Water boiling point vs pressure practical challenge workbook. Physics Olympiad training camp

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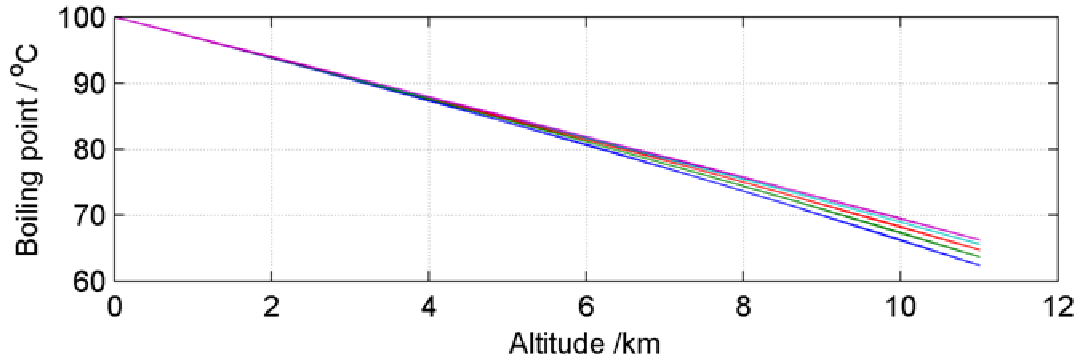
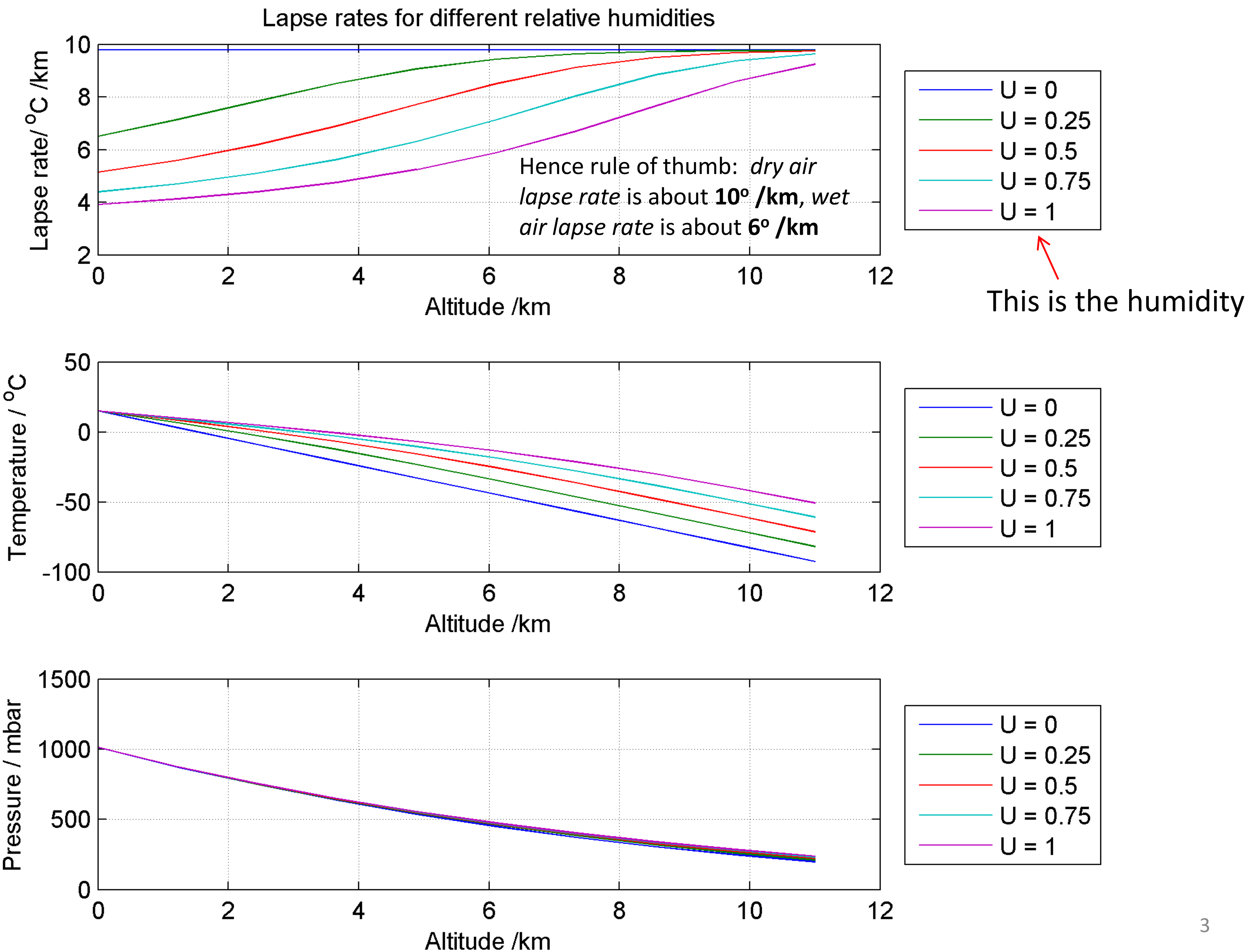
PART 1 – COMPLETE THIS AND SUBMIT RESULTS FROM QUESTION 7 BEFORE LOOKING AT PART 2

