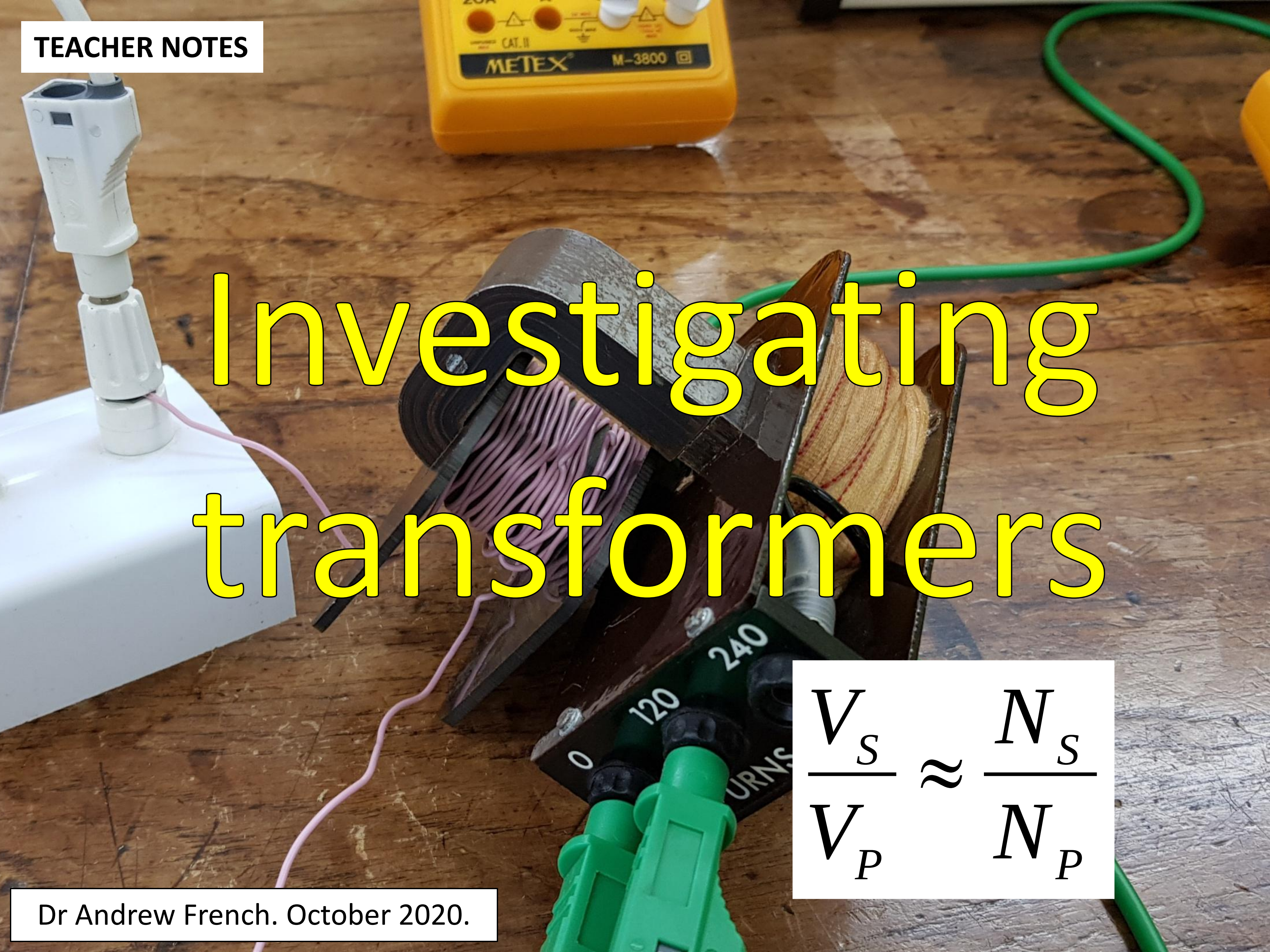
TEACHER NOTES

Investigating transformers

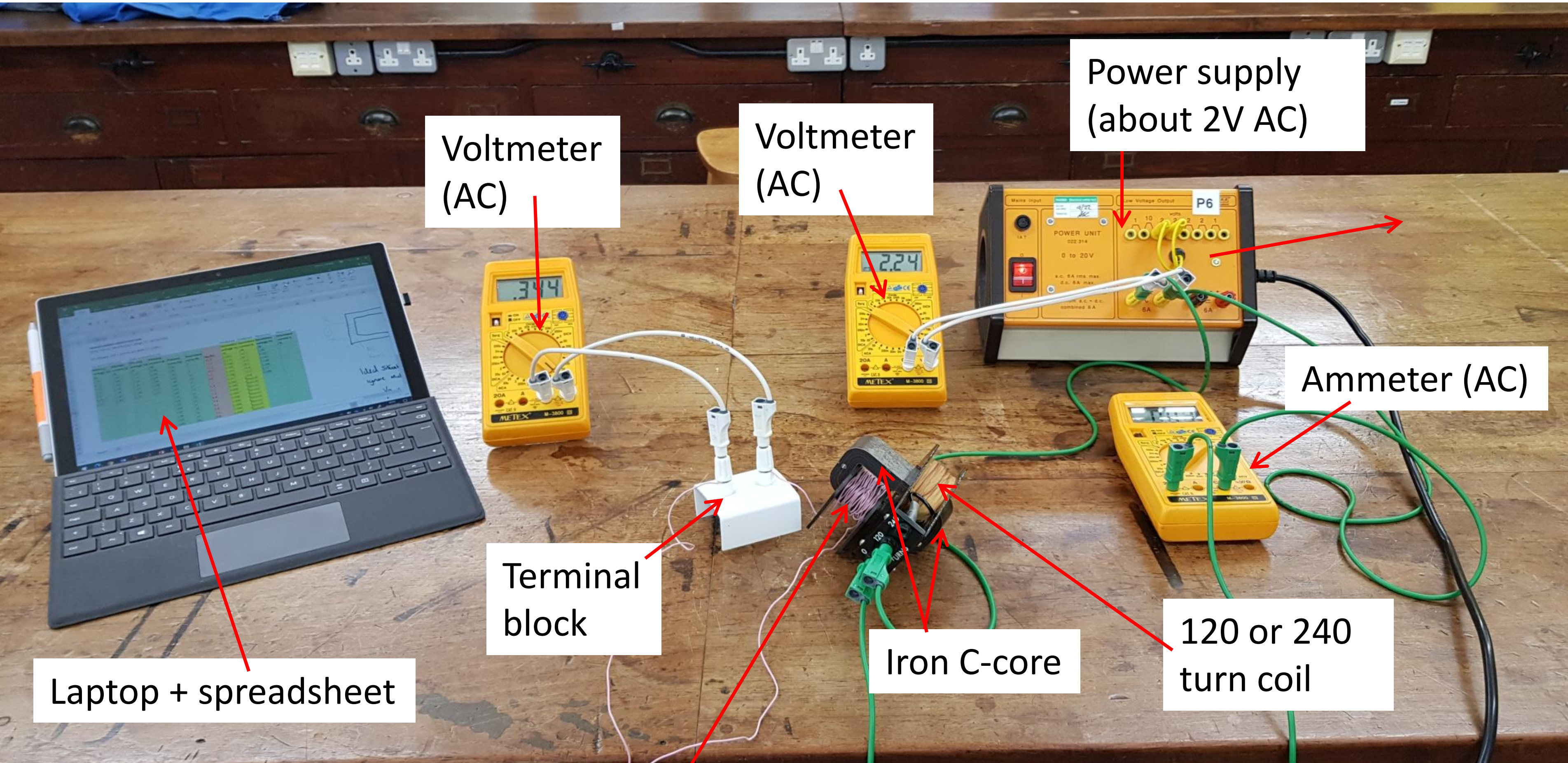
$$\frac{V_S}{V_P} \approx \frac{N_S}{N_P}$$

Dr Andrew French. October 2020.



