Vectors & Vector Addition

Vector: a quantity with both magnitude & direction

Examples: displacement, velocity, acceleration
All Forces, including weight
Momentum

Why are they important?

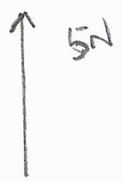
Analyse a game of snooker?

What happens? Clearly, without a direction you cannot answer this question. Are they going to collide? Are they going in opposite directions? Direction is critical.

Visual representation of vectors:

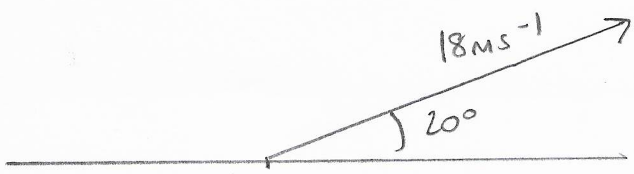
Vectors can be represented as an arrow in an associated direction

For example: A 5N force is exerted vertically upwards.



Note: vectors can be drawn to scale e.g. 1cm : 1N - more later.

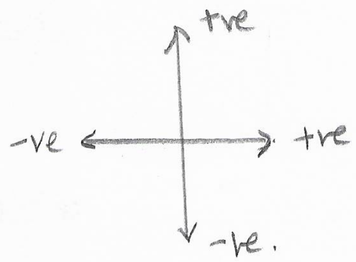
A ball is fired an a velocity of $18ms^{-1}$ at an angle of $20°$ from the horizontal



Cartesian Convention

The cartesian convention is that up is positive and to the right is positive (like a graph)

Therefore 'g' is an acceleration downwards, so would be



$$g = -10\,ms^{-2}$$

the negative denotes downwards.

Addition of Vectors

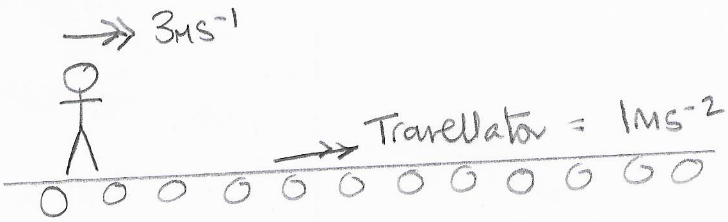
We will look at several methods:

1. Colinear vectors
2. Perpendicular vectors
3. Non-perpendicular vectors.

Colinear Vectors

Colinear means "in the same line"

Imagine a man walking on a travellator at the airport.

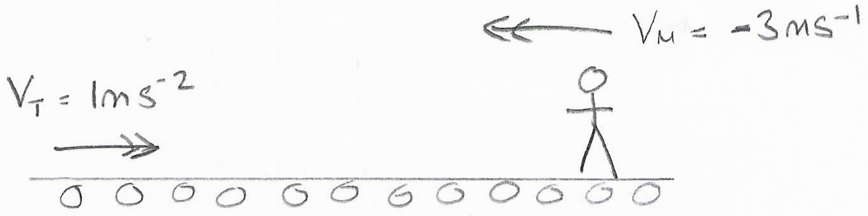


The man and the travellator are in the same line & both moving to the right

$\therefore$ Resultant velocity $= +3\,ms^{-1} + 1\,ms^{-1}$

$= \underline{4\,ms^{-1}}$

On reaching the end, the man turns around & walks against the flow of the travellator

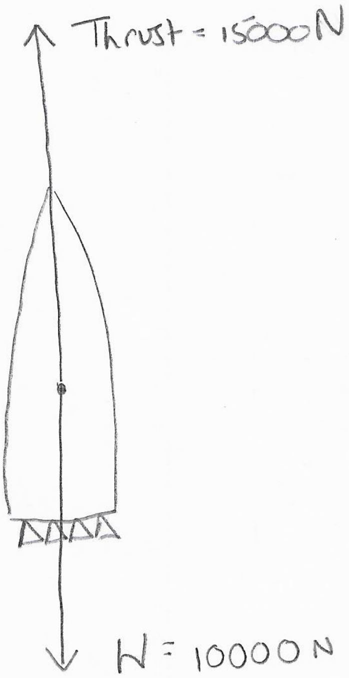


Therefore, the resultant velocity is

$$V_r = 1ms^{-1} - 3ms^{-1}$$
$$= \underline{\underline{-2ms^{-1}}}$$

the minus denotes that he is moving to the left.

