	8	0
.26	1.633628	-0.62791	9.980267								4.990134	4.764536	3.7426	5.843024	2.495067	6.921512	1.247533	8	0
.28	1.759292	-1.87381	9.822873								4.911436	4.29732	3.683577	5.531547	2.455718	6.765773	1.227859	8	0
0.3	1.884956	-3.09017	9.510565								4.755283	3.841186	3.566462	5.227458	2.377641	6.613729	1.188821	8	0
.32	2.010619	-4.25779	9.048271								4.524135	3.403328	3.393101	4.935552	2.262068	6.467776	1.131034	8	0
.34	2.136283	-5.35827	8.443279								4.22164	2.99065	3.16623	4.660433	2.11082	6.330217	1.05541	8	0
.36	2.261947	-6.37424	7.705132								3.852566	2.60966	2.889425	4.40644	1.926283	6.20322	0.963142	8	0
.38	2.38761	-7.28969	6.845471								3.422736	2.266368	2.567052	4.177578	1.711368	6.088789	0.855684	8	0
0.4	2.513274	-8.09017	5.877853								2.938926	1.966186	2.204195	3.977458	1.469463	5.988729	0.734732	8	0
.42	2.638938	-8.76307	4.817537								2.408768	1.71385	1.806576	3.809233	1.204384	5.904617	0.602192	8	0
.44	2.764602	-9.29776	3.681246								1.840623	1.513338	1.380467	3.675559	0.920311	5.837779	0.460156	8	0
.46	2.890265	-9.68583	2.486899								1.243449	1.367813	0.932587	3.578542	0.621725	5.789271	0.310862	8	0
.48	3.015929	-9.92115	1.253332								0.626666	1.27957	0.47	3.519713	0.313333	5.759857	0.156667	8	0
0.5	3.141593	-10	-7.7E-15								-3.8E-15	1.25	-2.9E-15	3.5	-1.9E-15	5.75	-9.6E-16	8	0
.52	3.267256	-9.92115	-1.25333								-0.62667	1.27957	-0.47	3.519713	-0.31333	5.759857	-0.15667	8	0
.54	3.39292	-9.68583	-2.4869								-1.24345	1.367813	-0.93259	3.578542	-0.62172	5.789271	-0.31086	8	0
.56	3.518584	-9.29776	-3.68125								-1.84062	1.513338	-1.38047	3.675559	-0.92031	5.837779	-0.46016	8	0
.58	3.644247	-8.76307	-4.81754								-2.40877	1.71385	-1.80658	3.809233	-1.20438	5.904617	-0.60219	8	0
0.6	3.769911	-8.09017	-5.87785								-2.93893	1.966186	-2.20419	3.977458	-1.46946	5.988729	-0.73473	8	0
.62	3.895575	-7.28969	-6.84547								-3.42274	2.266368	-2.56705	4.177578	-1.71137	6.088789	-0.85568	8	0
.64	4.021239	-6.37424	-7.70513								-3.85257	2.60966	-2.88942	4.40644	-1.92628	6.20322	-0.96314	8	0
.66	4.146902	-5.35827	-8.44328								-4.22164	2.99065	-3.16623	4.660433	-2.11082	6.330217	-1.05541	8	0
.68	4.272566	-4.25779	-9.04827								-4.52414	3.403328	-3.3931	4.935552	-2.26207	6.467776	-1.13103	8	0
0.7	4.39823	-3.09017	-9.51057	-1.7039	-8.32174	-0.31763	-7.13292	1.068644	-5.9441	2.454915	-4.75528	3.841186	-3.56646	5.227458	-2.37764	6.613729	-1.18882	8	0
.72	4.523893	-1.87381	-9.82287	-0.63959	-8.59501	0.59464	-7.36715	1.828867	-6.1393	3.063093	-4.91144	4.29732	-3.68358	5.531547	-2.45572	6.765773	-1.22786	8	0

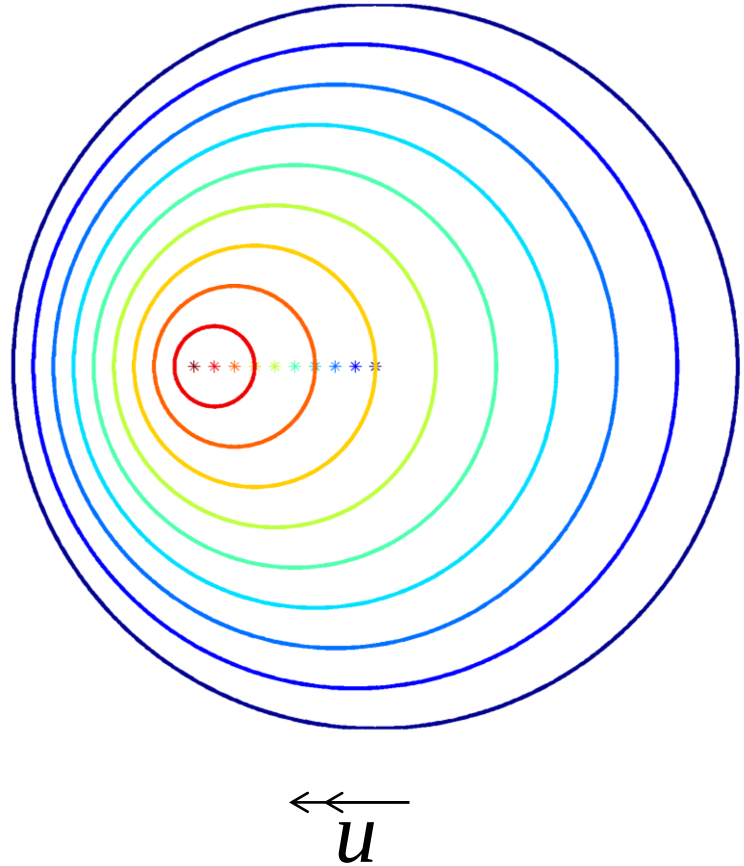



M = v/c = 0.4
M = v/c = 0.5
M = v/c = 0.6
MATLAB model
M = v/c = 0.7
M = v/c = 0.8
M = v/c = 1

Doppler shift method for measuring radial velocity

$$c = f\lambda$$

If an object emitting radiation at frequency f moves radially *towards* an observer at velocity u, the observer will measure a *slightly higher frequency* of radiation as the emitted waves 'bunch up'.



You'll see a minus sign in u otherwise

Velocity of emitter *towards* observer

Frequency of emitted radiation

frequency change

$$\Delta f = \frac{u}{c} f$$

Speed of radiation

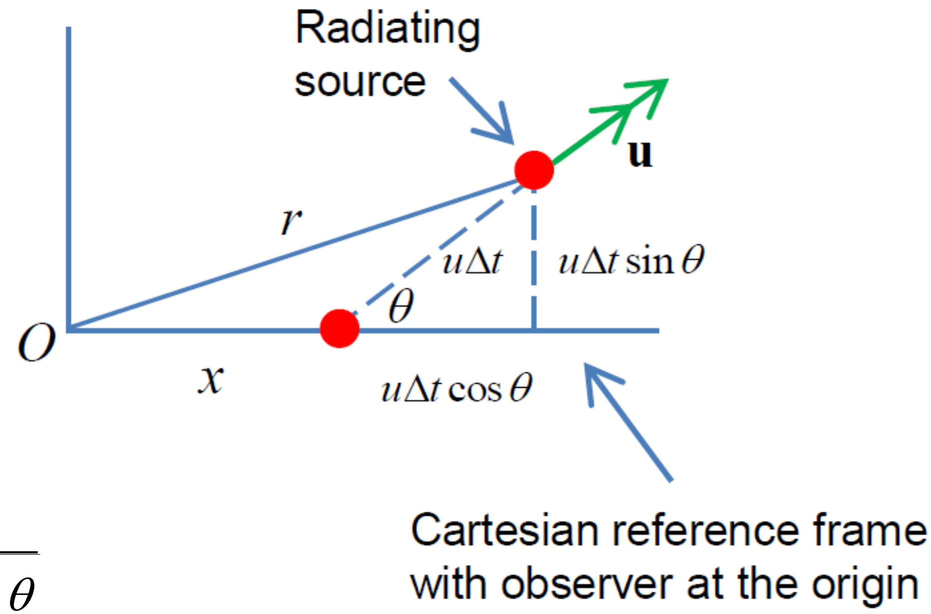
Note this formula is 'Classical'. It is valid when $u << c$, otherwise a **relativistic version** must be used

Christian Doppler
1803-1953

The period T of waves received by an observer (in the x direction) at the frame origin O is:

$$\boxed{T = \Delta t + \frac{r - x}{w}}$$

- Δt: time between wave crests *at source*
- w: wave speed
- $r - x$: extra distance travelled by source between wave crests



From geometry:

$$r = \sqrt{(x + u\Delta t\cos\theta)^2 + u^2\Delta t^2 \sin^2\theta}$$

$$r = \sqrt{x^2 + u^2\Delta t^2\cos^2\theta + 2ux\Delta t\cos\theta + u^2\Delta t^2\sin^2\theta}$$

$$r = \sqrt{x^2 + u^2\Delta t^2 + 2ux\Delta t\cos\theta}$$

$$r = x\sqrt{1 + 2\cos\theta\frac{u\Delta t}{x} + \left(\frac{u\Delta t}{x}\right)^2}$$

If $u\Delta t \ll x$ $\quad r \approx x\sqrt{1 + 2\cos\theta\frac{u\Delta t}{x}} \approx x\left(1 + \cos\theta\frac{u\Delta t}{x}\right) = x + u\Delta t\cos\theta$

$$\therefore \boxed{r - x \approx u\Delta t\cos\theta}$$

Hence frequency of radiation received at O is $F = 1/T$ where:

$$\frac{1}{F} = \Delta t + \frac{u \Delta t \cos\theta}{w} = \Delta t\left(1 + \frac{u\cos\theta}{w}\right)$$

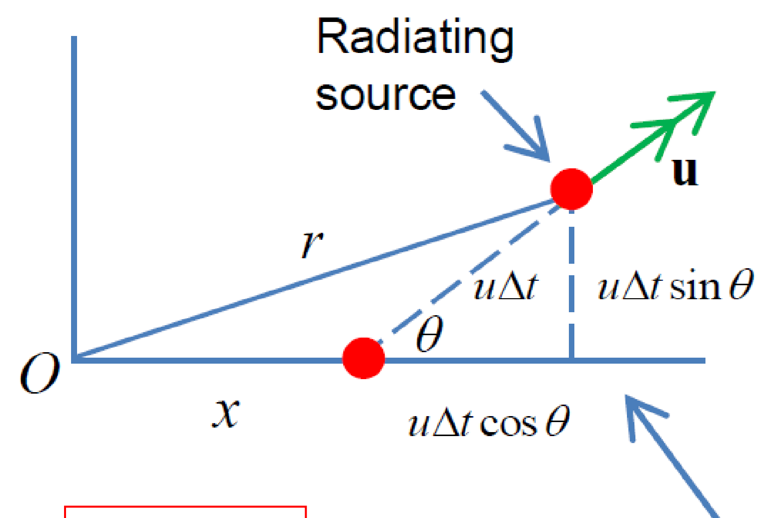
In a *Classical* scenario, where u, w are much less than the speed of light:

$$f = 1/\Delta t$$

Received frequency

$$\therefore F = \frac{1}{1 + \frac{u}{w}\cos\theta} f$$

Emitted frequency



Cartesian reference frame with observer at the origin

If $w \gg u,\ \theta \approx 0$

$$F \approx f - \frac{u}{w} f$$

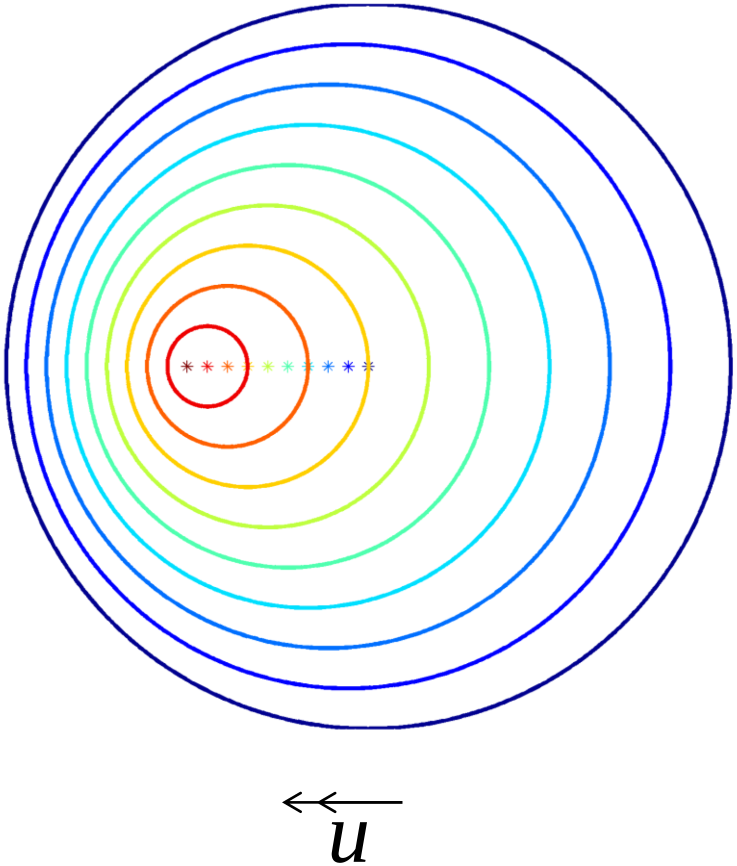
i.e. a **Doppler Shift** of:

$$\Delta f = \frac{u}{w} f$$

Doppler shift method for measuring radial velocity

$$c = f\lambda$$

If an object emitting radiation at frequency f moves radially *towards* an observer at velocity u, the observer will measure a *slightly higher frequency* of radiation as the emitted waves 'bunch up'.



You'll see a minus sign in u otherwise → Velocity of emitter *towards* observer

Frequency of emitted radiation

frequency change

$$\Delta f = \frac{u}{c} f$$

Speed of radiation

Note this formula is 'Classical'. It is valid when $u \ll c$, otherwise a **relativistic version** must be used

Christian Doppler
1803-1953

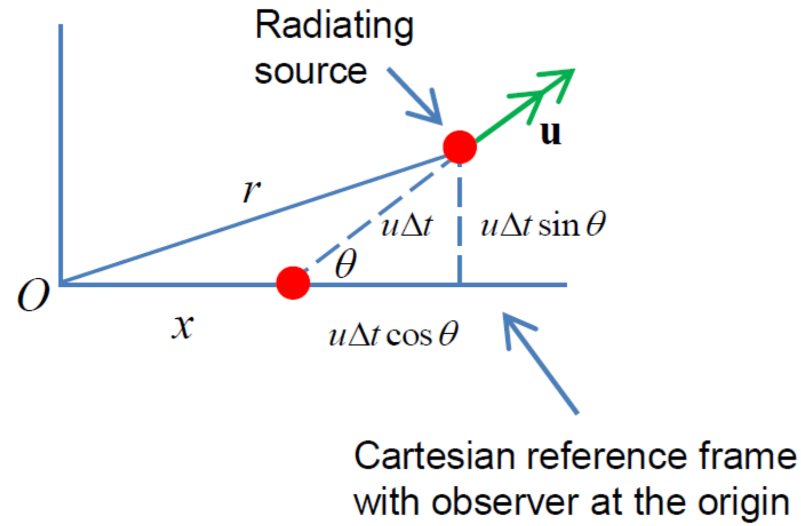
Wavelength shift and Redshift

$$F = \frac{1}{1 + \frac{u}{w}\cos\theta} f$$

emitted frequency

Received frequency at O

wave speed



$$F = \frac{w}{\lambda_o}, \quad f = \frac{w}{\lambda_e}$$

$$\therefore \frac{\lambda_o}{\lambda_e} = 1 + \frac{u}{w}\cos\theta$$

Define **REDSHIFT**

$$z = \frac{u\cos\theta}{w}$$

$$\therefore 1 + z = \frac{\lambda_o}{\lambda_e}$$

Recession velocity from observer

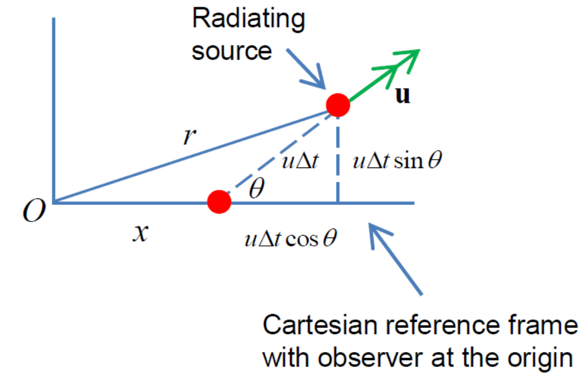
Observed wavelength

$$\frac{\lambda_o}{\lambda_e} = 1 + \frac{u}{w}\cos\theta$$

Emitted wavelength

wave speed

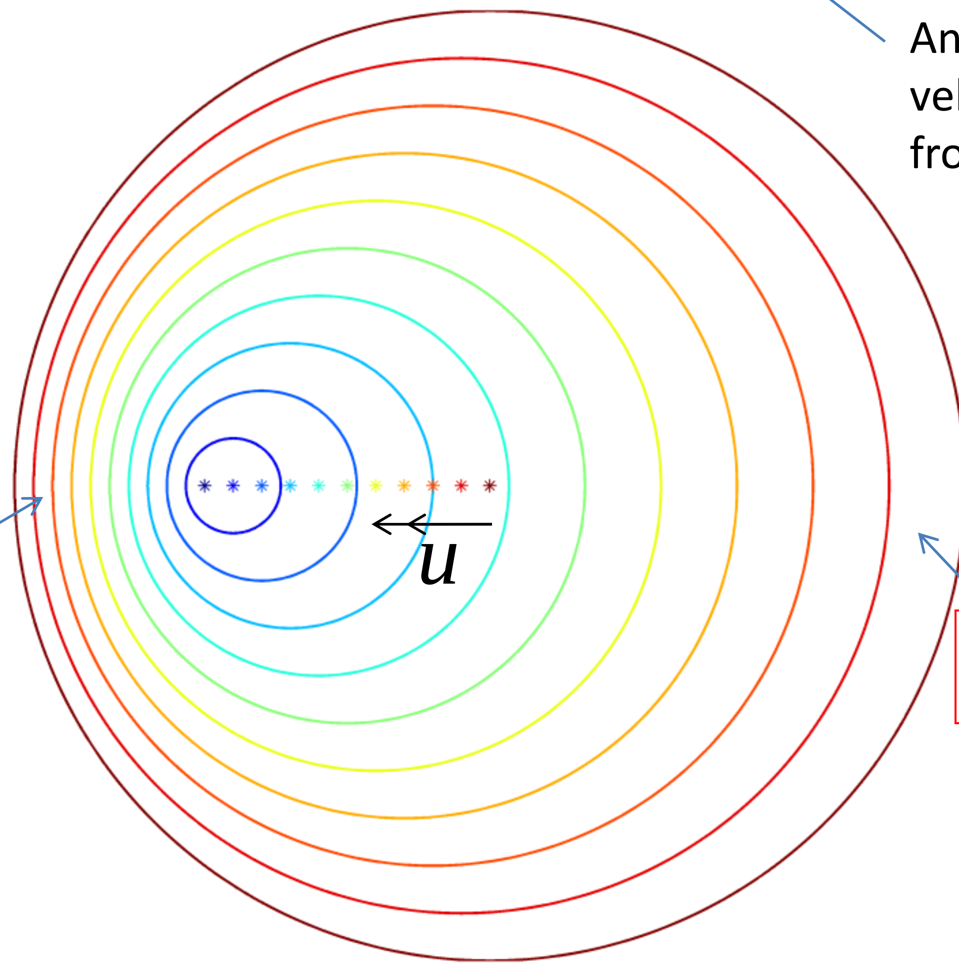
Angle of source velocity from horizontal



In this example:

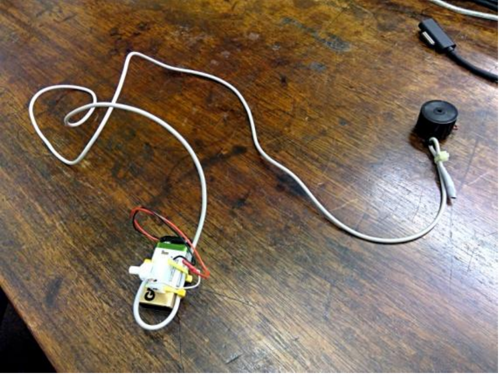
$$\theta = 0$$

$$u = 0.6w$$



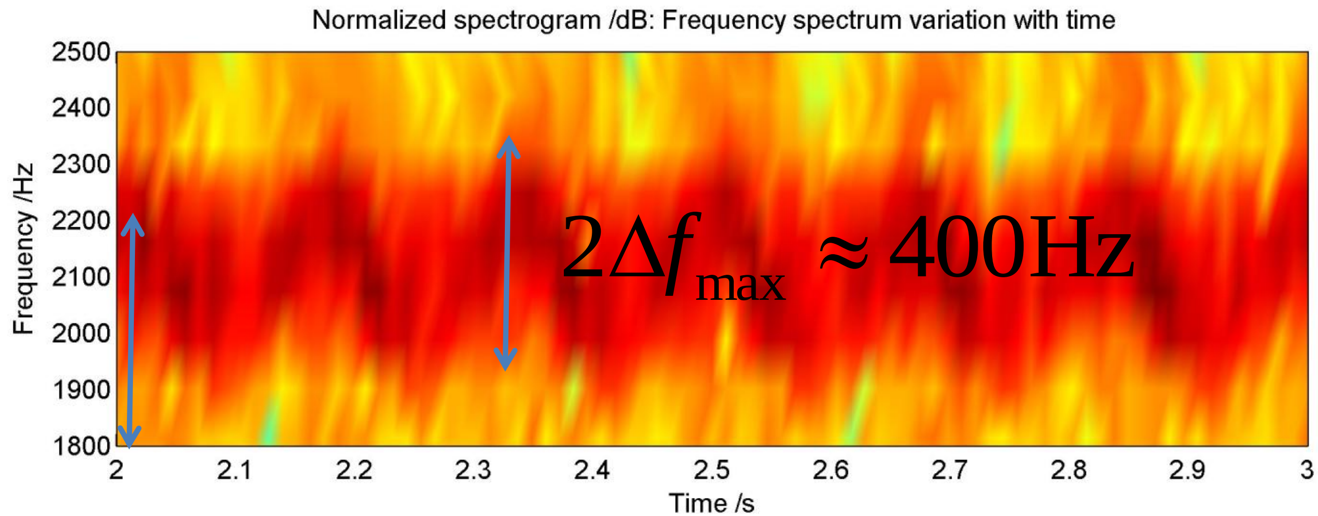
$$\lambda_o = \lambda_e(1 - 0.6)$$

$$\lambda_o = \lambda_e(1 + 0.6)$$

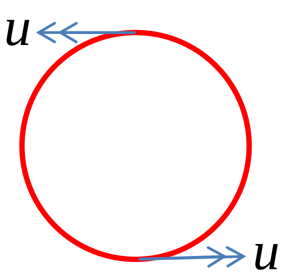


$$r \approx 0.50\text{m}$$ Radius of circle of whirling speaker

$$w = 340\text{ms}^{-1}$$ Speed of sound in air



Whirl a loudspeaker

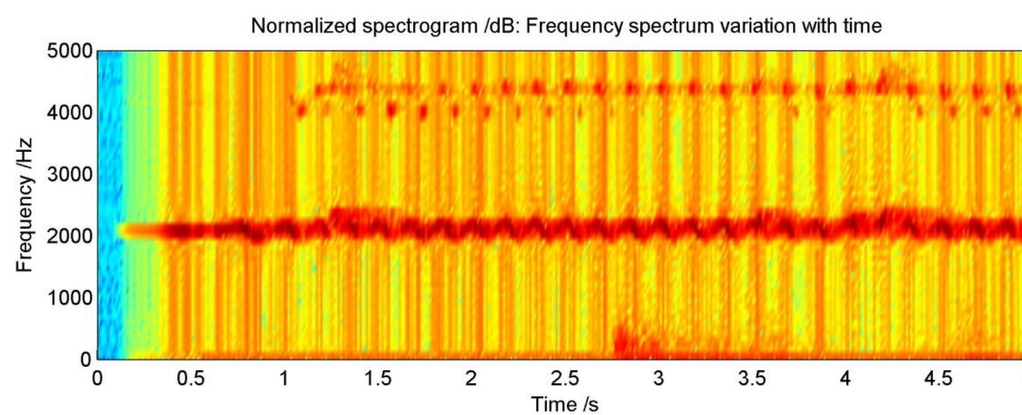


Record a *spectrogram* to determine frequency variation vs time.