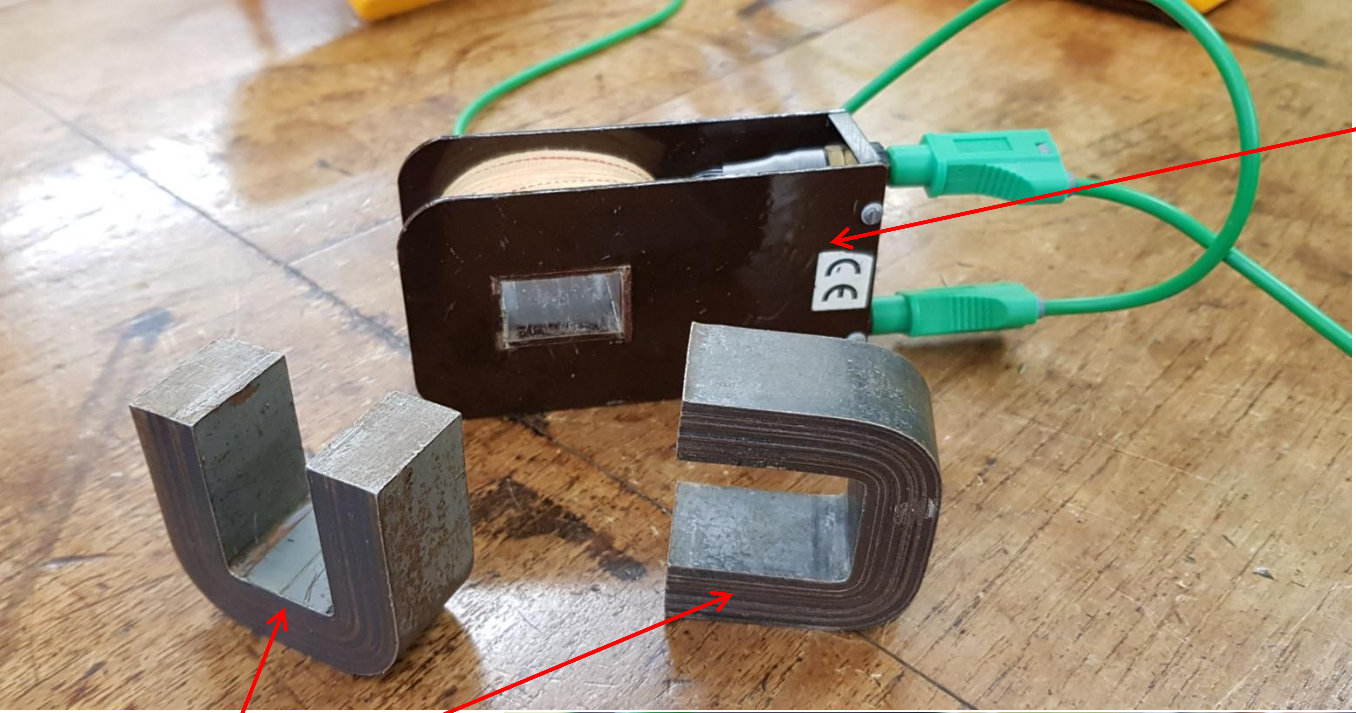
Equipment

For the 120 turn setting in particular, it is possible for large (AC) currents to flow in this system. Hence keep the (50Hz) power supply voltage to about 2V AC. Large currents (but low voltages) will be associated with the secondary coil, so don't connect an ammeter to this side of the circuit.
A primary RMS voltage of 2V will result in a sufficiently strong magnetic field to keep the C-cores together, although you will need to hold them close with your hand to prevent the 50Hz rattling and buzzing of the cores due to 'magnetostriction'.



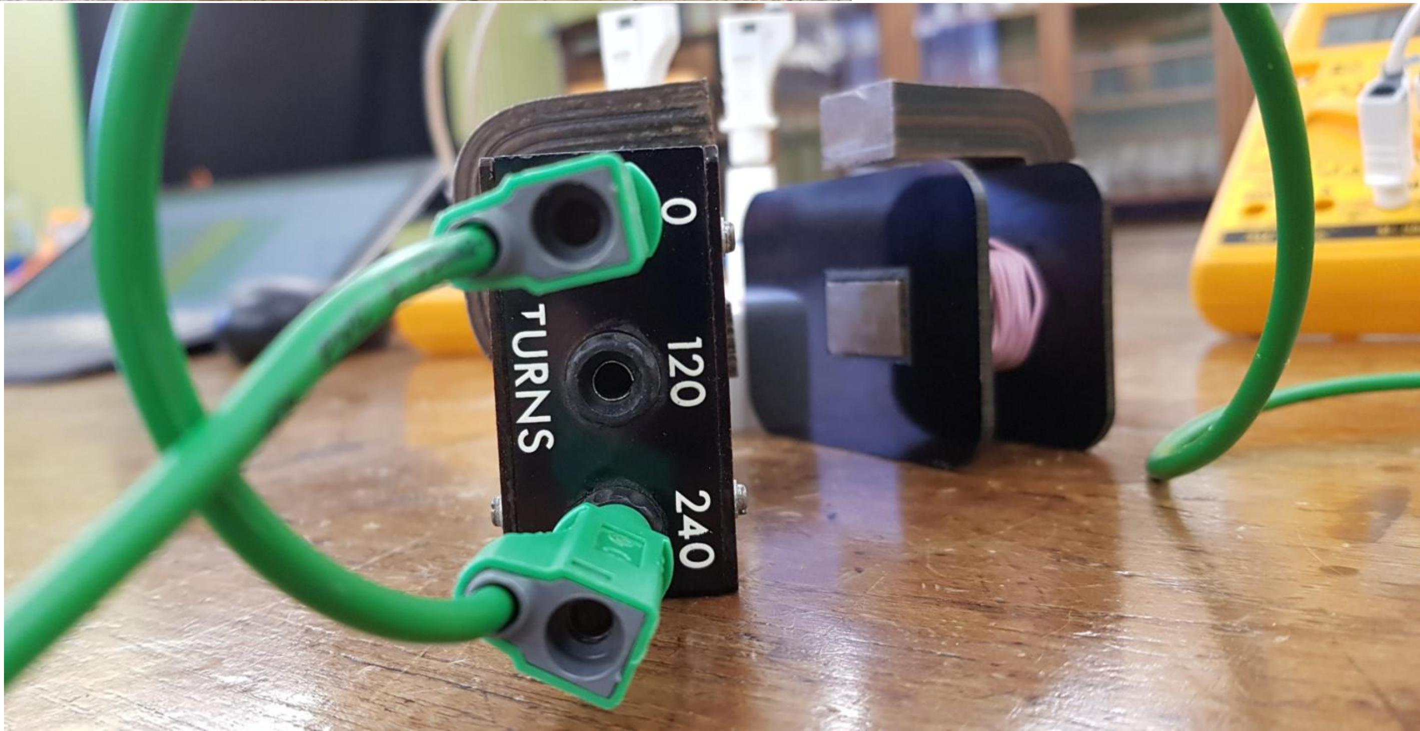
Pink wire wound N_s times around a plastic coil holder

Keep the 50Hz RMS voltage of the (AC) from the power supply to be just over 2V.
For the 120 turn setup, this results in a primary current of about an 1.0A. Over the course of about half an hour, the primary coil will become quite warm, but not excessively so. Note the power supply current limit is 6A, but much higher currents may be induced in the secondary coil. The fuse of an ammeter in the secondary circuit is likely to blow, so don't use a secondary ammeter. ***Only use an ammeter in the primary circuit.***