An example with forces:



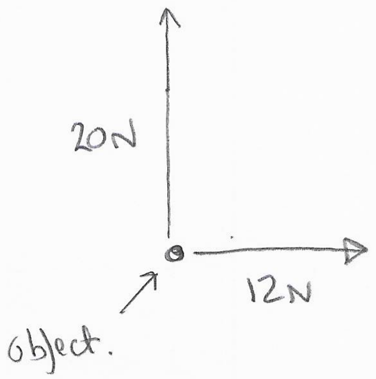
A mini space rocket has a weight of 10,000N. To take off the Thrust must be greater than the weight.

The forces are co-linear ∴ the Resultant Force (unbalanced force) acting on the rocket is:

$$+15000N - 10000N = \underline{+5000N}$$

weight is negative because it is downwards.

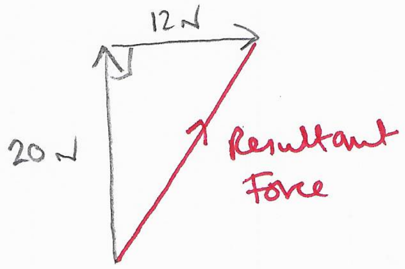
Perpendicular Addition of vectors



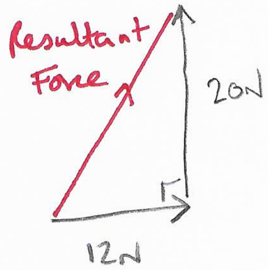
If two forces are acting on an object, we can determine the single force that could be applied that would have the same effect as these two independent forces. This is known as the RESULTANT FORCE.

Redraw the diagram by adding the forces tip to tail.

e.g.

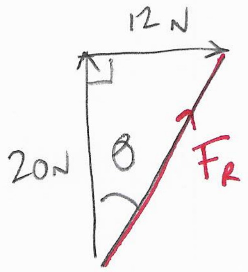


or



As you can see, it does not matter which way around I draw the forces, the resultant force is the same in both diagrams.

We can determine the resultant force using phythagorus' theorum and the angle using trigonometry



$$F_R = \sqrt{20^2 + 12^2}$$

$$F_R = 23.32\text{N}$$

$$\tan\theta = \frac{12}{20}$$

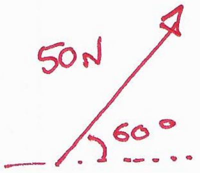
$$\theta = \tan^{-1}\left(\frac{12}{20}\right)$$

$$\theta = 31^\circ$$

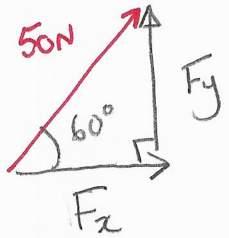
$F_R = 23.32$ N @ 31° from the vertical.

Conversely, we can resolve vector components from a force.

Note: whilst refered to in terms of x & y planes, vectors are coordinate independent



In this case, our 50N force could be replaced by a horizontal force & a vertical forces, F_x & F_y.



Using trigonometry

$$\sin 60^\circ = \frac{F_y}{50N}$$

$$F_y = 50 \times \sin 60$$

$$F_y = 43.3N.$$

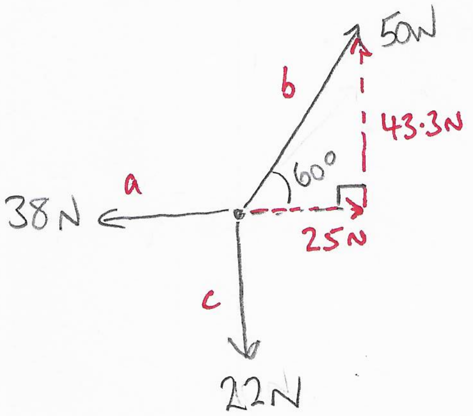
$$\cos 60 = \frac{F_x}{50N}$$

$$F_x = 25N.$$

Apply a horizontal force of 25N & a vertical force of 43.3N would be the same affect as applying a 50N force at 60° from the horizontal.