$$\Delta f_{\max} \approx \frac{u}{w} f$$

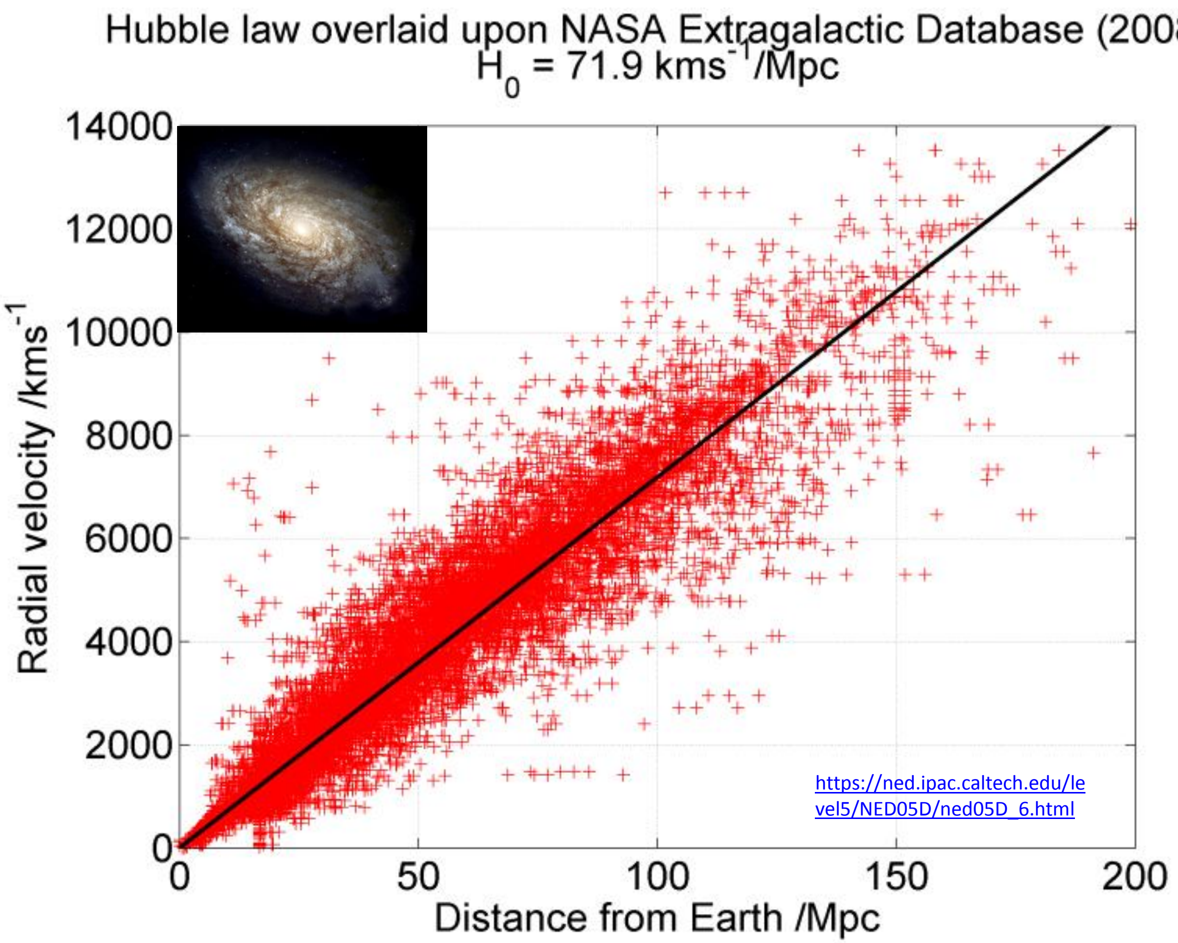
$$\therefore u \approx \frac{200}{2100} \times 340\text{ms}^{-1}$$

$$\therefore \boxed{u \approx 33\text{ms}^{-1}}$$ speaker speed



Doppler Shift is a highly useful tool in ***Cosmology***, since by comparing the *spectra* of light from stars to the light emitted by their constituent gases such as hydrogen or helium *in the laboratory*, we can infer the recession speed of the stars. The *general trend of galaxies to be red-shifted* is the key piece of evidence to suggest the *Big Bang Theory* of the evolution; i.e. that it the **Universe is expanding** and 'began' as an extraordinarily hot and dense 'singularity.'

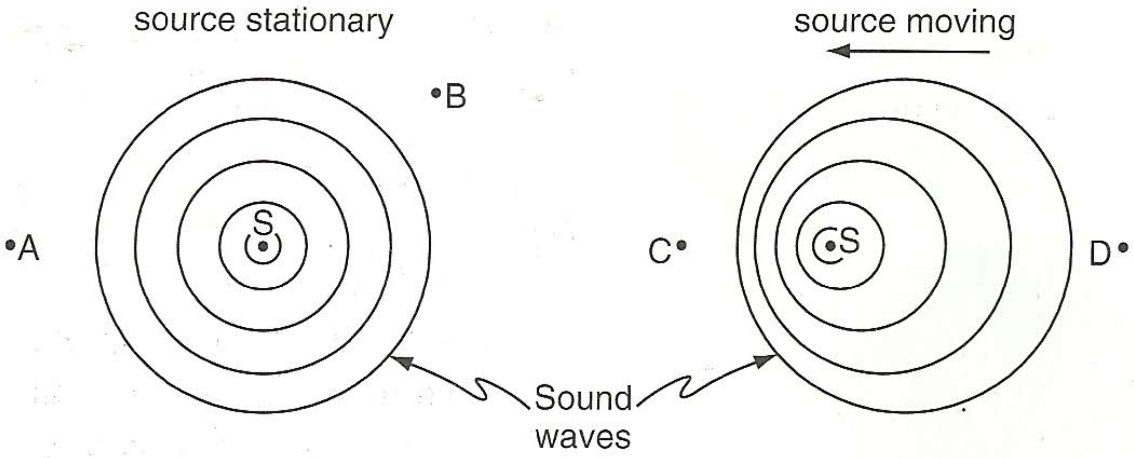
15,231 measurements of 4672 galaxies



$$\left(\frac{v}{\text{kms}^{-1}}\right) \approx 71.9\left(\frac{d}{\text{Mpc}}\right)$$

$$1\text{Mpc} = 3.26\times10^{6}\,\text{Ly} = 3.09\times10^{22}\,\text{m}$$

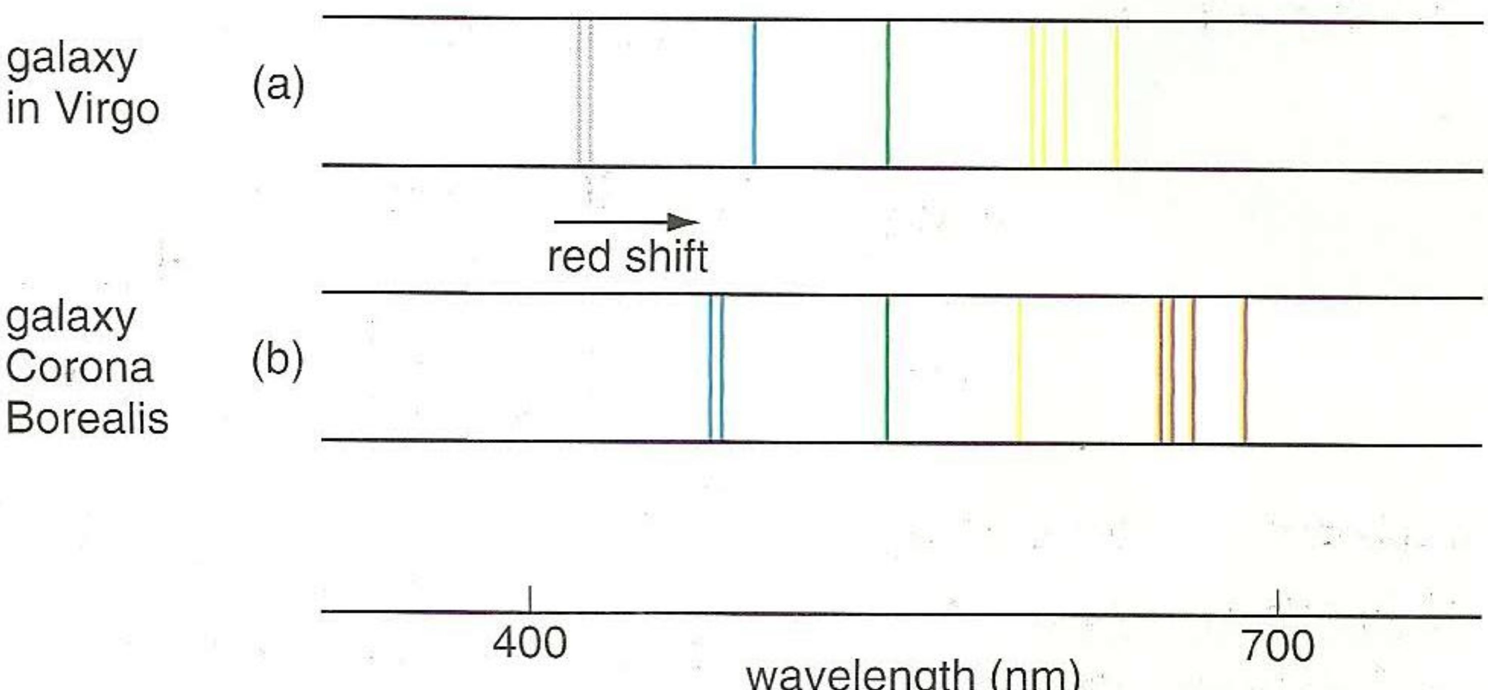
Edwin Hubble
1889-1953



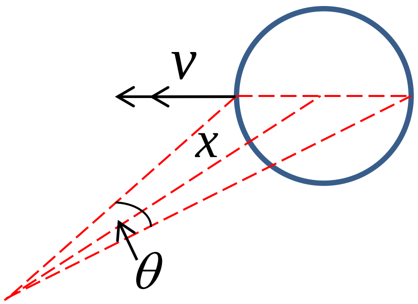
Redshift z is the fractional change in wavelength of light due to the Doppler effect

$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emmitted}}}{\lambda_{\text{emmitted}}}$$

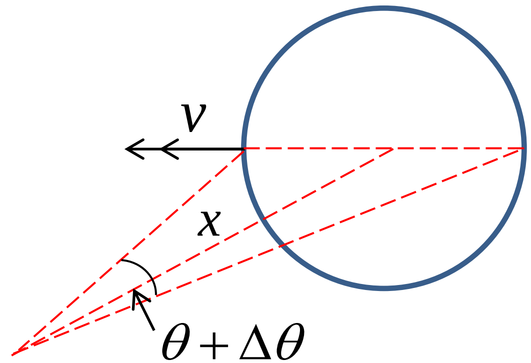
Emission spectra of light from stars. Assume mostly **Hydrogen**.
Compare to Hydrogen spectra measured in an Earth laboratory.