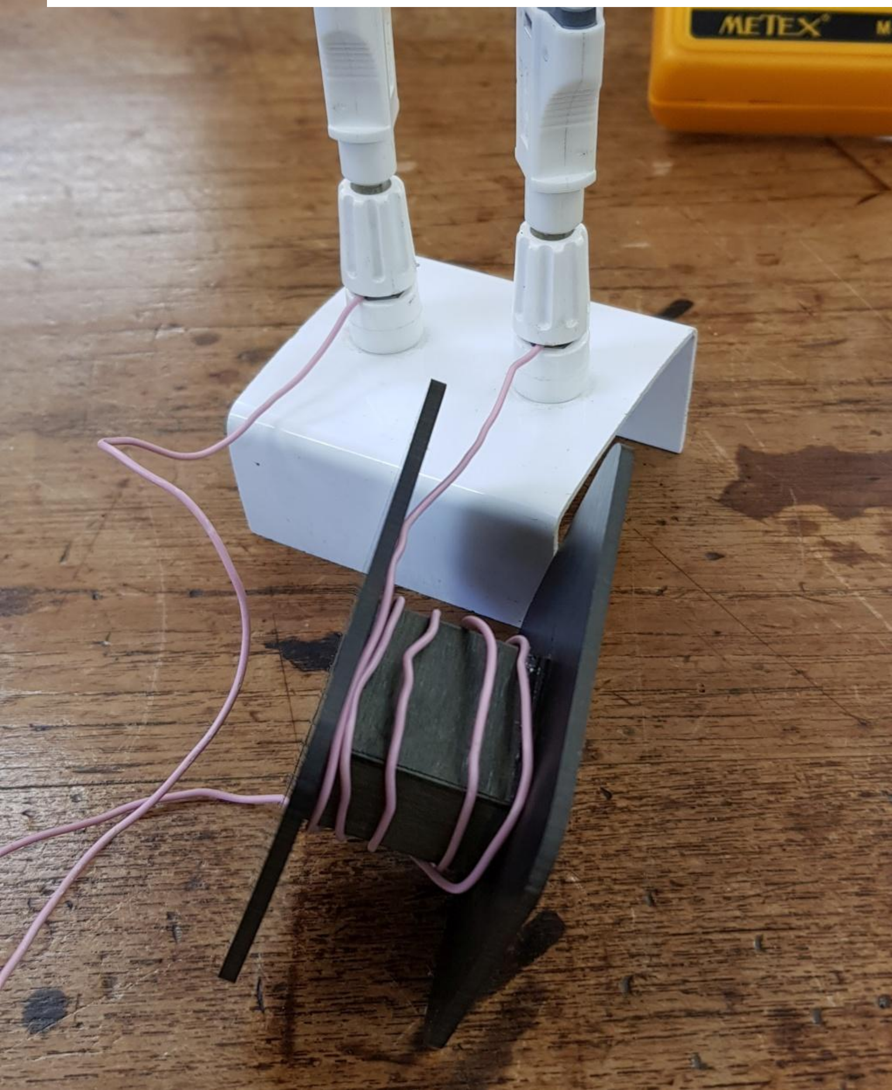


120 or 240 turn primary coil

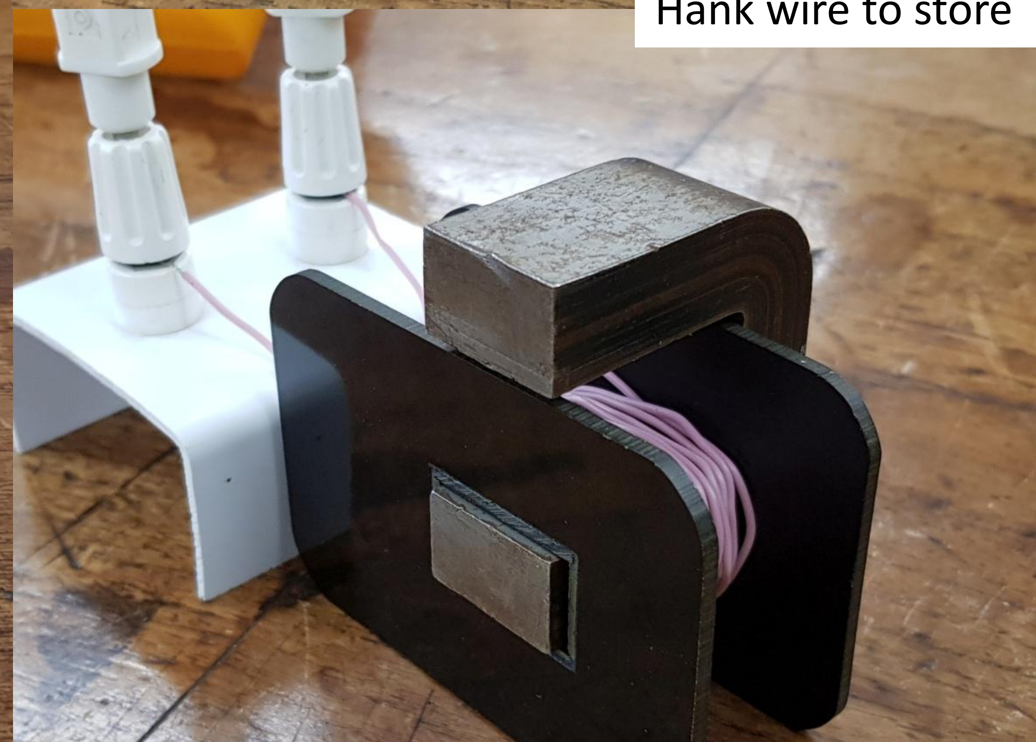
Pair of iron C-cores



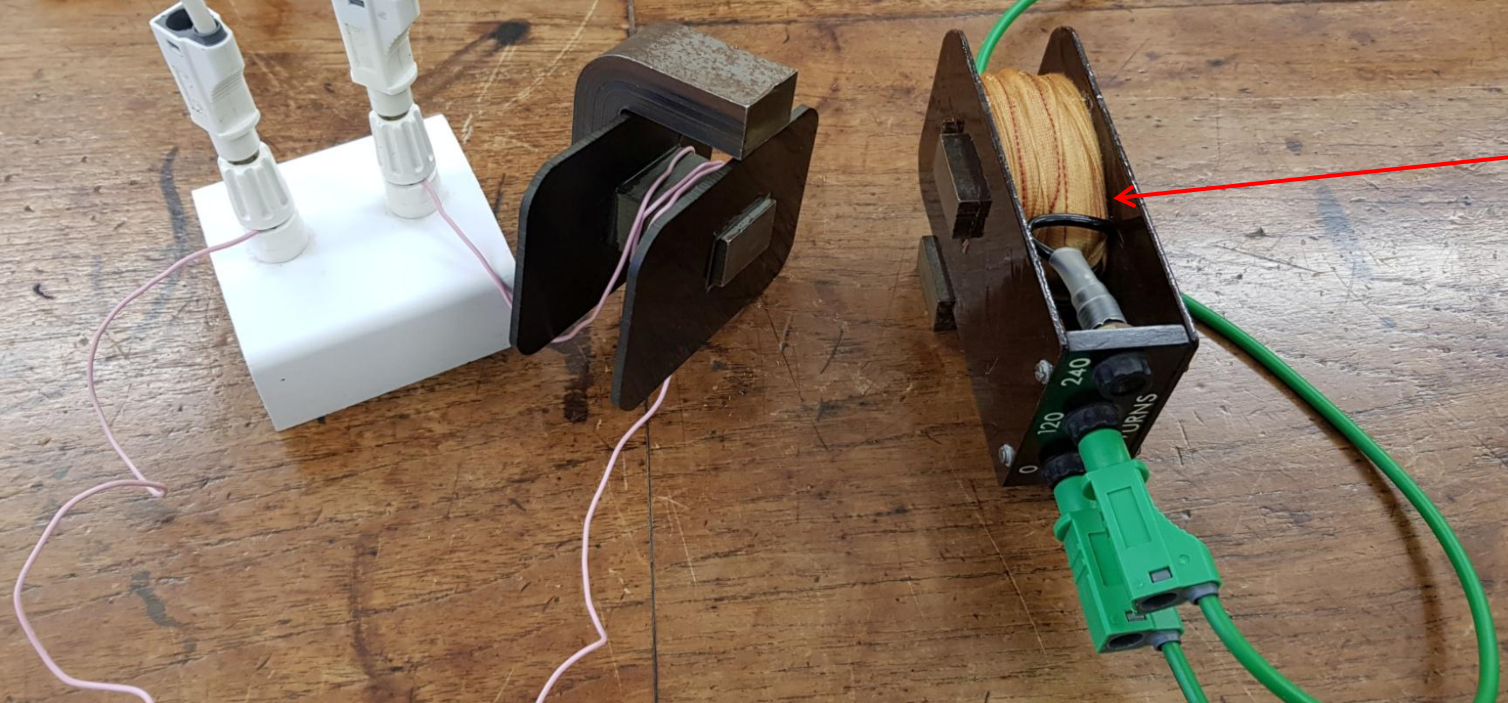
Connect a wire loop to the terminal block. Wind five turns round a plastic holder, then increase turns by two, make measurements (see following slide), and then repeat till wire is fully wound.