Why is the helpful? If forces are applied at angles, they can be resolved into horizontal and vertical vectors. They are then co-linear so can be added/subtracted

An example:



Three forces are applied to an object. What is the resultant force on the object.

From above, we know that Force b can be resolved into:

$F_y = 43.3N$ & $F_x = 22N$

Force a & F_x are now colinear:

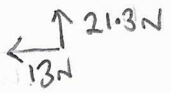
$\therefore$ In the x plane $= 25N - 38N$

$= -13N$

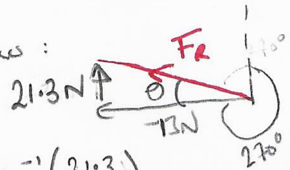
Force c & F_y are colinear:

$43.3N - 22N = 21.3N$

So the resultant is



Redraw:



$F_R = \sqrt{21.3^2 + 13^2}$

$F_R = 24.95N$

@ 328.6°

$\theta = \tan^{-1}\left(\frac{21.3}{13}\right)$

$\theta = 58.6°$

Scale Drawings

Vector additions can be solved using SCALE DRAWING. This is when each vector is represented to scale e.g. 1cm : 1N, and the drawn in such a way that the RESULTANT FORCE can be determined :

The Method :

1. Study the question and identify the forces.
2. Calculate an suitable scale e.g. 1cm : 5N ; 1cm : 500 N etc.
3. Determine the length in cm of each vector
4. Visualise the added 'tip to tail' - work out how you will fit the answer in the available space.
5. Draw the first vector.
6. Add subsequent vectors 'tip to tail'.
7. Draw in resultant force, calculate length in cm, use scale to convert to Newtons, ms^{-1} etc.
8. Determine angle.
9. Write out resultant vector : magnitude & angle.

You need :

1. Ruler
2. Sharp pencil
3. Protractor
4. Rubber...

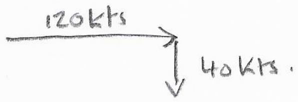
The process is about precision : be accurate & neat.

Scale Drawing - Example 1.

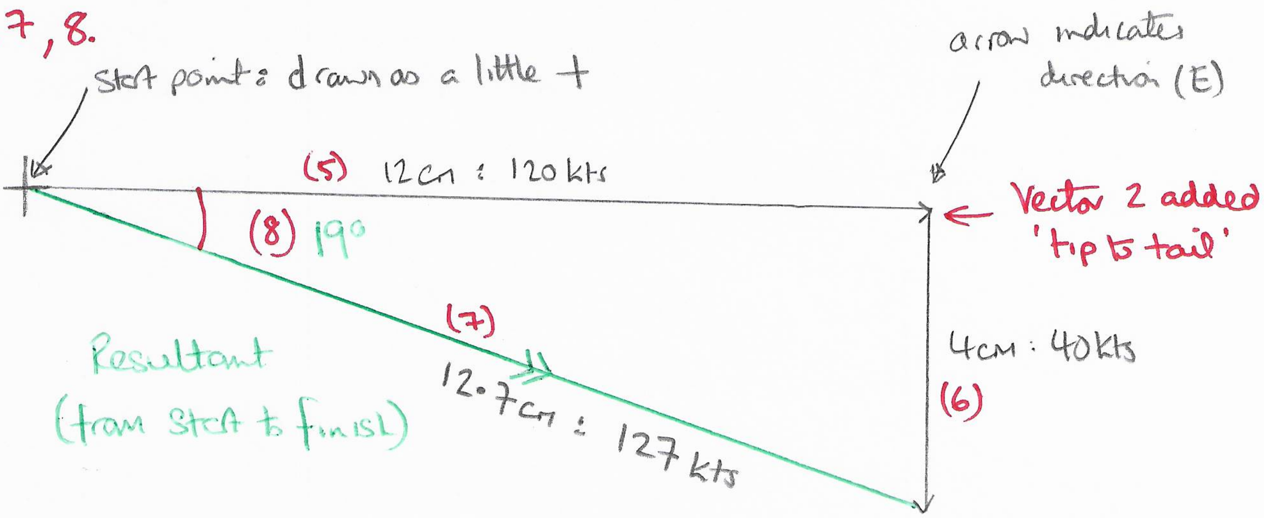
A helicopter is flying due East at 120 knots. The is a stray crosswind from the North (i.e. N → S) of 40kts. What is the aircraft's actual groundspeed & direction?