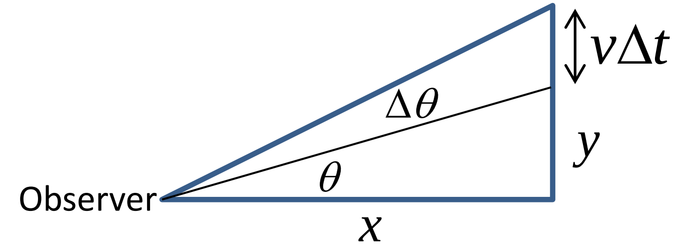
Using **radial velocity** calculation (via Doppler shift) to calculate distances of stars



Radially expanding gas cloud at time *t*



Radially expanding gas cloud at time $t + \Delta t$



$$y = x \tan\theta$$

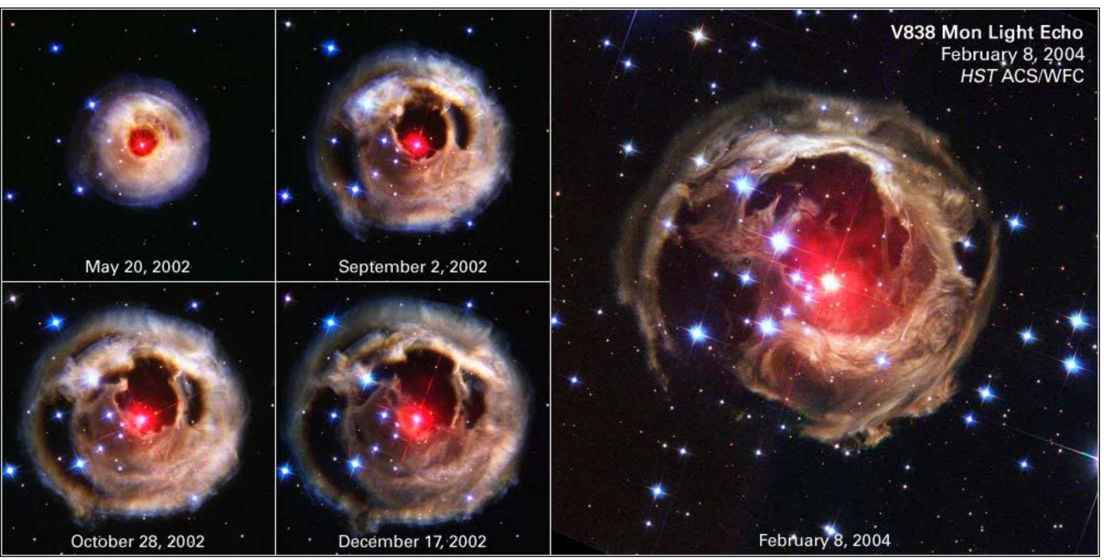
$$\tan(\theta + \Delta\theta) = \frac{v\Delta t + y}{x}$$

$$x = \frac{v\Delta t + x\tan\theta}{\tan(\theta + \Delta\theta)}$$

$$x\left(\tan(\theta + \Delta\theta) - \tan\theta\right) = v\Delta t$$

$$x = \frac{v\Delta t}{\tan(\theta + \Delta\theta) - \tan\theta}$$

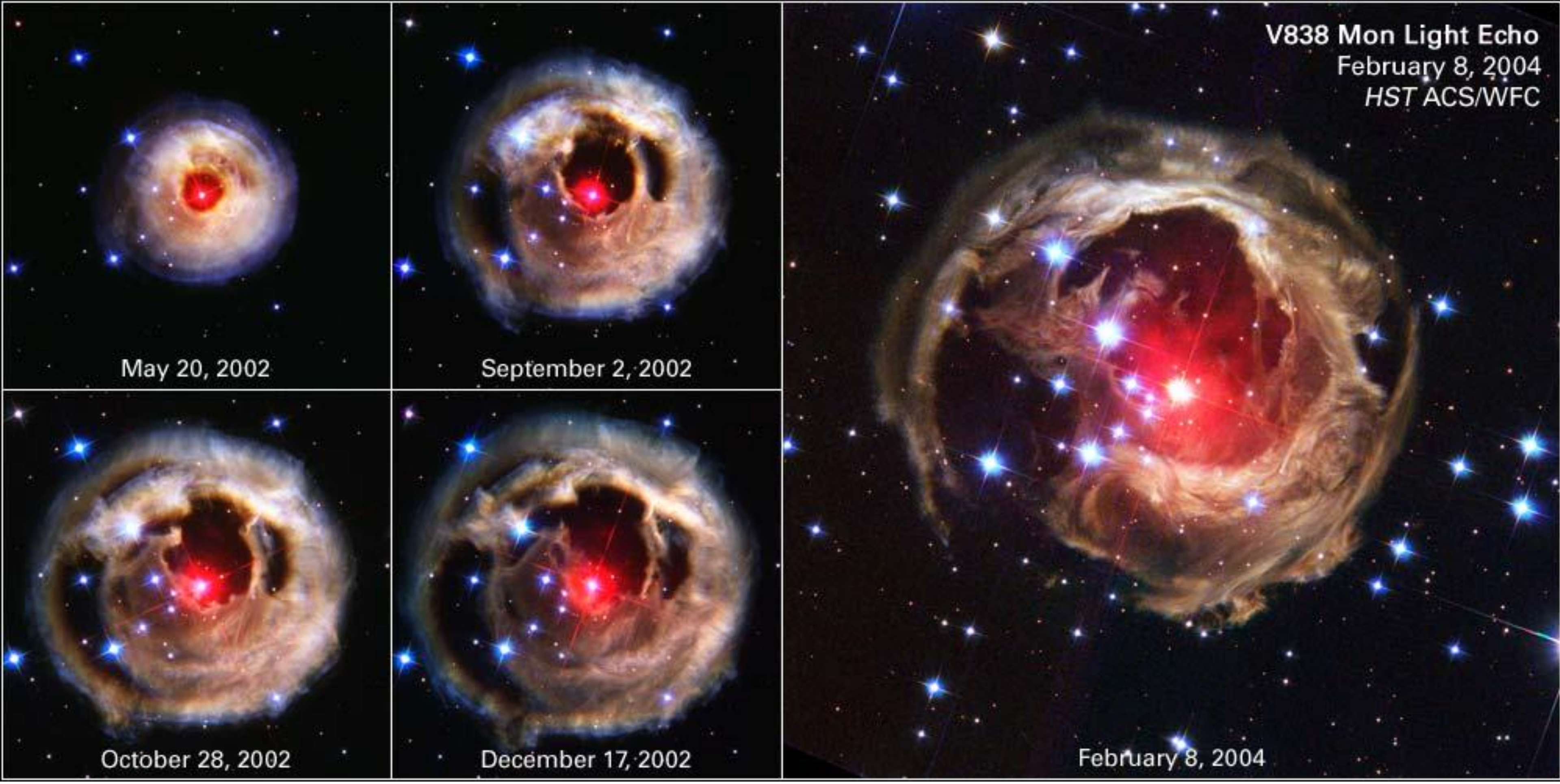
$$\boxed{x \approx \frac{v\Delta t}{\Delta\theta}}$$



*Measure *v* from Doppler shift of spectrum

*Measure angular change $\Delta\theta$ between observations

*Hence obtain distance of star at centre of expanding gas cloud



*Measure v from Doppler shift of spectrum
*Measure angular change $\Delta\theta$ between observations
*Hence obtain distance of star at centre of expanding gas cloud

$$\Delta f = \frac{v}{c} f$$

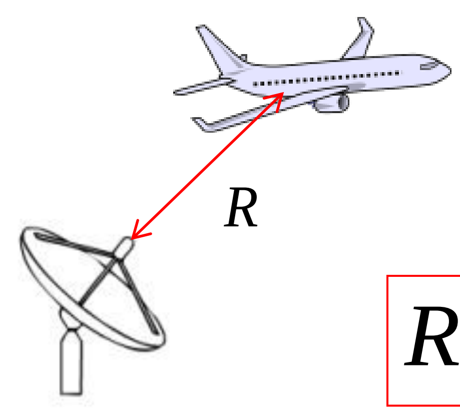
$$x \approx \frac{v\Delta t}{\Delta\theta}$$

The key challenge here is to work out what the **emission frequency** f should be, in order to work out the Doppler shift

RAdio Detection And Ranging

Radars detect the presence of a physically remote object via the reception and processing of **backscattered electromagnetic waves**.

Unlike optical systems, (which are responsive to frequencies ≈10^{15}Hz), Radar is typically associated with frequency bands ranging from a **few MHz** (High Frequency or HF band) up to **hundreds of GHz** (mm wave).



$$R = \tfrac{1}{2} c \Delta t$$

$$c = 2.998 \times 10^{8}\,\mathrm{ms}^{-1}$$

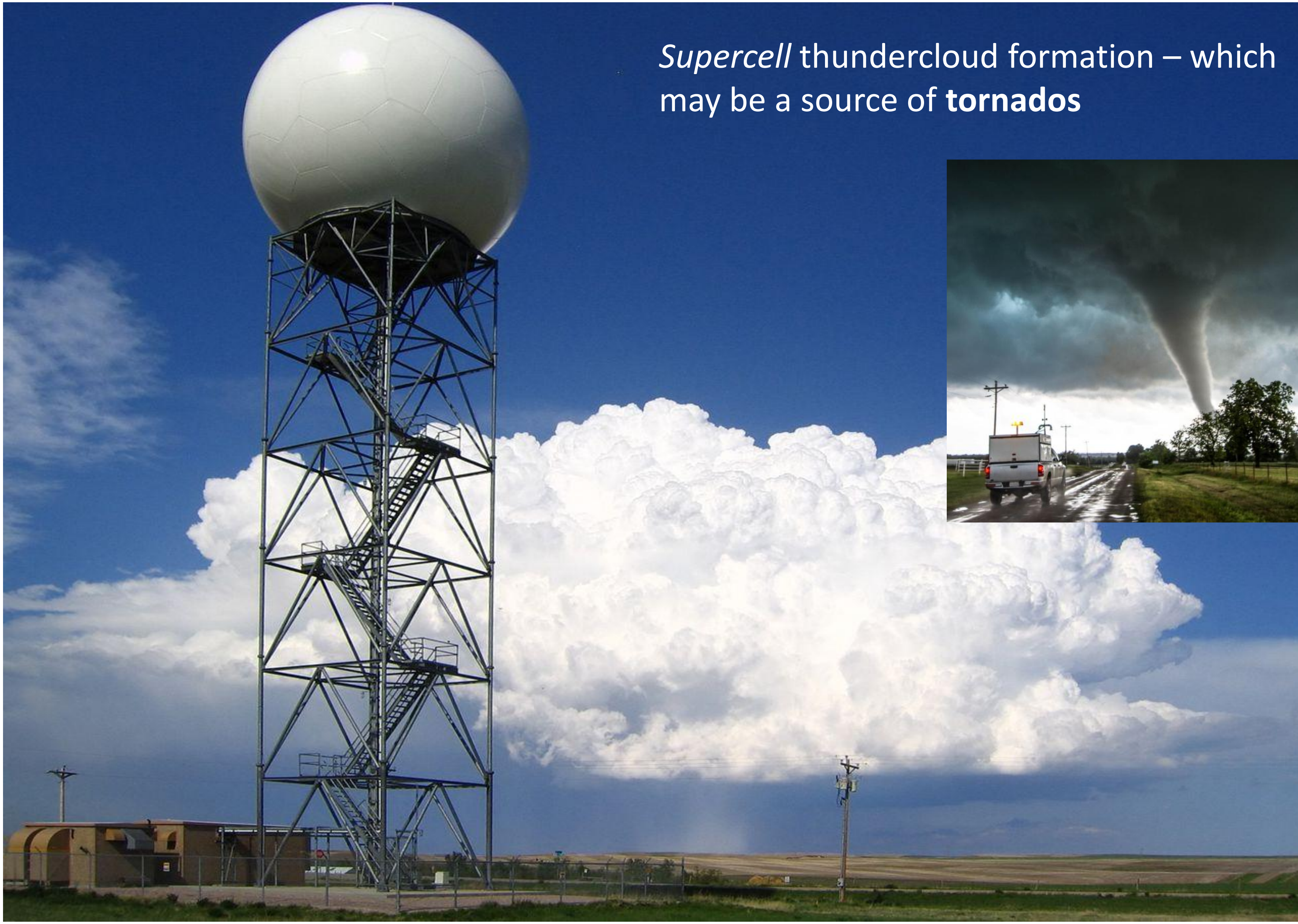
- Most targets of interest (especially those constructed from **metal**) are **highly reflective** at Radar frequencies.