Hank wire to store

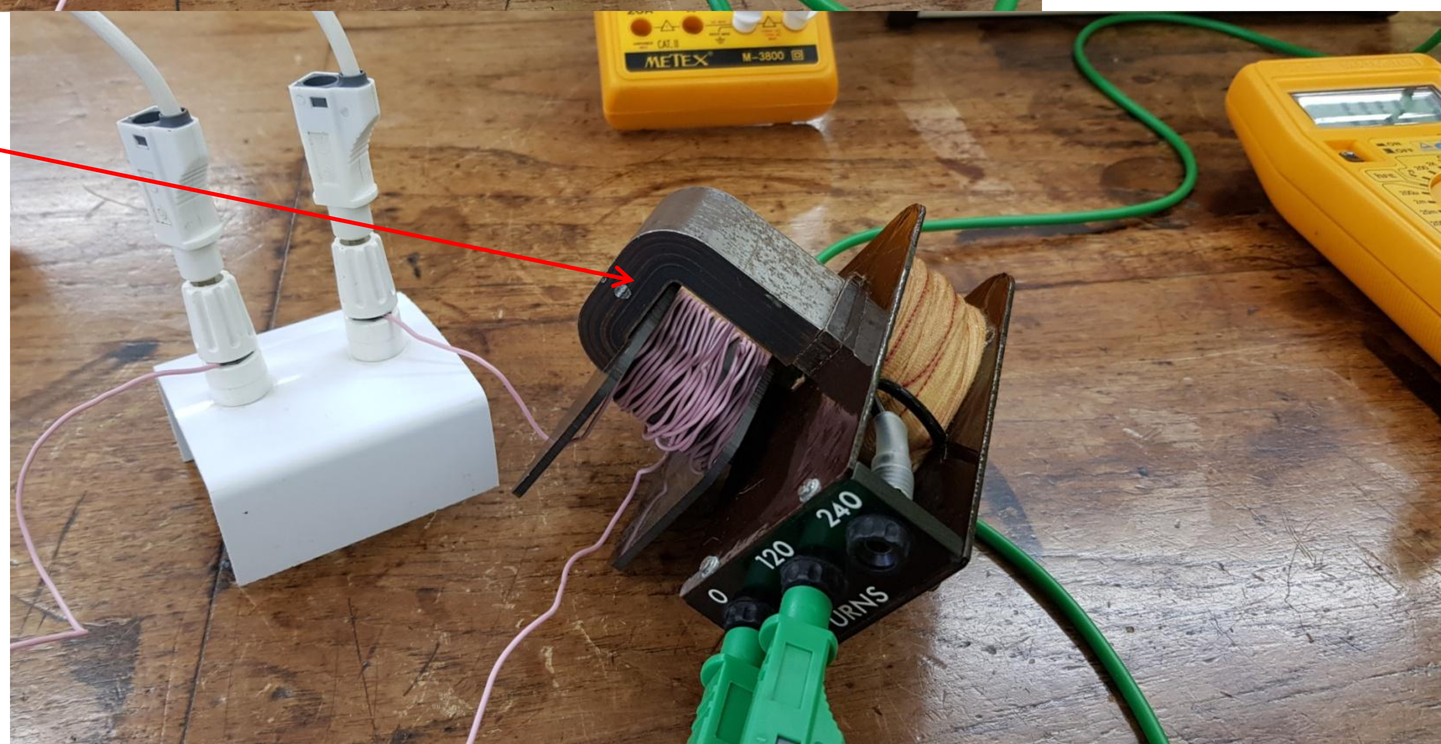


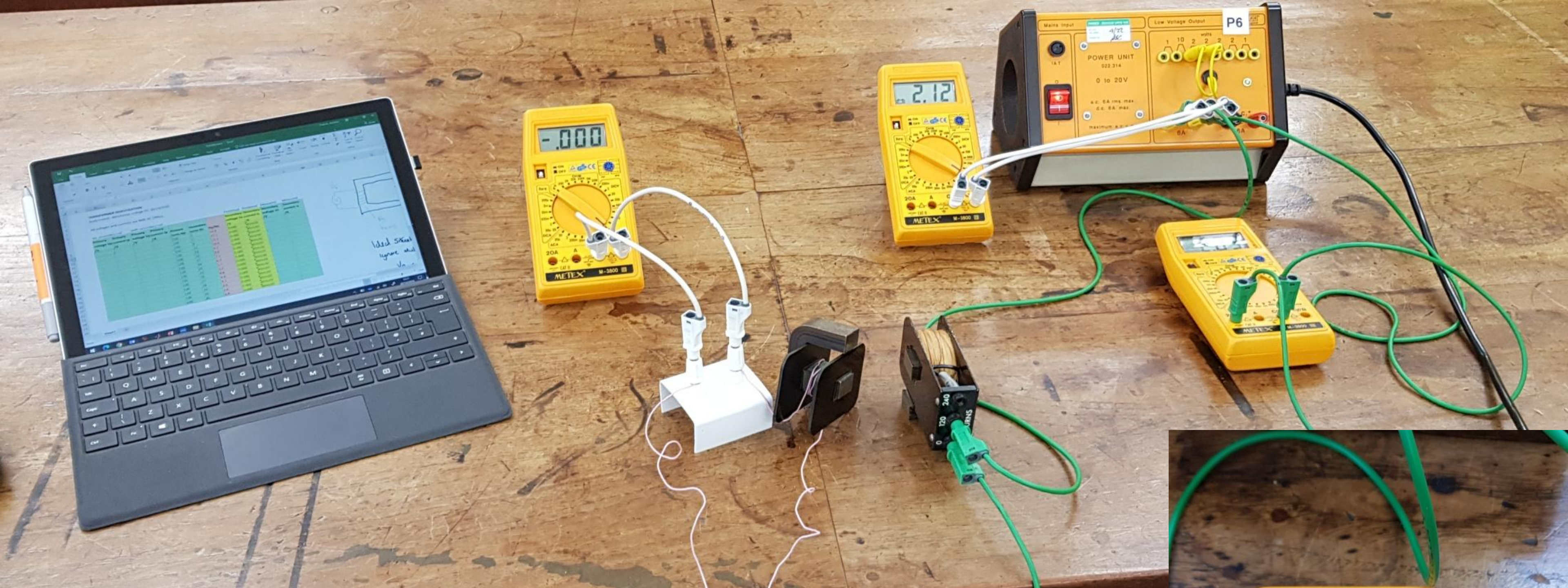
Wire fully wound (about thirty turns) and C-core placed within plastic holder.



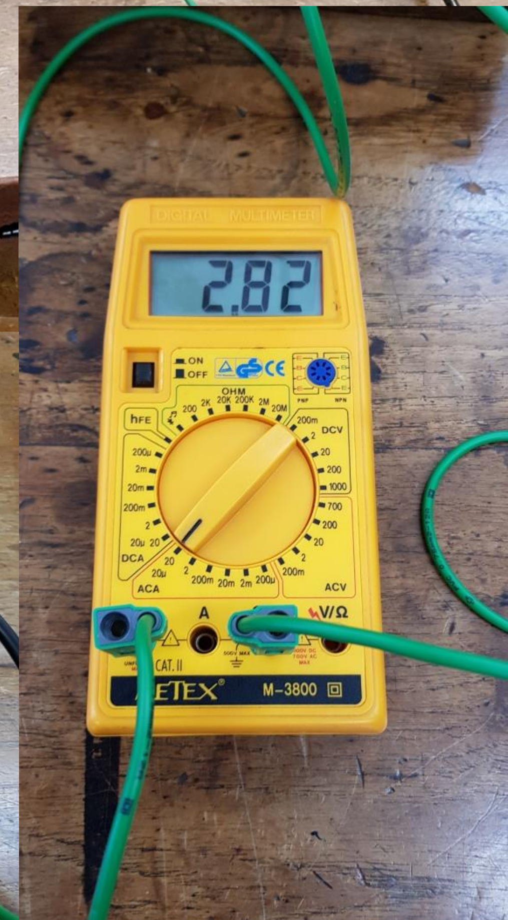
Magnetic circuit broken. Current only flows in **primary coil**.

