Wave-Particle duality and Electron diffraction

Louis de-Broglie proposed that all objects with total momentum p will have an 'associated wavelength.' If this wavelength is on a similar scale to an *aperture*, then *significant diffraction effects should be observed*. Although the electron had been established as a *particle*, i.e. an object localized in space with a defined mass and velocity, Germer and Davisson showed that 'electron waves' can be diffracted by the atomic lattice associated with a disk of carbon atoms. *Wave-particle duality* is certainly measurable for electrons i.e. *both models* are appropriate descriptions of its physical characteristics.

$$e = 1.6021766208(98)\times10^{-19}\,\text{C}$$
$$c = 2.99792458\times10^{8}\,\text{ms}^{-1}$$
$$h = 6.626070040(81)\times10^{-34}\,\text{kgm}^2\text{s}^{-1}$$
$$m_e = 9.10938356(11)\times10^{-31}\,\text{kg}$$

Louis de Broglie
1892 – 1987
Nobel Prize 1929

The de Broglie formula expressing the relationship between wavelength and momentum can be 'derived' by comparing Einstein's famous mass-energy relationship with the classical expression for the momentum of a particle travelling at the speed of light. Although the argument is flawed, i.e. Special Relativity shows that *only massless particles* can travel at the speed of light, it does produce the correct result! The de Broglie analysis also alludes to another truth, that although photons are indeed *massless*, they do possess *momentum*

$E = mc^2$ Energy

$p = mc$ Momentum

$\therefore E = pc$

$E = hf$ Planck formula

$c = f\lambda$ Wave velocity

$$\therefore E = \frac{hc}{\lambda}$$

$$\therefore pc = \frac{hc}{\lambda}$$

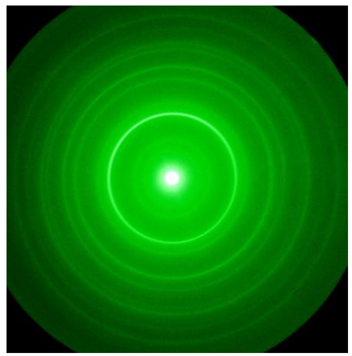
$$\therefore \lambda = \frac{h}{p}$$

de Broglie relation

Classical calculation for electron wavelength resulting from electron accelerated by voltage V

$$eV = \tfrac{1}{2}m_e v^2$$
$$p = m_e v$$
$$\therefore eV = \tfrac{1}{2}m_e\left(\frac{p}{m_e}\right)^2 = \frac{p^2}{2m_e}$$
$$p = \sqrt{2m_e eV}$$
$$\lambda = \frac{h}{p}$$
$$\lambda = \frac{h}{\sqrt{2m_e eV}}$$

Electron wave diffraction rings



Relativistic calculation for electron wavelength resulting from electron accelerated by voltage V

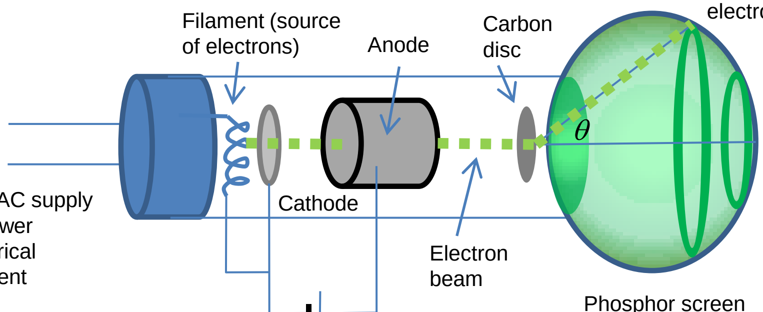
$E = m_e c^2 + eV$ Total energy including rest mass

$E^2 = p^2c^2 + m_e^2c^4$ Energy-momentum invariant

$$(m_e c^2 + eV)^2 = p^2c^2 + m_e^2c^4$$
$$m_e^2c^4 + 2m_e c^2 eV + e^2V^2 = p^2c^2 + m_e^2c^4$$
$$p^2 = \frac{2m_e c^2 eV + e^2V^2}{c^2}$$
$$p = \sqrt{2m_e eV + \frac{e^2V^2}{c^2}} = \sqrt{2m_e eV}\sqrt{1 + \frac{eV}{2m_e c^2}}$$
$$\lambda = \frac{h}{p}$$
$$\lambda = \frac{h}{\sqrt{2m_e eV}}\frac{1}{\sqrt{1 + \frac{eV}{2m_e c^2}}}$$

Lester Germer (right) with Clinton Davisson in 1927. Nobel Prize 1937.

Voltage = 5kV
Classical electron wavelength = 0.017344nm
Relativistic Factor (RF) = 0.99756
Electron wavelength = 0.017302nm
Diffraction ring 1 : theta = 9.96°
Diffraction ring 2 : theta = 20.25°
Diffraction ring 3 : theta = 31.27°
Diffraction ring 4 : theta = 43.80°
Diffraction ring 5 : theta = 59.89°



Electron diffraction rings

$$\theta = \sin^{-1}\left(\frac{n\lambda}{d}\right)$$

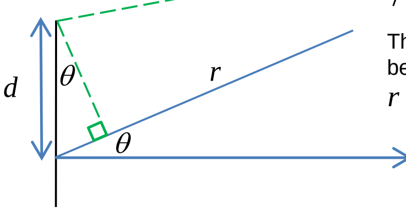
d is the atomic spacing of carbon atoms

For *constructive interference* (i.e. a bright ring)

$$\Delta\phi = kd\sin\theta = 2\pi n$$

Wavenumber $k = \dfrac{2\pi}{\lambda}$ n is an integer

$$2\pi n = \frac{2\pi}{\lambda}d\sin\theta$$
$$n\lambda = d\sin\theta$$



*Perhaps the de Broglie relation should be taken as *axiomatic*