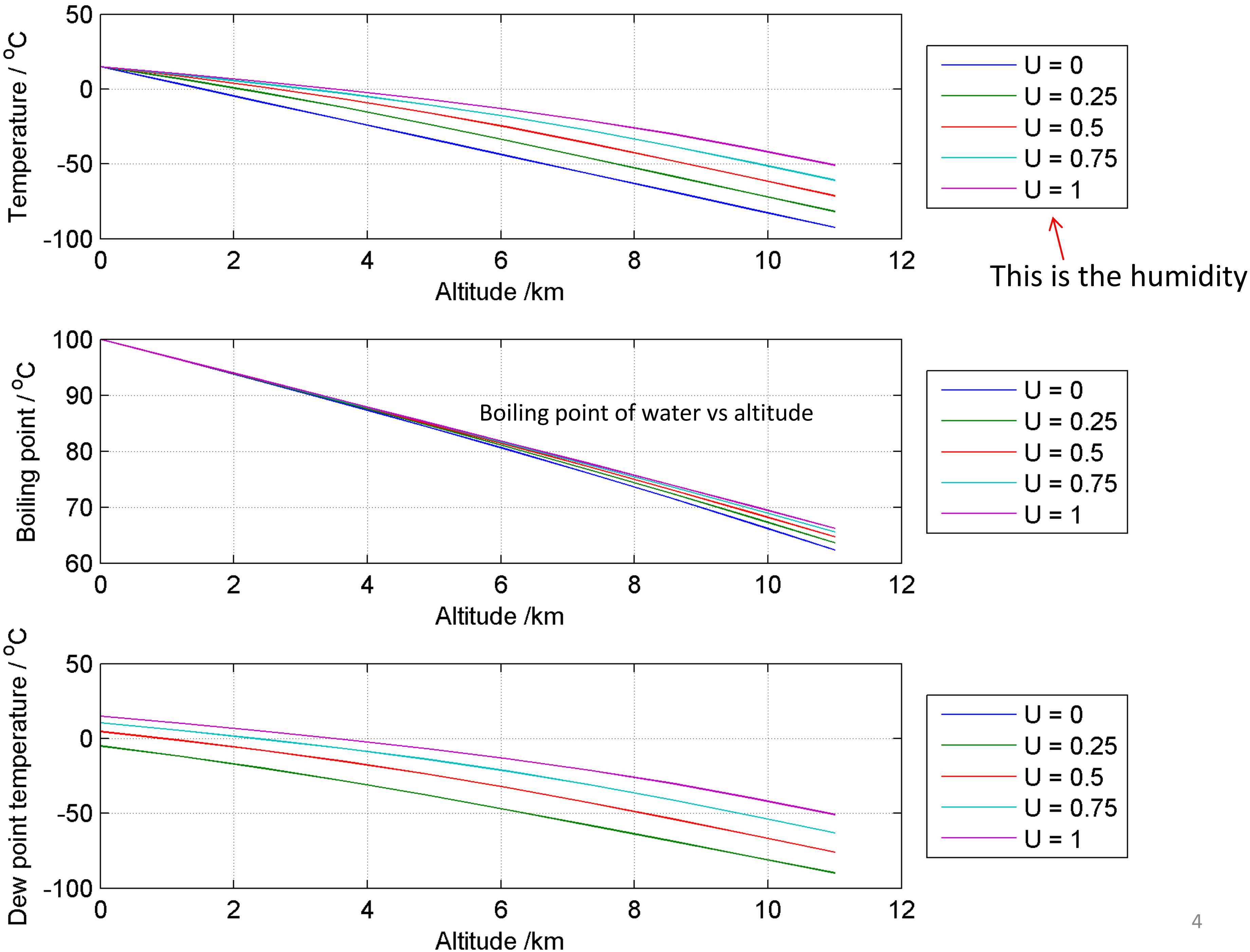
BPhO
British Physics Olympiad

Motivation: The boiling point of water *decreases* as atmospheric pressure *decreases*. You can observe this on a mountaineering expedition as you ascend in altitude.

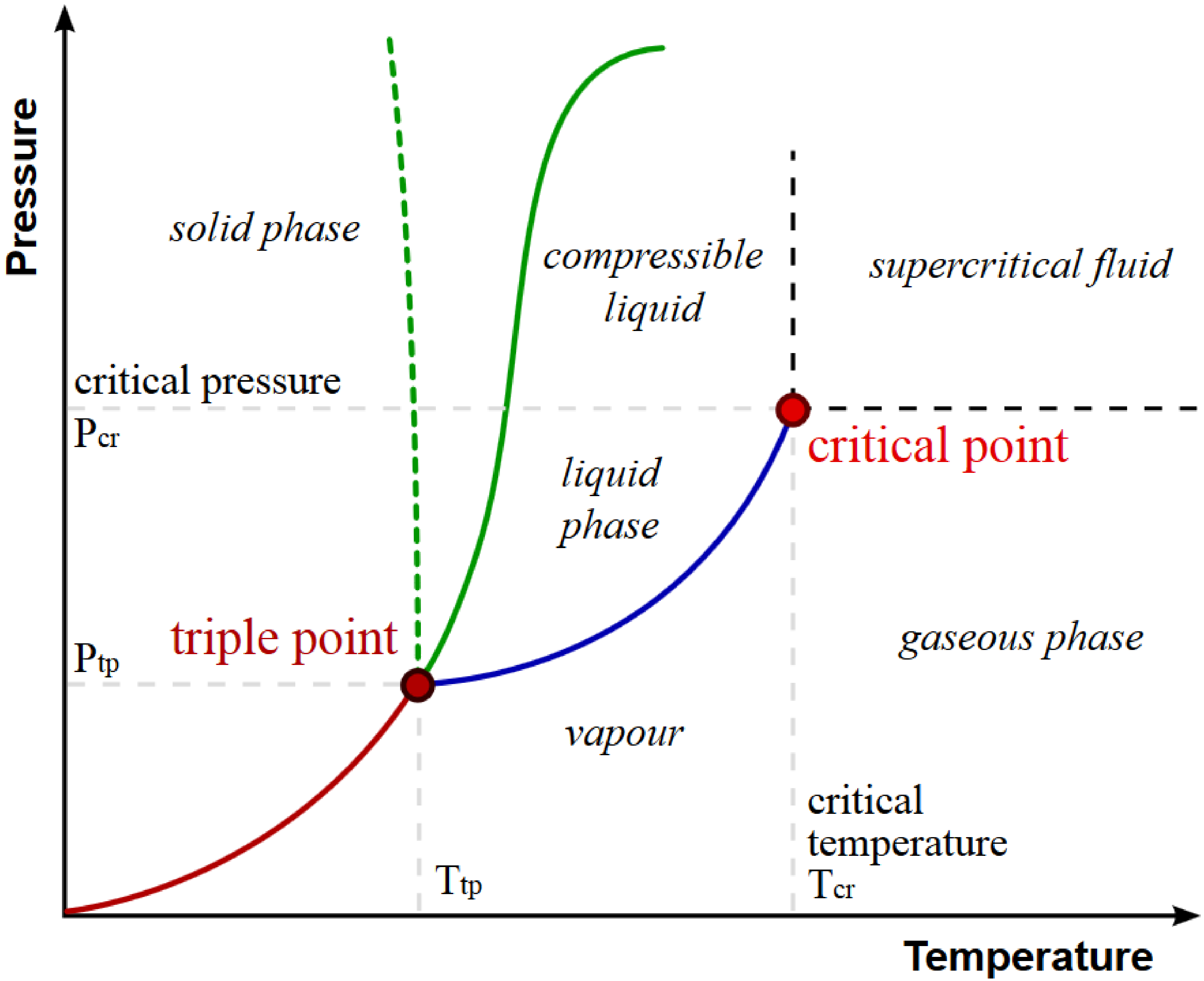


K2. August 2018

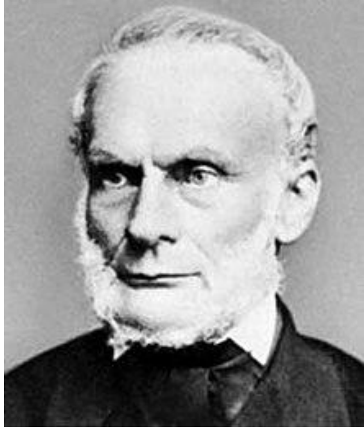

Lapse rates for different relative humidities
Lapse rate/ °C /km
Altitude /km
Hence rule of thumb: dry air lapse rate is about 10° /km, wet air lapse rate is about 6° /km
U = 0
U = 0.25
U = 0.5
U = 0.75
U = 1
This is the humidity
Temperature / °C
Altitude /km
U = 0
U = 0.25
U = 0.5
U = 0.75
U = 1
Pressure / mbar
Altitude /km
U = 0
U = 0.25
U = 0.5
U = 0.75
U = 1