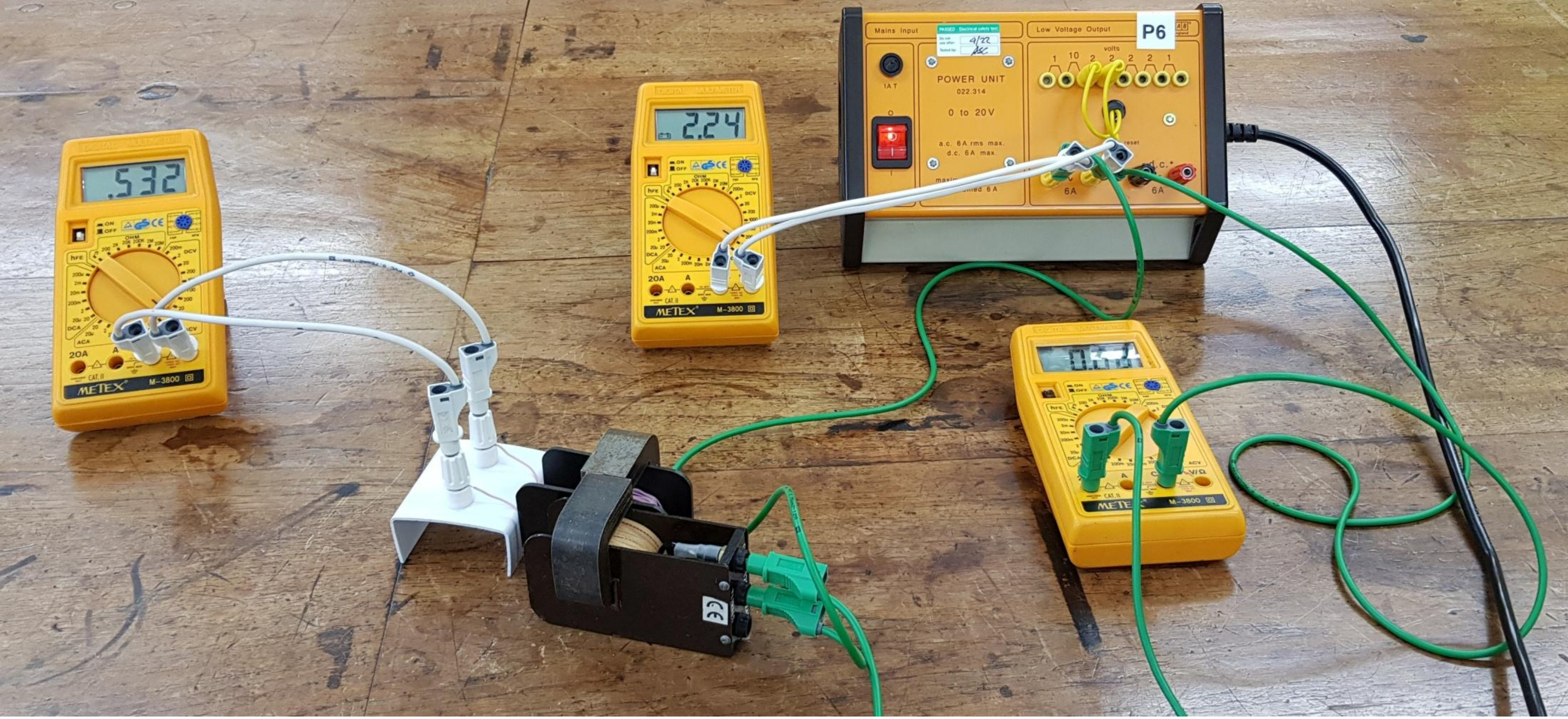
Magnetic circuit completed by pushing together the C-cores. A voltage will be induced across the secondary coil and AC currents will flow. i.e. electrical power will be transferred from the primary to the secondary coil.





For each experiment run, firstly record the RMS primary voltage and current without connection to the secondary circuit.



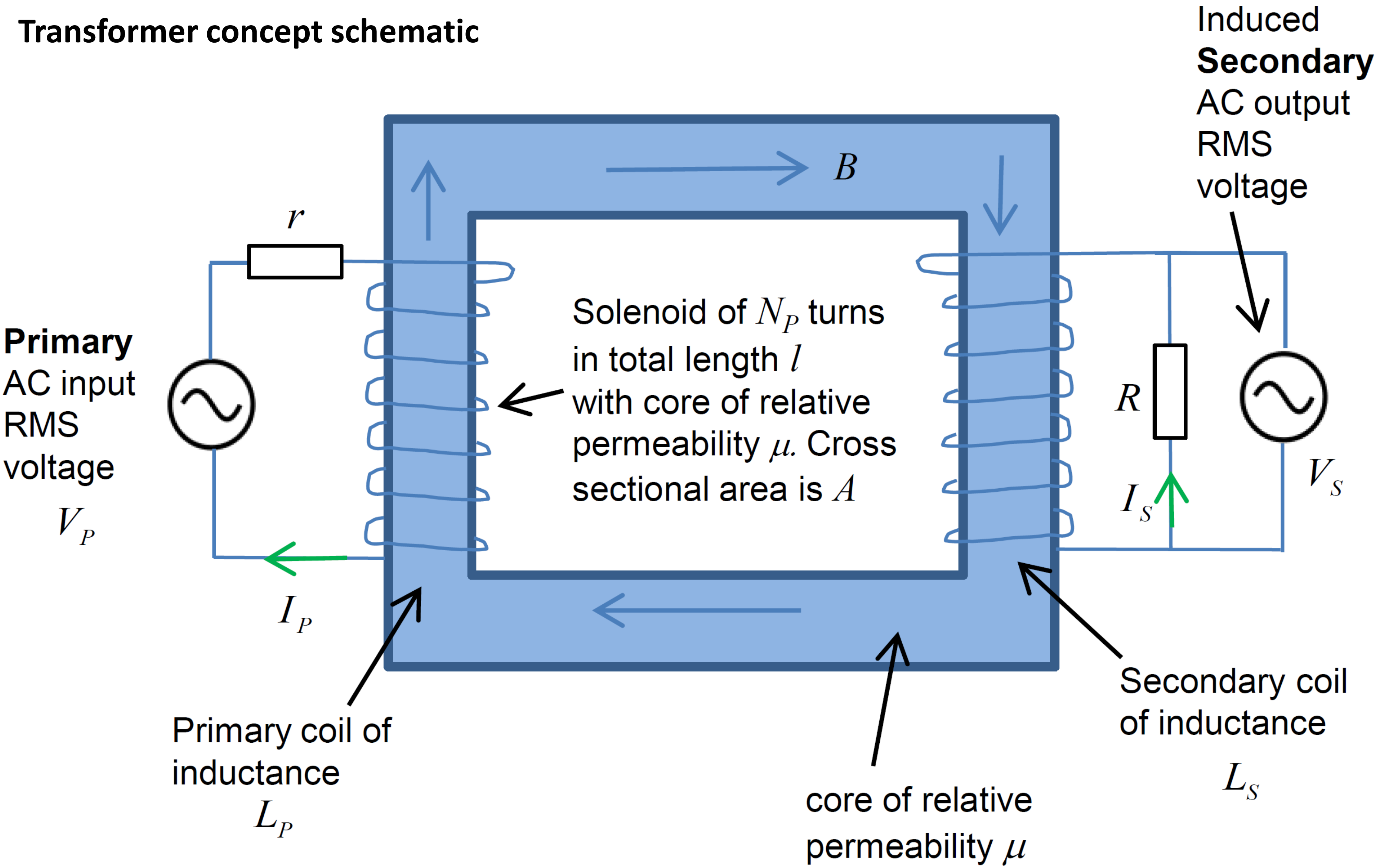


For each value of N_s, the number of secondary coil windings, record the primary voltage and current, and the induced secondary voltage. **Make sure the multimeters are in AC mode!**

You should observe the primary current to drop to near zero, and also perhaps a small change in the primary voltage.

Repeat the experiment for secondary windings in the range 5,7,9....29. Keep the number of primary coil turns at 120. Then repeat for 240 turns, this time unwinding from 29 to 5 secondary turns in steps of two turns as before.

Transformer concept schematic



Note in our case we assume the resistances r and R are small (only a few ohms).