The red pen represents the steps of the methodology.

1. 120 kts East
   40 kts South
2. Scale 1cm : 10kts
3. 120kts – 12cm ; 40kts – 4cm.
4. Will roughly look like

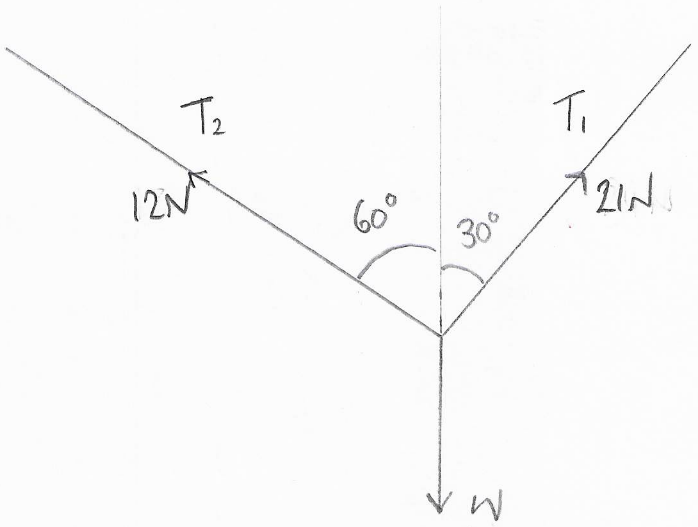


5, 6, 7, 8.