Temperature / °C
50
0
-50
-100
0
2
4
6
8
10
12
Altitude /km
U = 0
U = 0.25
U = 0.5
U = 0.75
U = 1
This is the humidity
Boiling point / °C
100
90
80
70
60
Boiling point of water vs altitude
Altitude /km
U = 0
U = 0.25
U = 0.5
U = 0.75
U = 1
Dew point temperature / °C
50
0
-50
-100
Altitude /km
U = 0
U = 0.25
U = 0.5
U = 0.75
U = 1

To **boil** water, it must undergo a *phase transition* from liquid to gas. This requires a certain amount of heat, *the latent heat of vaporization*, to break the inter-molecular bonds inherent in the water.



The gradient of a phase transition line in the p,T diagram is given by the *Clausius-Clapeyron equation:*



Rudolph Clausius
1822-1888

Pressure in Pascals (Pa)

Latent heat /Jmol[-1]

$$\frac{dp}{dT} = \frac{L_{vap}}{T\Delta V}$$

Temperature in Kelvin

Volume change of 1 mole of substance during the phase transition

In a **liquid to gas** transition we can assume the volume change is sufficiently large as to ignore the original fluid volume. If one assumes the resulting gas is ***ideal***:

$$\Delta V = \frac{RT}{p}$$

Hence:

$$\frac{dp}{dT} = \frac{L_{vap}}{RT^2} p$$

Benoît
Clapeyron
1799-1864

We can use this relationship to determine the liquid-to-gas line in the p,T diagram if the latent heat of vaporization is assumed to be temperature independent. (In reality this is not the case, but is less of an issue at low temperatures – see next slide).

QUESTION 1: Derive the relationship between boiling point T and pressure p from:

$$\frac{dp}{dT} = \frac{L_{vap}}{RT^2} p$$

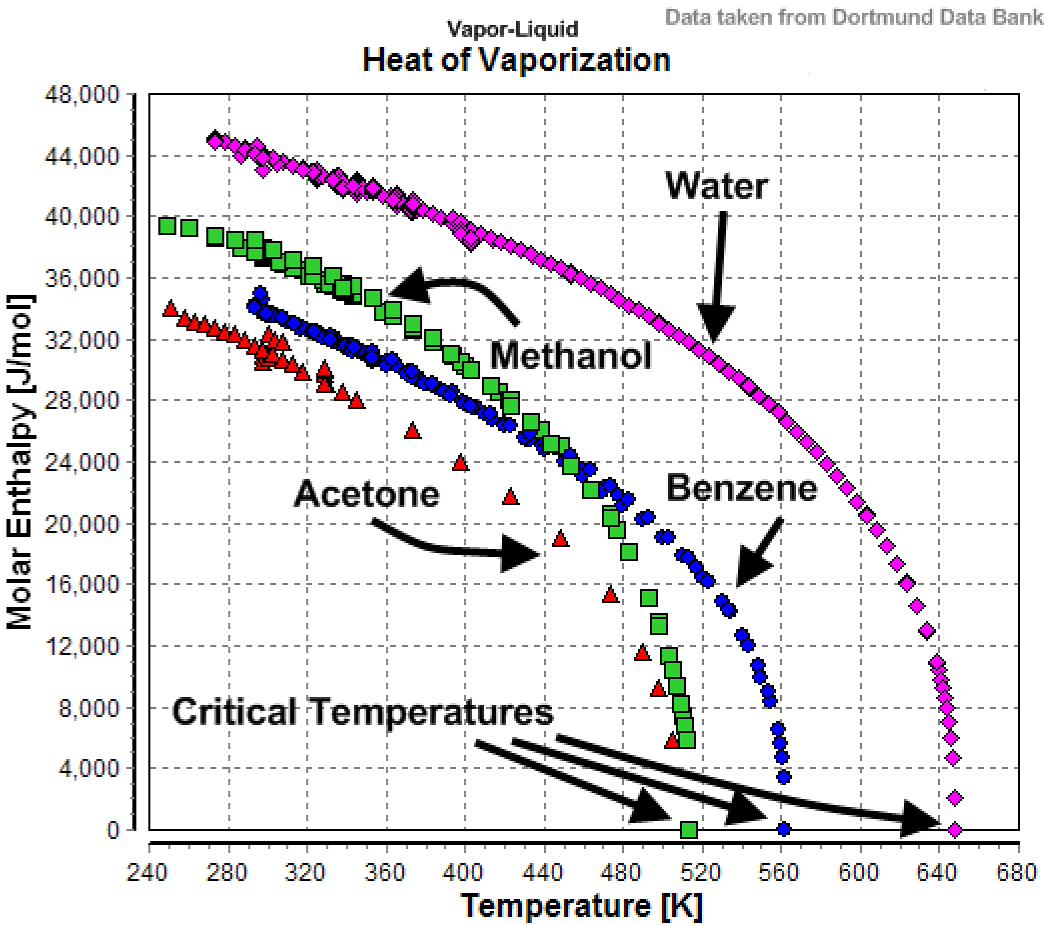
i.e. show that:

$$T_{boil} = \left(\frac{1}{T_*} - \frac{R}{L_{vap}} \ln\left(\frac{p}{p_*} \right) \right)^{-1}$$

Since the temperature corresponds to the liquid-to-gas transition line, we can therefore derive an expression for the **boiling point** of a liquid relative to ambient temperature T and pressure p, as long as one fixed boiling point T_* and corresponding pressure p_* is known.

$$T_{boil} = \left(\frac{1}{T_*} - \frac{R}{L_{vap}} \ln\left(\frac{p}{p_*} \right) \right)^{-1}$$

Boiling point of water at different atmospheric pressures /mbar.