Simplistic derivation of ideal transformer equation

Magnetic flux linked by primary and secondary coils is:

$$\Phi_S = N_S BA; \quad \Phi_P = N_P BA$$

This ignores the back EMF from the secondary coil, and any mutual induction.

$$V_S = -\frac{d\Phi_S}{dt} = -N_S A \frac{dB}{dt}$$

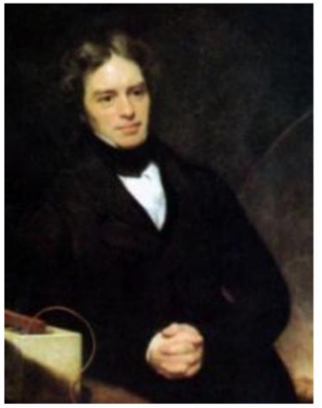
i.e. via Faraday's Law

$$V_P = -\frac{d\Phi_P}{dt} = -N_P A \frac{dB}{dt}$$

$$\therefore \boxed{\frac{V_S}{V_P} = \frac{N_S}{N_P}}$$

"Rate of change of magnetic flux linked is proportional to EMF induced"

The C-core pairing means the area A is the *same* for both primary and secondary sides of the transformer magnetic circuit.



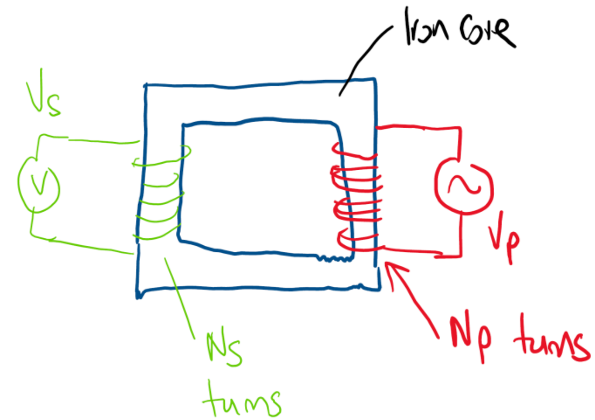
Michael Faraday
1791-1867

TRANSFORMER INVESTIGATION

Andy French. Winchester College P5. 30/10/2020

All voltages and currents are RMS AC (50Hz).

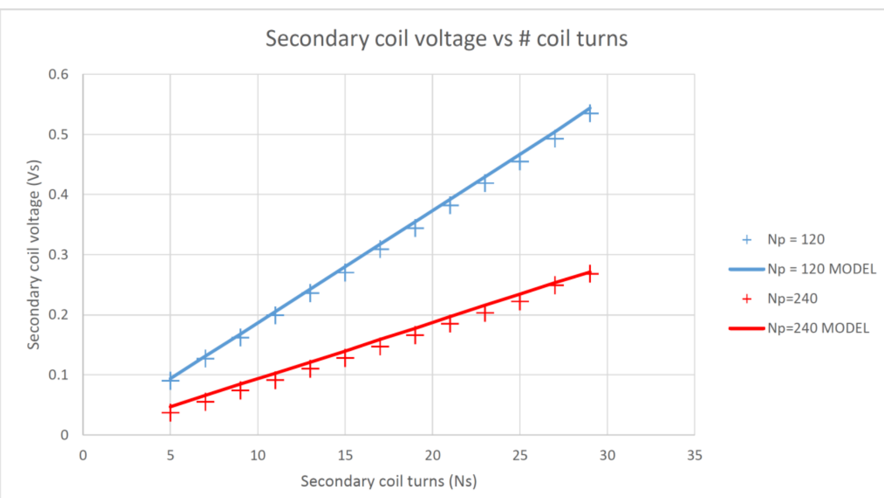
No secondary coil		With secondary coil					Predicted	Predicted	Measured
Primary voltage Vp /V	Primary current Ip /A	Primary voltage Vp /V	Primary current Ip /A	Primary turns Np	Secondary turns Ns	Np/Ns	Secondary voltage Vs /V	Secondary current Is /A	Secondary voltage Vs /V
2.12	2.82	2.24	0.08	120	5	24.0	0.093	67.7	0.09
2.12	2.84	2.24	0.08	120	7	17.1	0.131	48.7	0.127
2.12	2.82	2.24	0.09	120	9	13.3	0.168	37.6	0.162
2.12	2.82	2.24	0.09	120	11	10.9	0.205	30.8	0.199
2.12	2.85	2.24	0.08	120	13	9.2	0.243	26.3	0.236
2.12	2.85	2.24	0.17	120	15	8.0	0.280	22.8	0.27
2.12	2.82	2.24	0.07	120	17	7.1	0.317	19.9	0.309
2.12	2.86	2.24	0.07	120	19	6.3	0.355	18.1	0.344
2.12	2.87	2.24	0.07	120	21	5.7	0.392	16.4	0.382
2.12	2.88	2.24	0.07	120	23	5.2	0.429	15.0	0.419
2.12	2.88	2.24	0.07	120	25	4.8	0.467	13.8	0.455
2.12	2.89	2.24	0.06	120	27	4.4	0.504	12.8	0.493
2.12	2.90	2.25	0.07	120	29	4.1	0.544	12.0	0.535
2.22	0.97	2.24	0.00	240	29	8.3	0.271	8.0	0.268
2.22	0.99	2.25	0.00	240	27	8.9	0.253	8.8	0.249
2.22	0.99	2.25	0.00	240	25	9.6	0.234	9.5	0.222
2.23	0.99	2.25	0.00	240	23	10.4	0.216	10.3	0.203
2.22	0.98	2.25	0.00	240	21	11.4	0.197	11.2	0.185
2.21	0.99	2.24	0.00	240	19	12.6	0.177	12.5	0.166
2.21	1.00	2.24	0.00	240	17	14.1	0.159	14.1	0.147
2.21	0.99	2.23	0.00	240	15	16.0	0.139	15.8	0.128
2.2	0.98	2.23	0.00	240	13	18.5	0.121	18.1	0.11
2.2	0.98	2.23	0.00	240	11	21.8	0.102	21.4	0.091
2.2	0.99	2.25	0.00	240	9	26.7	0.084	26.4	0.074
2.22	1.00	2.25	0.00	240	7	34.3	0.066	34.3	0.055
2.22	1.00	2.25	0.00	240	5	48.0	0.047	48.0	0.037



Ideal situation (no losses ignore mutual induction etc)

$$\frac{V_p}{V_s} \simeq \frac{N_p}{N_s}$$ (Faraday's law but no back EMF etc)

$$I_p V_p = I_s V_s$$ (100% power transmission)



For the predicted secondary voltages and currents, the primary voltage is taken **as that measured when the C-cores are connected.** Note this appears to be *always higher* than when the secondary circuit is disconnected.

If 100% of power is transmitted to the secondary coil, **then very large currents would be predicted.** Although a lack of a high current (secondary) ammeter means we cannot measure this current, one might anticipate it to be lower than predicted, given the heating of the iron cores implies significant energy losses. (We have also not considered mutual induction effects).