- Radar can be used in **darkness** and can **penetrate haze, fog, snow and rain.**

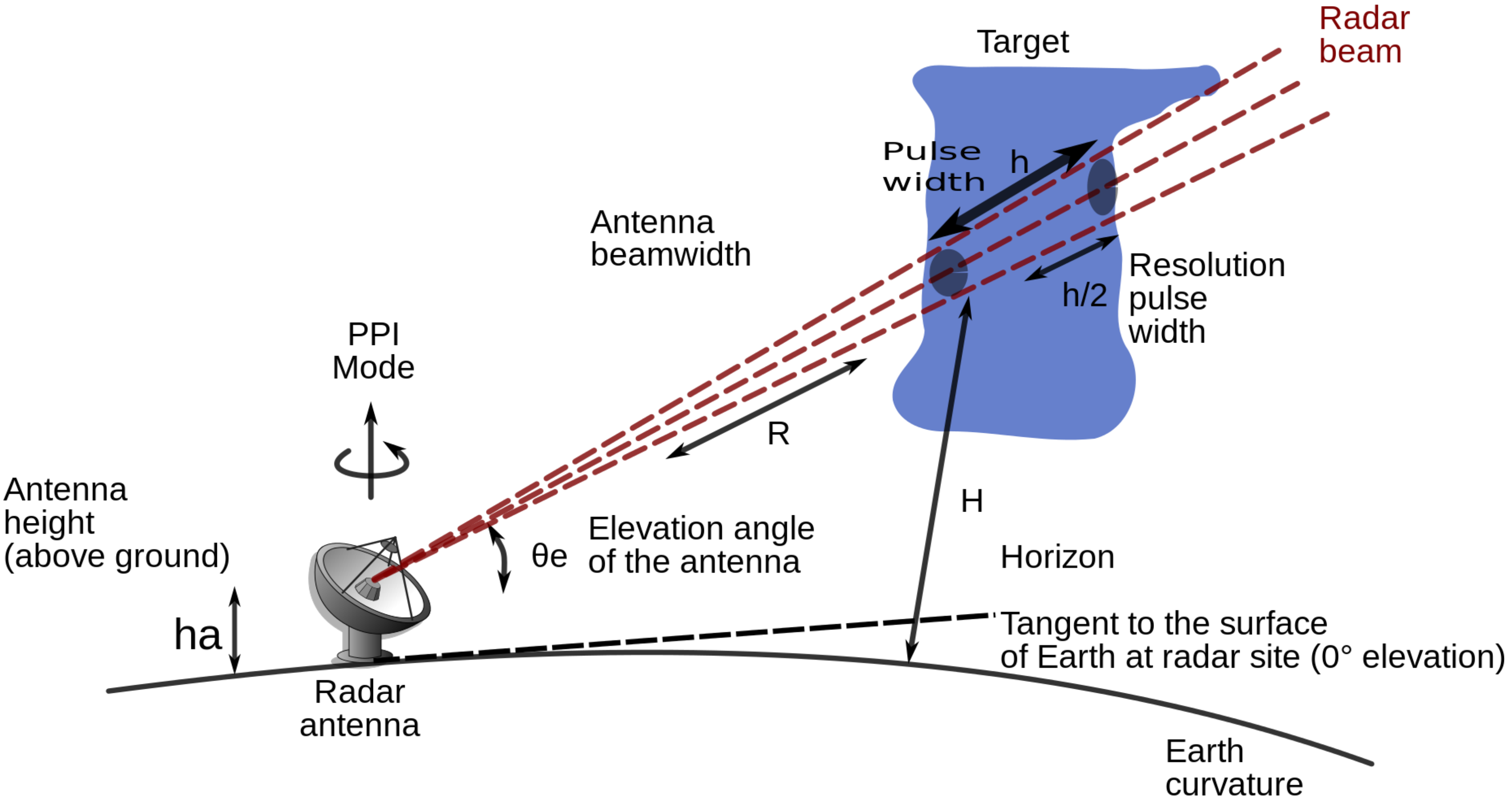
- **Atmospheric propagation attenuation is much less severe** for Radar than higher frequency electromagnetic disturbances. This means Radar can be used for **long range surveillance**. A **military** air defence system may have an operational range of **hundreds of km**.

- Radar has been used to successfully measure the distance between the Earth and other planets in the solar system. Note Mars is 56 million km from Earth!

I told you it would useful!

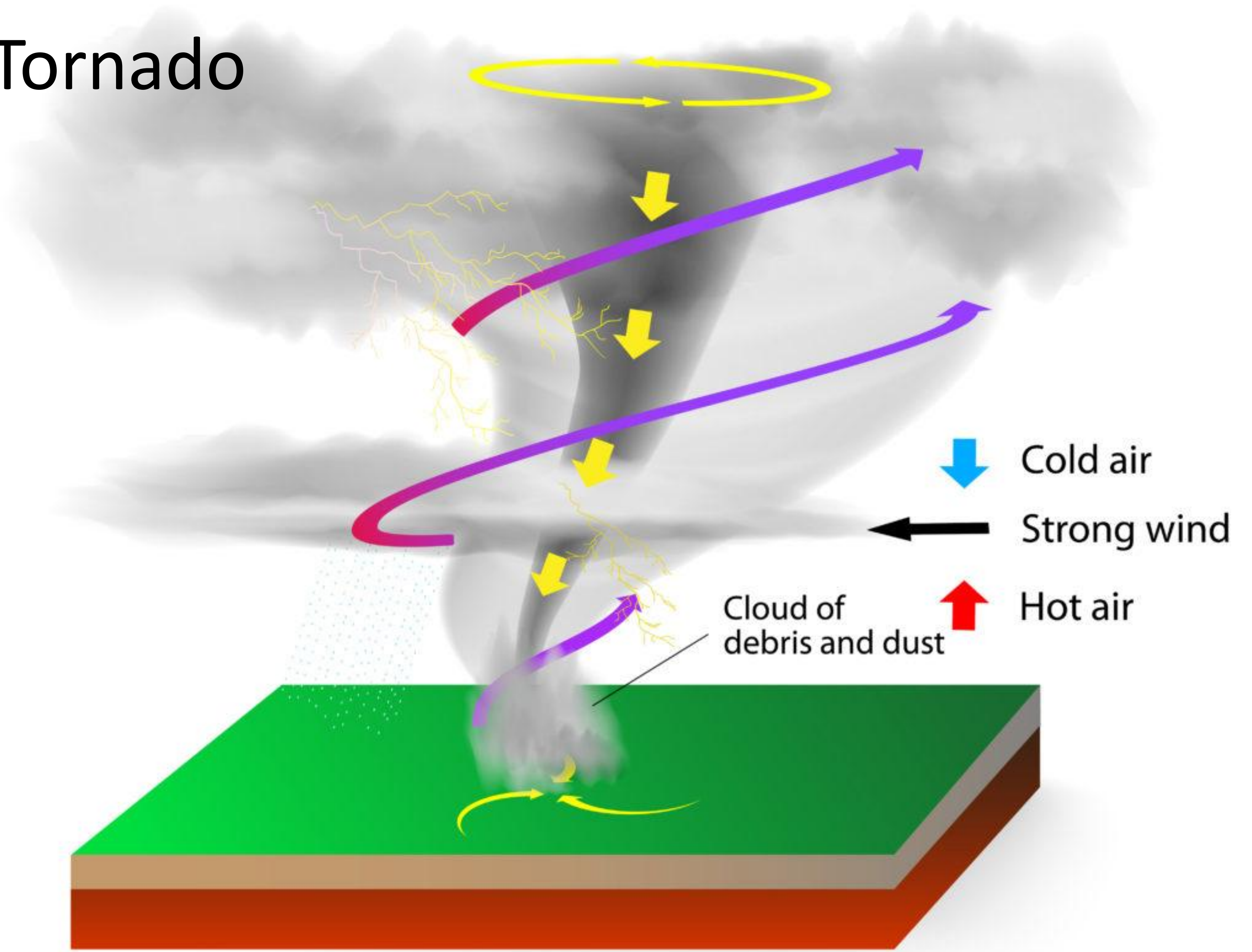


Doppler radar in Oklahoma, USA.



Radar
beam
Target
Pulse
width
h
Antenna
beamwidth
Resolution
pulse
width
h/2
PPI
Mode
R
Antenna
height
(above ground)
H
Elevation angle
of the antenna
θe
Horizon
ha
Tangent to the surface
of Earth at radar site (0° elevation)
Radar
antenna
Earth
curvature