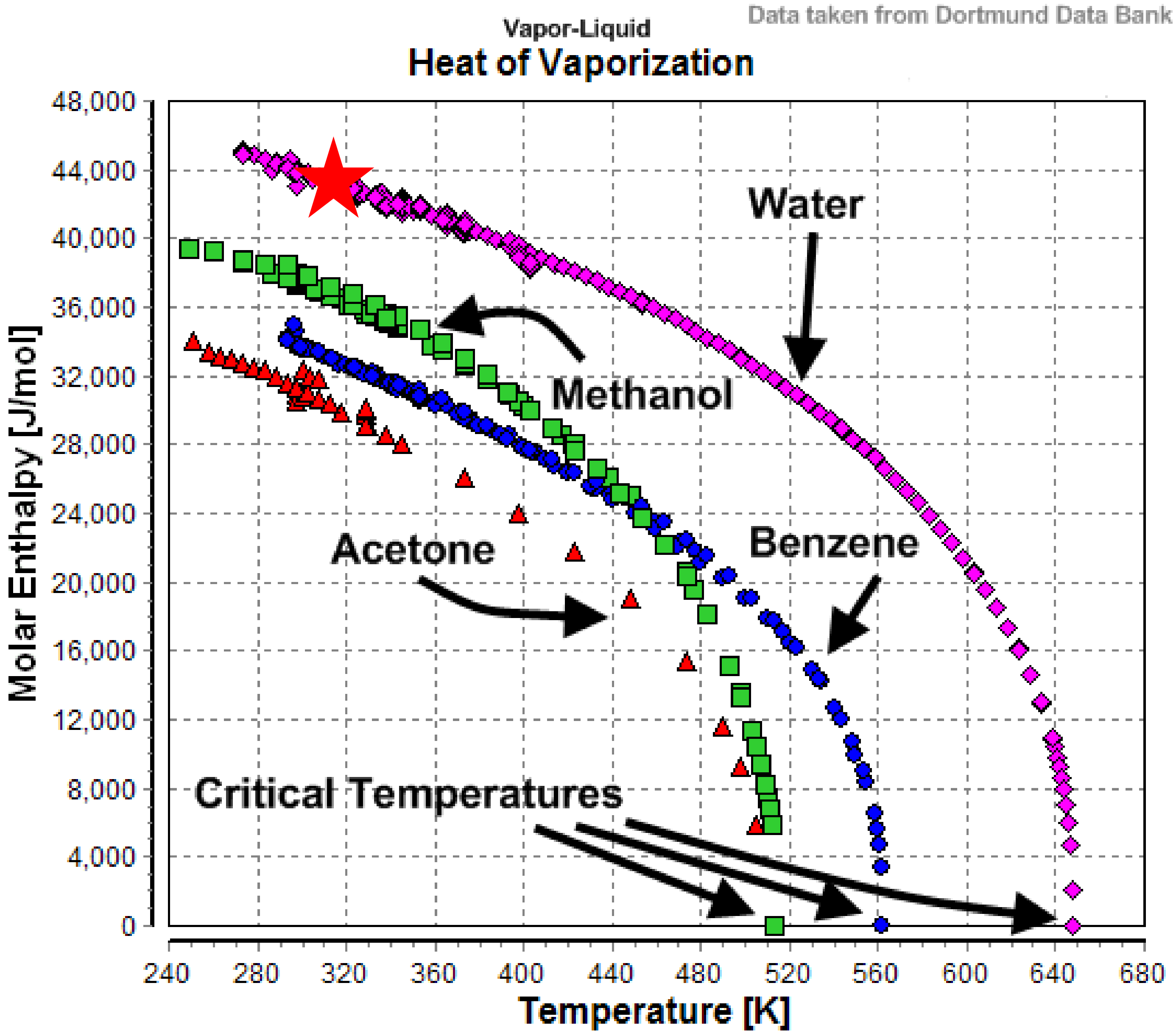
Latent heat of vaporization of water is L_{vap} = 43.8 kJ mol^{-1} at 100°C and 1013.25 mbar ambient air pressure.

At 'typical' troposphere temperatures, a higher value of 45.07 kJ mol^{-1} is used in some calculations, such as the lapse rate model.

http://en.wikipedia.org/wiki/Lapse_rate
http://en.wikipedia.org/wiki/Enthalpy_of_vaporization

http://en.citizendium.org/wiki/Heat_of_vaporization

At around ambient temperatures of about 30 to 40 degrees C (i.e. 303K to 313K) molar latent heat of vaporization of water is about **43.8 kJ/mol**



QUESTION 2: Derive the *Boltzmann-Factor* relationship between pressure p and temperature T

$$T_{boil} = \left(\frac{1}{T_*} - \frac{R}{L_{vap}} \ln\left(\frac{p}{p_*} \right) \right)^{-1} \longrightarrow p = p_* e^{\frac{L_{vap}}{RT_*}} e^{-\frac{L_{vap}}{RT_{boil}}}$$

Ludwig Boltzmann
1844-1906

QUESTION 3: Now *explain* why the 'linearized' version is useful in an experimental context. Sketch an *appropriate graph* to illustrate how p, T_{boil} data can be used to:
(i) Show that our model is plausible; (ii) Determine L_{vap}

$$p(T_{boil}) = p_* e^{\frac{L_{vap}}{RT_*}} e^{-\frac{L_{vap}}{RT_{boil}}}$$

$$\underbrace{\ln\left(\frac{p_*}{p}\right)}_{y} = \frac{L_{vap}}{R} \underbrace{\frac{1}{T_{boil}}}_{x} \underbrace{- \frac{L_{vap}}{R} \frac{1}{T_*}}_{c}$$

$$y = mc + c$$

$$m = \frac{L_{vap}}{R}, \quad c = -\frac{L_{vap}}{R} \frac{1}{T_*}$$

Now we shall build an experimental rig!

