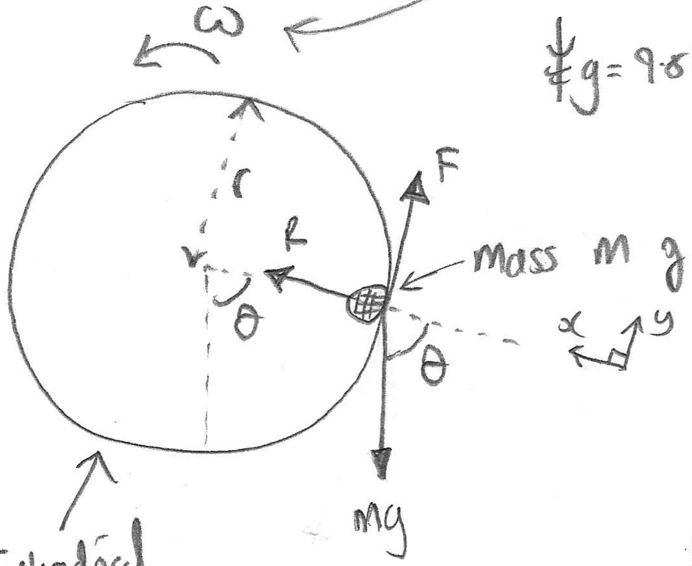
WASHING MACHINE

Constant, so no tangential acceleration



← mass m of washing *

Cylindrical drum

$g = 9.81\,\mathrm{N/kg}$

Newton II:

(Radially inwards)

$/\!/x: \quad \boxed{mr\omega^2 = R - mg\cos\theta}$ ①

$/\!/y: \quad \boxed{0 = F - mg\sin\theta}$ ②

(tangential to drum)

No slip: $\boxed{F \leq \mu R}$ ③

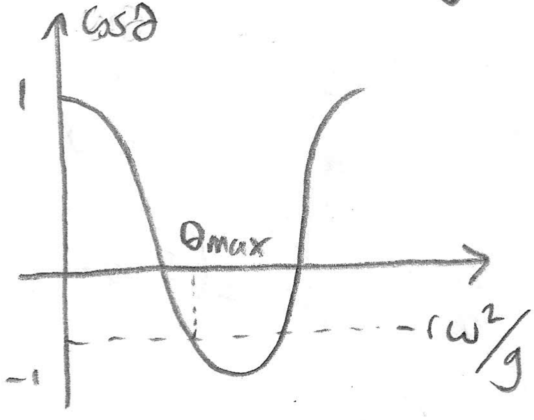
In contact with drum $\boxed{R > 0}$

* Ignore extension – treat as a point mass on surface of drum
obviously this is a somewhat crude model!

So $\boxed{\begin{aligned} R &= mr\omega^2 + mg\cos\theta \\ F &= mg\sin\theta \end{aligned}}$

For contact: $mr\omega^2 + mg\cos\theta > 0$

so $\cos\theta > -\dfrac{r\omega^2}{g}$



Now $\cos\theta \geqslant -1$

So if $\dfrac{r\omega^2}{g} > 1$ this

means $\cos\theta > -\dfrac{r\omega^2}{g}$ is true

$\forall\, \theta$. ie $R > 0 \ \forall\, \theta$, so

contact with drum is maintained.

$\therefore$ for $R > 0 \ \forall\, \theta$

$\Rightarrow \boxed{\omega > \sqrt{\dfrac{g}{r}}}$

[$r = 0.3\,\mathrm{m}$, $g = 9.81\,\mathrm{N/kg}$ $\Rightarrow \omega > 5.72\,\mathrm{rads^{-1}}$

$\Rightarrow \omega > \boxed{54.6\ \mathrm{RPM}}$]

Now for no slip: $\boxed{F \leq \mu R}$

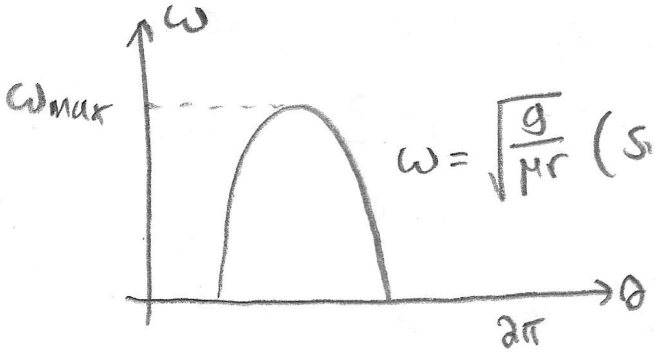
$\therefore \quad mg\sin\theta \leq \mu\left(mr\omega^2 + mg\cos\theta\right)$

$$\therefore \quad g\sin\theta \leq \mu\left(r\omega^2 + g\cos\theta\right)$$

$$\therefore \quad \omega^2 \geq \frac{g}{\mu r}\sin\theta - \frac{g\cos\theta}{r}$$

$$\omega^2 \geq \frac{g}{\mu r}\left(\sin\theta - \mu\cos\theta\right)$$

$$\boxed{\omega \geq \sqrt{\frac{g}{\mu r}\left(\sin\theta - \mu\cos\theta\right)}}$$



$$\omega = \sqrt{\frac{g}{\mu r}}\left(\sin\theta - \mu\cos\theta\right)^{\frac{1}{2}}$$

real!

[only valid when
$\sin\theta - \mu\cos\theta > 0$
$\sin\theta > \mu\cos\theta$
$\boxed{\tan\theta > \mu}$]

$$\left.\frac{d\omega}{d\theta}\right|_{\omega=\omega_{max}} = 0$$

So if $\omega > \omega_{max}$ then no slip $\forall\theta$.

So for no slip and contact $\forall\theta$, ω is $>$ than max of $\sqrt{g/r}$ and $\sqrt{g/r}\,\frac{1}{\sqrt{\mu}}(1+\mu^2)^{\frac{1}{4}}$

$$\frac{d\omega}{d\theta} = \sqrt{\frac{g}{\mu r}}\,\frac{1}{2}\left(\sin\theta - \mu\cos\theta\right)^{-\frac{1}{2}}\left(\cos\theta + \mu\sin\theta\right)$$

So $\dfrac{d\omega}{d\theta} = 0$ when $\cos\theta = -\mu\sin\theta$

So $$\omega_{max} = \sqrt{\frac{g}{\mu r}}\left(\sin\theta - \mu(-\mu\sin\theta)\right)^{\frac{1}{2}}$$

$$\omega_{max} = \sqrt{\frac{g}{\mu r}}\left(\sin\theta + \mu^2\sin\theta\right)^{\frac{1}{2}} = \sqrt{\frac{g}{\mu r}}\sqrt{\sin\theta}\left(1+\mu^2\right)^{\frac{1}{2}}$$

Now $\cos^2\theta = \mu^2\sin^2\theta$

$$1 - \sin^2\theta = \mu^2\sin^2\theta$$
$$1 = (\mu^2+1)\sin^2\theta$$
$$\frac{1}{1+\mu^2} = \sin^2\theta$$

$$\therefore \quad \sin\theta = \frac{1}{(1+\mu^2)^{\frac{1}{2}}}$$

$$\therefore \quad \boxed{\omega_{max} = \sqrt{\frac{g}{\mu r}}\left(1+\mu^2\right)^{\frac{1}{4}}}$$

So for $r = 0.3\,\mathrm{m}$, $\mu = \frac{1}{\sqrt{3}}$ $\quad \omega_{max} = \boxed{77.2\ \mathrm{RPM}}$