Assuming 100% power transfer

$$I_S V_S = I_P V_P$$

$$\therefore I_S = \frac{I_P V_P}{V_S}$$

$$\therefore \boxed{I_S = I_P \frac{N_P}{N_S}}$$

TRANSFORMER INVESTIGATION

Andy French. Winchester College P5. 30/10/2020

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Primary voltage Vp /V	**Primary current Ip /A**	**Primary voltage Vp /V**	**Primary current Ip /A**	**Primary turns Np**	**Secondary turns Ns**	**Np/Ns**	**Secondary voltage Vs /V**	**Secondary current Is /A**	**Secondary voltage Vs /V**
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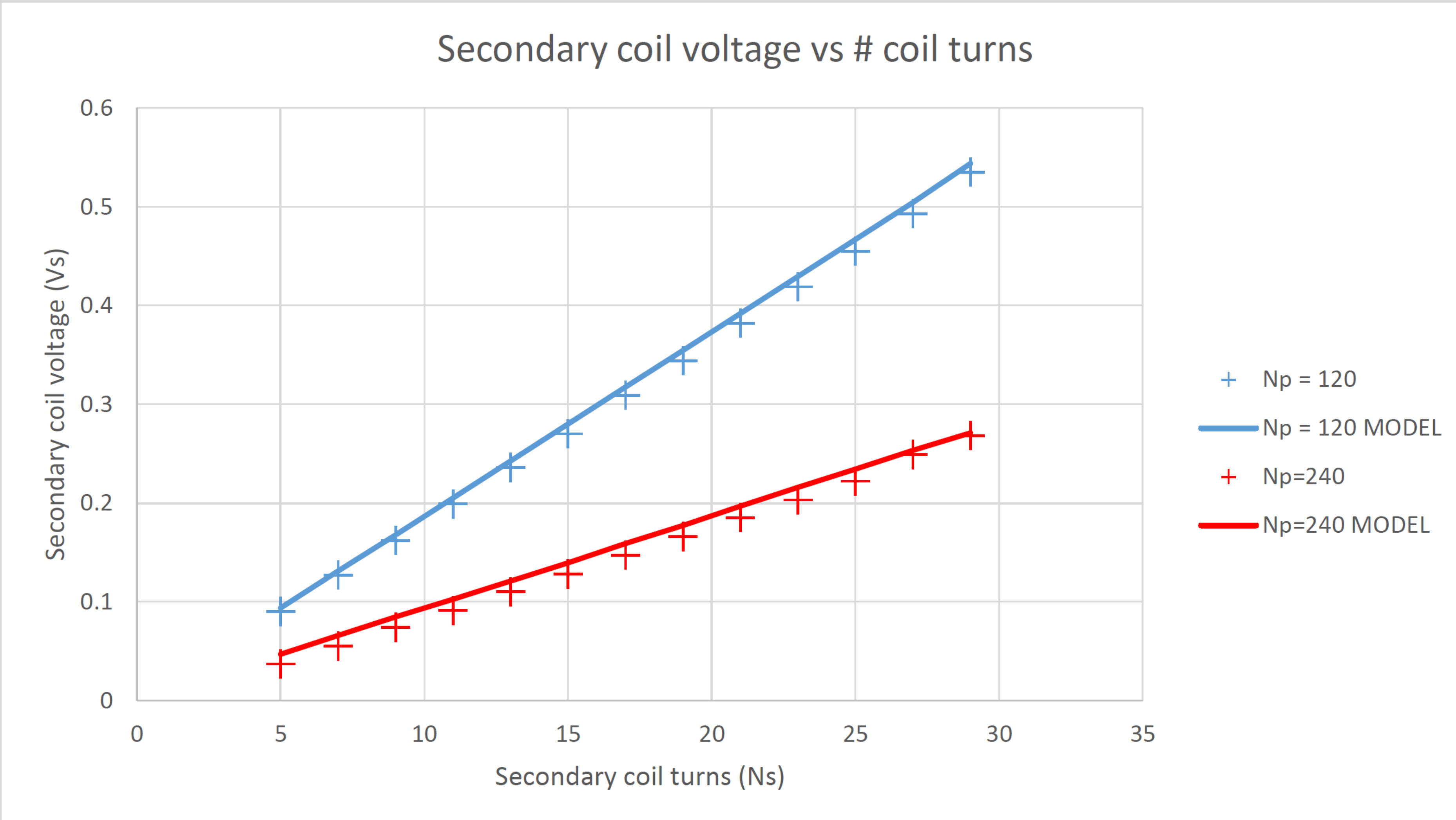
2.21	0.99	2.23	0.00	240	15	16.0	0.139	15.8	0.128
2.2	0.98	2.23	0.00	240	13	18.5	0.121	18.1	0.11

Note when the cores are connected the primary voltage rises. So the induced current could in principle be *higher* that what is presented. But this is probably not the case (see later!)

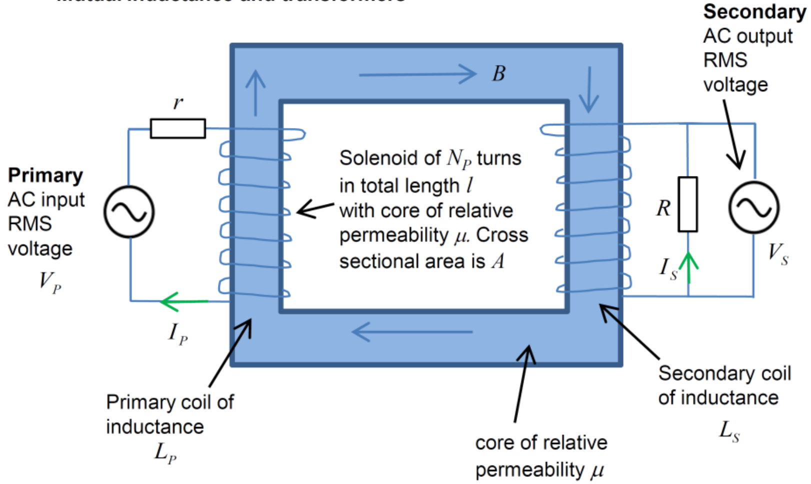


A strong linear correlation is in evidence for both 120 and 240 primary turns, indicating the ‘transformer equation’ is a reasonable model. However, the predicted secondary voltage is slightly higher, *by a possibly systematic amount*, than the measured voltage.

Model:

$$\boxed{V_S = \frac{V_P}{N_P} N_S}$$

Mutual inductance and transformers



For *coupled* inductors, connected within a magnetic circuit mediated by a soft magnetic core such as iron (i.e. where it will lose its magnetism once current is switched off in the solenoids), if we apply Kirchoff's Second Law:

Applied EMF minus back EMF + EMF due to mutual induction*

$$\boxed{\begin{aligned} V_P - L_P\frac{dI_P}{dt} + M\frac{dI_S}{dt} &= I_P r \\ M\frac{dI_P}{dt} - L_S\frac{dI_S}{dt} &= I_S R = V_S \end{aligned}}$$

Primary solenoid

Secondary solenoid

M is the **mutual inductance** between the solenoid coils. Let us consider steady state solutions to the coupled differential equations above. It makes sense that the secondary AC output is at the same frequency, but different phase and amplitude as the input.

Actual values will be the *real parts* of these

$$V_P = V_0 e^{i\omega t}$$

$$I_P = \frac{V_0}{r}Ae^{i\omega t}$$

$$I_S = \frac{V_0}{R}Be^{i\omega t}$$

Note constants A and B will be **complex numbers** to account for phase differences

Substituting these into Kirchoff's Second Law and dividing by $V_0 e^{i\omega t}$

$$1 - \frac{L_P i\omega A}{r} + \frac{Mi\omega B}{R} = A$$

$$\frac{Mi\omega A}{r} - \frac{L_S i\omega B}{R} = B$$

$$\therefore B = \frac{iM\omega A}{r\left(1+\frac{L_S i\omega}{R}\right)} = \frac{iM\omega RA}{r\left(R+L_S i\omega\right)}$$

Substituting for B in the Primary coil equation

$$1 - \frac{L_P i\omega A}{r} + \frac{Mi\omega}{R}\left(\frac{iM\omega RA}{r\left(R+L_S i\omega\right)}\right) = A$$

$$1 - \frac{L_P i\omega A}{r} - \frac{M^2\omega^2 A}{r\left(R+L_S i\omega\right)} = A$$

$$\therefore 1 = A\left(1 + \frac{L_P i\omega}{r} + \frac{M^2\omega^2}{r\left(R+L_S i\omega\right)}\right)$$

$$\boxed{\begin{aligned} \therefore A &= \left(1 + \frac{L_P i\omega}{r} + \frac{M^2\omega^2}{r\left(R+L_S i\omega\right)}\right)^{-1} \\ \therefore B &= \frac{iM\omega RA}{r\left(R+L_S i\omega\right)} \end{aligned}}$$

The **power** dissipated in the loads r and R is given by**

$$\boxed{\begin{aligned} P_P &= \tfrac{1}{2}\mathrm{Re}\left(I_P r \times I_P^*\right) \\ P_S &= \tfrac{1}{2}\mathrm{Re}\left(I_S R \times I_S^*\right) \end{aligned}}$$