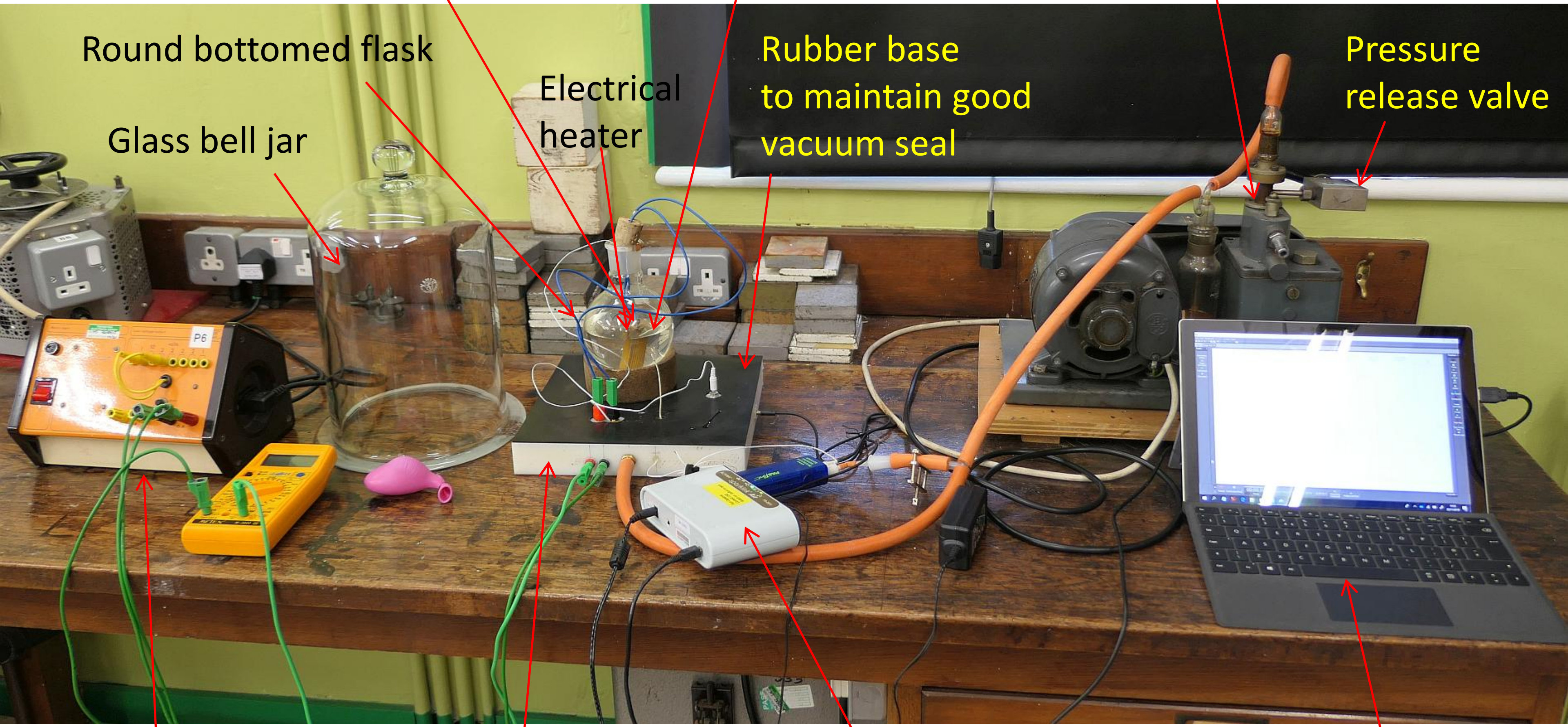
[Watch the video]

Note a slightly different vacuum pump in the video

About 400ml of water

Thermocouple in water

Vacuum pump



12V DC power supply to electrical heater (about 4.8A)

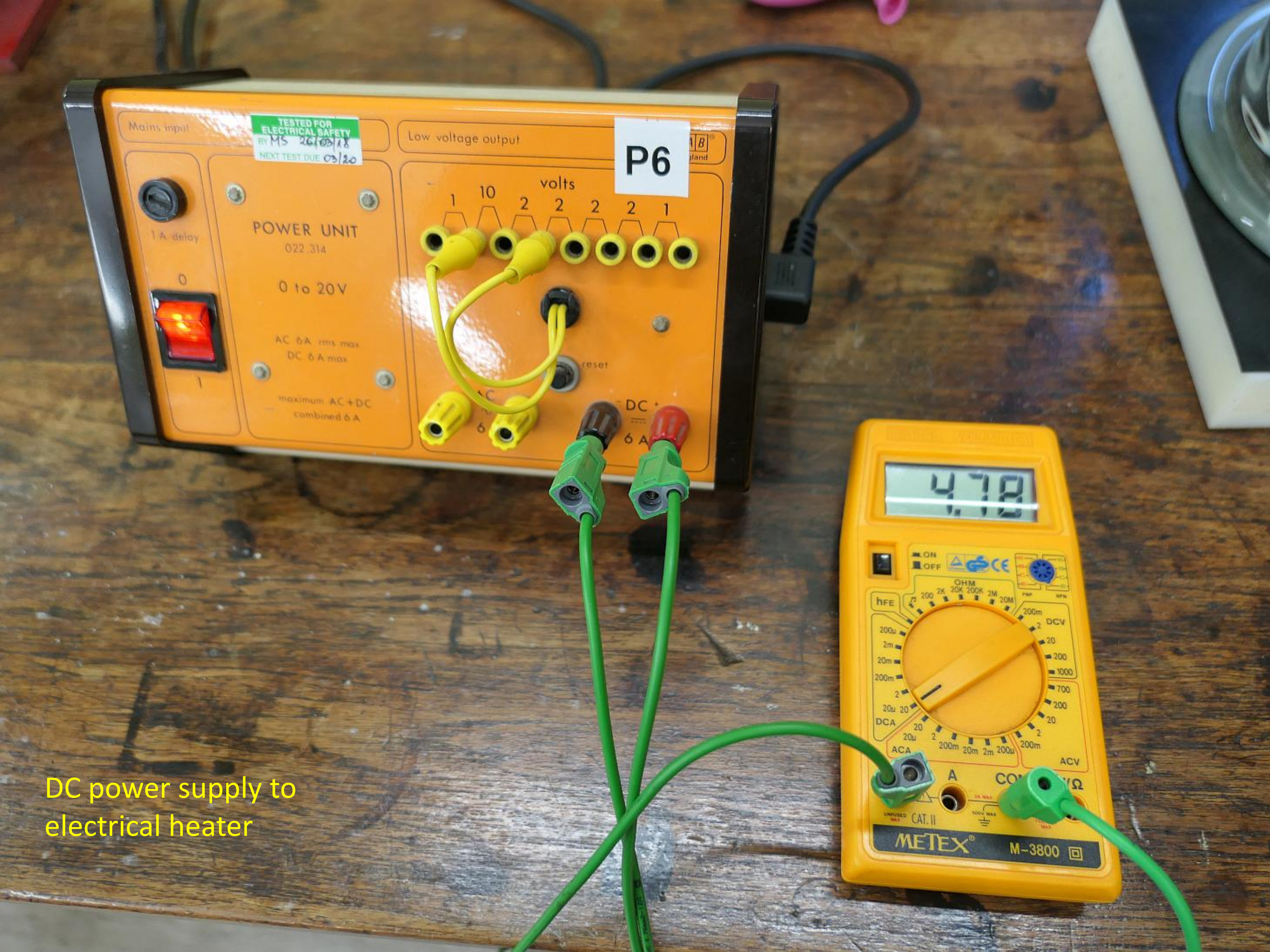
Pump valve, +/- power socket + 2x thermocouple connections drilled into plastic base

PASCO datalogger USB hub (pressure unit + 2x thermocouple connections)