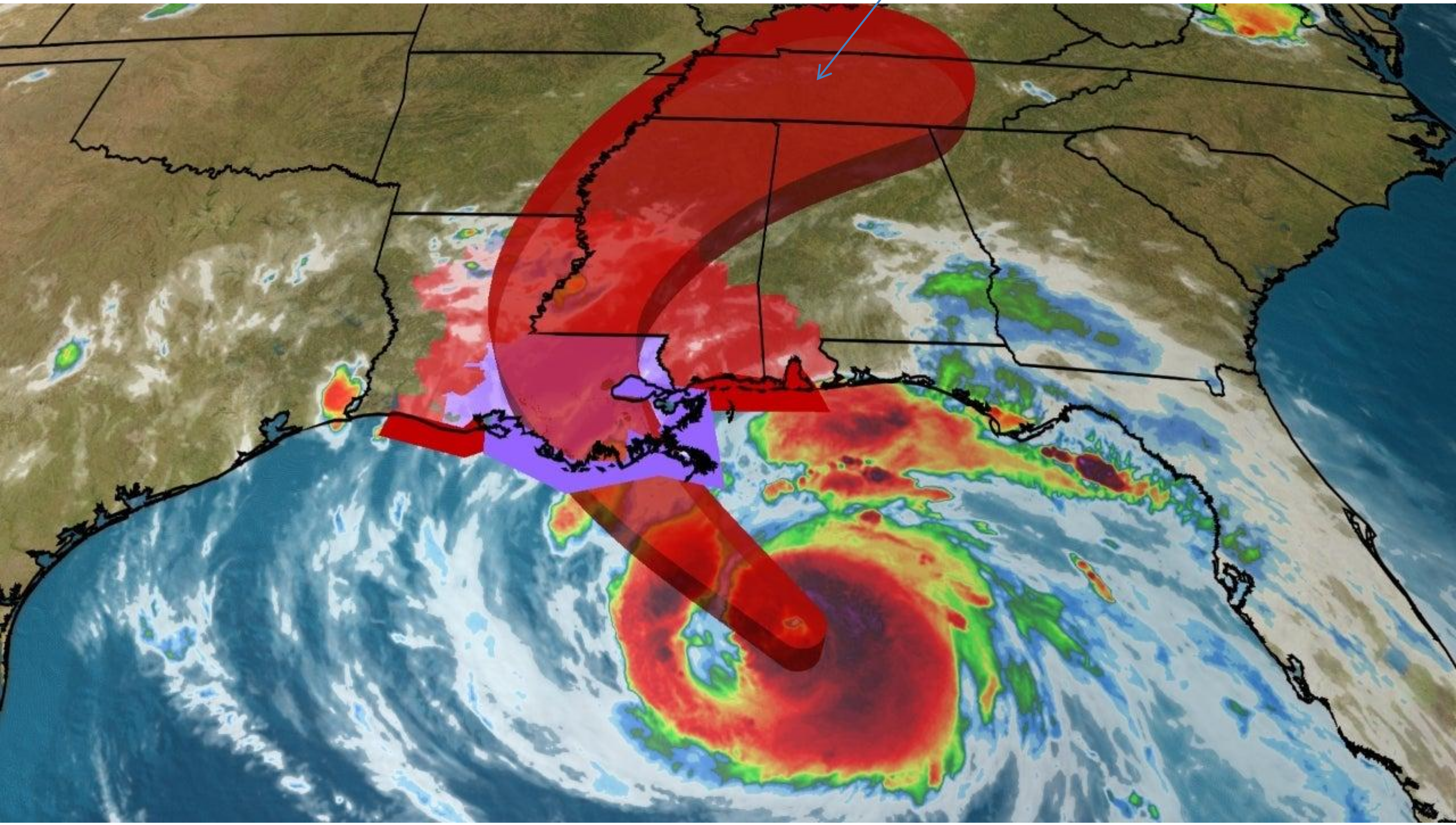
Tornado



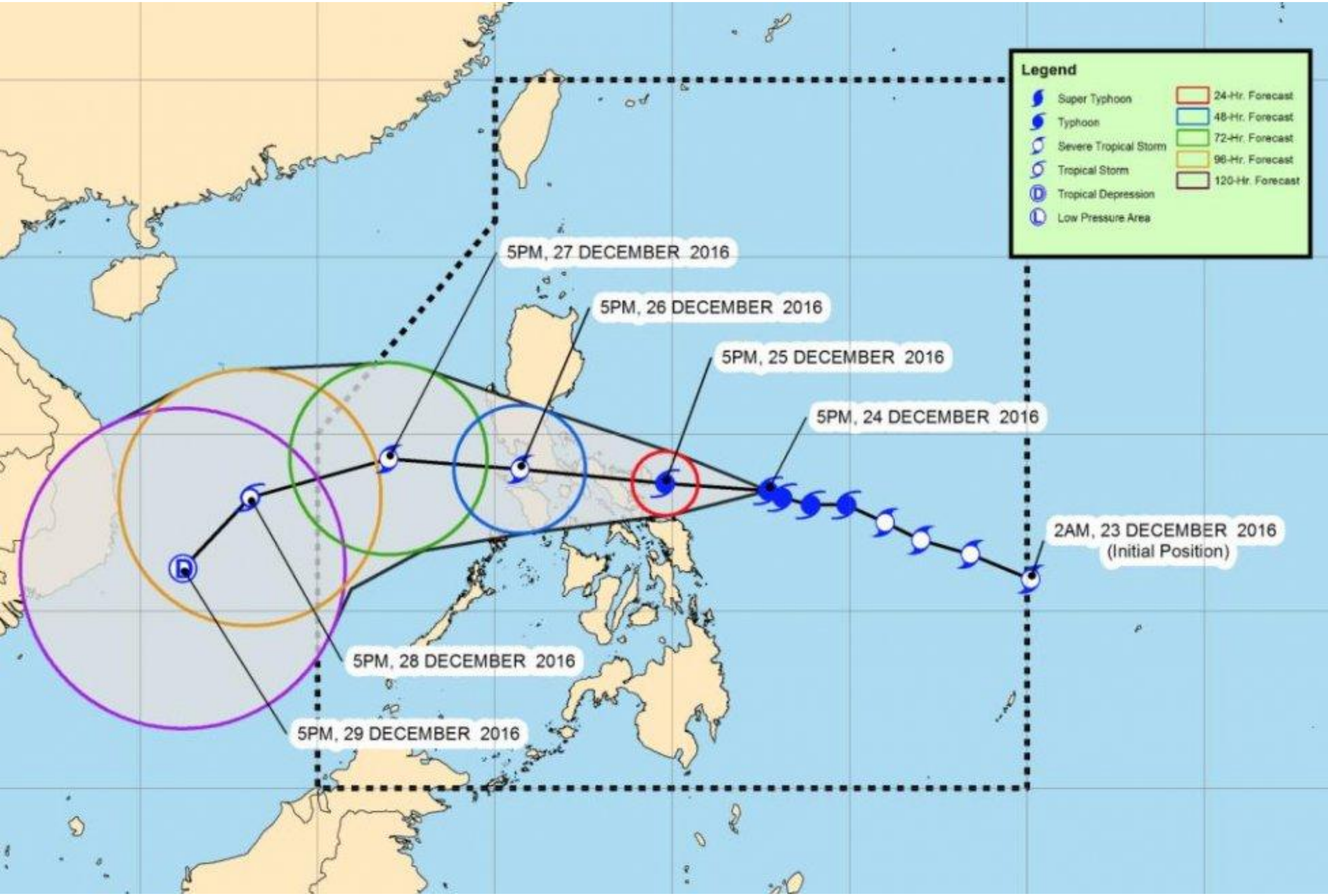
Doppler radar can be used to determine **wind speed**. i.e. reflections from moving clouds are processed. This is an important tool in **meteorology**, particularly in forecasting extreme weather events such as hurricanes and tornados.

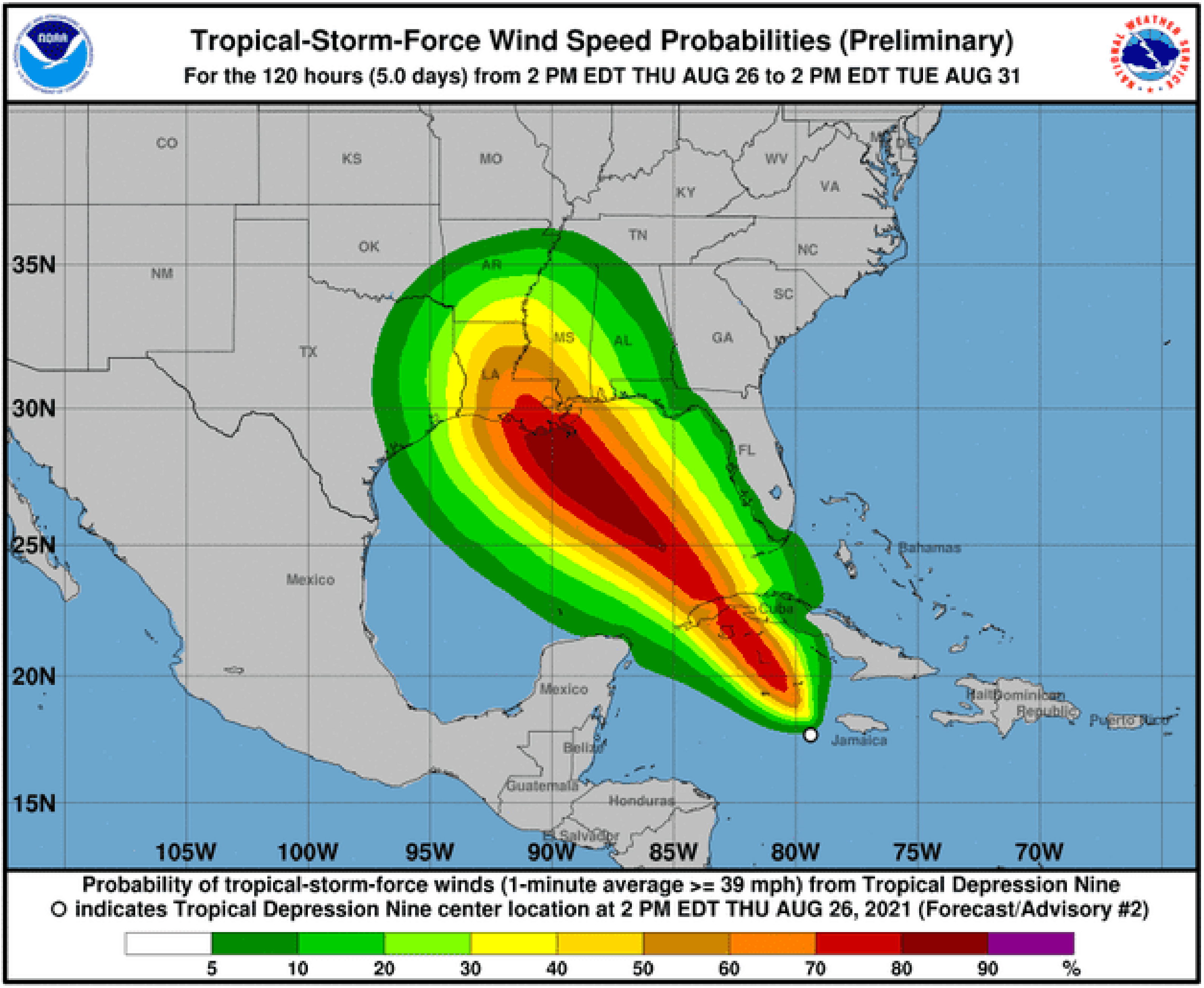
Colour scale gives an indication of wind speed. Red means *very* high winds.

Predicted hurricane track

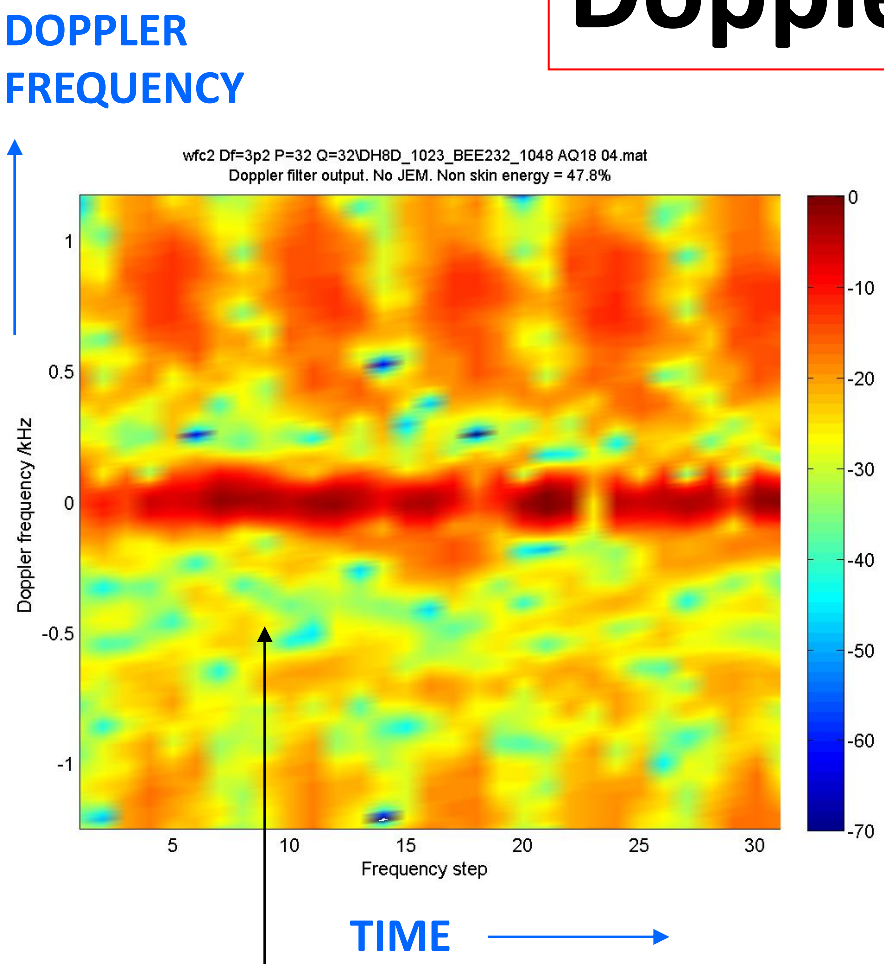


Combine Doppler radar and satellite imagery to help predict the likely trajectory of a hurricane. Live measurements help to correct a mathematical model.


