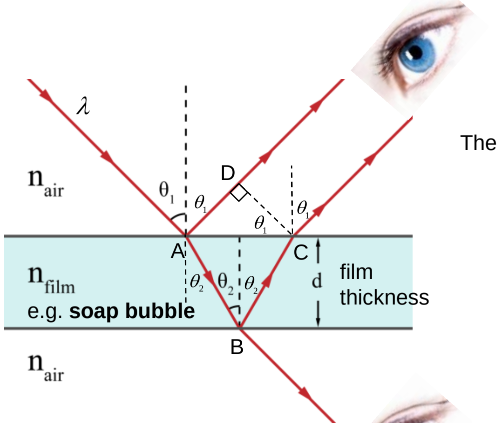
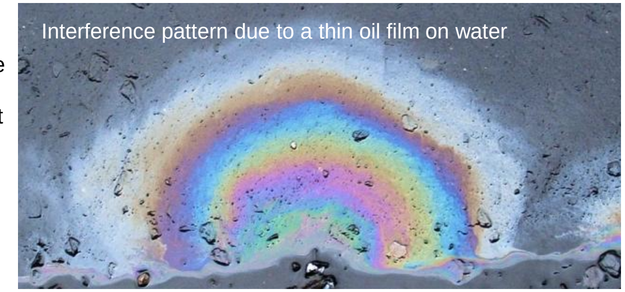
Thin film interference



Reflected light off a thin film such as **oil on water** or a **soap bubble** will often exhibit an **interference pattern**. If white light is used (i.e. a *spectrum* of different wavelengths from blue to red), this pattern may form a rather beautiful set of multi-coloured **fringes**. The reason for the fringes is constructive (or destructive) interference of waves that reflect off the uppermost surface, and those which take a path which reflects of the lower surface of the film.



The **phase difference** between the AD and the ABC paths for the thin film geometry on the left is:

$$\Delta\phi = \frac{2\pi}{\lambda_{film}}AB + \frac{2\pi}{\lambda_{film}}BC + \pi - \frac{2\pi}{\lambda}AD$$

$$AB = BC$$

$$AB\cos\theta_2 = d \quad \therefore AB = \frac{d}{\cos\theta_2}$$

$$AD = AC\sin\theta_1$$

$$AC = 2AB\sin\theta_2 = \frac{2d\sin\theta_2}{\cos\theta_2}$$

$$\therefore AD = \frac{2d\sin\theta_2\sin\theta_1}{\cos\theta_2}$$

Note the π phase shift due to the air-film interface reflection (but *not* the film-air interface reflection)

π radians phase shift for reflections off a higher n boundary (approximately!)

$$\frac{c}{n} = f\lambda$$

Wave-speed c/n
Refractive index n

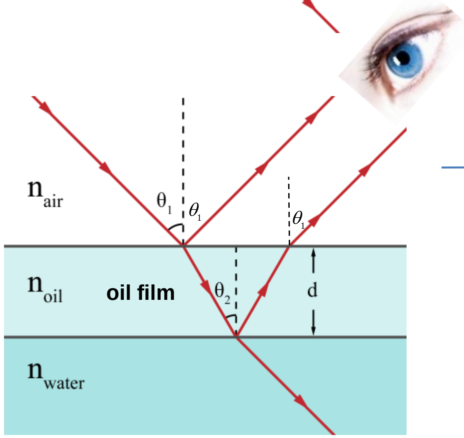
$$\therefore \frac{c}{f} = n\lambda$$

$$\therefore n_{film}\lambda_{film} = 1\times\lambda$$

Frequency the *same* across a boundary

$$\therefore \lambda_{film} = \frac{\lambda}{n_{film}}$$

$$\therefore \Delta\phi = \frac{2\pi n_{film}}{\lambda}\times 2\times\frac{d}{\cos\theta_2} + \pi - \frac{2\pi}{\lambda}\frac{2d\sin\theta_2\sin\theta_1}{\cos\theta_2}$$



We would expect the **oil on water scenario** to have the *same* interference pattern as the soap bubble since one expects the refractive index of oil to be *greater* than that of water.

$$2n_{oil}d\cos\theta_2 = \left(m - \tfrac{1}{2}\right)\lambda$$

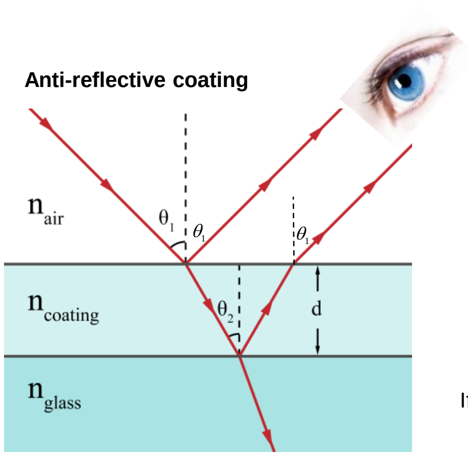
$$\theta_1 = \sin^{-1}\left(n_{oil}\sqrt{1 - \frac{\left(m-\frac{1}{2}\right)^2\lambda^2}{4n_{oil}^2 d^2}}\right)$$