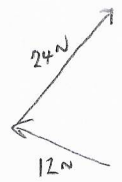
9. Answer: 127kts @ 109°

Scale Drawing 1 Example 2



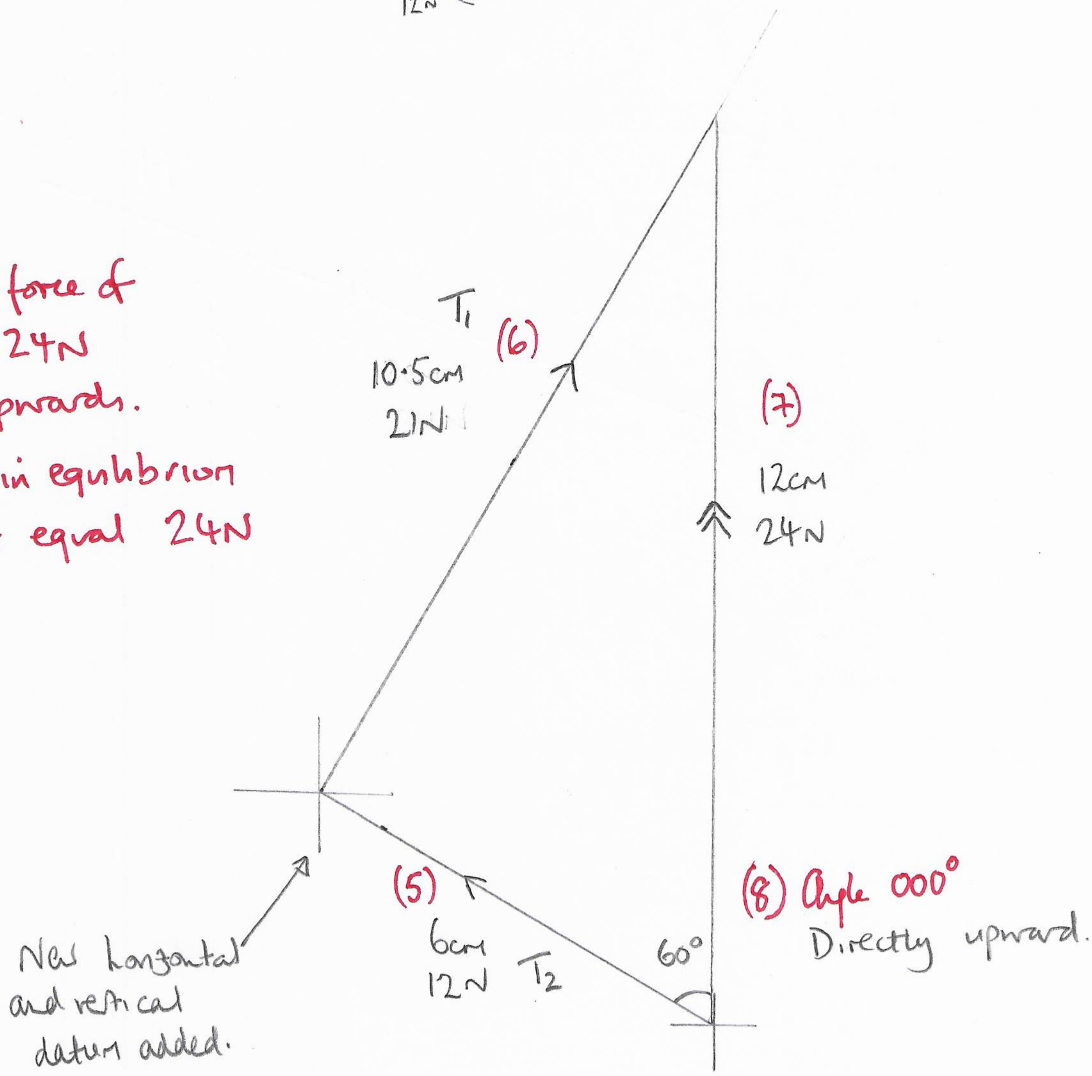
A weight, W, is suspended as shown. Calculate the resultant force upwards provided by T_1 & T_2.
You may assume that the system is in equilibrium.

1. $T_2 = 12N$; T_1 $21N$
2. Scale ⇒ 1cm : 2N
3. $T_1 = 6cm$; $T_2 = 10.5cm$.
4. Will roughly look like :



5, 6, 7, 8

9. Resultant force of T_1 & T_2 is 24N directly upwards.
If system is in equilibrium the W must equal 24N downwards.

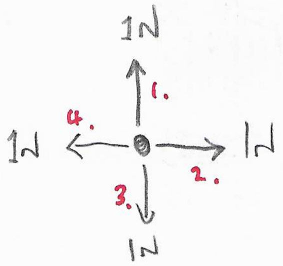


Systems in Equlibrium.

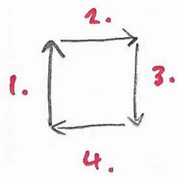
For a system to be in equlibrium, there are two conditions:

1. There must be no resultant force acting on it
2. There must be no resultant turning moment acting on it.

Example 1:



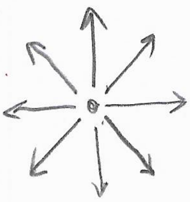
Draw the forces 'tip to tail'



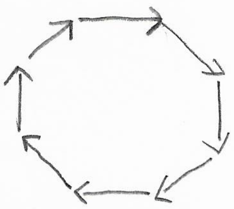
It is a closed loop ∴ no resultant force acts.

Example 2:

all forces are 1N

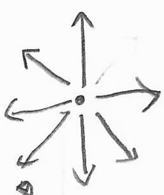


redraw:

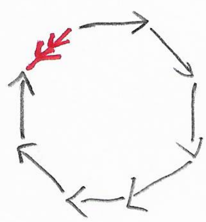


Closed system no resultant force.

Example 3:



This force is not balanced.



System not in equlibrium. Object moves in direction of resultant force