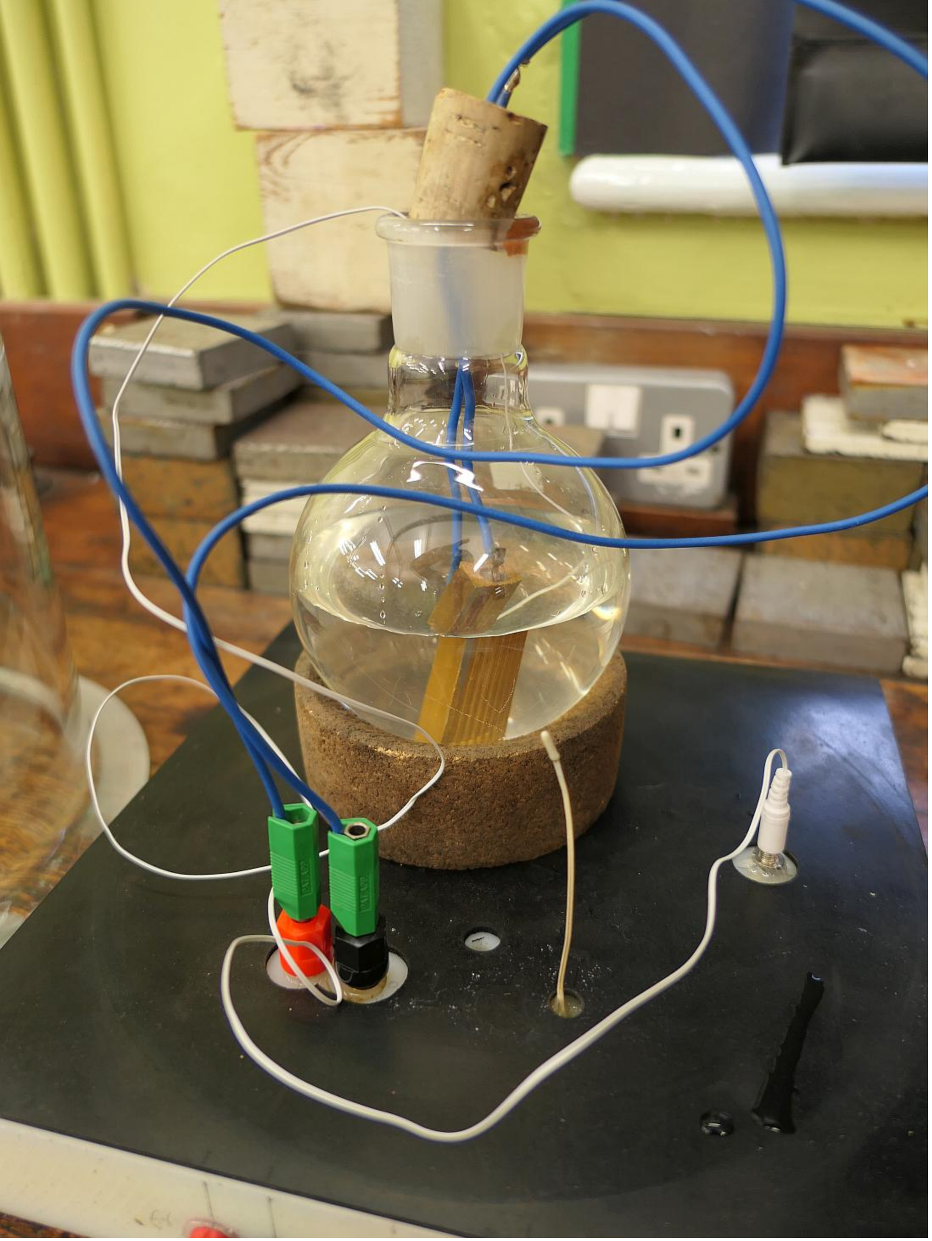
Laptop running CAPSTONE connected to USB hub

Vacuum pump, connected to bell jar. About 3% of atmospheric pressure is possible.

Turn pressure valve *slowly* to return air to bell jar between experiments.



DC power supply to electrical heater



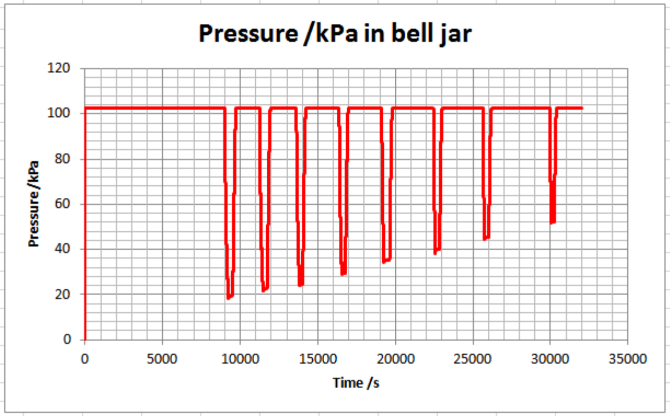
Heat water up to a desired temperature using electrical heater. (It is pretty slow, i.e about 10s per deg C, so you can leave it on)

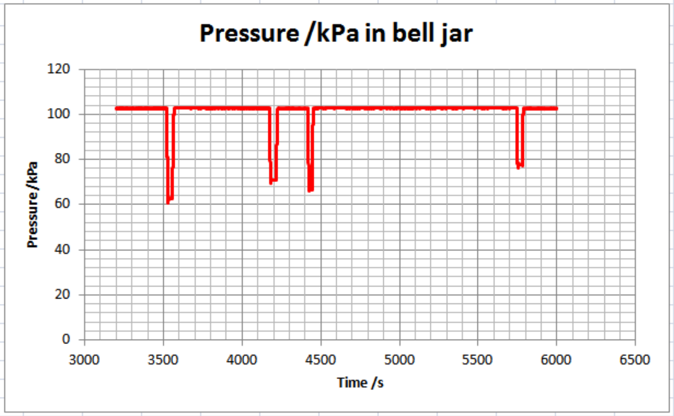
Then pump out air till water spontaneously boils. At this point turn off the pump and record pressure and temperature using the datalogger display on the PC

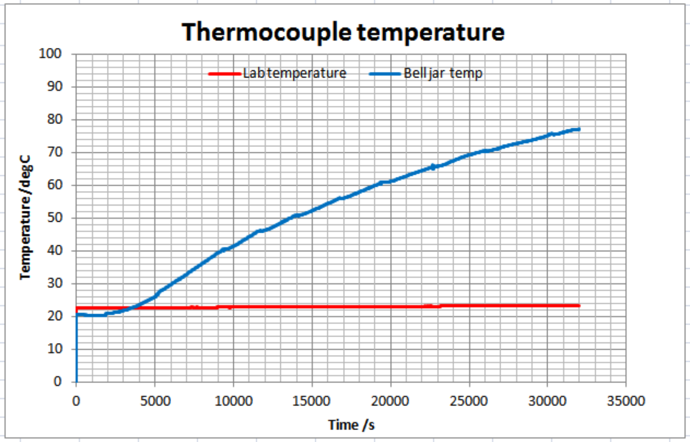
QUESTION 4: (i) Sketch a diagram of the experimental equipment, clearly labelling each part. (ii) Write a series of bullet point instructions how you would conduct the experiment. (You might wish to review the video again first). (iii) Look carefully at the video and indicate: (a) How long the experiment will take; (b) What is the data rate of the datalogger? (c) How many data points will you have to deal with? (d) What is the largest source of error?