Snell's Law of Refraction: $n_{film}\sin\theta_2 = 1\times\sin\theta_1$

$$\therefore \Delta\phi = \frac{4\pi n_{film}d}{\lambda\cos\theta_2} + \pi - \frac{4\pi d\sin\theta_2\left(n_{film}\sin\theta_2\right)}{\lambda\cos\theta_2}$$

$$\Rightarrow \Delta\phi = \frac{4\pi n_{film}d}{\lambda}\left(\frac{1-\sin^2\theta_2}{\cos\theta_2}\right) + \pi = \frac{4\pi n_{film}d\cos\theta_2}{\lambda} + \pi$$

Anti-reflective coating



For an **anti-reflective coating** (e.g. on a pair of glasses), the refractive index of the coating is chosen to be greater than air, but less than glass. Hence there is a π phase change from reflections at both the air-coating interface and the coating-glass interface.

Hence for constructive interference:

$$2n_{coating}d\cos\theta_2 = m\lambda$$

$$\theta_1 = \sin^{-1}\left(n_{coating}\sqrt{1 - \frac{m^2\lambda^2}{4n_{coating}^2 d^2}}\right)$$

If $d = \frac{\lambda}{4n_{coating}}$ $\therefore 2n_{coating}\frac{\lambda}{4n_{coating}}\cos\theta_2 = m\lambda$

$\therefore \cos\theta_2 = 2m$ i.e. *no solutions**. So 'quarter-wave thickness' coatings can eliminate reflections!

For **constructive interference**: $\Delta\phi = 2\pi m$

(where m is an integer)

$$\therefore 2\pi m = \frac{4\pi n_{film}d\cos\theta_2}{\lambda} + \pi$$

$$\therefore 2n_{film}d\cos\theta_2 = \left(m - \tfrac{1}{2}\right)\lambda$$

$$\therefore \cos\theta_2 = \frac{\left(m-\frac{1}{2}\right)\lambda}{2n_{film}d}$$

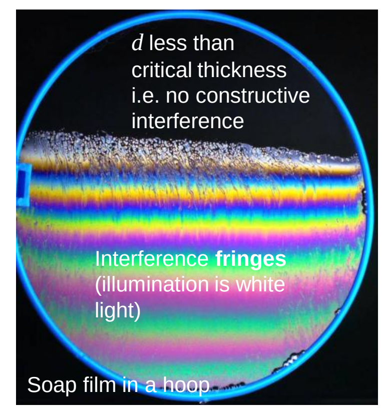
$$\sin\theta_2 = \sqrt{1-\cos^2\theta_2}$$

$$\therefore \theta_1 = \sin^{-1}\left(n_{film}\sqrt{1 - \frac{\left(m-\frac{1}{2}\right)^2\lambda^2}{4n_{film}^2 d^2}}\right)$$

$$n_{film}\sin\theta_2 = \sin\theta_1$$

So different wavelengths result in bright fringes at different angles from normal incidence.

As d becomes thinner, there reaches a point where constructive interference is impossible. **Below the critical thickness the film will therefore look black**, as can be seen in soap film demonstrations.



Images on this page from https://en.wikipedia.org/wiki/Thin-film_interference

*since m is integer

Newton's Rings are a special type of **interference pattern** formed due to an *extra optical path* resulting from the air gap between a spherical lens and a flat reflective surface (which in addition, imparts a 180° (π radians) phase shift to reflections).

From geometry:

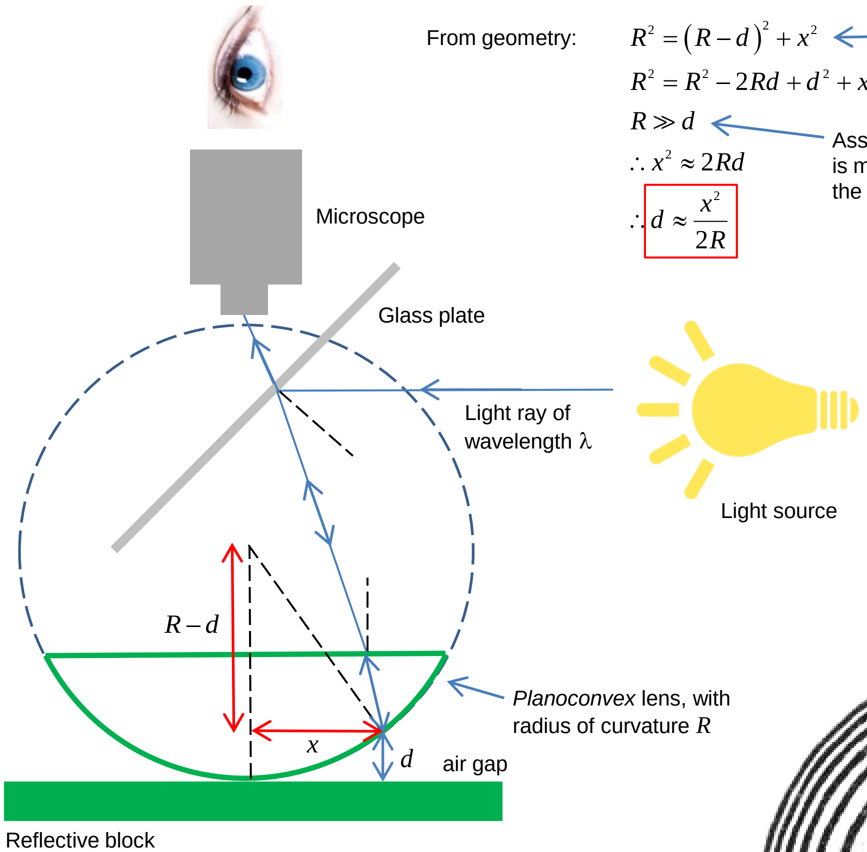
$$R^2 = (R-d)^2 + x^2$$ ← Pythagoras' Theorem

$$R^2 = R^2 - 2Rd + d^2 + x^2$$

$$R \gg d$$ ← Assume the air gap is much smaller than the lens radius of curvature

$$\therefore x^2 \approx 2Rd$$

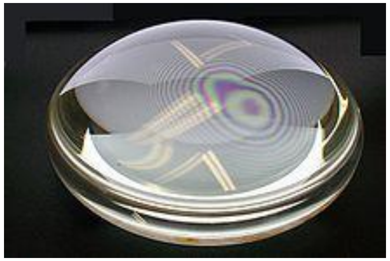
$$\therefore d \approx \frac{x^2}{2R}$$



Note in Hecht and Woan the fringe radius is stated as

$$x = \sqrt{R\lambda\left(n + \tfrac{1}{2}\right)}$$

This is not a problem! In our case the first bright fringe is when $n = 1$, in their case it is when $n = 0$.



The **phase difference** between the ray which reflects off the lower surface of the lens and the ray which reflects off the block is:

$$\Delta\phi = \frac{2\pi}{\lambda} \times 2d + \pi$$

$$\therefore \Delta\phi \approx \frac{2\pi}{\lambda} \times \frac{x^2}{R} + \pi$$

Note block reflection imparts a phase shift of π

For *constructive interference*

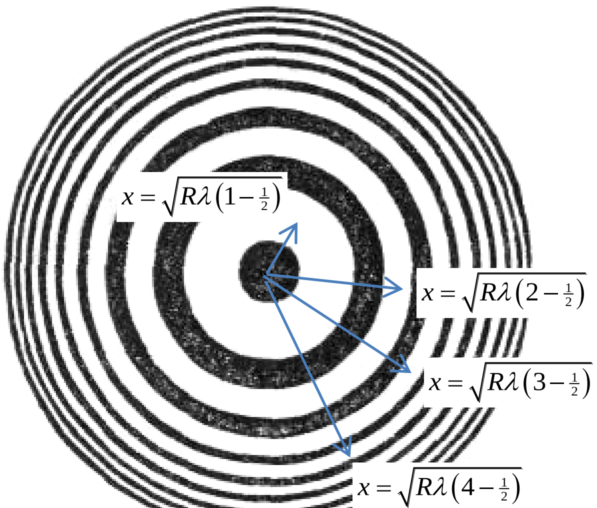
$$\Delta\phi = 2\pi n$$

where n is an integer

$$\therefore \frac{2\pi}{\lambda} \times \frac{x^2}{R} + \pi = 2\pi n$$

$$\therefore x = \sqrt{R\lambda\left(n - \tfrac{1}{2}\right)}$$

For *monochromatic* light the microscope image would look a bit like the figure below. The bright fringes correspond to distance x where constructive interference occurs



If **white light** is used, coloured fringes will appear at slightly different positions due to the wavelength dependence of

$$x = \sqrt{R\lambda\left(n - \tfrac{1}{2}\right)}$$