It can be shown that the **Ideal Transformer Equation** is applicable in a high frequency, low secondary current and high coupling situation:

$$\omega \gg 1$$
$$R \gg r$$
$$k \to 1$$

$$\boxed{\left|\frac{V_P}{V_S}\right| \to \sqrt{\frac{L_P}{L_S}} = \frac{N_P}{N_S}}$$

in our case since the coil lengths and areas are the same

**

$$P = \overline{\mathrm{Re}(V)\times\mathrm{Re}(I)}$$

$$V = V_R + iV_I$$

$$I = I_R + iI_I$$

$$VI^* = \left(V_R + iV_I\right)\left(I_R - iI_I\right)$$

$$VI^* = V_R I_R + V_I I_I + i\left(V_I I_R + V_R I_I\right)$$

$$\therefore \boxed{\tfrac{1}{2}\mathrm{Re}\left(VI^*\right) = \tfrac{1}{2}\overline{V_R I_R} + \tfrac{1}{2}\overline{V_I I_I}}$$

The inductances are given by (see page 1):

$$\boxed{L_P = \frac{\mu\mu_0 N_P^2 A}{l} \qquad L_S = \frac{\mu\mu_0 N_S^2 A}{l} \qquad \therefore \frac{L_P}{L_S} = \left(\frac{N_P}{N_S}\right)^2}$$

Define a dimensionless **coupling constant** from which we can define the **mutual inductance**

$$\boxed{M = k\sqrt{L_P L_S}} \qquad k \geq 0$$

Now consider the quantity

$$E = \tfrac{1}{2}L_P I_P^2 + \tfrac{1}{2}L_S I_S^2 + MI_S I_P$$

$$\therefore \frac{dE}{dt} = L_P I_P\frac{dI_P}{dt} + L_S I_S\frac{dI_S}{dt} + MI_S\frac{dI_S}{dt} + MI_P\frac{dI_P}{dt}$$

which is essentially *the rate of power flowing in the inductors*. This must be greater than zero, hence:

$$\tfrac{1}{2}L_P I_P^2 + \tfrac{1}{2}L_S I_S^2 + MI_S I_P \geq 0$$

$$\tfrac{1}{2}\begin{pmatrix} I_P & I_S \end{pmatrix}\begin{pmatrix} L_P & M \\ M & L_S \end{pmatrix}\begin{pmatrix} I_P \\ I_S \end{pmatrix} \geq 0$$

$$\therefore \begin{vmatrix} L_P & M \\ M & L_S \end{vmatrix} \geq 0$$

$$\therefore L_P L_S - M^2 \geq 0$$

$$\therefore M \leq \sqrt{L_P L_S}$$

$$\therefore \boxed{0 \leq k \leq 1}$$

So the maximum value for the coupling constant is *unity*.

Simplistic derivation of ideal transformer equation

Magnetic flux linked by primary and secondary coils is:

$$\Phi_S = N_S BA; \quad \Phi_P = N_P BA$$

$$V_S = -\frac{d\Phi_S}{dt} = -N_S A\frac{dB}{dt}$$

$$V_P = -\frac{d\Phi_P}{dt} = -N_P A\frac{dB}{dt}$$

i.e. via Faraday's Law

$$\therefore \boxed{\frac{V_S}{V_P} = \frac{N_S}{N_P}}$$

Incorporating mutual inductance

From Eclecticon note: [Electromagnetic induction generators & transformers]()

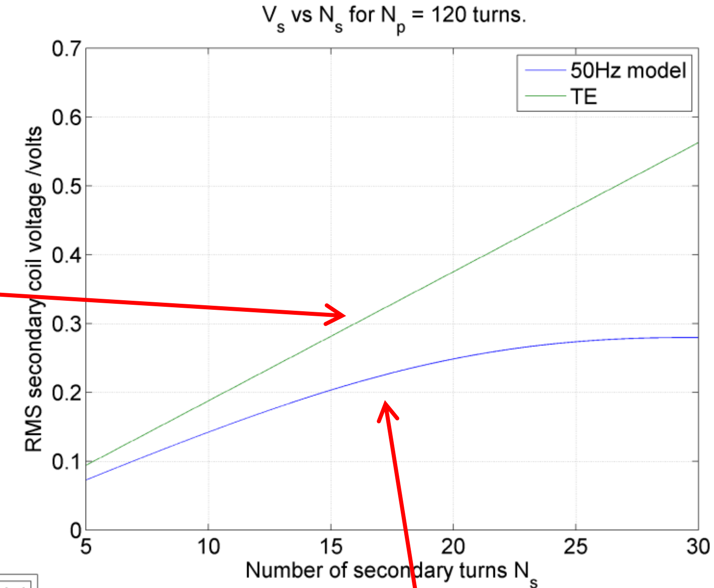
```matlab
%transformer
% Model of a double C-core transformer. The primary coil has a fixed
% number of turns, but the number of secondary windings can be varied.

function transformer

%Secondary turns
Ns = linspace(5,30,1000);

%Number of primary turns
Np = 120;

%Input voltage (RMC AC) to primary coil
Vp_rms = 2.25;

%Relative permeability of iron C-cores
mu = 1000;

%Resistance of primary coil /ohms
r = 5;

%Resistance of seondary coil /ohms
R = 1;

%Mutual inductance coupling constant
k = 0.9;

%AC frequency /Hz
f = 50;

%Primary coil length /m
lp = 1.5/100;

%Secondary coil length /m
ls = 2.0/100;

%C-core cross sectional area /m^2
A = 2.0e-2 * 1.5e-2;

%Permeability of free space
mu0 = 4*pi*1e-7;
```

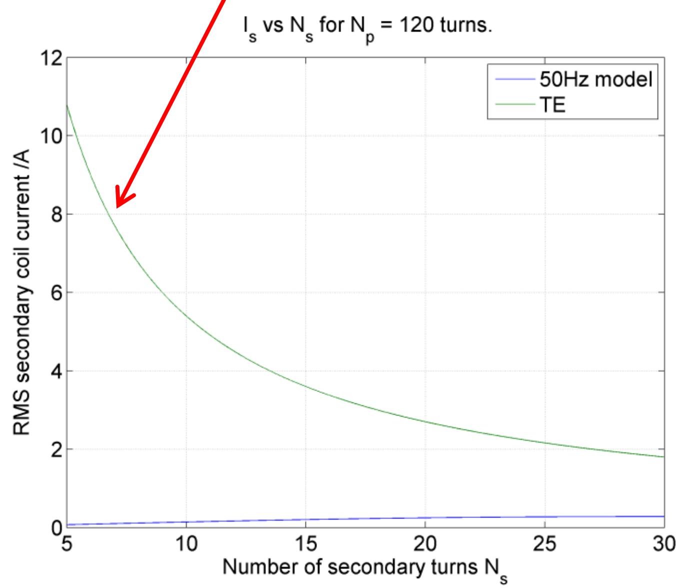
MATLAB model

Transformer Equation (TE) model

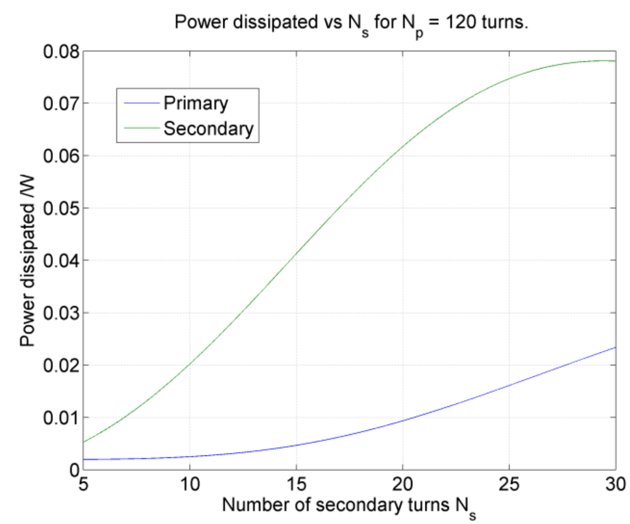
$$V_S = \frac{V_P}{N_P} N_S$$

$$I_S = I_P \frac{N_P}{N_S}$$





Note in our case the TE model works well for Vs prediction. This means so some of the model input parameters must be incorrect (r, R in particular, also k and μ).



Using the estimates for parameters on the left, one expects a *much* lower secondary current than the transformer equation suggests. i.e. anticipate the mutual induction effects to be significant.