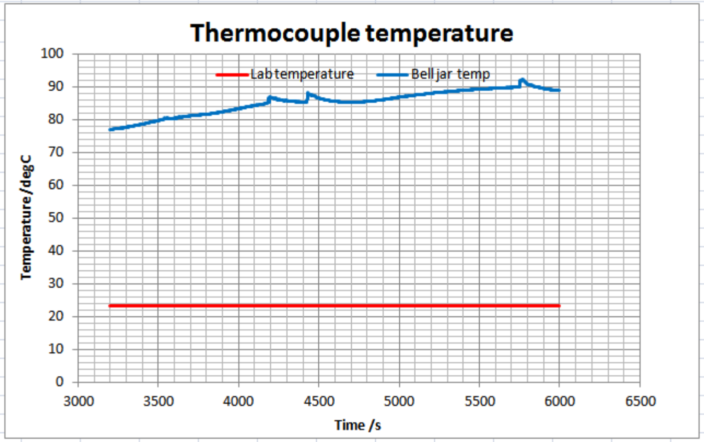
Screenshot of the first 4.7s of data logging (after the data has been copied into Excel:

	A	B	C	D
1	**Lab**	**In bell jar**	**In bell jar**	
2	Temperature (°C)	Temperature (°C)	Absolute Pressure (kPa)	Time (s)
3	22.6	20.47	102.434	0
4	22.6	20.47	102.445	0.1
5	22.6	20.47	102.423	0.2
6	22.6	20.47	102.436	0.3
7	22.6	20.47	102.41	0.4
8	22.6	20.47	102.423	0.5
9	22.6	20.47	102.451	0.6
10	22.6	20.47	102.418	0.7
11	22.6	20.47	102.462	0.8
12	22.6	20.47	102.432	0.9
13	22.6	20.47	102.469	1
14	22.6	20.47	102.491	1.1
15	22.6	20.47	102.436	1.2
16	22.6	20.47	102.471	1.3
17	22.6	20.47	102.458	1.4
18	22.6	20.47	102.473	1.5
19	22.6	20.47	102.447	1.6
20	22.6	20.47	102.467	1.7
21	22.6	20.46	102.438	1.8
22	22.6	20.46	102.458	1.9
23	22.6	20.46	102.482	2
24	22.6	20.47	102.46	2.1
25	22.6	20.47	102.469	2.2
26	22.6	20.47	102.438	2.3
27	22.6	20.47	102.469	2.4
28	22.6	20.47	102.46	2.5
29	22.6	20.47	102.469	2.6
30	22.6	20.46	102.447	2.7
31	22.6	20.47	102.46	2.8
32	22.6	20.47	102.432	2.9
33	22.6	20.46	102.425	3
34	22.6	20.46	102.416	3.1
35	22.6	20.47	102.48	3.2
36	22.6	20.47	102.445	3.3
37	22.6	20.46	102.445	3.4
38	22.6	20.46	102.445	3.5
39	22.6	20.47	102.454	3.6
40	22.5	20.46	102.454	3.7
41	22.6	20.47	102.432	3.8
42	22.6	20.46	102.436	3.9
43	22.5	20.47	102.434	4
44	22.6	20.46	102.467	4.1
45	22.5	20.47	102.449	4.2
46	22.5	20.47	102.484	4.3
47	22.6	20.47	102.46	4.4
48	22.5	20.47	102.469	4.5
49	22.6	20.47	102.429	4.6
50	22.6	20.47	102.478	4.7





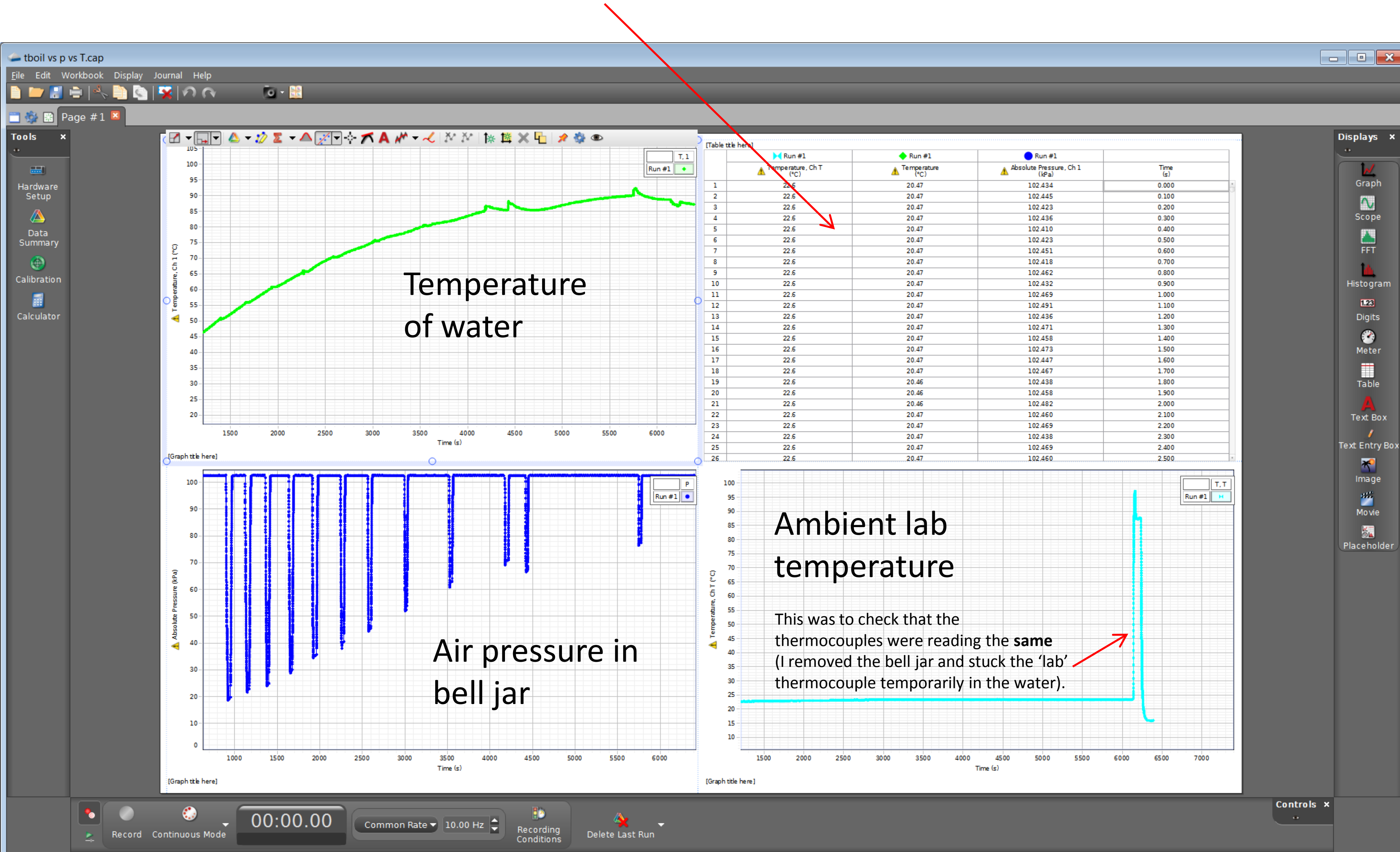




QUESTION 5: Explain what is happening! (i) Why does the pressure vs time graph have the comb shape? (ii) How could we use the graph to get T_{boil} vs p ? (ii) Why does the time between the 'pressure dips' increase?