Legend
Super Typhoon
Typhoon
Severe Tropical Storm
Tropical Storm
Tropical Depression
Low Pressure Area
24-Hr. Forecast
48-Hr. Forecast
72-Hr. Forecast
96-Hr. Forecast
120-Hr. Forecast
5PM, 27 DECEMBER 2016
5PM, 26 DECEMBER 2016
5PM, 25 DECEMBER 2016
5PM, 24 DECEMBER 2016
2AM, 23 DECEMBER 2016
(Initial Position)
5PM, 28 DECEMBER 2016
5PM, 29 DECEMBER 2016


Tropical-Storm-Force Wind Speed Probabilities (Preliminary)
For the 120 hours (5.0 days) from 2 PM EDT THU AUG 26 to 2 PM EDT TUE AUG 31
CO
KS
MO
WV
VA
KY
TN
NC
OK
AR
NM
SC
MS
AL
GA
TX
LA
FL
Bahamas
Mexico
Cuba
Mexico
Haiti
Dominican Republic
Puerto Rico
Jamaica
Belize
Guatemala
Honduras
El Salvador
35N
30N
25N
20N
15N
105W
100W
95W
90W
85W
80W
75W
70W
Probability of tropical-storm-force winds (1-minute average >= 39 mph) from Tropical Depression Nine
O indicates Tropical Depression Nine center location at 2 PM EDT THU AUG 26, 2021 (Forecast/Advisory #2)
5
10
20
30
40
50
60
70
80
90
%

Doppler spectra



Doppler spectrum for 32 pulse, 32 frequency step 2.5kHz PRF

Dash8 six blade propeller aircraft

Identify the aircraft from the Doppler spectra of propellers, jet engines..

Note **the factor two** in the Doppler formula. This is because the ultrasound is **re-radiated by the moving source** and receiver by the (stationary) transducer.

$$I_r = I_0 \times \left(\frac{Z_1 - Z_2}{Z_1 + Z_2} \right)^2$$

Coupling gel is used to prevent reflection at air-skin boundary

MEASURING BLOOD FLOW VIA ULTRASOUND

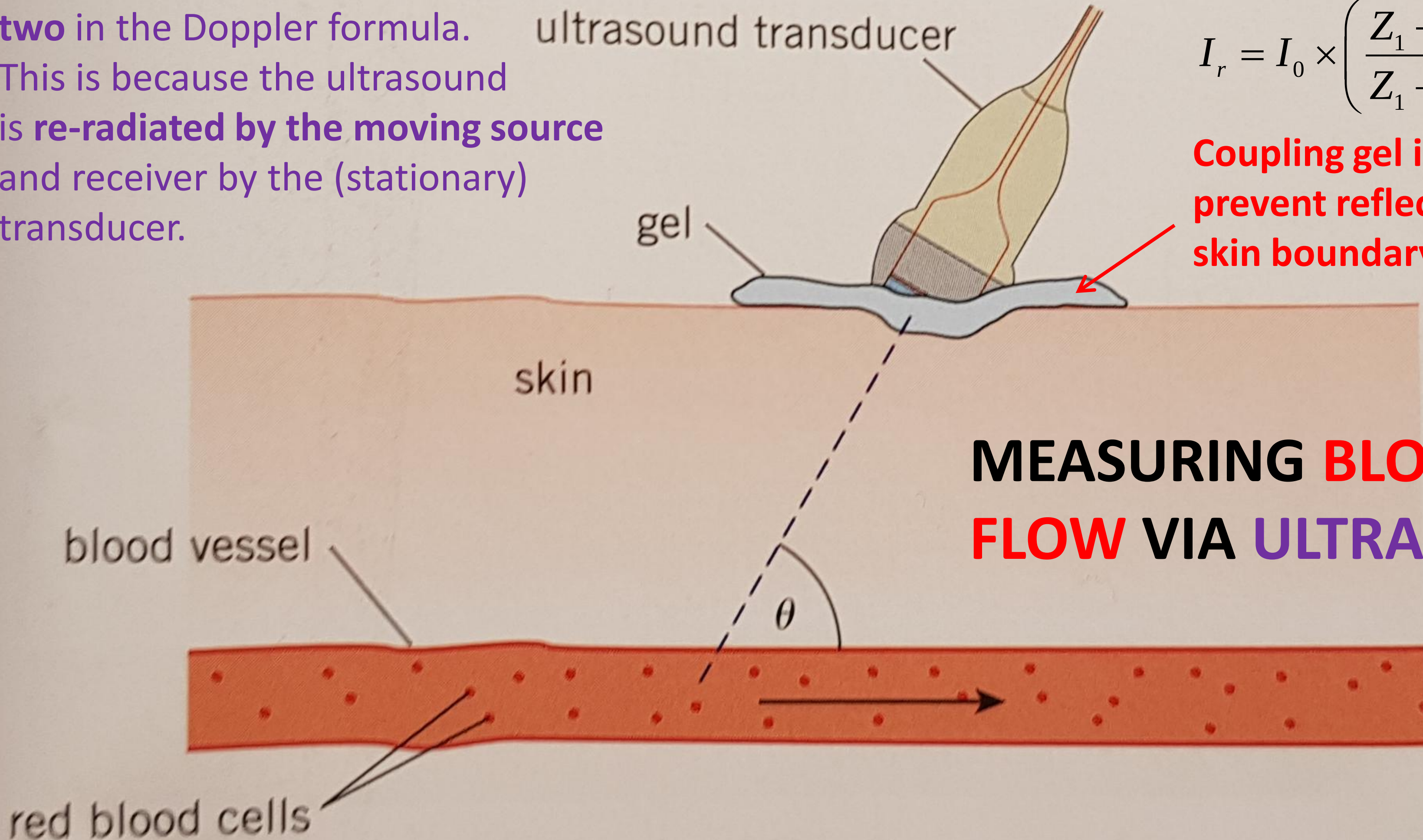
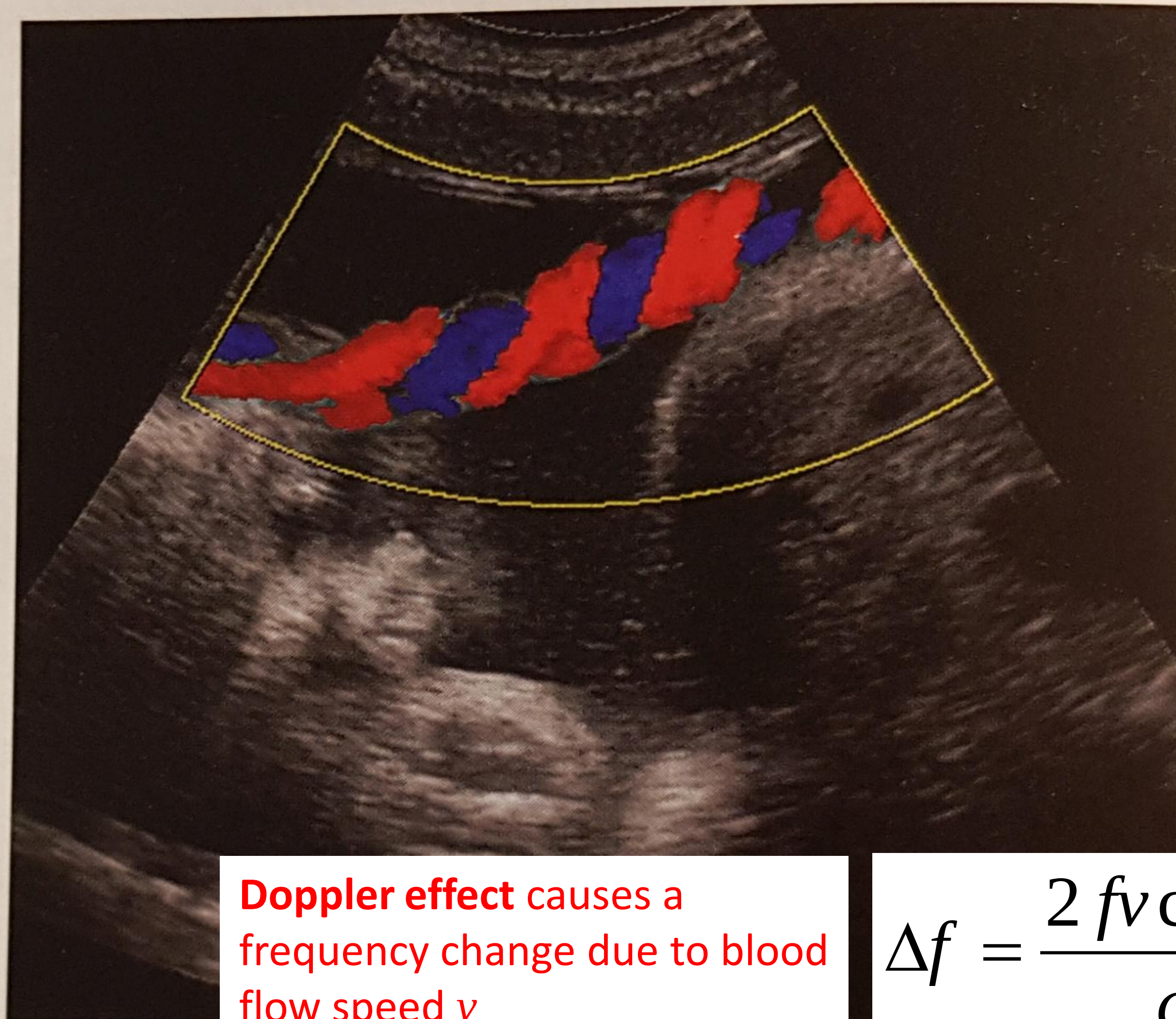