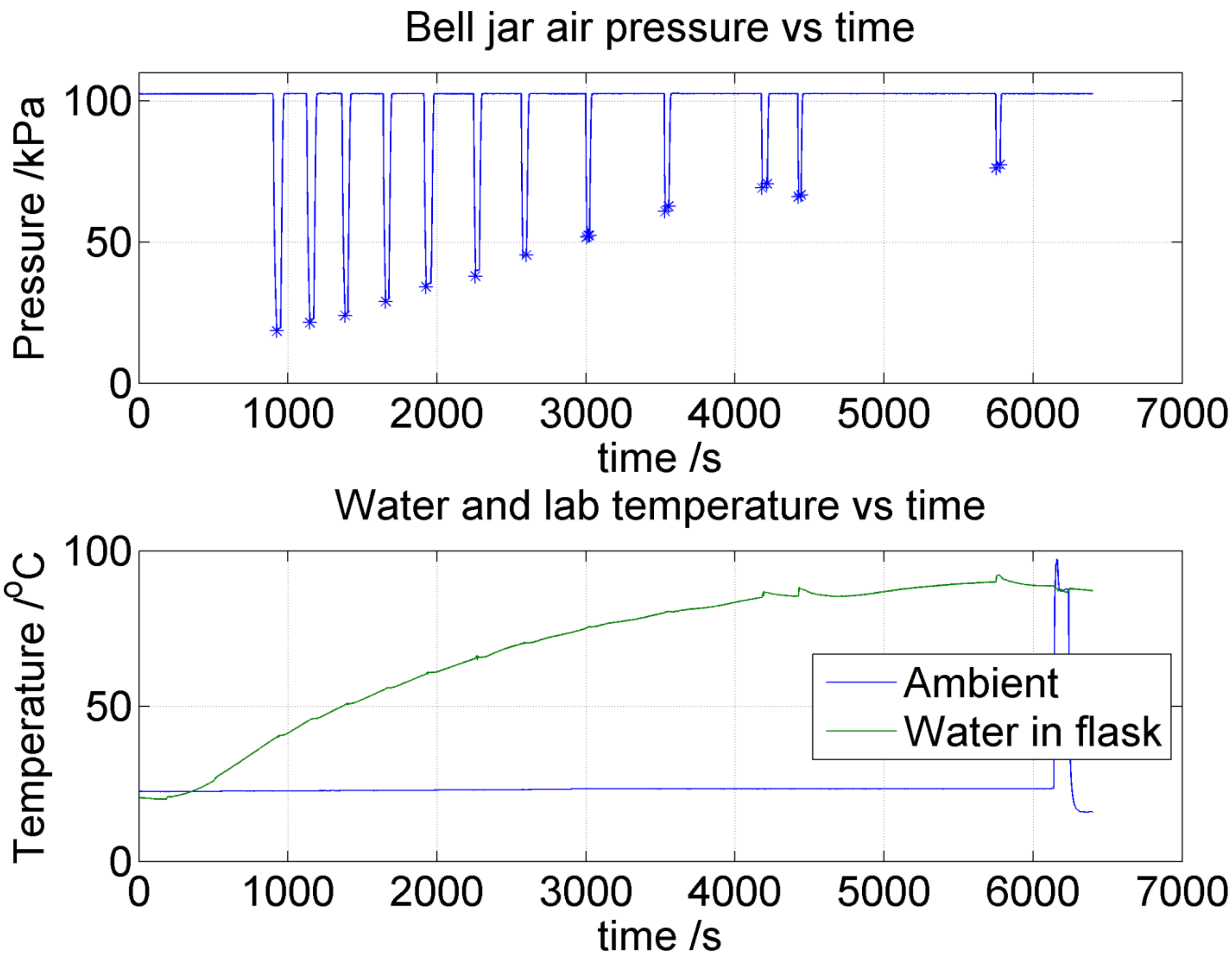
Screenshot of the experiment from PASCO Capstone datalogging software

You export data by selecting all and copying **(ctrl+a, ctrl+c)** from this table. Then paste this into Excel (or for larger data sets, a text file).



The **raw data** from **PASCO Capstone** was copied and pasted into **Excel**, and then *ingested* into **MATLAB** for analysis, and automatic graph production.

This is an example of a *data processing pipeline.* A great incentive to learn the basics of **`coding`**.



QUESTION 6: Suggest (in words i.e. ‘psuedocode’) an *algorithm* how a computer program could find the * local minima of pressure i.e. the pressure that the water boils.

$$p(T_{boil}) = p_* e^{\frac{L_{vap}}{RT_*}} e^{-\frac{L_{vap}}{RT_{boil}}}$$

$$p_* = 101.325\text{kPa}$$
$$T_* = 373\text{K}$$

$$\underbrace{\ln\left(\frac{p_*}{p}\right)}_{y} = \frac{L_{vap}}{R} \underbrace{\frac{1}{T_{boil}}}_{x} \underbrace{- \frac{L_{vap}}{R}\frac{1}{T_*}}_{c}$$

$$y = mc + c$$

$$m = \frac{L_{vap}}{R}, \quad c = -\frac{L_{vap}}{R}\frac{1}{T_*}$$

p/kPa	$T_{boil}/^{o}\text{C}$
18.468	39.97
21.496	45.64
23.897	50.50
28.881	55.58
34.053	60.42
37.919	65.42
45.400	70.38
51.720	75.18
52.297	75.58
52.237	75.78
60.770	80.13
62.742	80.51
69.144	85.10
70.650	86.63
66.039	85.44
66.683	87.64
76.205	89.99
77.338	91.98

QUESTION 7:

(i) **Plot p vs T_{boil} as + marks on a graph. *Don't join them up.* (We have a *model* to underlay!) Are *error bars* appropriate?**

(ii) **Now tabulate y and x using our linearized version. NOTE YOU WILL NEED TO CONVERT TEMPERTURES INTO KELVIN.**

(iii) **From a *line of best fit* (you can do this by eye), determine the specific (molar) latent heat of vaporization L_{vap} in kJ/mol**

(iv) **Now work out via the *model* (using your value of L_{vap}) what p should be given a sensible range of T between 35°C and 100°C.**

(v) **Overlay this smooth curve on your data points.**

Submit your analysis, and then if time allows, look at **PART 2**