Figure 3 *Ultrasound transducer used to determine the speed of blood flow*

Doppler effect causes a frequency change due to blood flow speed *v*

$$\Delta f = \frac{2 f v \cos\theta}{c}$$



Doppler effect causes a frequency change due to blood flow speed v

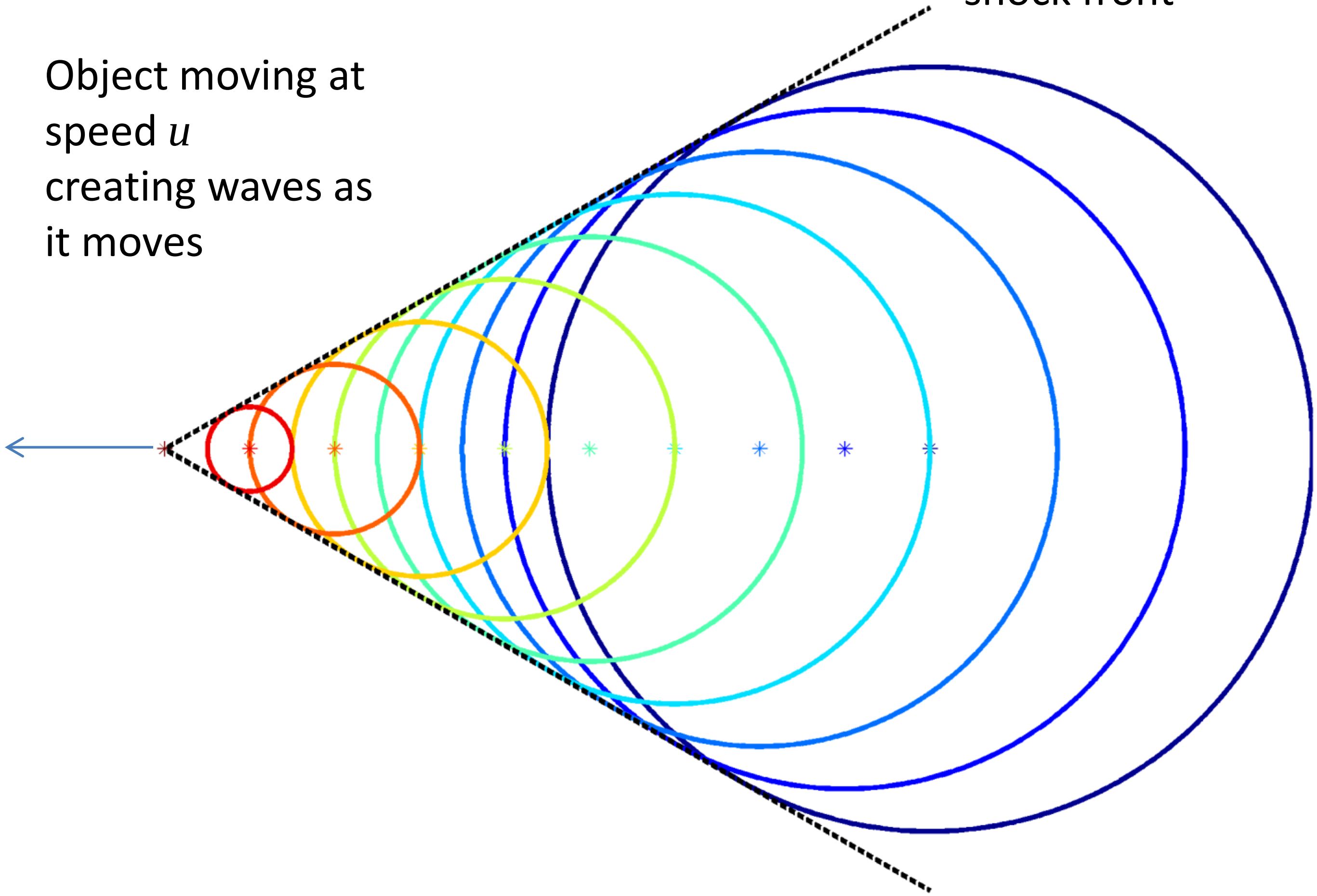
$$\Delta f = \frac{2 f v \cos\theta}{c}$$

▲ **Figure 2** *Coloured Doppler ultrasound scan showing umbilical blood flow – the fetus is lying across the bottom left, and oxygenated (arterial) blood, which is flowing from mother to fetus, is red, whilst deoxygenated (venous) blood, which is flowing from fetus to mother, is blue*

Mach's construction: Shock Waves

Ernst Mach
1838-1916

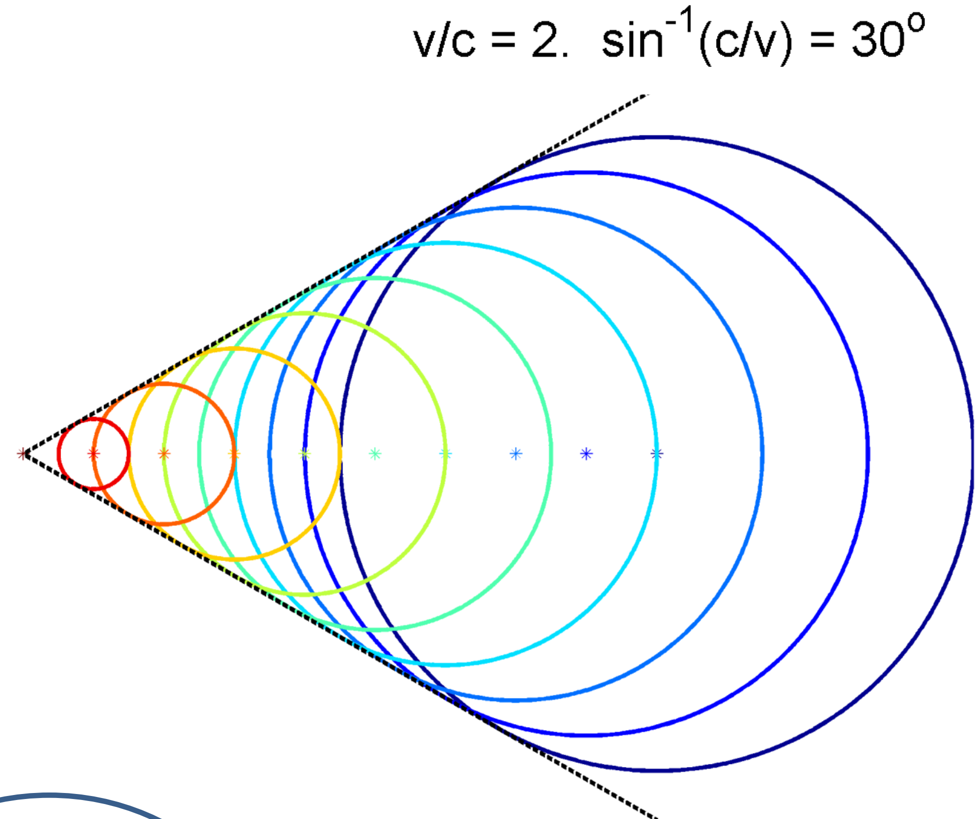
Object moving at speed u creating waves as it moves



'Infinitesimally thin' spherical shells of disturbance are created continuously as the object moves. They radiate out at the wave speed c

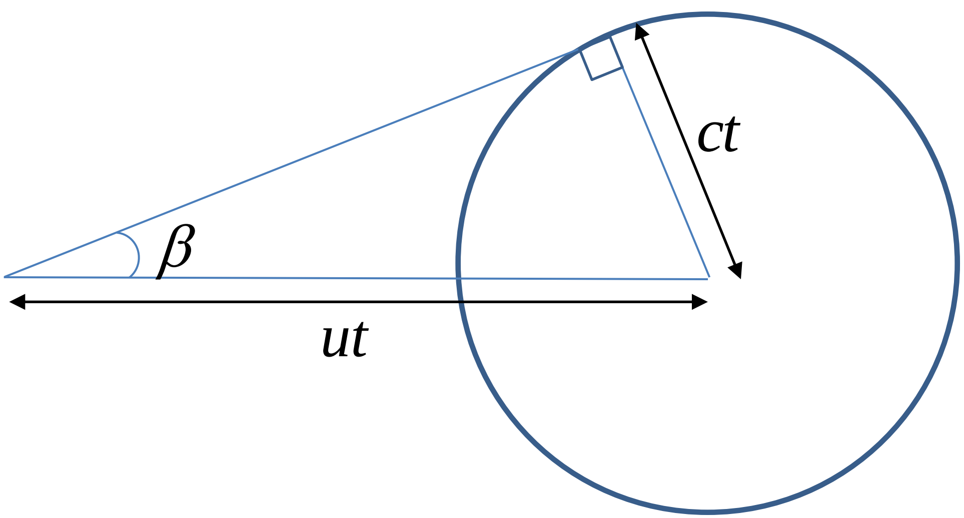
Mach's construction

c is the wave speed
u is the velocity of the object making the waves



Mach number

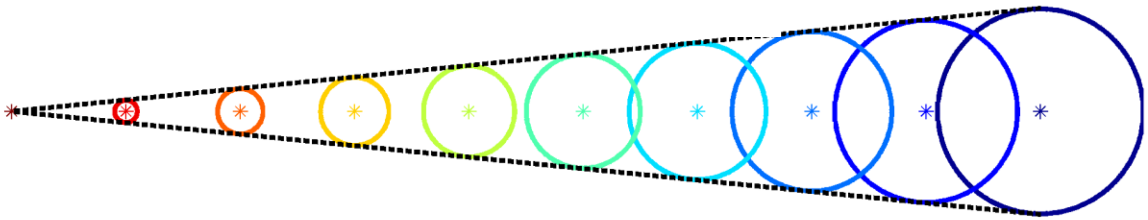
$$M = \frac{u}{c}$$

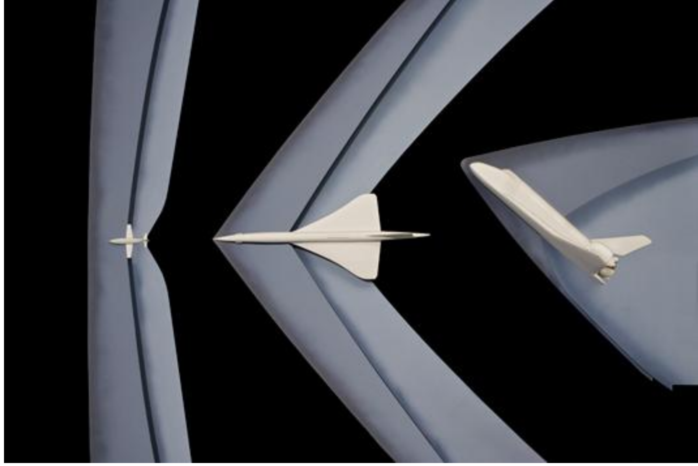


$$ut \sin\beta = ct$$

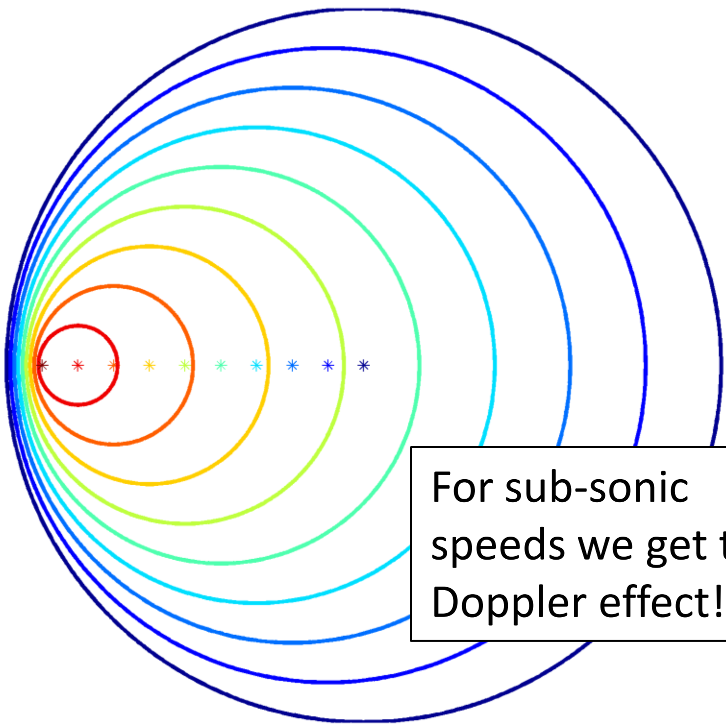
$$\therefore \beta = \sin^{-1}\left(\frac{c}{u}\right) = \sin^{-1}\frac{1}{M}$$

v/c = 10. $\sin^{-1}(c/v) = 5.7392^{o}$





v/c = 0.9. $\sin^{-1}(c/v) = NaN^{o}$



For sub-sonic speeds we get the Doppler effect!

v/c = 0.5. $\sin^{-1}(c/v) = NaN^{o}$